INFLUENCE OF CLAMPING LOSS AND ELECTRICAL DAMPING ON NONLINEAR DISSIPATION IN MICROMECHANICAL RESONATORS

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ABSTRACT

We study the influence of clamping loss, electrical damping, and transduction nonlinearity on the measured nonlinear dissipation in encapsulated micromechanical wheel resonators. Our measurements suggest that nonlinear dissipation may arise from the same phonon scattering origins as thermoelastic dissipation in flexural mode resonators ranging in size from carbon nanotubes to guitar strings. These results point to future investigations of thermal bath engineering to probe the origins of nonlinear dissipation in micro- and nanomechanical resonators.

KEYWORDS

nonlinear dissipation, quality factor, MEM resonator, thermoelastic dissipation, phonon scattering

INTRODUCTION

Micro- and nano-electromechanical (MEM/NEM) resonators underlie a variety of oscillators and resonant sensors, such as atomic force microscopes, gyroscopes, and accelerometers. Perhaps the most straightforward method for improving the signal-to-noise ratio (SNR) in oscillators and resonant sensors is to increase the vibration amplitude of the underlying MEM/NEM resonator. Increasing the vibration amplitude beyond the nonlinear threshold results in nonlinear behavior, i.e. amplitude-dependent resonance frequency (Duffing nonlinearity) and amplitude-dependent damping (nonlinear dissipation) [1, 2, 3]. These nonlinearities can be incorporated into the equation-of-motion describing the forced resonator displacement x as:

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x + \alpha x^3 + \eta x^2 \dot{x} = f + f_{th}, \qquad (1)$$

where $\gamma = \omega_0/Q$ is the damping rate, $\omega_0/2\pi$ is the resonance frequency, Q is the quality factor, α is the Duffing coefficient, η is the nonlinear dissipation coefficient, f is the external coherent forcing, and f_{th} describes the incoherent thermomechanical noise force with white amplitude-spectral-density (ASD) $F_{th} = \sqrt{4k_B T\omega_0 m^{-1} Q^{-1}}$. By operating the MEM/NEM resonator in particular regions of the nonlinear response, the SNR of the corresponding oscillator or resonant sensor can exceed the SNR in the linear regime [4, 5]. Recent advances in ultra-high-Q MEM/NEM resonator enables exceedingly weak forces to drive the vibrational mode into the nonlinear regime [6, 7], potentially enabling thermomechanical fluctuations alone to induce nonlinear behavior [8]. Furthermore, in a variety of emerging MEM/NEM oscillator and resonant

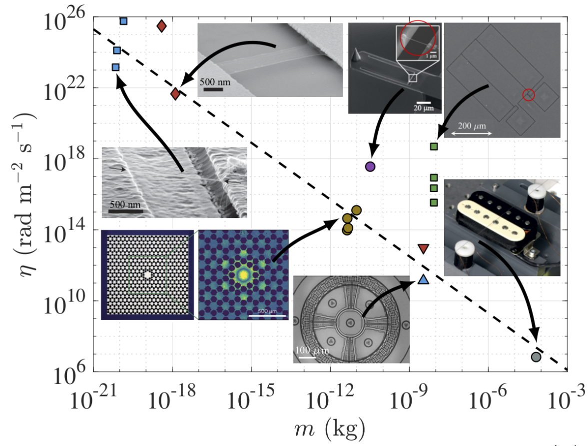

Figure 1: Scaling of nonlinear dissipation (η) in MEM/NEM resonators with resonator lumped mass (m): carbon nanotubes [17], graphene [17], localized membrane modes [18], atomic force microscope with embedded nanotube [19], microcantilever with high stress beam [15], wheel resonator (this work), and guitar string [20]. η scales as $\approx m^{-1}$ over 15 orders of m.

sensor topologies, nonlinearities are crucial for the performance [9, 10, 11, 12, 13, 14, 15, 16].

Nonlinear dissipation contributes damping that increases as the vibrational field amplitude increases. In optical resonators, nonlinear dissipation is a well-characterized nonlinearity that limits the optical field intensity during lasing [21]. In mechanical resonators, nonlinear dissipation is thought to arise from the scattering of phonons in the forced mode of interest off of other phonon modes in the thermal bath [22]. The thermal bath can be comprised of phonon modes within or near the resonator, in the resonator supports, or even quasi-modes in the surrounding gas molecules. Nonlinear dissipation has been measured in a variety of mechanical resonators, including carbon nanotubes [17], graphene resonators [17], soft-clamped membrane modes [18], atomic force microscope cantilevers with embedded nanotubes [19], microcantilevers with high stress beams [15], guitar strings [20], and metal plates and cylinders [23]. In these devices, the constituent degrees-of-freedom in the thermal bath can be inferred from the linear dissipation mechanisms that limit Q in the forced mode, such as gas damping, thermoelastic dissipation (TED), clamping loss, or electrical dissipation. For thermoelastic dissipation, the thermalized modes are located primarily in the resonator body and form the thermal bath. For clamping loss, vibrational modes outside of the resonator body form the thermal bath. For electrical dissipation, the ther-

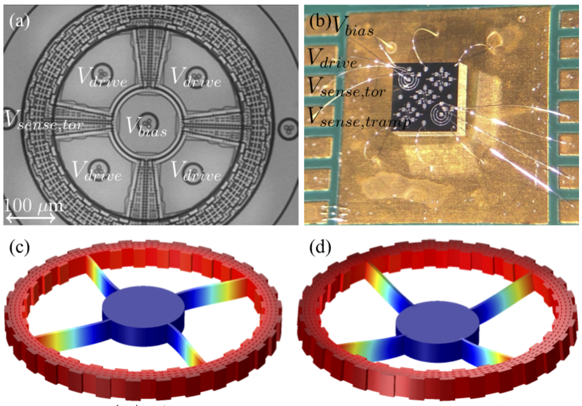

Figure 2: (a) An x-ray microscope image of the wheel resonator which delineates the electrical transduction. (b) A suspended chip with devices "floated" via surrounding aluminum wire bonds. A simulation of the (c) torsional and (d) trampoline mode shapes.

mal bath is comprised of thermalized free electrons. Alter *et al.* previously showed that nonlinear dissipation exists in TED-limited silicon micromechanical resonators even in perfect vacuum [24, 25], suggesting that nonlinear dissipation may be linked to TED and arises from phonon-scattering off vibrational modes in the resonator body.

In this work, we probe the origins of nonlinear dissipation by engineering the thermal bath in flexural mode micromechanical wheel resonators, depicted in Fig. 2. The resonators are fabricated from single-crystal-silicon and polysilicon in individually vacuum-encapsulated chips [26]. Adjusting the bias voltage tunes the electrical damping in the low-frequency flexural modes [27], and modifying the chip boundary conditions with the frame tunes the clamping loss [28]. We additionally show that nonlinearity in the position transduction at large vibration amplitudes can strongly influence the measured nonlinear dissipation values. Our results suggest that nonlinear dissipation may be linked to thermoelastic dissipation.

EXPERIMENT AND DISCUSSION

We combine a thermomechanical-noise-limited displacement calibration approach [29] with the nonlinear ringdown technique [30] to measure nonlinear dissipation and Duffing nonlinearity in the torsional mode of micromechanical wheel resonators as a function of bias voltage (tuning electrical damping), chip boundary conditions (tuning clamping loss), and capacitive transduction nonlinearity. Our measurements indicate that tuning electrical damping and clamping loss does not appreciably influence the nonlinear dissipation, suggesting that nonlinear dissipation primarily arises from phonon scattering off thermalized vibrational modes within the vibrating resonator body. By first extracting the amplifier responsivity from the thermomechanical noise measurements, our nonlinear dissipation measurements can be compared to the other measurements of nonlinear dissipation. Comparing to the literature indicates a general dependence of nonlinear dissipation on resonator lumped mass over sixteen orders of magnitude, as shown in Fig. 1, suggesting that TED is linked to nonlinear dissipation in flexural modes of atomic tubes up to the macroscale, and is remarkably scale-independent when normalized by the mass involved in vibrations.

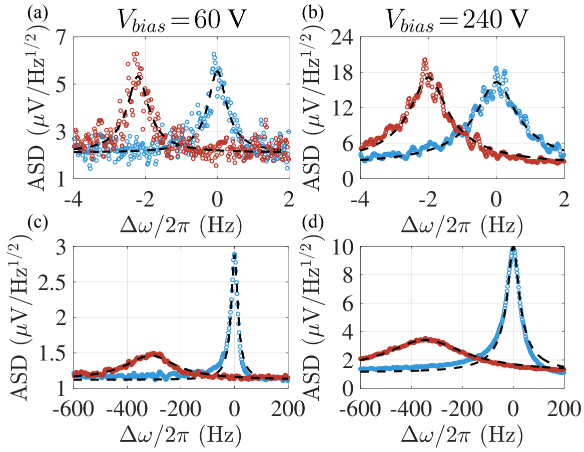

Figure 3: The measured thermomechanical noise amplitude spectral density (ASD) for the (a,b) torsional and (c,d) trampoline modes in the fixed (red circles) and floating (blue circles) configurations, at (a,c) 60 V bias and (b,d) 240 V bias, for a polysilicon wheel microresonator.

Table 1: Quality factors and amplifier responsivities for the polysilicon wheel resonator.

V_{bias} torsional	Q_{fix}	Q_{float}	$\mathfrak{R}_{fix}$ (V/μm)	$\mathfrak{R}_{float}$ (V/μm)
60 V	96.9k	114k	1.56	1.51
240 V	56.8k	53.3k	7.02	6.92
V_{bias} tramp.	Q_{fix}	Q_{float}	$\mathfrak{R}_{fix}$ (V/μm)	$\mathfrak{R}_{float}$ (V/μm)
60 V	1.93k	17.4k	41.8	39.4
240 V	1.28k	9.33k	156	178

Figure 3 presents the thermomechanical noise spectrum of the torsional and trampoline modes in the wheel resonator depicted in Fig. 2, at two different bias voltages, in the fixed and floating chip configuration. Utilizing a best-fit of the thermomechanical noise ASD yields the measured Q values and amplifier responsivity values summarized in Table 1. Tuning the bias voltage has a large impact on the Q for both the torsional mode and trampoline mode, indicating increased phonon-electron interactions for both modes. Floating the chip dramatically reduces the clamping loss for the trampoline mode but has little impact on the torsional mode, providing evidence that the torsional mode is limited primarily by TED and electrical damping.

Measuring the ringdowns in Fig. 4 in the fixed and floating configuration shows that varying the clamping loss in the auxiliary trampoline mode does not appreciably influence nonlinear dissipation in the tor-

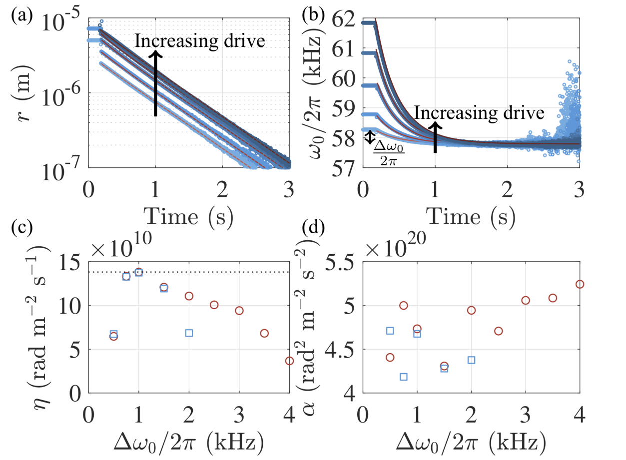

Figure 4: The polysilicon torsional mode (a) vibration amplitude (r) and (b) resonance frequency ($\omega_0/2\pi$) during a ringdown. Extracted (c) nonlinear dissipation coefficient (η) and (d) Duffing nonlinearity (α) as a function of resonance frequency upshift ($\Delta\omega_0/2\pi$) at the onset of the ringdown, in the fixed (red circles) and floating (blue squares) configurations.

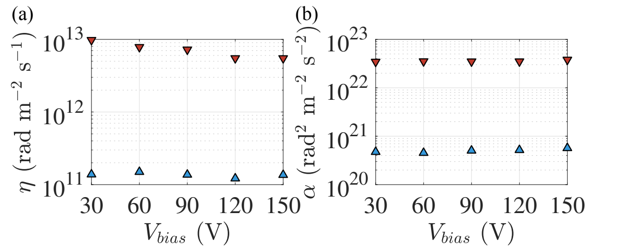

Figure 5: The extracted (a) nonlinear dissipation coefficient (η) and (b) Duffing nonlinearity (α) as a function of bias voltage (V_{bias}), for the polysilicon (upward blue triangles) and single crystal silicon (downward red triangles) resonators.

sional mode, suggesting that nonlinear dissipation in the torsional mode arises primarily from phonon scattering off higher frequency thermalized modes. Surprisingly, performing the ringdown from progressively larger initial amplitudes results in a monotonically decreasing extracted nonlinear dissipation value, which arises from nonlinearity in the capacitive displacement measurement. Fig. 5 shows that increasing the bias voltage does not increase the nonlinear dissipation via increased electrical damping for either single-crystal-silicon or polysilicon wheel resonators, but instead causes the extracted nonlinear dissipation to decrease erroneously due to increased capacitive nonlinearity.

Comparing the extracted nonlinear dissipation for these polycrystalline-silicon and single-crystal-silicon resonators in Fig. 5 to the other measurements from the literature in Fig. 1 suggests that, with the exception of resonators employing an embedded element [19, 15], mechanical nonlinear dissipation follows an inverse dependence on resonator mass. Nonlinear dissipation is likely to be linked to a variety of dissipation mechanisms via the phonon scattering picture, such as thermoelastic dissipation (this work and likely all of the works in Fig. 1), gas damping [24, 16], and even Akhiezer dissipation [22]. Phonon-electron scattering is known to yield exceedingly linear viscous electrical damping [31], and these measurements provide additional evidence that nonlinear dissipation is not linked to electrical damping. The observation that embedding high-stress elements into the resonator [19, 15, 16] increases the nonlinear dissipation beyond what would be expected from mass-scaling in Fig. 1 suggests that higher peak stresses enable more efficient phonon scattering, likely off thermalized modes located within or in close proximity to the high-stress elements. The phonon scattering picture underlying nonlinear dissipation also suggests that reducing the temperature of the thermal bath of phonon modes will result in reduced nonlinear dissipation through reduced phonon scattering; recent measurements tentatively indicate this to be the case.

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