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Human spatial learning strategies in wormhole virtual environments

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ABSTRACT
Humans can learn spatial information through navigation in the environment. The nature of these spatial representations is constantly debated, including whether they conform to Euclidean geometry. The present study examined the types of Euclidean representations people may form while learning virtual wormhole mazes. Participants explored Euclidean or non-Euclidean tunnel mazes and drew maps of the landmark layout on a 2D canvas. The results showed that people have different, consistent strategies, some mainly preserving distance information while others mainly preserving turning angles. The straightness of the segments was mostly preserved. These results suggest that representations of non-Euclidean space may be highly variable across individuals, and possible Euclidean solutions need to be carefully examined before testing Euclidean vs alternative models.

KEYWORDS
Spatial representation; Euclidean geometry; wormhole; cognitive map; virtual reality

1. Introduction

Spatial knowledge is central to the way in which humans perceive and reason about the physical world, and people develop a rich set of skills to learn the spatial environment. For example, people can recognize places, remember landmarks, recall past navigation experiences such as turns made at particular intersections, returning home using path integration, etc. Among these skills, the ability to construct a representation of the environment to guide flexible path planning (such as novel shortcuts) has often been considered the highest achievements of spatial cognition. However, the nature of such representations has been the subject of constant debate (Bennett, 1996; Burgess, 2006; Gallistel, 1989; O'Keefe & Nadel, 1978; Shelton & McNamara, 2001; Tolman, 1948; Wang, 2012, 2016; Wang & Spelke, 2002; Warren, 2019; Wehner & Menzel, 1990).

One of the main issues about these cognitive maps is whether they conform to Euclidean geometry, and finding experimental paradigms that can address this question has been very challenging. For example, some recent studies

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provided evidence against the traditional Euclidean map hypothesis using the wormhole technique in Virtual Reality. In particular, Warren, Rothman, Schnapp and Ericson (2017) examined how people perform route and shortcut tasks in Euclidean vs Non-Euclidean environments with wormholes. Participants explored various locations in both Euclidean and non-Euclidean Virtual Environments (VEs). The non-Euclidean VEs contained wormholes which linked spatially distant parts of the environment via a teleportation mechanism. During the test phase, participants completed a route task and a shortcut task. In the route task, participants walked to a target location along corridors in the virtual maze. In the shortcut task, participants walked to a target location in a direct path after features of the environment were removed. Results showed comparable learning for both environments. Shortcuts for the non-Euclidean group were biased toward the wormhole location, which was taken as indicating a violation of the metric assumptions of positivity and triangle inequality. Moreover, shortcuts to multiple targets showed "folds," i.e., ordinal reversals, which was again taken as evidence against the Euclidean map hypothesis.

There is a potential caveat in this approach, however. That is, given the non-Euclidean nature of the wormhole environment, it is not clear what the Euclidean map should look like. Without knowing how participants may interpret such an extraordinary space in a Euclidean fashion, it is not clear what the predictions of the Euclidean map hypothesis should be. For example, Figure 1(a) shows a simple maze with one wormhole in it (P-P'). According to Warren et al. (2017), the Euclidean map hypothesis should predict shortcut directions from H in the order of A, B, C, as indicated in the drawing of Figure 1(a). However, Figure 1(b) shows a possible mental representation of such a maze, which is not veridical but perfectly Euclidean. Such a representation could be derived if the recalibration process (Wang, 2016) or regularization process (Warren, 2019) of the path integration system attributes the discrepancy/error to mis-perception of the turning angles of segment ABC on the left and makes the adjustment by increasing/decreasing the angles to create a corrected representation (see Appendix for a detailed walk-through of how such a representation is constructed). This potential Euclidean map of the wormhole maze actually predicts shortcut directions in the order of A, C, B, which contains the same type of ordinal reversals observed in Warren et al. (2017). This example shows that different Euclidean interpretations of the wormhole mazes can make drastically different predictions of the spatial judgment tasks, and rips and folds themselves do not necessarily disprove the Euclidean map hypothesis.

The lack of unique Euclidean solution to represent a non-Euclidean environment also lies in the fact that for a given virtual maze, the position and number of wormholes cannot be defined uniquely in the inertial reference frame. Figures 1(c,d) show two design blueprints (i.e., the spatial model the

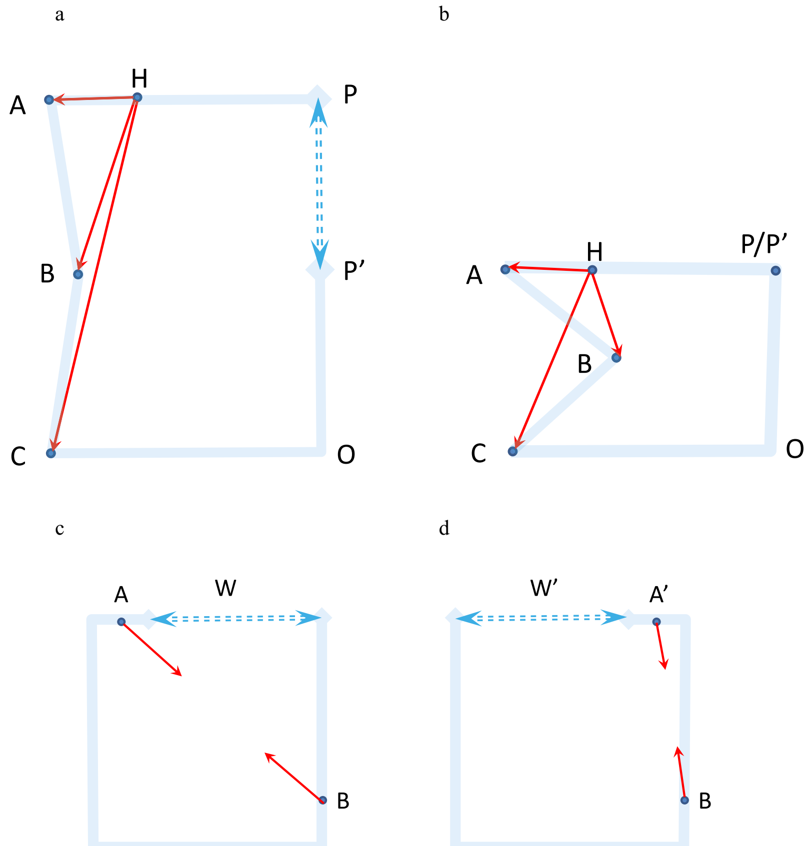

Figure 1. An example of a Euclidean map for a wormhole environment that contains folds (panels a & b), and an example of two isomorphic design blueprints for the same virtual wormhole maze with different shortcut directions in the inertial reference frame (panels c & d). Panel (a) shows a wormhole tunnel containing 5 segments. The navigators will be transported instantaneously to P′ when they reach P, and vice versa (wormhole effect). Using the coordinates of A, B, C, H in this inertia reference frame, the pointing directions from H to A, B, and C has the ordinal relationship of HA-HB-HC. Panel (b) shows a potential Euclidean representation people may construct (e.g., by someone prioritizing distance information) for the wormhole environment in panel (a) that preserves all the distance information while sacrificing the turning angles (see more discussion in the results section below about strategy and cognitive style). In this representation, the ordinal relationship is HA-HC-HB, which is a "fold" according to Warren et al. (2017). Therefore Euclidean representations can also produce rips and folds. Panels (c, d) are different design blueprints of the same wormhole environment. In both cases, the navigator is transported instantaneously across the wormhole (W or W′), resulting in identical navigation experience of a maze with three regular length legs and a shortened leg connected by 90° turns. Using the inertial coordinates in the design blueprints (i.e., positions on the paper), the directions of B from A are different, again suggesting wormhole environments don't have unique Euclidean predictions.

programmer uses to create the wormhole maze in virtual reality) of the same virtual maze, with drastically different target locations in the inertial reference frame. That is, if one cuts off a piece of the maze at one end of a wormhole (e.g., the small segment containing target A at the left side of wormhole W), and attaches it to the other end to close it, one will remove the original wormhole (e.g., wormhole W) and create a new set of wormhole(s) (e.g., wormhole W' in Figure 1(d)). The new design blueprint will generate exactly the same virtual environment as the original one, and an observer traveling in the maze will have no means to differentiate the two. In other words, the design blueprint is merely a convenient way for the programmer to construct a 3D model to generate the relevant computer graphics, and there are many design blueprints corresponding to a given virtual wormhole environment, each with different structure and target layout in the inertial reference frame.

The lack of unique design blueprint for a given wormhole environment means that the target positions in the inertial reference frame in a specific design blueprint the experimenter happens to choose are merely accidental and arbitrary, and calculations of the shortcut errors can be drastically different across different versions of the design blueprints. For example, the shortcut directions from Figure 1(c) are about 45 deg diagonal, while those from Figure 1(d) are about 15 deg from vertical, even though both should be considered the Euclidean map prediction according to Warren et al. (2017). This example shows that using the target coordinates in the inertial reference frame of a given design blueprint can be very misleading.

The difficulty in defining the Euclidean map for a non-Euclidean wormhole environment highlights the importance of understanding how people accommodate conflicting/inconsistent perceptual information during spatial learning in such environments. A few studies have examined the type of information preserved in human spatial learning of non-Euclidean space using virtual reality (VR) technology. For example, Kluss, Marsh, Zetzsche and Schill (2015; also see Zetzsche, Wolter, Galbraith, & Schill, 2009) had participants navigate in a rectangular or triangular tunnel maze that was either Euclidean or non-Euclidean (through portals) in nature. After exploration, participants reproduced the maze by walking along the learned paths around the tunnel maze. The results showed that people preserved the number of segments and the turning angle between segments. These results were taken as evidence that people do not need to convert the impossible VEs into a "possible" Euclidean format.

There are two potential limitations in these studies. One limitation is the reproduction task. It has been shown that people can remember and reproduce motion profiles such as velocity/acceleration over time (Berthoz et al., 1995). Therefore, it is possible that participants simply reproduced the motion sequences stored in the motor system, which is not a demonstration of the type of cognitive map representation examined here. The second limitation is that different types of spatial information (such as distance vs turning angles) were not examined in the same study, therefore it is difficult to know whether people simultaneously preserved both types of information or not.

The present study aimed to further examine what types of potential Euclidean representations people may construct when learning non-Euclidean virtual environments using impossible spaces similar to those used in Kluss et al. (2015). Participants were tested across four VEs, three of which contained wormhole path segments, resulting in violations of Euclidean geometry. Participants' spatial knowledge was assessed using a map-drawing task, wherein participants illustrated a configuration of landmarks to indicate the global shape of each environment. Basic geometric properties of the environment, such as distances, angles, and straightness were then derived for subsequent analysis to see what type of information is preserved and what is not when people have to produce a Euclidean map.

2. Methods

2.1. Participants

The experiment was conducted in the Virtual Reality and Spatial Cognition Lab in the Department of Psychology at the University of Illinois at Urbana-Champaign. A total of 29 participants completed the experiment. All participants were screened for color blindness using pseudoisochromatic color plates and had self-reported normal or corrected-to-normal visual acuity. All participants received course credit for their participation. This research complied with the American Psychological Association Code of Ethics and was approved by the Institutional Review Board at the University of Illinois at Urbana-Champaign. Informed consent was obtained from each participant.

2.2. Virtual environments (VE)

Four virtual environments were constructed using Hammer World Editor software from Valve Corporation (see Figure 2(a)). Each VE consisted of four hallways joined at orthogonal angles to form a closed loop populated by twelve landmarks including the four corners. The starting location was indicated by a checkered pattern and each subsequent corner was marked by a unique number, i.e., 01, 02, and 03, respectively. The remaining eight landmarks consisted of four colors (i.e., blue, red, yellow, or black) and four tiled patterns (i.e., vertical stripes, horizontal stripes, triangles, or circles) that marked the VE walls at arbitrary locations. Landmarks were positioned such that only two landmarks could appear between two corners. One environment formed a square shape (Euclidean space), while the other three contained a contracted or expanded path segment so that the overall shape of the environment violated the principles of Euclidean geometry. A portal was introduced between seams of the contracted or expanded path segment to a b

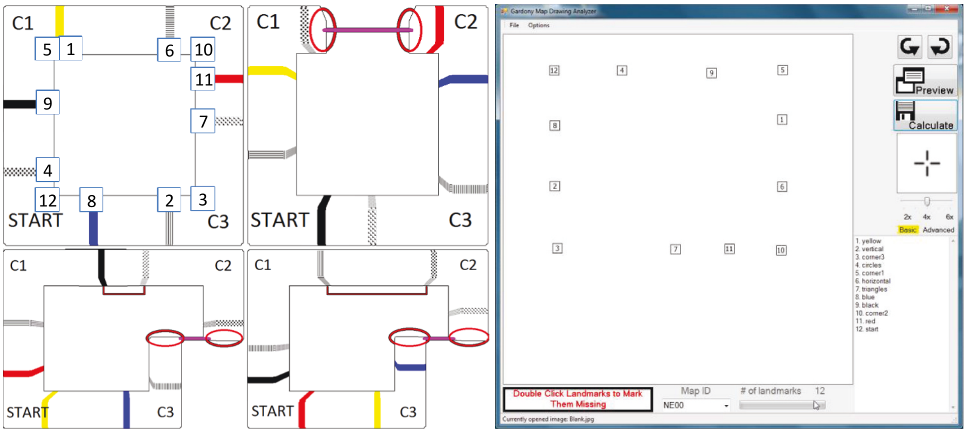

Figure 2. (a) Schematic depiction of the four VEs used in the experiment in clockwise order: Euclidean, contracted, expanded (v1), and expanded v2. The maps represent an overhead, perspective view of the mazes, and the bent portion of each landmark is on the inner vertical wall. Wormholes are indicated by red circles. An array of numbered tiles corresponding to the GMDA landmark list is placed next to each landmark in the Euclidean map for easier comparison to Figure 2(b). The number-landmark pairing was different for each environment. (b) a screen shot of a map drawing produced by one participant for the Euclidean environment, as an illustration of the user interface for the Gardony Map Drawing Analyzer (GMDA) depicting interactable numbered tiles representing landmarks from the VE. Note the drawing was rotated 90° clockwise relative to the Euclidean map in Figure 2(a). Participants could place the tiles anywhere on the canvas.

achieve this effect. For the contracted VE, one path segment was reduced by fifty percent of the original path length. For the two expanded VEs, one path segment was increased by fifty percent of the original path length. Additionally, the positions of two landmarks for the expanded path segment were manipulated across the two expanded VEs. This manipulation was done for a theoretical reason that is beyond the scope of the present paper and the two VEs were combined in the data analyses. Geometry in each VE used unlit shaders (i.e. no lighting or shading was present in the scene); however, the walls and ceiling were colored in contrast with the floor to indicate where surfaces met to form edges. Participants freely traversed each VE from a first-person perspective on a desktop computer using a keyboard and mouse. The player camera was controlled by a mouse and movement was controlled using the up, down, left, and right arrow keys.

2.3. Gardony Map Drawing Analyzer (GMDA)

The GMDA is a software that analyzes digital conversions of hand drawn sketch maps (Gardony, Taylor & Bruny, 2016). The interface consists of a 700 × 700 (pixel) canvas in which participants can drag-and-drop numbered tiles into a configuration that represents the global shape of an environment (see Figure 2(b)). The software calculates pairwise landmark comparisons to estimate canonical and bidimensional regression coefficients, including Euclidean distance and angle information for each pair. For this study, only observed distance and observed angle were used for analysis.

2.4. Procedure

The experiment was divided into two repeating phases: a study phase and a test phase. During the study phase, participants were assigned to one of the four VEs in pseudo-random order. That is, the Euclidean maze was always tested first, while the three non-Euclidean mazes were tested in a random order. Participants were required to traverse the environment for at least three minutes prior to testing but were allowed additional time if requested. For the test phase, participants were asked to draw a map of the studied environment using the GMDA. To do this, participants dragged-and-dropped labeled markers into a configuration that best represented the environment that they experienced. This process was repeated until each participant had navigated and mapped all four VEs. The entire experiment lasted approximately sixty minutes.

2.5. Data analysis

Three basic geometric properties of the environment were examined as the dependent measures, namely the angle, distance, and straightness of path segments depicted in the map-drawing task. The angle refers to the corner angles and is operationalized in terms of the angle formed by a corner landmark (a vertex) and its two flanking landmarks (the legs) (see Figure 3(b)). Four angle values were computed from the map drawings, one for each corner. The derived angle was then subtracted from 90° to calculate an error term (absolute error) for each corner. This measure was computed to quantify how much participants preserved the size of the turning angle of the virtual spaces they learned through navigation.

Distance information was operationalized in terms of the ratio of a path segment divided by the perimeter of the sketch map (hereafter referred to as normalized distance); the normalization was applied to equate sketch maps that would otherwise vary in scale, while preserving comparisons for relative segment length features. A path segment was defined as the sum of the subsegments between landmarks that lie between two adjacent corners (i.e., the sum of the distance between 2–4, 4–8, 8–10 in Figure 3(c)). The perimeter was defined as the total distance between all landmarks that formed the boundary of the sketch map. Normalized distance was also computed for the four original maze maps to derive the correct distance ratio. The difference of the

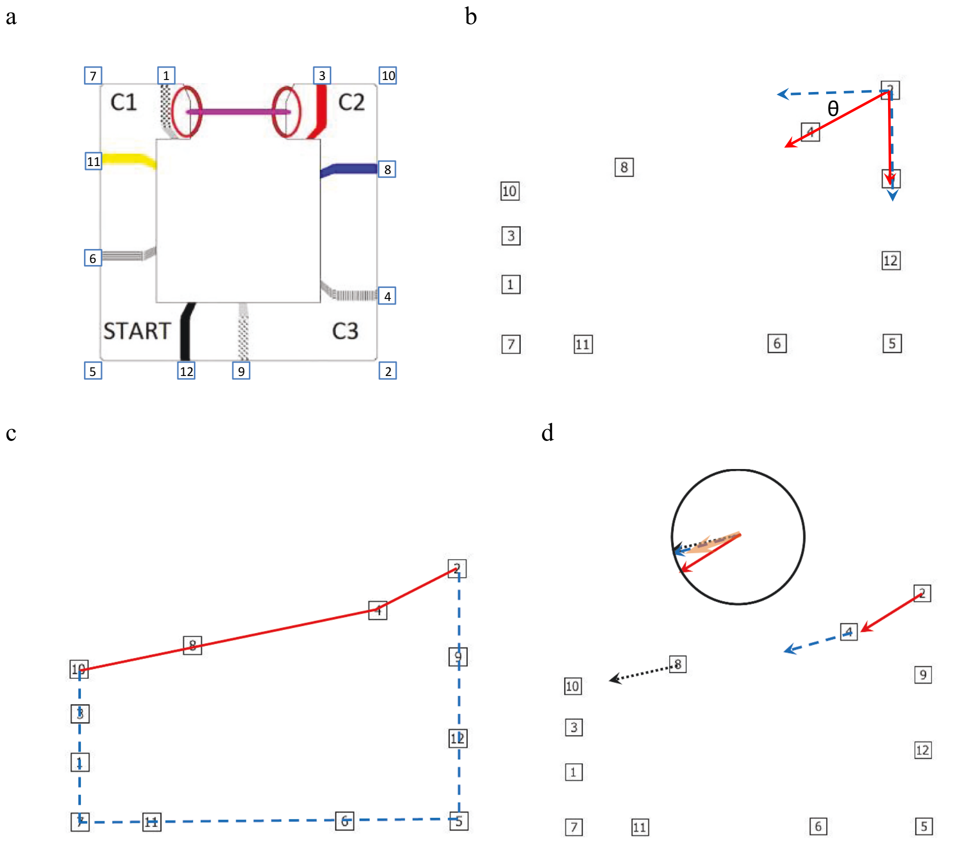

Figure 3. (a) a map of the contracted maze with corresponding landmark labels. (b) A sample drawing of the contracted maze (rotated 90° counter-clockwise relative to the map) illustrating the calculation of the angular error θ, which is the difference between the observed angle formed by a corner and its two adjacent landmarks (solid) and the actual turning angle (90°, dashed). The angular error was calculated for each corner and the four absolute errors were averaged as the mean angular error for each drawing. (c) illustration of the normalized distance calculation for a given segment as the path segment length (solid) divided by the perimeter (solid + dashed). The distance error is defined as the normalized distance for the drawing minus the normalized distance for the corresponding map. The distance error was calculated for each segment, and the four absolute errors were averaged as the mean distance error for each drawing. (d) an illustration of the straightness index calculation. Mean vector (thick shaded arrow) is derived from unit vectors between adjacent landmarks along a path segment (solid, dashed, and dotted lines). When the vectors are not aligned, as shown in the circle at the top, the mean vector is smaller than 1, and the segment is not straight. The more aligned the vectors, the larger the mean vector (up to 1), and the straighter the line is.

normalized distance between the observed (drawing) and target maps was calculated as an error term for each of the four path segments. This measure was computed to characterize how much participants preserved the distance relations in the four learned environments.

Straightness was operationalized in terms of a mean vector derived from the direction between landmarks along a path segment (see Figure 3(d)). The magnitude of the mean vector is one type of measurements of angular dispersion and ranges from 0 (uniform dispersion) to 1 (complete concentration in one direction) (Mahan, 1991). If the path segment is straight, then the 3 direction vectors should be collinear, and the mean vector length should be 1. Thus, as the value of the mean vector increases, a given path segment is said to be increasing in straightness, and vice versa. This measure was computed to quantify how much participants preserved the linearity/straightness of path segments in their map drawings.

3. Results

Data from each dependent measure were analyzed separately in a one-way repeated measures ANOVA with environment type as the factor. Significant results were followed up with post-hoc pairwise t-tests with Bonferroni corrections to identify the sources of the effects. Bayes Factor analysis was also conducted (with default prior $r = 0.707$ using the BayesFactor package in R) for each pairwise comparison to evaluate the relative evidence for the alternative vs null hypothesis as a Bayesian alternative to the classical hypothesis testing. Moreover, the proportion of participants having at least one ordinal error for landmark identity per VE was higher than expected. These errors occurred equally often in the Euclidean environment as in the non-Euclidean environments, and were not related to the nature of the environment. Therefore, dependent measures were calculated for landmark positions independent of landmark identity. Data from all participants ($N = 29$) were included in the final analysis. Significance was tested at $\alpha = 0.05$.

The angular error analysis examined whether the turning angles (90°) were preserved in the map drawings in the non-Euclidean environments as well as in the Euclidean environment. As shown in Figure 4(a), mean absolute angular error was significantly different among the three VE types; $F(2, 54) = 14.582$, $p < .001$. Paired t-tests comparing Contracted to Euclidean, Expanded to Euclidean, and Contracted to Expanded all revealed statistically significant differences in terms of mean absolute angular error; $t(27) = 4.407$, $p < .001$, BF>100; $t(28) = 2.672$, $p = .037$, BF = 3.8 and $t(27) = 3.361$, $p < .01$, BF = 16.1, respectively. Thus, participants were worse in preserving the corner angle in the non-Euclidean mazes than in the Euclidean maze. Moreover, they were more likely to preserve the angular properties of the corners of the environment when the space had been expanded rather than contracted by a similar factor.

The distance error analysis examined whether the distance information was preserved in the map drawings in the non-Euclidean environments as well as in the Euclidean environment. As shown in Figure 4(b), mean distance error was significantly different among the three VEs; $F(2, 54) = 17.108$, $p < .001$. Paired t-tests

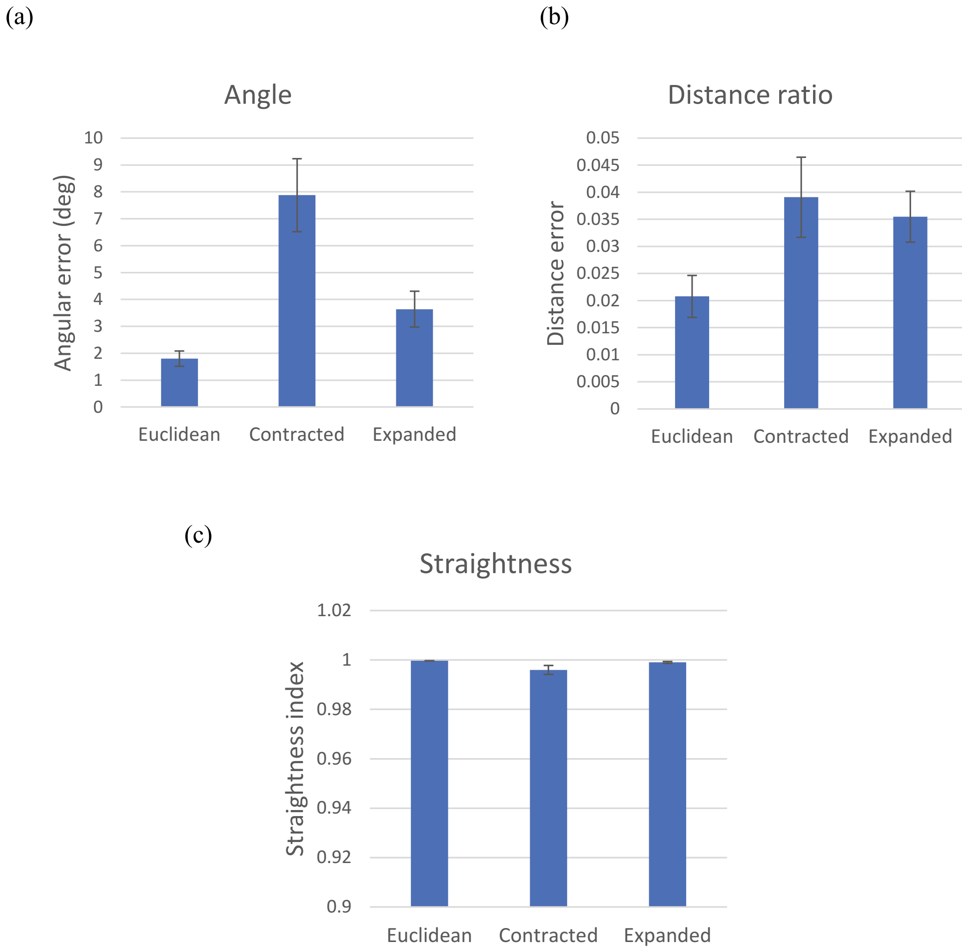

Figure 4. (a) Mean absolute angular error for the map drawing task across environment type. (b) Mean absolute distance error for the map drawing task across environment type. (c) The straightness index (average mean vector length) for the map drawing task across environment type. Note the floor of the y-axis is raised to make the difference between conditions more discernible. The error bars are between-subject standard errors.

comparing Contracted to Euclidean and Expanded to Euclidean both revealed significant differences in terms of mean distance error; $t(27) = 4.732$, $p < .001$, BF>100 and $t(28) = 5.647$, $p < .001$, BF>100, respectively, but not between Contracted and Expanded ($t(27) = 1.488$, $p = .445$, BF = 0.54). Thus, participants were worse in preserving the distance information for the non-Euclidean environments compared to the square (Euclidean) environment, regardless of whether the manipulated path segment was contracted or expanded.

The straightness analysis examined whether the straightness of the segments was preserved in the map drawings in the non-Euclidean environments as well as in the Euclidean environment. The repeated measures ANOVA on the straightness index showed significant effect of environment type ($F(2, 54) = 3.348$, $p = .043$)

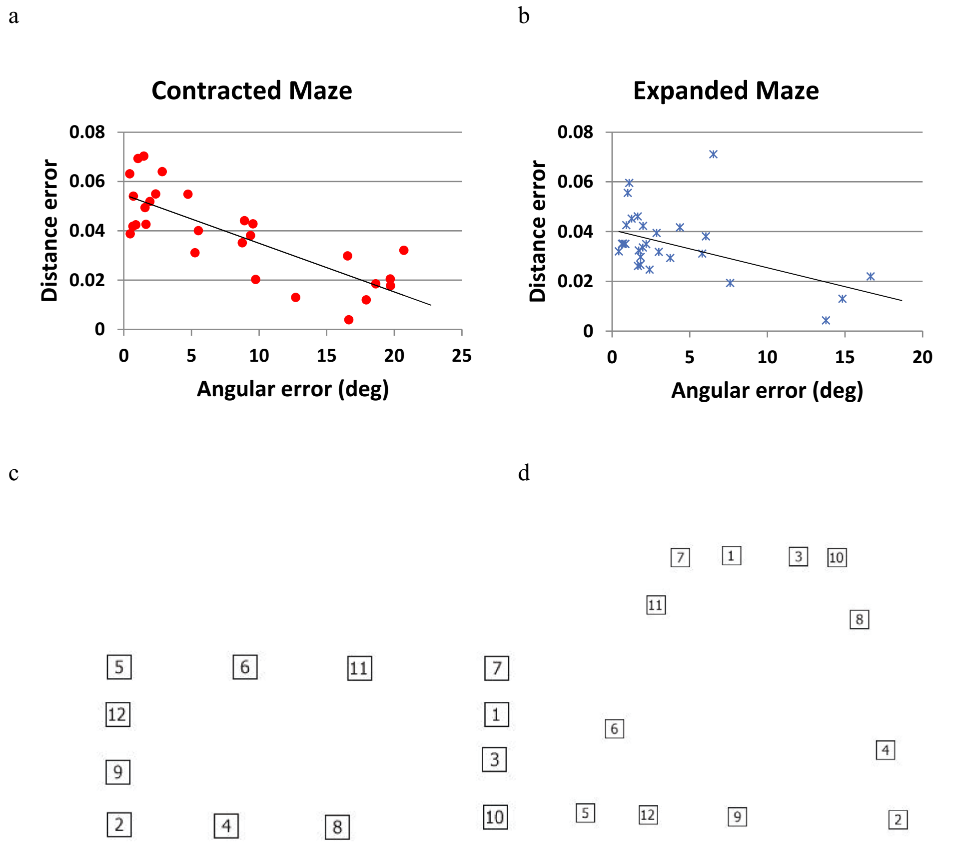

Figure 5. (a) Correlation between mean absolute angular error and distance error for the contracted maze. (b) Correlation between mean absolute angular error and distance error for the expanded maze. (c) participant #100 in the contracted maze condition preserved the turning angles but had high distance error. (d) participant #401 in the contracted maze condition preserved the distance relationship but showed large errors in the turning angles.

(Figure 4(c)). However, paired t-tests did not show significant difference between any environment pairs, with or without Bonferroni corrections (all ts <2.025, ps>.158 with Bonferroni corrections, BFs = 0.64, 0.95 and 1.18 for Contracted vs Expanded, Expanded vs Euclidean, and Contracted vs Euclidean, respectively). Overall, these results suggest that participants generally preserved the straightness of the maze segments for the non-Euclidean environments as well as the square (Euclidean) environment, regardless of whether the manipulated path segment was contracted or expanded.

To find out whether there was any relationship between participants' preservation of the distance relationship and the corner angles, follow-up correlation analyses were conducted on the angle and distance data to examine whether participants employed different strategies in drawing their spatial maps. As can be seen in Figures 5(a,b), there was a significant negative correlation between the mean absolute angular error and distance error (for the Contracted and Expanded VEs: $r = -.793$, $p < .001$ and $r = -.499$, $p < .01$, respectively). This correlation means that as angular error increases, distance error decreases, and vice versa. This pattern of results suggested that some participants prioritized preserving the linear distance information, while others prioritized preserving angular relationships. Two representative map drawings were shown in Figures 5(c,d) for these two types of prioritizations.

Moreover, there is also a significant correlation between the angular error and the straightness index (for Contracted maze: $r = -0.536$, $p < .01$; for Expanded maze: $r = -0.530$, $p < .01$), but not between the distance error and the straightness index (for the Contracted maze: $r = .213$, $p = .276$; for Expanded maze: $r = 0.185$, $p = .336$). These data suggest that people showing higher angular error also tended to have less straight segments, but there is no relationship between the distance error and straightness of the map drawings.

Additional correlation analyses were conducted on the angle, straightness and distance data across environment types to see whether participants were consistent in applying these strategies across the non-Euclidean environments. Results indicated a significant correlation between the Contracted and Expanded environments in terms of the mean absolute angular error and the distance errors ($r = .503$, $p < .01$ and $r = .583$, $p < .001$ after removing one outlier, respectively). There was no correlation between the straightness index in the Contracted and Expanded mazes ($r = -.066$, $p = .737$). Overall, participants did seem to apply a consistent cognitive style in producing Euclidean maps for the non-Euclidean environments.

4. General discussion

The present study examined the types of Euclidean representations people may form while learning virtual wormhole mazes that violate the principles of Euclidean geometry. Participants explored a simple square tunnel maze (Euclidean) and three non-Euclidean mazes, one with one of the legs contracted and two with one of the legs expanded by wormholes. After learning each maze, they drew a map of the landmark layout on a 2D canvas. Examination of the map drawings showed that when confronted with spatial information that is inconsistent with Euclidean space, certain geometric properties are emphasized more often than others. Some participants were more likely to distort representations of angularity while minimizing distortions of distance information, while the opposite was true for others. Across all conditions, the representation of straightness was generally preserved. That is, the linearity, or straightness, of path segments was consistent with what was observed during navigation, at least in simple rectilinear mazes tested in the present study, although those with larger angular errors also tended to have slightly less straight segments. It can also be seen by correlations for angular and distance errors between Contracted and Expanded mazes that participants generally applied a consistent cognitive strategy across the different environment types. These results suggest that representations of non-Euclidean space may be highly variable across individuals, and possible Euclidean solutions need to be carefully examined before testing different models.

The main challenge of learning wormhole environments in a Euclidean fashion is that it is not possible to construct a Euclidean representation of a non-Euclidean environment with all spatial information retained veridically, therefore whatever Euclidean representation one might form of a wormhole environment must have some spatial relations distorted. In other words, a non-Euclidean environment will create spatial conflicts that are not compatible with a Euclidean system, such as a segment of a maze cannot be so long given the spatial structure of other parts of the maze, a corner angle cannot be so small, etc. However, which part of the maze is "wrong" is ambiguous; it could be that this segment is too long, or the segment on the opposite side is too short. How to make accommodations is the fundamental problem on what type of Euclidean representation people may form while learning a non-Euclidean space.

For example, while encountering conflicting information during navigation in these wormhole mazes, the veridicality of at least one of the parameters – angle, distance, or straightness – needs to be distorted to preserve consistency in the others. If individuals prioritize distance relations and straightness in their representation, then concomitant changes in angularity must be made. If individuals prioritize angularity and straightness in their representation, then concomitant changes in distance relations must be made. If individuals prioritize angularity and distance relations in their representation, then concomitant changes in straightness must be made. The exact trade-off people chose to make would reflect their individual difference and cognitive style.

The selection of the trade-off strategies does not necessarily require conscious awareness of the conflict. That is, violation of the Euclidean principles may be detected by the path integration/spatial learning system, and the conflict resolved through the recalibration process to generate a non-veridical but coherent Euclidean representation, all outside people's conscious awareness of these processes. Therefore, lack of explicit awareness of the violation does not necessarily mean people are insensitive to Euclidean principles. Whether people can detect the violation is an interesting question by itself, but it does not say much about whether people follow Euclidean principles in their spatial representations or not.

Another important theoretical concept needing clarification is the distinction between Euclidean vs. non-Euclidean geometry and metric vs non-metric space (e.g., Montello, 1992). A metric space satisfies the following properties: 1) the distance from A to B is zero if and only if A and B are the same point; 2) the distance between two distinct points is positive (positivity); 3) the distance from A to B is the same as the distance from B to A (symmetry); and 4) the distance from A to B is less than or equal to the distance from A to B via any third point C (triangle inequality). A Euclidean space is a type of metric space that also satisfies the parallel postulate, therefore a space can be Euclidean, non-Euclidean but metric, or non-metric at all. A spatial representation that does not conform to Euclidean geometry can have violations specific to Euclidean metric (e.g., parallel postulate), or violations of general metric principles that are not specific to Euclidean geometry (e.g., symmetry or triangle inequality). Therefore, it is important to distinguish between Euclidean vs non-Euclidean and metric vs non-metric spaces. When the experimental evidence only involves violation of the general metric properties, it is more appropriate to call it "non-metric" than non-Euclidean, and the theoretical distinction should be referred to as metric vs non-metric instead of Euclidean vs non-Euclidean.

There are many tasks that can be used to examine people's spatial representations. Map drawing tasks have been commonly used to assess people's spatial knowledge of an environment (Axia, Bremner, Deluca & Andreasen, 1998; Coluccia, Bosco & Brandimonte, 2007; Coluccia, Iosue & Brandimonte, 2007; Pazzaglia & Taylor, 2007; Sugimoto & Kusumi, 2014; van Asselen, Fritschy & Postma, 2006). Because our research goal was to examine "*IF* people were to form Euclidean representations for non-Euclidean environments, what would these representations look like," we deliberately chose a map drawing task to force participants to produce a map in a Euclidean space. If participants actually formed Euclidean representations of the non-Euclidean environments, then the map they produced most likely reflected their actual underlying representation. If they did not have Euclidean representations of the non-Euclidean environments, then the map they produced would reflect the most likely Euclidean representation they *would have* constructed if they were to form one, either through conscious, high-level strategies or through unconscious, implicit corrections.[1] Regardless of whether participants had Euclidean representations or not, the drawings they produced can provide an estimation of the most likely Euclidean representation people may have for non-Euclidean environments.

Besides map drawing, various other spatial tasks such as JRD and in-environment target localization are also commonly used to examine spatial representations. However, they do not necessarily produce Euclidean maps, and therefore do not suit our research goal in the present study. Like the map drawing task, these tasks are also subject to influences of the responding

[1]Because the same type of high level strategies can also be used naturally without a map drawing task (e.g., participants may detect a conflict while learning a non-Euclidean maze and spontaneously adjust their mental map of the space with these strategies), we consider such strategies to be legitimate cognitive processes/mechanisms for constructing a Euclidean representation of non-Euclidean space and not an artifact introduced by the map drawing task per se.

processes beyond the representation itself such as inferences, reasoning, and strategies, and violations of Euclidean principles do not necessarily reflect the nature of the underlying spatial representations (McNamara & Diwadkar, 1997; Montello & Battersby, 2022; Sampaio & Wang, 2009; Widdowson & Wang, 2022, etc.). Moreover, like the map drawing task, these tasks also do not necessarily rely on existing representations, and solutions may be calculated/ constructed a posteriori during the responding process. Therefore, these tasks are not necessarily more valid than the map drawing task. Nevertheless, forcing people to produce Euclidean representations in a map drawing task limits the scope of what one can conclude about the geometry of human spatial knowledge. Future research combining and comparing other tasks with the map drawing task can be fruitful in further understanding the nature of people's spatial representations.

Because the goal of the present research was not about pinpointing *whether* people form Euclidean representations, these data do not (and were not meant to) directly address the Euclidean vs alternative theories. Instead, our findings help shed light on what the hypothetical Euclidean representation should be. As discussed in the introduction, there are many possible Euclidean representations one may construct based on path integration information for a non-Euclidean environment, depending on how the recalibration/regularization process attributes sources of the errors and how corrections are made. The present study showed that people treated different geometrical features of the environment differently. That is, the straightness of the path was generally preserved, while the distance ratio and turning angles were preserved by some participants but not the others, resulting in different, consistent cognitive styles/strategies across participants. The priority of the spatial information people preserved might reflect the salience and/or reliability of the information they perceived. For example, the straightness of the path was perceptually salient and un-mistakable, while the size of the turning angle and distance ratio between paths required measurements over time/movements, and therefore less certain/less reliable. As a result, people may give more priority in preserving the straightness of the path when constructing a Euclidean map for non-Euclidean environments. If this hypothesis is true, then people's different style/strategy on preserving distance vs angle information may reflect their perceptual ability in measuring these two spatial properties. That is, those preserving distance ratio may be better at linear distance estimation, while those preserving turning angles may be better at estimating turns, and each group preserved the spatial information they were most confident about. Moreover, which information people choose to preserve may depend on the nature of the environment such as complexity and the source of the information. For example, when body-based cues are the primary source of self-motion information, straightness of the path is also a property that has to be measured over time, and may not be prioritized as in purely visual navigation. Future research is need to further test these hypotheses.

The present study used the desktop VR system with purely visual navigation due to difficulties in implementing non-Euclidean environments in Euclidean physical space. Path integration can use multiple cues, such as visual, vestibular, proprioceptive, motor command, etc., and performance is generally better when more cues are available. Although it has been shown that body-based cues are important sources of information for path integration and can improve navigation performance, it has also been shown that path integration and spatial learning tasks can be performed effectively with visual information alone, especially when stable landmarks are present, as in the present study (e.g., Riecke, Veen & Bülthoff, 2002). Moreover, desktop VR has been widely used to study spatial representations, spatial learning and navigation, both in basic research and in applied settings such as education and training (e.g., Hegarty, Montello, Richardson, Ishikawa & Lovelace, 2006; Jansen-Osmann, 2002; Otto et al., 2003; Wiener & Mallot, 2006; Zhao et al., 2020). In addition, performance of the Euclidean condition in the present study also showed that participants were capable of learning the environment based on pure visual navigation. Thus, we believe the usage of the desktop VR in the present study is valid and the findings are meaningful. Future research incorporating body-based cues can further examine the effects of different self-motion cues on the type of information people would preserve when constructing Euclidean representations of non-Euclidean environments.

In summary, our study examined what Euclidean representations of non-Euclidean environments should be *if* people were to conform to Euclidean principles by forcing them to produce Euclidean maps of non-Euclidean wormhole environments using a map drawing task. The results suggested that people preserve certain geometrical properties more than others when encountering conflicting spatial information that violates Euclidean geometry, and people may prioritize different spatial information differently according to their own strategy and cognitive style. These findings suggest that possible Euclidean solutions need to be carefully examined before testing Euclidean vs alternative models of spatial representations.

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Appendix: A Euclidean representation with rips and folds

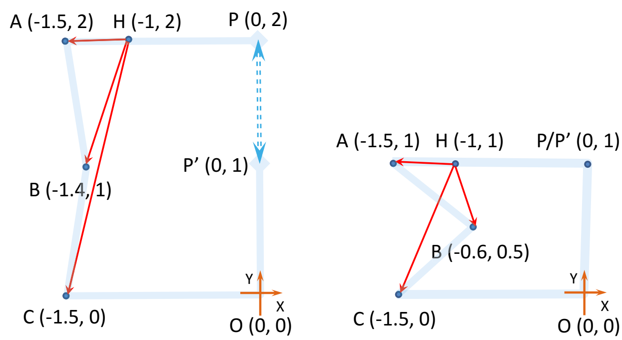

Figure A1. (left) a design blueprint of a wormhole tunnel maze with a wormhole between P and P′, so that the navigator is transported instantaneously to P′ when arriving at P, and vice versa. The numbers in the parenthesis are the x and y coordinates of each point in the inertial reference frame. (right) a possible Euclidean representation of the wormhole maze that preserves the length of each segment but sacrifices the turning angles at corners A, B and C. This representation contains “rip and fold” (i.e., change in ordinal relationship in pointing directions from H to A, B and C) comparing to the design blueprint representation in the left panel.

Suppose a navigator is placed in a wormhole tunnel maze depicted in Figure A1 (left panel) and navigates around the maze while trying to learn the landmark locations using the path integration system. For convenience, the following example is based on an allocentric reference frame using Cartesian coordinate system, although an egocentric updating system as described in Wang (2016) and other coordinate systems will generate the same results.

As the navigator moves from the origin O to turning points C, B, A, and passes by landmark H, everything is normal as in an ordinary Euclidean environment. Assuming the navigator uses a Cartesian coordinate system with origin at O, s/he can build a representation of these locations with the coordinates shown in the left panel. However, when the navigator reaches P, the wormhole will instantaneously transport him/her to P’, creating a non-perceptible translation of 1 m in the negative Y direction. In other words, the position of the navigator will change from (0, 2) to (0, 1) instantaneously with no perceptual information indicating this change, therefore the path integration system still has his/her position at (0, 2). From that moment on the navigator will continue the trip on the other side of the wormhole as if the gap does not exist. Perceptually, the navigator simply walks to corner P, turns around by 90°, and sees the origin O 1 m away (instead of 2 m away in a normal Euclidean space).

Because of the wormhole effect, the navigator now has a conflict between the perceptual system and the path integration system when s/he reaches the origin O. According to the path integration system, which is calculated based on self-motion information along the journey, the origin should still be 1 m away. However, the perceptual system indicates that the navigator is already at O. To resolve this conflict, the navigator has to make some assumptions about where this discrepancy came from and make a correction accordingly. This correction process is known as the resetting/recalibration process (e.g., Wang, 2016) or regularization process (Warren, 2019), and is part of the normal operation of the path integration system to correct for accumulated errors in ordinary Euclidean space.

The conflict between the perceptual and path integration system during wormhole navigation is also similar to the situation of cue conflict, where a set of landmarks is moved/manipulated during navigation without the navigator's knowledge. How the navigator resolves the conflict depends on where the path integration system attributes the source of the error. For example, if the path integration system assumes that the error mainly came from the distance estimation in the last leg, i.e., the navigator actually moved 2 m, while the odometer somehow only recorded 1 m, then it could make a correction by simply resetting the representation of the navigator's location to (0, 0) upon arriving at O, and leaving the representations of all other landmarks in the previous legs unaffected. As a result, this navigator will represent the maze as start corner O (0, 0), first corner C (−1.5, 0), second corner B (−1.4, 1), third corner A (−1.5, 2), landmark H (−1, 2), and fourth corner P (0, 2). When asked to point to A, B, and C from H, this navigator will point in the order of A, B, and C counterclockwise. This is the "Euclidean prediction" assumed by Warren et al. (2017), which is based on the design blueprint representation.

However, this is not the only way a navigator may resolve the conflict and build a Euclidean representation of this wormhole maze. For example, if the path integration system attributes the error/conflict to misperception of the turning angles at corners A, B and C, instead of to misperception of distance, then it can make some adjustments/corrections to the angles at these corners while preserving all the distance information. This strategy was actually observed in some participants in the present study (see the results section). A resulting representation from this recalibration process is depicted in the right panel. This representation is not veridical, but perfectly legitimate and Euclidean. When asked to point to A, B and C from H, this navigator will point in the order of A, C and B counterclockwise, which constitutes a "rip and fold" according to the definition in Warren et al. (2017). This example shows that Euclidean representations of non-Euclidean environments based on the path integration system can contain rips and folds, therefore the presence of rips and folds does not necessarily rule out the Euclidean hypothesis.

It should be noted that these are not the only two solutions of the recalibration process. In fact, there are infinite ways the recalibration process could operate, including which distance and angles to adjust and by how much, resulting in an infinite number of potential Euclidean representations a navigator may form of non-Euclidean environments.

Moreover, the lack of unique Euclidean representation for non-Euclidean environments does not depend on the type of cues available for path integration, and the presence of body-based cues is not necessarily helpful for establishing a unique solution from all these possibilities. For example, when the navigator moves from O to C to B to A to H, the body-based cues will indicate the same self-motion information and result in the same coordinates for these locations. Moreover, when the navigator arrives at P, s/he will be visually transported to P' in the virtual maze, but physically remain at the same location in the real environment because it is not possible to instantaneously transport one physically to P'. As a result, when the navigator continues to move from P/P' toward O, s/he will move 1 m when arriving at O, both visually and physically. That is, regardless of whether the path integration system uses visual information or body-based cues, the vector summation process will always result in the same error of 1 m when the perceptual system indicates that the navigator has already reached O. Therefore body-based cues can provide important information for path integration and improve performance, but they cannot solve the fundamental issue on the lack of unique Euclidean representation for non-Euclidean environment.