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Human navigation in curved spaces

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ABSTRACT

Navigation and representations of the spatial environment are central to human survival. It has often been debated whether spatial representations follow Euclidean principles, and a number of studies challenged the Euclidean hypothesis. Two experiments examined the geometry of human navigation system using true non-Euclidean environments, i.e., curved spaces with non-Euclidean geometry at every point of the space. Participants walked along two legs in an outbound journey, then pointed to the direction of the starting point (home). The homing behavior was examined in three virtual environments, Euclidean space, hyperbolic space, and spherical space. The results showed that people's responses matched the direction of Euclidean origin, regardless of the curvature of the space itself. Moreover, participants still responded as if the space were Euclidean when a learning period was added for them to explore the spatial properties of the environment before performing the homing task to ensure violations of Euclidean geometry were readily detected. These data suggest that the path integration / spatial updating system operates on Euclidean geometry, even when curvature violations are clearly present.

1. Introduction

Spatial knowledge is central to the way in which humans perceive and reason about the physical world. However, the underlying representation of spatial knowledge remains an undetermined issue. Broadly speaking, spatial representations might conform to at least two mathematical systems: Euclidean or non-Euclidean. In Euclidean geometry, spatial information must abide by a set of five axioms that describe the behavior of points and straight lines in a plane. The fifth axiom concerns the behavior of parallel lines and states that the sum of the interior angles of a triangle must be equal to two right angles (180°); stated differently, given a line L and a point P not on L, there can be one and only one line parallel to L that passes through point P. Relaxing this statement engenders alternative (non-Euclidean) geometries, in which the sum of the interior angles of a triangle is larger, or smaller, than 180°, e.g. elliptic and hyperbolic geometry, respectively (Coxeter, 1998) (see Fig. 1).

The most familiar Euclidean space is a metric space defined by a distance function for which for every pair of points x, y is assigned a number $d(xy)$, known as their distance, equal to the length of the line segment $\overline{xy}$ connecting them. The metric distance function must also satisfy assumptions of positivity: $d(x,x) = 0$ and $d(x,y) > 0$ if $x \neq y$; symmetry: $d(x,y) = d(y,x)$; the triangle inequality: $d(x,y) + d(y,z) \geq d(x,z)$; and segmental additivity: $d(x,y) + d(y,z) = d(x,z)$. Thus, if $x = (x_1,x_2)$ and $y = (y_1,y_2)$, then the Euclidean distance in a two-dimensional plane is given by the Pythagorean theorem, $d(x,y) = \sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2}$.

Given that humans have evolved in a Euclidean world, one might assume that the structure of spatial knowledge is also Euclidean. An important mechanism for building spatial knowledge is path integration, which has been found across the animal kingdom and regarded as one of the most basic, fundamental building blocks for spatial representations (Alyan & McNaughton, 1999; Collett & Collett, 2000; Loomis et al., 1993; Mittelstaedt & Mittelstaedt, 1980; Saint Paul, 1982; Wang, 2012; Wang, 2016; Warren, 2019; Wehner & Srinivasan, 1981). A Euclidean spatial representation, or 'cognitive map', could emerge from idiothetic path integration based on vestibular and motor feedback during navigation. Distances and angles would be assigned coordinates within an inertial coordinate reference frame and used to estimate trajectories between locations in the cognitive map (Warren, 2019). A Euclidean cognitive map is therefore capable of supporting judgments of straight-line distances, relative directions, and novel shortcuts (Gallistel,

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1989; Peer, Brunec, Newcombe, & Epstein, 2021; Tolman, 1948; Wang, 2016, 2017; Wang & Spelke, 2002). Behavioral studies have confirmed these abilities for human participants in a variety of real and simulated environments (Ishikawa & Montello, 2006; Shelton & McNamara, 2001; Street & Wang, 2014, 2016; Thorndyke & Hayes-Roth, 1982;, Wang, 2004; Wang & Brockmole, 2003; etc.).

Evidence from animal studies corroborates these assumptions and suggests that grid, place, and head-direction cells in the hippocampal formation may reflect the neural basis of cognitive maps (O'Keefe & Nadel, 1978; Peer et al., 2021; Taube, Muller, & Ranck, 1990). For example, in a study by O'Keefe and Speakman (1987), rats were placed in a four-arm maze and trained to navigate to a goal location marked by specific landmarks, which varied from trial to trial. Microelectrodes were surgically implanted and used to record pyramidal cells in the CA1 and CA3 regions of the hippocampus. These cells showed patterns of activation corresponding to the position of the rat in the previously learned environment irrespective of orientation. These cells are referred to as 'place cells' and are involved in the storage and integration of spatial representations of the environment. Similar cellular networks are believed to underlie spatial representations in humans as well (Miller et al., 2013).

Nonetheless, some research has challenged the idea of Euclidean geometry in human spatial representations. Evidence for non-Euclidean spatial representation came from two main lines of research. The first evidence is from the well-known phenomenon of systematic distortions in spatial memory (Hirtle & Jonides, 1985; Huttenlocher, Hedges, & Duncan, 1991; Sampaio & Wang, 2009, 2017; Stevens & Coupe, 1978). For example, a large set of studies have shown distortions in mental representations for space, potentially violating the Euclidean metric axioms described above. In a study by Stevens and Coupe (1978), participants estimated directions between location pairs of US cities. Results showed distortions toward the direction of the superordinate relationship, e.g. participants thought Nevada was east of California, therefore all Nevada cities were also east of California cities (false); suggesting that superordinate category of 'state' biases the subordinate category of 'city'.

These results are echoed in a study by Moar and Bower (1983) showing inconsistency in spatial knowledge for city pairs in the UK. Participants judged from memory the relative direction between nine locations in Cambridge, forming three triads. Responses were recorded in terms of the angular direction depicted in a perspective taking and spatial orientation task. Results showed angles were consistently orthogonalized, i.e. biased in the direction of 90°. Moreover, the sum of the derived angles for each of the three triads was consistently greater than 180°. These results suggest that mental representations for space can be spatially inconsistent in terms of angular and directional properties.

Landmarks, too, seem to have a distorting effect on spatial representations. For example, participants were asked to estimate distances between pairs of campus/city locations with either a memorable landmark or an unknown location as a reference point. When the landmark was the reference point, other locations were judged as being closer to it than vice versa (McNamara & Diwadkar, 1997; Sadalla, Burroughs, & Staplin, 1980). Other evidence shows similar violations, such that when participants were asked to estimate straight-line distance between points on a route, distance estimates are greater when a route contains a barrier or detour, compared to when the route is relatively direct (Thorndyke, 1981). Participants also exaggerated distance between cities closer to their perspective compared to cities further from their perspective (Holyoak & Mah, 1982). Collectively these distortions suggest that the underlying representation guiding spatial memory may contain violations of Euclidean assumptions and cognitive maps may not be veridical representation of the actual space. However, there was also evidence that these distortions may occur at the retrieval stage and not in the spatial representation itself (Sampaio & Wang, 2009), or are due to general principles of human memory and judgment such as contextual scaling (McNamara & Diwadkar, 1997).

The second type of evidence came from a set of recent studies examining human spatial learning of non-Euclidean space using virtual reality (VR) technology. For example, in a study by Galbraith, Zetzsche, Schill, and Wolter (2009; also see Kluss, Marsh, Zetzsche, and Schill, 2015), participants navigated in a rectangular or triangular virtual tunnel maze that was either Euclidean or non-Euclidean in nature; non-Euclidean mazes were closed, continues spaces with violations of Euclidean geometry globally, e.g. quadrilateral with interior angles exceeding 360°. After an exploration phase, participants navigated the shortest path between two points for various starting locations and completed a two-alternative forced choice test, asking them to indicate among image pairs which image depicted the non-Euclidean environment. Interestingly, participants navigated Euclidean and non-Euclidean VEs equally well and were unable to identify which VE was geometrically impossible. Thus, in this case, successful navigation did not depend on an accurate, Euclidean map-like representation. Participant's failure to detect the geometric inconsistency of the impossible environments suggests that geometric violations of Euclidean assumptions can be pushed quite far, before hindering navigation.

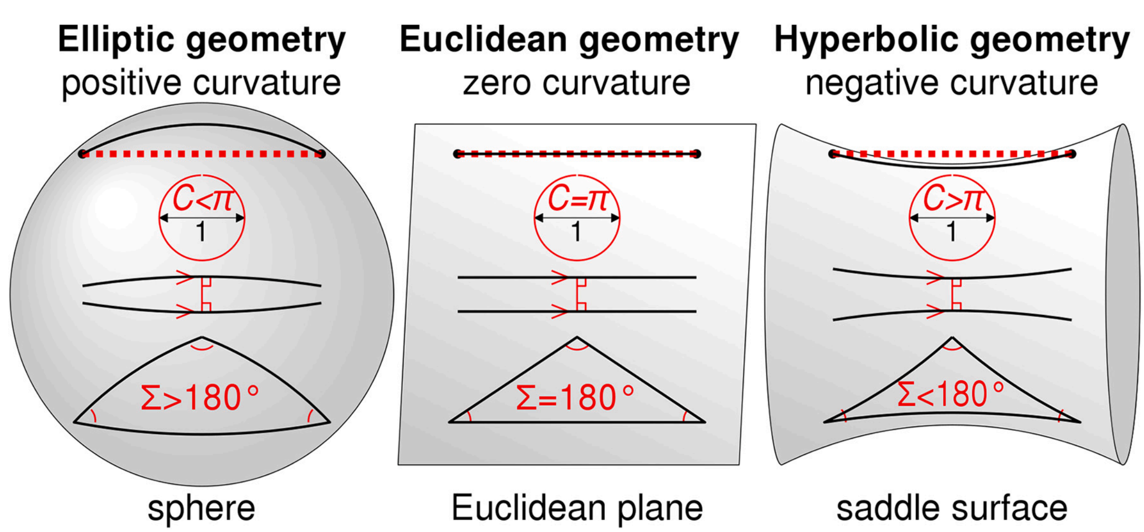

Fig. 1. Graphical illustration of the properties of Euclidean and non-Euclidean geometries. By Cmglee - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=94781281

A Euclidean or non-Euclidean representation was explicitly tested in a recent study by Warren, Rothman, Schnapp, and Ericson (2017). Participants were trained to navigate various locations in both Euclidean and non-Euclidean VEs. The non-Euclidean VEs contained wormholes which linked spatially distant parts of the environment via a teleportation mechanic. During the test phase, participants completed a route task and shortcut task. During the route task, participants walked to a target location under full-cue conditions along previously learned routes in the maze. In the shortcut task, participants walked to a target location directly under reduced cue conditions, e.g. features of the environment were removed. Results showed comparable learning for both environments. Moreover, shortcuts for the non-Euclidean group were biased toward the wormhole location, which was taken as evidence for the violation of Euclidean assumptions of positivity and triangle inequality.

These data do not provide definitive evidence for Euclidean vs non-Euclidean geometry, due to the intrinsic ambiguity of the metric properties of these types of spaces. For example, because the perceptual experience across the portal was continuous, the observer had no means of determining where exactly the portal was. As a result, the locations of the targets are essentially ambiguous, e.g., it is not clear whether a landmark is on one side of the portal or the other side. Moreover, shortcut/pointing task in such environment is not well-defined mathematically. In an ordinary Euclidean space, pointing/shortcut direction is defined as the straight line connecting the observer and the target. However, a “straight line” in tunnel mazes with portals is un-defined, because the line has to pass through the “empty” space outside the tunnels, where the geometry is un-defined and straightness has no clear meaning. Without concrete theoretical predictions on how such ambiguities were resolved by the participants, it is difficult to test the Euclidean vs non-Euclidean hypotheses.

Nonetheless, given the above findings, it is at least reasonable to consider the possibility that human spatial representation is supported by a non-Euclidean geometry. For example, a non-Euclidean framework could account for a variety of distortion effects by relaxing Euclidean assumptions (e.g. the triangle inequality, positivity, and symmetry). The present study sought to examine whether people are capable of taking geometry of the space (i.e., the curvature) into account when performing path integration tasks using true non-Euclidean virtual spaces, for the first time to our knowledge, that are curved and non-Euclidean locally at every point in space. Because the geometry of such environments is fully defined in every point of the space, homing/pointing task is also well-defined mathematically, allowing easy comparison between predictions of Euclidean and non-Euclidean hypotheses.

2. Experiment 1

Although the VEs used in previous research can be considered non-Euclidean in a generic sense, the fundamental shape of the environment is undetermined via the introduction of a portal. In this way, the environments used in Warren et al. (2017) can more precisely be described as ‘impossible spaces’ as opposed to a truly non-Euclidean space, as described by Lobachevskian or Riemannian geometry, which are well-defined. In Experiment 1 a truly non-Euclidean space was rendered in real-time to a VR headset, allowing users to explore an infinite 3D space modeled after Euclidean or non-Euclidean assumptions.

2.1. Methods

2.1.1. Participants

The experiment was conducted in the Virtual Reality and Spatial Cognition Lab in the Department of Psychology at the University of Illinois at Urbana-Champaign. A total of 24 participants completed the experiment, eight in each environment type (flat: 2 males; hyperbolic: 0 male; spherical: 3 males). The participant number was based on expected effect size for the spherical space, which has the most salient violations of Euclidean geometry and was the primary interest of the study. The mean separation between the two hypotheses (flat origin vs spherical origin, see explanations in the methods section below) was ~40 degrees. Based on our previous studies using similar task (i.e., path completion task in virtual Euclidean environment similar to the flat condition), the standard deviation of signed errors was ~20 degrees. Therefore we expected an effect size of ~2. Based on this estimation, 5 participants are needed to achieve a power of 80% at $p = .05$. We used $N = 8$ to ensure there’s sufficient power to at least differentiate whether people use Euclidean vs curved geometry in the spherical environment where curvature is most likely to be detected.

All participants were screened for color blindness using pseudoisochromatic color plates and had normal or corrected-to-normal visual acuity. All participants were screened for stereo depth perception using a random dot stereogram viewed through red-cyan anaglyph glasses. This research complied with the American Psychological Association Code of Ethics and was approved by the Institutional Review Board at the University of Illinois at Urbana-Champaign. Informed consent was obtained from each participant.

2.1.2. Virtual environment (VE)

The HTC Vive VR headset was used to display experimental stimuli using a Windows 10 desktop computer (i7-5820K, 3.3 GHz CPU; 32 GB RAM; NVIDIA GeForce GTX 980 Ti graphics card). The headset contained two low-persistence AMOLED displays (1080 × 1200 per eye, 110° hFOV, 90 Hz) and included two handheld motion-tracked controllers equipped with a track pad, grip buttons, and a dual-stage trigger. The system achieves six degrees of freedom positional tracking for both the headset and controllers within a 3 m × 3 m × 3 m volume.

The VE consisted of an open-world flight simulator depicting a three-dimensional Euclidean or non-Euclidean universe with different Earth-like planets floating sparsely in space. Two non-Euclidean environments – spherical (S^3) and hyperbolic (H^3) – and one Euclidean control were implemented based on previous work by Weeks (2006) (see Fig. 2 and supplemental material for video demos of sample trials). Both the spherical and hyperbolic environments had radius of 10 m. Participants could navigate the VE by turning their body and aiming a virtual beam pointer in the direction they wanted to travel and by pressing (and holding) a button on the controller to move forward at a constant velocity. An established point-to-origin paradigm known as the triangle-completion task was used to assess participant’s spatial representation in VR (Klatzky, Loomis, Beall, Chance, & Golledge, 1998; Loomis et al., 1993; Wan, Wang, & Crowell, 2012; etc.). The triangle-completion task requires participants to travel through a two-segment path defined by three points: origin (A), midpoint (B), and endpoint (C), respectively. For the present study, the initial physical heading was always aligned with respect to the outbound path, $\overline{AB}$. The participant travels from A to B, turns and travels from B to C, and then responds by turning to face the origin of locomotion.

Rendering in curved space produces some surprising optical properties that we will now discuss. Recall that an ordinary sphere is the 2D surface of a 3D ball, defined as $x^2 + y^2 + z^2 = 1$. Extending the equation of a sphere into a higher dimensional space generates the hypersphere, the 3D surface of a 4D ball, defined as $x^2 + y^2 + z^2 + w^2 = 1$. To imagine the optics of the hypersphere, consider the following example. You are standing at the north pole of a hypersphere and throw a ball in front of you. As the ball moves away, it appears to shrink in size, reaching its smallest apparent size near the equator. As the ball continues past the equator toward the south pole, the apparent size begins to increase – the increase in apparent size of the ball at this point is abnormal from what we would expect in a Euclidean space. The reason for this phenomenon is because light in the hypersphere travels in great circles (geodesics), rather than straight lines. Your line of sight propagates across the surface of the region in front of you (anterior hemisphere), converges at the antipodal point (south pole), propagates back across the region behind

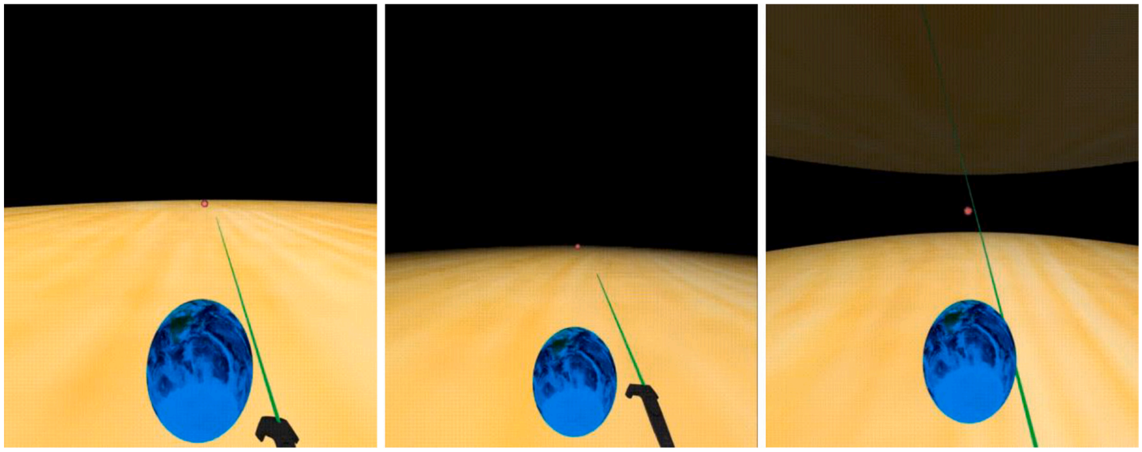

Fig. 2. Perspective view of the VEs used in Experiment 1. From left, Euclidean (flat), hyperbolic, and spherical. Participants stood on a ground plane at the start of the trial, but could change altitude during the experiment. Note the apparent size of the first landmark (the small globe the virtual wand points at) and the posterior hemisphere appearing above for the spherical space.

you (posterior hemisphere), and re-converges at the origin (north pole). Thus, in an empty region of the hypersphere, you would expect to see the back of your own head fully occupying your field of view above the horizon.

2.1.3. Procedure

The triangle-completion task was manipulated across two within-subjects factors: path ratio and turning angle. The ratio of the distance between $\overline{AB}$ and $\overline{BC}$ was manipulated within subjects to be either 1:2 or 2:1. For half of the trials $\overline{AB}$ was 10 m and $\overline{BC}$ was 20 m and for the other half $\overline{AB}$ was 20 m and $\overline{BC}$ was 10 m. The angle of rotation defined by the turn at point B was manipulated in 30° increments from −150° to +150° resulting in ten different turning angles: ±30°, 60°, 90°, 120°, 150°. The turning angle and path length were factored together to produce 20 unique paths. Each path combination was tested three times (in random order) for a total of 60 trials.

Participants were randomly assigned to one of three VE conditions – Euclidean, hyperbolic, or spherical – and asked to complete 60 trials of virtual triangle-completion task. During each trial participants traveled a three-point path marked by colored Earth-like planets appearing at the origin (blue), midpoint (red), and endpoint (yellow) of each path. Participants navigated by turning their body to aim a virtual beam pointer with a handheld controller and pressed a button to travel forward in any direction at a constant velocity. At the endpoint of each path, participants were asked to turn and point to the origin of locomotion (hidden) and submit their response by pressing a button on the controller. The entire experiment lasted approximately 60 min.

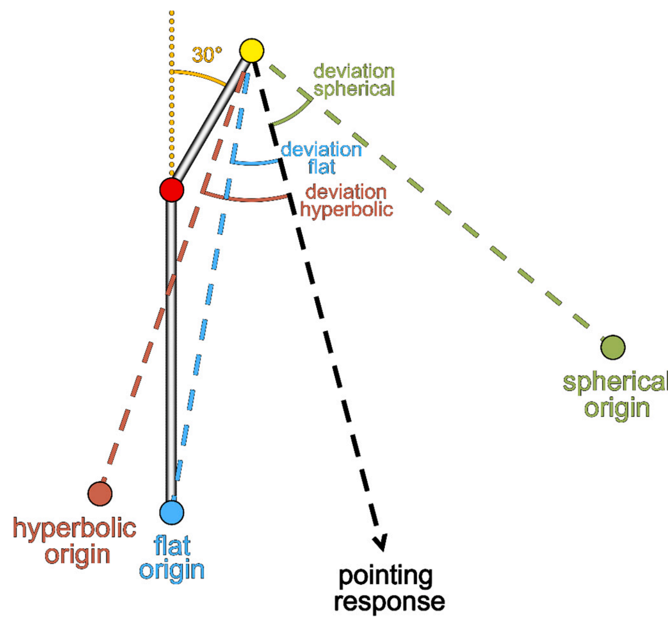

Fig. 3. An illustration of the angular deviation measurements. The angular deviations were generated per trial by comparing the pointing direction to the expected homing direction for flat, hyperbolic and spherical geometry to derive the deviation-flat, deviation-hyperbolic, and deviation-spherical, respectively.

2.1.4. Data analysis

Performance was examined in terms of the signed angular deviation between the pointing direction and the expected homing direction for a given geometry (i.e., flat origin, hyperbolic origin, and spherical origin). Due to different curvature of the spaces, the origin would be in different directions for the same path traveled in different spaces. For example, after walking 20 m, turning 30 deg. to the right, then walking 10 m (see Fig. 3), the origin would be at ~160 deg. to the right if the trip was in a Euclidean space (i.e., the *flat origin*), but at ~105 deg. if the trip was in a spherical space (i.e., the *spherical origin*). If a participant taking this path in a spherical space responded by pointing 160 deg. to the right, then s/he must have mistakenly treated the space as flat instead of spherical. In contrast, if the participant responded 105 deg., then s/he must have been able to take the curvature of the space into account and updated the position of the origin according to the spherical geometry. Thus, to examine whether participants were able to take the curvature of the space into account, we compared whether their pointing direction was closer to the flat origin or the spherical/hyperbolic origin.

More specifically, the pointing direction was represented as a vector defined by the user's head position and orientation at the time the response was submitted for each trial, projected onto the surface formed by the position of the first two landmarks and the participant. The expected homing direction for a given geometry was represented as a vector, defined by the expected origin and participants' head position at the time of response. In order to test which geometry participants used to perform the task, the *angular deviation* for a given geometry was calculated for each path as the signed angle between the pointing direction and the corresponding expected homing direction, both in the simulated environment and for the corresponding flat environment as comparison (see Fig. 3). These measures indicate how close their responses were to what one would expect if they used a given geometry. For example, a small deviation-flat and larger deviation-hyperbolic / deviation-spherical means the response was very close to what one would expect if the participant treated the environment as flat.

2.2. Results

The following analyses examined the effect of the curvature of 3D space on spatial updating for a virtual homing task. Underlying spatial representations for each environment were assessed in terms of the signed angular deviation between the participants' responses relative to where the path-origin should be for a Euclidean, hyperbolic, or spherical environment – smaller deviation indicates better resemblance. For the purposes of analysis, trials evaluating a turning angle of the same magnitude (e.g. $\pm 30^\circ$) were combined for all trials defined by the same path ratio. Pointing responses for the spherical and hyperbolic environments were examined in separate repeated measures ANCOVAs on pointing errors with the within subject factor of error type (deviation-flat vs. deviation-hyperbolic/spherical). Because it is well known that people have systematic biases in these homing tasks as a function of the target direction (e.g., Loomis et al., 1993), the absolute value of the correct homing direction was included as a covariate to control for variation in pointing bias as a function of the direction of the origin. For the Euclidean environment, a linear regression was run for deviation-flat as a function of the target direction to verify whether there was indeed systemic bias in their responses, as shown in previous research.

For participants that experienced a Euclidean (flat) environment ($N = 8$), there was a significant correlation between the pointing error (deviation-flat, mean = 5.93, 95% CI = [−0.54, 12.40]) and the target direction ($r = -0.573$, $p < .001$). As can be seen in Fig. 4a, people showed systematic biases in their pointing responses, over-estimating small target angles and under-estimating large target angles. These biases replicated the classical findings in previous research (Loomis et al., 1993).

A repeated measures ANCOVA with error type (deviation-flat vs deviation-hyperbolic-flip) as the within subject factor, and target direction as a covariate was applied to data for participants that experienced the hyperbolic environment ($N = 8$). The analysis revealed that pointing responses deviated significantly more from a hyperbolic (M = −8.42°, 95% CI [−14.63°, −2.21°]) representation compared to a flat (M = 6.02°, 95% CI [−1.03°, 13.07°]) representation; $F(1, 78) = 51.64$, $p < .001$, partial $eta^2 = 0.40$ (see Fig. 4b). There was also a significant effect of target direction ($F(1, 78) = 25.09$, $p < .001$) and an interaction between error type and target direction ($F(1, 78) = 54.84$, $p < .001$).[1] These data suggest that people's pointing responses resembled those of a Euclidean space more than those of a hyperbolic space.

Data for participants that experienced the spherical environment ($N = 8$) were analyzed in the same way as for the hyperbolic environment using a repeated measures ANCOVA, except with deviation-spherical instead of deviation-hyperbolic. Pointing responses deviated significantly less from a flat (M = 27.09°, 95% CI [16.44°, 37.74°]) compared to a spherical (M = 65.36°, 95% CI [54.11°, 76.60°]) representation; $F(1, 78) = 89.52$, $p < .001$, partial $eta^2 = 0.53$ (see Fig. 4c). There was also a significant effect of target direction, $F(1, 78) = 34.49$, $p < .001$ and a significant interaction between the error type and target direction, $F(1, 78) = 7.76$, $p < .01$. These data suggest that people's responses resembled those of a Euclidean space more than those of a spherical space.

To further examine whether deviations from the flat origin differed across the three environments, an ANCOVA was run with deviation-flat as the dependent variable, target direction as the covariate, and environment type as the factor. There was no significant difference among the environment types, $F(2, 236) = 2.78$, $p = .064$, partial $eta^2 = 0.02$. There was an effect of target direction, $F(1, 236) = 103.12$, $p < .001$. These results suggest that participants' responses conformed to Euclidean geometry to the same degree in the three environments, despite the difference in the actual geometry.

2.3. Discussion

There was no evidence in the results that people utilized the curvature of three-dimensional space in a path integration task. Pointing responses in both the spherical and hyperbolic environments resembled (i. e. showed less deviation from) the Euclidean (flat) space rather than the actual corresponding curved spaces. Responses also showed typical systematic biases of under−/over-estimation for larger / smaller target angles, respectively, replicating classical findings in previous research. These results suggest that when confronted with spatial information indicating violations of Euclidean geometry, participants failed to take the curvature of the space into account to complete tasks requiring spatial updating. These results are consistent with the hypothesis that human spatial representations and spatial processing are fundamentally Euclidean.

There are two major concerns with this experiment, however. The first issue is that people may not be able to detect violations of the Euclidean geometry due to their limitations in the perceptual system. For example, detection of object size, speed, and acceleration all has limitations in resolution, thus participants may not be able to discriminate the perceptual difference between a Euclidean space and a curved space. This is particularly concerning for the hyperbolic space, where the perceptual deviations from the Euclidean geometry were mostly quantitative and relatively subtle. The second issue is that people may need time to familiarize with the novel curved space in order to act properly. Experiment 2 addressed these issues by allowing participants to have an exploration/learning period for them to get to know the test space more thoroughly before performing the task. Moreover, we used the environment and paths that maximized the perceptual salience of the curved nature of the space to make sure violations of Euclidean geometry were readily perceivable.

3. Experiment 2

In this experiment, participants were provided a 5-min learning phase during which they were allowed to freely explore a non-Euclidean environment before their knowledge of that same environment was tested. Given the high curvature of the spherical space (10 m radius), a 20 m journey (equal to the longer leg of the testing path) would cover 1/3 of the spherical space around the perimeter, and participants could easily make multiple trips around the spherical space during the learning period to experience multiple qualitative violations of the Euclidean geometry. Therefore if curvature of space is a parameter in people's spatial representation, they should be able to detect it and take it into account when performing the homing task.

3.1. Materials and methods

The experiment focused on the Riemannian (spherical) space given that the perceptual deviations from the Euclidean geometry were most salient for that space, with not only quantitative but various qualitative violations of Euclidean geometry. Moreover, the predicted outcomes for the two types of geometry were very distinctive, making it easier to experimentally differentiate them. By implication, if participants cannot detect the curved nature of the spherical space, it is unlikely that they would for the hyperbolic space. Additionally, because there were large inherent systematic biases in the 2:1 paths that can potentially complicate the analysis on the effect of curvature of space, the design of the triangle-completion task trial structure was altered to focus on 1:2 paths by discarding 2:1 paths and doubling the per combination trial count

[1] The interaction between error type and target direction means that the deviation of the responses from one origin does not follow the same trend as that from the other origin, which basically means that the two origins themselves are not parallel, therefore deviation of the response from one is non-parallel to the other, resulting in an interaction.

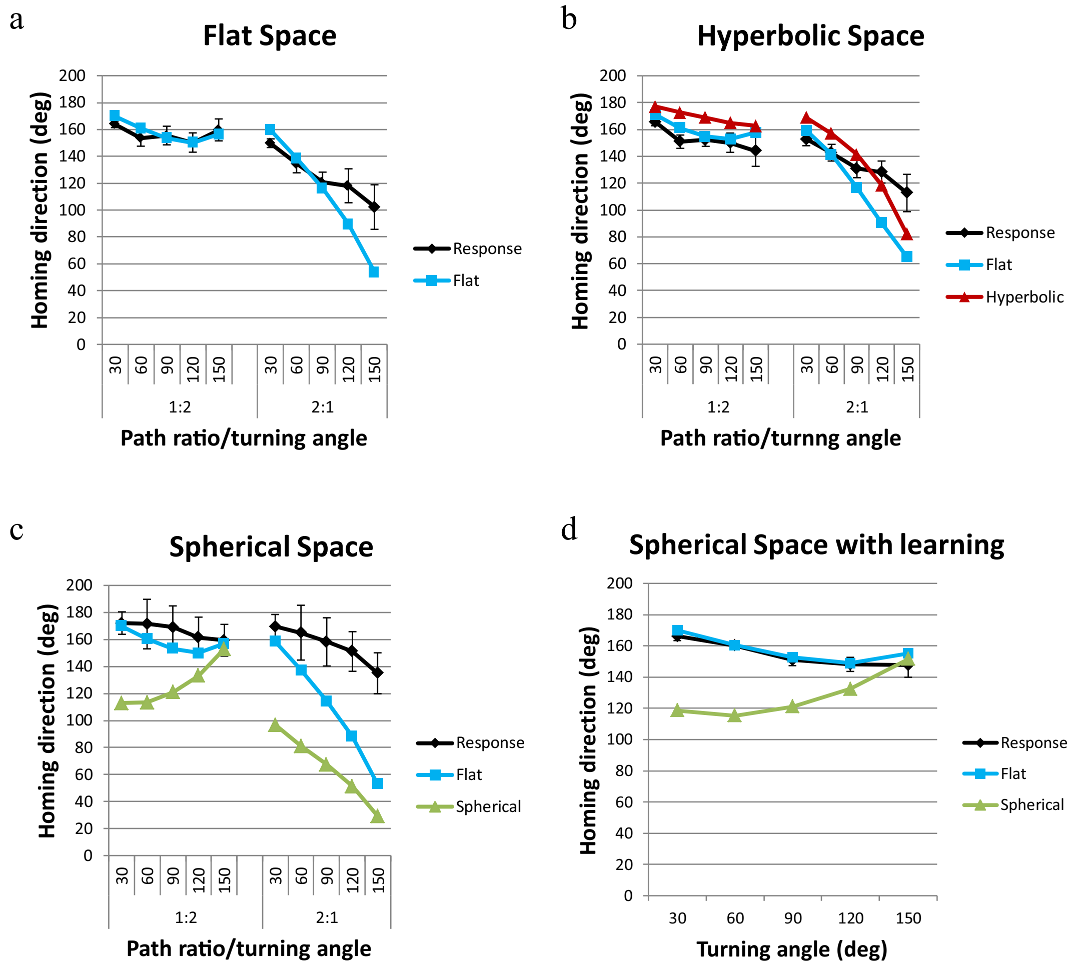

Fig. 4. (a) Response direction and the direction of the origin (egocentric homing direction) in the Euclidean space in Experiment 1. Note the systematic biases, especially for paths with a large turning angle for 2:1 path ratio. (b) Response direction and the direction of the origin for both the hyperbolic geometry and the Euclidean geometry in the hyperbolic space in Experiment 1. (c) Response direction and the direction of the origin for both the spherical geometry and the Euclidean geometry in the spherical space in Experiment 1. (d) Response direction and the direction of the origin for both the spherical geometry and the Euclidean geometry in the spherical space in Experiment 2. If participants were able to use the hyperbolic/spherical geometry, their responses should be closer to the hyperbolic/spherical origin line. In contrast, if their responses were closer to the flat origin line, then they must have failed to utilizethe curvature of the space and performed the task as if they were in a Euclidean space. The data showed they treated the space as Euclidean. The error bars are between-subject standard errors.

from three to six. Methods for Experiment 2 were otherwise identical to the spherical condition in Experiment 1.

3.1.1. *Participants*

The experiment was conducted in the Virtual Reality and Spatial Cognition Lab in the Department of Psychology at the University of Illinois at Urbana-Champaign. A total of 12 participants completed the experiment (4 males). All participants were screened for color blindness using pseudoisochromatic color plates and had normal or corrected-to-normal visual acuity. All participants were screened for stereo depth perception using a random dot stereogram viewed through red-cyan anaglyph glasses. The research complied with the American Psychological Association Code of Ethics and was approved by the Institutional Review Board at the University of Illinois at Urbana-Champaign. Informed consent was obtained from each participant.

3.1.2. *Procedure*

The procedure for Experiment 2 was identical to Experiment 1 with the exception that participants engaged in a 5 min learning phase before completing the triangle-completion task. Participants were told the virtual space was unusual and unlike the physical space we live in, but were not given specific instructions about the curved/spherical nature of the space. For the learning phase, participants were first introduced to the VR simulation and navigation controls. Then the experimenter guided the participants verbally from the first landmark (blue), to the second landmark (red), to the third landmark (yellow), and then participants were given five minutes to navigate freely in the environment. During the free exploration period, the second and third landmarks remained visible and stable, while the first landmark disappeared once the participants reached the second landmark (as in an experimental trial) before the learning period started. See supplemental material for video demos of the learning period.

3.1.3. *Data analysis*

Data processing and analysis for Experiment 2 were identical to Experiment 1 for the spherical space.

3.2. *Results and discussion*

The following analysis examined the effect of the curvature of 3D space on spatial updating for a virtual homing task. Participants' underlying spatial representation of the environment was assessed in terms of the signed angular deviation between the participants' responses relative to where the path-origin should be for a Euclidean or spherical environment – smaller deviation indicates better resemblance. For the purposes of analysis, trials evaluating a turning angle of the same magnitude (e.g. $\pm 30^\circ$) were combined.

Pointing responses were analyzed in a repeated measures ANCOVA to determine whether there were statistically significant differences in response deviations relative to flat origin vs spherical origin ($N = 12$). The absolute value of the correct homing direction was included as a covariate to control for systematic biases in pointing responses as a function of the target directions. There was statistically significant difference in response deviations between error types; pointing responses deviated significantly less from the flat (M = −2.75°, 95% CI [−7.20°, 1.70°]) origin compared to a spherical (M = 26.85°, 95% CI [20.16°, 33.53°]) origin; $F(1, 58) = 29.75$, $p < .001$, partial eta $^2 = 0.34$ (see Fig. 4d). Moreover, there was a significant effect of target direction, $F(1, 58) = 368.39$, $p < .001$, and a significant interaction between the error type and the target direction, $F(1, 58) = 24.14$, $p < .001$.

To further examine whether familiarization with the environment would allow people to use curvature in the homing task, the data were split in half and those from the second half of the testing stage were analyzed as before. Even after participants had spent more than half an hour in the spherical space exploring and performing the path completion task, their responses still conformed to the Euclidean geometry and not the spherical one. Pointing responses deviated significantly less from the flat (M = −1.50°, 95% CI [−6.97°, 3.96°]) origin compared to a spherical (M = 28.32°, 95% CI [21.03°, 35.62°]) origin; $F(1, 58) = 20.46$, $p < .001$, partial eta $^2 = 0.26$. Moreover, there was a significant effect of target direction, $F(1, 58) = 378.72$, $p < .001$, and a significant interaction between the error type and the target direction, $F(1, 58) = 16.18$, $p < .001$. These results replicated those of Experiment 1 and suggest that failure to take curvature of space into account was not a result of perceptual limitations or familiarity. Instead, these findings suggest that people's spatial representations in the path integration / spatial updating system follow Euclidean geometry, even when violations were clearly present.

4. General discussion

Two experiments examined people's path integration in true non-Euclidean environments, i.e., curved spaces with non-Euclidean geometry at every point of the space. Experiment 1 tested people's homing behavior in three environments, Euclidean space, hyperbolic space, and spherical space. Participants walked along two legs in an outbound journey, then pointed to the direction of the starting point (home). The length of the two segments and the turning angles were manipulated. The results showed that participants' responses matched the direction of Euclidean origin, regardless of the curvature of the space itself. Experiment 2 used the most perceptually salient space (i.e., spherical space) and added a learning period for participants to explore the spatial properties of the environment before performing the homing task to ensure violations of Euclidean geometry were readily detected. The results replicated that of Experiment 1 and participants still responded as if the space were Euclidean. These data suggest that the path integration / spatial updating system operates on Euclidean geometry, even when violations are clearly present.

Although the path integration task used in the present study does not involve cognitive map per se, these findings nonetheless may have implications on the geometrical assumptions in more complex spatial representations. It has been proposed that the path integration process is the building block of more complex spatial representations. For example, Wang (2016) proposed a process that multiple path integrators form a dynamic "cognitive map." Warren (2019) proposed that metric information in a labeled graph is established from basic path integration system. Therefore it is reasonable to assume that Euclidean principles operate in spatial representations in general, at least at the construction / processing level, and/or for local spatial relations. However, whether Euclidean rules are preserved in the global spatial relations depends on the form of the representation. For example, a single reference frame, coordinate-based cognitive map where each target is assigned a set of unique coordinates will ensure that Euclidean geometry is preserved for the entire map. Relaxation of these assumptions can lead to violations of global Euclidean geometry. For example, there will be no guarantee that Euclidean principles will hold globally when multiple reference frames are used for different targets, or when multiple (redundant) spatial coding exists for the same target associated with different context.

The learning period in Experiment 2 was intended to ensure that participants had sufficient opportunity to detect violations of the Euclidean geometry, not for familiarization in the sense of having hours or years of learning. Based on the split-data analysis results, it's unlikely that exposure to the environment alone will allow participants to do the task. Even with full knowledge of the nature of the space, the experimenters still had trouble understanding the perceptual phenomena when simply looking at the environments without additional help from drawing diagrams to work out the mathematical relations. Therefore the ability to perform spatial tasks in curved spaces likely requires much more elaborated training, possibly with the help of multiple tools such as diagrams, analogy, etc. Whether people can eventually do it, and what type of training is most effective are important questions that need future research.

We chose the path integration task because it is generally regarded as one of the most basic, simplest spatial tasks. When people failed to perform a task, the failure could come from any of the components of the task. For example, the homing task contains at least two main components, 1) to build a spatial representation of the curved space based on the perceptual information; and 2) to use this representation to perform the path integration task. If the failure lied in #1, then it means that people couldn't form a spatial representation of the curved space, even though violations of Euclidean geometry were plainly visible. If the failure lied in #2, then it means that the path integration/spatial updating system abides by Euclidean geometry even when a non-Euclidean representation exists for other purposes. The present study cannot pinpoint whether people failed in the first or the second component, and the data simply showed that the path integration/ spatial updating system operates on Euclidean geometry, at least when explicit training is absent. Whether this failure to use curvature of space is specific to the path integration/spatial updating system or for other spatial tasks in general remains a question for future research.

The present study is very different from previous studies on non-Euclidean representations using wormhole type spaces, both in terms of the theoretical questions addressed, and the research paradigm used. In terms of the theoretical questions, the present study examined different spatial systems than previous research, i.e., we were primarily interested in path integration/spatial updating, while previous research (e.g., Warren, 2019) was interested in cognitive map. Moreover, the present study examined a different aspect of non-Euclidean geometry, i.e., we emphasized on local geometry, while previous research focused on global geometry. As a result, the research paradigm and the type of virtual environments used are also totally different. Previous research used wormhole/teleportation techniques in their display, which is locally Euclidean and violation of Euclidean geometry only occurs at a global scale. In contrast, we used curved spaces, which are non-Euclidean both locally and globally. As discussed in the Introduction, wormhole type tunnel environments themselves do not have well-defined geometry in terms of curvature. As a result, pointing/shortcut tasks are not well-defined mathematically either. Therefore participants' pointing/shortcut responses reflect their implicit assumptions about the geometry of the unseen space between the tunnel walls, which to our knowledge has not been addressed in past research yet. Because of the theoretical questions and the specific spatial system/process we were interested in, theories focusing on violations of global Euclidean geometry such as labeled graph model are largely irrelevant to our study, and it is not clear that type of representation can provide a solution for the problems of curved spaces.

In the present study there was a combination of visual and proprioceptive/motor information, as participants actively turned their body to face different directions. However the body-based cues were restricted to rotation only, while translation was purely visual. It has been well established that body-based cues are important for spatial updating (e.g., Cherep, Kelly, Miller, Lim, & Gilbert, in press). We did not use physical walking as in some of the previous research (e.g., Warren et al., 2017) for two reasons. The first reason is that it's difficult to implement physical walking in our study, partly due to limitations of the physical space, since the lab is not large enough for full walking, and partly because of the nature of the environment. The second reason is that it is not clear that including physical walking is necessarily helpful for our non-Euclidean environments. Because physically walking in a Euclidean world while visually moving in a curved space would create cue conflicts between body-based cues and visual information of the non-Euclidean virtual world, and such conflicts might hinder learning rather than facilitate learning comparing to no body-based cues. Nevertheless, future research is needed to examine the effects of body-based cues in this type of true non-Euclidean environments.

The failure to take the curvature of the space into account in the present study does not necessarily mean people are never able to comprehend curved spaces. Because we were primarily interested in how people process non-Euclidean information naturally without specific instructions, as in previous studies using the impossible spaces (e.g., Warren et al., 2017), participants were not explicitly instructed about the curvature of space. Therefore these results suggest that curvature is not a pre-existing parameter of human spatial updating system that can be readily utilized even when perceptual information indicates the space is curved. In other words, the path integration/spatial updating system as is cannot handle curvature of space, however it remains an empirical question whether extensive training/tutoring can help expand the capacity of the system to accommodate curved spaces.

In summary, our study examined whether humans can comprehend and perform a basic navigation task based on non-Euclidean geometry using truly non-Euclidean, curved virtual spaces for the first time. The results provided evidence that people's spatial representations in the spatial updating system follow Euclidean geometry, even when violations were clearly present. Whether people can eventually take curvature of the space into account after extensive training remains a topic for future research.

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