Two-stage Stochastic Programming for Maintenance Optimization of Multi-component Systems

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SUMMARY

As the complexity of modern industrial systems increases faster than ever, it is imperative to a develop cost-effective maintenance plan to secure system safety while lowering maintenance cost for complex systems. However, most maintenance studies focus on single-component systems, which are not applicable to complex multi-component systems due to the various interactions among components, such as stochastic dependence, structure dependence and economic dependence. Economic dependence is the most commonly seen one among these interactions. Economic dependence means that any maintenance action incurs a system-dependence cost, regardless of the number of components maintained. Significant cost savings can be achieved by maintaining multiple components jointly instead of separately.

In this paper, we study the maintenance optimization problem of multi-component systems with economic dependence among components. The objective is to determine the maintenance actions at each decision stage over a finite planning horizon so that the total maintenance cost is minimized. Such a maintenance optimization problem is challenging due to the combinatorial maintenance grouping problem with the stochastic component failure process. We present a two-stage stochastic programming model for this problem, which analytically expresses the total cost as a function of maintenance decisions. Progressive hedging algorithm is applied to solve this problem. We conduct a case study by using real-world pavement deterioration data. Experiment results provide insights on how economic dependence affects single-component maintenance decision.

ACRONYMS

CM	corrective maintenance
PM	preventive maintenance
CR	corrective replacement
PR	preventive replacement

NOTATION

n	number of components
N	component set, $N = \{1, 2, \ldots, n\}$
T	length of planning horizon
T_s	planning horizon, $T_s = \{1, 2, \ldots, T\}$
q	number of individuals
R	individual set, R = $\{1, 2, \ldots, q\}$
Ω	scenario set
I_{ir}	individual r of component i
T_{ir}	lifetime of individual r of component i
T_{ir}^{ω}	Lifetime T_{ir} in scenario ω
T'	Extended planning horizon, $T' = \max_{i,r,\omega} T_{ir}^{\omega}$
$c_{i,\mathrm{PR}}$	lifetime of individual r of component i
$c_{i,\mathrm{CR}}$	lifetime T_{ir} in scenario ω
$C_{i,\mathrm{PR}}$	total PR cost incurred by individuals of component i in the planning horizon T_s
$C_{i,\mathrm{CR}}$	total CR cost incurred by individuals of component i in the planning horizon T_s
C_s	total setup cost in the planning horizon T_s
$Q(\mathbf{x}, \omega)$	objective value of the second stage
d	setup cost
$\eta(t)$	shape parameter of gamma process
γ	rate parameter of gamma process
x_i	equal to 1 when an individual of component i is replaced at the first stage, 0 otherwise
$\tilde{x}_{it}^{r\omega}$	equal to 1 when I_{ir}
z_t	equal to 1 when there is at least one individual maintained at stage t, 0 otherwise
z_t^{ω}	equal to 1 when there is at least one individual maintained at time t in scenario ω, 0 otherwise

1 INTRODUCTION

Effective maintenance is of vital importance for complex, capital intensive and hazardous industries. Inappropriate maintenance may result in catastrophic failures, such as the loss of Piper Alpha oil platform [1]. Therefore, efficient maintenance planning is essential for the complex systems that consist of multiple components.

However, most studies in the literature focus on single-component system, which is not applicable to multi-component systems due to the various dependence among components, namely, stochastic, structural and economic dependence [2]. Among these three dependences, economic dependence is the most commonly seen, and is considered in this paper.

Typically, systems with economic dependence incurs a system-dependent cost that is known as setup cost when there is any maintenance taking place. This setup cost is considerably large in many industries. For example, plant shutdown is required for the maintenance of critical components in a chemical plant. The downtime loss due to the production loss range from $5,000 to $10,000 per hour [3]. The existence of economic dependence implies a joint maintenance of multiple components instead of separately so that setup cost can be shared.

Multi-component maintenance optimization problem joins the stochastic processes regarding the failures of the components with the combinatorial problems regarding the grouping of maintenance activities [4]. The explicit analytical model is therefore complex and sometimes impossible to derive. In the literature, existing models are built on special system assumptions [5],restricted grouping policies [6], or resort to simulation tools [7] to reduce mathematical difficulties. From a solution perspective, most solution methods can only handle a small number of components because of the exponential growth of the problem size [8]. A widely adopted approach to coordinating maintenance activities is to group components with some grouping rules, which can be further divided into direct grouping and indirect grouping. In direct grouping, all components are partitioned into some fixed groups in which components are always maintained jointly [9]. However, this approach is essentially a set-partitioning problem which is NP (nondeterministic polynomial)-complete. An indirect grouping strategy groups preventive maintenance (PM) activities by making PM interval a multiple of a basis interval, so the maintenance of different components can coincide. However, such grouping methods ignore the maintenance opportunities generated by corrective maintenance (CM) at failure [10]. Recently, Patriksson et al. [10] models maintenance optimization problems as a stochastic integer program. The integer L-shaped method proposed in their paper becomes prohibited when the problem scale gets larger.

In this paper, we develop a multi-component maintenance optimization model in a finite-time horizon without any restriction on the types of maintenance activities that can be grouped. The problem is formulated as a two-stage stochastic linear model. We use the progressive hedging algorithm to solve our model. We conduct a case study of road maintenance by using real-world data. Experiment results provide insights on how economic dependence affects single-component maintenance decision.

The remainder of paper is organized as follows. The proposed two-stage stochastic programming model is introduced in section 2. In section 3, we present a numerical example to provide some insights of the optimal maintenance decision for a practical road maintenance case. We conclude this research and discuss the future work in section 4.

2 MODEL DEVELOPMENT AND ALGORITHM

In this section, we propose a two-stage stochastic program to model the maintenance optimization problem for multi-component systems. To the best of our knowledge, this is among the very first efforts that develop an analytical model to such a problem. Here we consider the system of interest consists of $N = \{1, \dots, n\}$ types of component. We consider two types of maintenance activities, preventive replacement (PR) and corrective replacement (CR) with corresponding costs are $c_{i,\text{PR}}$ and $c_{i,\text{CR}}$ respectively ($c_{i,\text{PR}} < c_{i,\text{CR}}$) for component $i \in N$. Each physical instance $r \in R$ that replaced in the r^{th} replacement of a component is called an individual I_{ir}, where $R = \{1, 2, \dots, q\}$. Throughout this paper, we use component only when referring to its type, and refer to physical components as individuals. The system setup cost is denoted by d at any maintenance occasion regardless of the number of individuals replaced.

The maintenance decision process can be divided into two stages. The first stage decision is to select a group of individuals for PR at the current decision time. The second stage decision is to group the individuals for PR at future decision time. We consider a discrete finite planning horizon $\mathcal{T} = \{0, 1, \dots, T\}$. At the first stage, i.e., $t = 0$, we first observe the failure state ξ_i for the individual of component $i \in N$. If the individual of component $i \in N$ is functioning, then $\xi_i = 0$, otherwise $\xi_i = 1$. We then make the decision for the first stage:

$$x_i = \begin{cases} 1, \text{if the individual of component } i \\ \text{is replaced at } t = 0, & i \in N \\ 0, \text{otherwise.} & i \in N \end{cases}$$

and

$$z = \begin{cases} 1, \text{if any maitenance occurs at } t = 0, i \in N \\ \qquad 0, \text{otherwise.}\ i \in N \end{cases}$$

The second stage decisions are the maintenance decisions at time $t \in \mathcal{T} \setminus \{0\}$ that are made after the individual failure states are revealed at $t = 1$. Because of the randomness of component lifetime, we model the lifetime of each component with an appropriate distribution and randomly generate lifetimes for all its individuals. A combination of lifetimes of all individuals of all components is referred to as a scenario. By using this scenario generation method, the lifetimes of individuals are deterministic for a given scenario. For each scenario $\omega \in \Omega$, the decision variables are defined as:

$$\tilde{x}_{it}^{r\omega} = \begin{cases} 1, \text{if } I_{ir} \text{ is replaced at or before time } t \text{ in scenario } \omega, \\ & i \in N, t \in \mathcal{T}, r \in R, \omega \in \Omega \\ 0, \text{otherwise.} & i \in N, t \in \mathcal{T}, r \in R, \omega \in \Omega \end{cases}$$

and

$$z_t^{\omega} = \begin{cases} 1, \text{if any maintenance occurs at time } t \text{ in scenario } \omega \\ & , t \in \mathcal{T}, \omega \in \Omega \\ 0, \text{otherwise.} & t \in \mathcal{T}, \omega \in \Omega \end{cases}$$

The <u>deterministic equivalent form (DEF)</u> of proposed two-stage stochastic model is described as follows.

minimize

$$\sum_{\omega \in \Omega} p(\omega) \left(\sum_{i \in N} \left(\underbrace{\sum_{i \in R} c_{i,\text{PR}} Y_i^{r\omega}}_{C_{i,\text{PR}}} + \underbrace{\sum_{i \in R} c_{i,\text{CR}} (1 - Y_i^{r\omega}) - c_{i,\text{CR}} (1 - \tilde{x}_{iT}^{r\omega})}_{C_{i,\text{CR}}} \right) + \underbrace{\sum_{t \in \mathcal{T}} d z_t^{\omega}}_{C_\text{s}} \right) \quad \text{(1a)}$$

subject to

$$\tilde{x}_{it}^{r\omega} \le \tilde{x}_{i,t+1}^{r\omega}, i \in N, t \in \mathcal{T}\backslash\{T\}, r \in R, \omega \in \Omega \quad (1b)$$

$$\tilde{x}_{i,t+1}^{r+1,\omega} \le \tilde{x}_{i,t}^{r\omega},\ i \in N, t \in \mathcal{T}\backslash\{T\}, r \in R\backslash\{q\}, \omega \in \Omega \quad (1c)$$

$$\sum_{r\in R}\left(\tilde{x}_{it}^{r\omega} - \tilde{x}_{i,t-1}^{r\omega}\right) \le z_t^{\omega}, i \in N, t \in \mathcal{T}\backslash\{0\}, \omega \in \Omega \quad (1d)$$

$$\tilde{x}_{it}^{r\omega} \le \tilde{x}_{i,T_{i,r+1}^{\omega}}^{r+1,\omega}, \quad i \in N, t \in \left\{0, \dots, T - T_{i,r+1}^{\omega}\right\}, r \in R\backslash\{q\}, \omega \in \Omega \quad (1f)$$

$$\tilde{x}_{iT_{i1}^{\omega}}^{1\omega} = 1, i \in \{j \in N | T_{j1}^{\omega} \le T\}, \omega \in \Omega \quad (1g)$$

$$\tilde{x}_{i0}^{r\omega} = 0, i \in N, r \in R\backslash\{1\}, \omega \in \Omega \quad (1h)$$

$$x_i = \tilde{x}_{i0}^{1\omega}, i \in N, \omega \in \Omega \quad (1i)$$

$$x_i \ge \xi_i,\ i \in N \quad (1j)$$

$$Y_i^{1\omega} = 1 - w_{iT_{i1}^{\omega}}^{1\omega}, i \in N, \omega \in \Omega \quad (1k)$$

$$Y_i^{r\omega} = \frac{\left(\sum_{t=T_{ir}^{\omega}}^{T+T_{ir}^{\omega}} |y_{it}^{r\omega}| + \sum_{t=0}^{T_{ir}^{\omega}-1} w_{it}^{r\omega}\right)}{2}, i \in N, r \in R\backslash\{1\}, \omega \in \Omega \quad (1l)$$

$$y_{it}^{r\omega} = w_{it}^{r\omega} - w_{i,t-T_{ir}^{\omega}}^{r-1,\omega}, i \in N, r \in R\backslash\{1\}, t \in \{T_{i,r}^{\omega}, T'\}, \omega \in \Omega \quad (1m)$$

$$w_{it}^{r\omega} = \tilde{x}_{it}^{r\omega} - \tilde{x}_{i,t-1}^{r\omega}, i \in N, r \in R, t \in \mathcal{T}\backslash\{0\}, \omega \in \Omega \quad (1n)$$

$$w_{i0}^{r\omega} = \tilde{x}_{i0}^{r\omega}, i \in N, r \in R, \omega \in \Omega \quad (1o)$$

$$w_{it}^{r\omega} = 0,\ i \in N, r \in R, t \in \{T+1, \dots, T'\}, \omega \in \Omega \quad (1p)$$

$$\tilde{x}_{it}^{r\omega}, x_i, z_t^{\omega}, w_{it}^{r\omega}, Y_{it}^{r\omega} \in \{0,1\}, i \in N, r \in R, t \in \mathcal{T}, \omega \in \Omega \quad (1q)$$

2.1 *Objective function*

The total cost in function (1a) consists of (1) sum of PR and CR costs incurred by individuals of component i in the planning horizon, denoted by $C_{i,\mathrm{PR}}$ and $C_{i,\mathrm{CR}}$ respectively and (2) total system setup cost C_{S}. Probability of scenario $\omega \in \Omega$ is denoted by $p(\omega)$. Auxiliary variable $Y_i^{r\omega}$ indicates individual I_{ir} in scenario ω takes PR when equals to 1 and CR when equals to 0. However, to derive $Y_i^{r\omega}$ from variables $\tilde{x}_{it}^{r\omega}$ is non-trivial because $\tilde{x}_{it}^{r\omega}$ has no indication of maintenance type. Next, we will show how we get $Y_i^{r\omega}$.

Variable $Y_i^{r\omega}$ is defined in constraints (1k) and (1l). From (1n), auxiliary variable $w_{it}^{r\omega}$ equals to 1 when I_{ir} is replaced at time t and 0 otherwise. For an individual I_{ir}, one way to determine its replacement type is to examine the time interval between the replacements of individuals $I_{i,r-1}$ and I_{ir}, as shown in Figure 1. Suppose that individuals $I_{i,r-1}$ and I_{ir} are replaced at time t_1 and t_2, (i.e., $w_{it_1}^{r-1,\omega} = 1$ and $w_{it_2}^{r\omega}$) respectively. If the difference between t_2 and t_1 equals to the lifetime of I_{ir}, namely, T_{ir}^{ω}, the $I_{i,r}$ is replaced at the end of its lifetime and the replacement type CR. Otherwise, the replacement is PR. Therefore, we have $\sum_{t=0}^{T}|y_{it}^{r\omega}| = \sum_{t=0}^{T}\left|w_{it}^{r\omega} - w_{i,t-T_{ir}^{\omega}}^{r-1,\omega}\right| = 0$ for CR (Figure 1(a)) and $\sum_{t=0}^{T}|y_{it}^{r\omega}|/2 = 1$ for PR (Figure 1(b)), where $y_{it}^{r\omega}$ is defined in constraint (1m).

However, there is a boundary issue in $\sum_{t=0}^{T}|y_{it}^{r\omega}|$. From constraint (1m) $y_{it}^{r\omega} = w_{it}^{r\omega} - w_{i,t-T_{ir}^{\omega}}^{r-1,\omega}$, time t can only be no smaller than T_{ir}^{ω}. Therefore, the summation $\sum_{t=0}^{T}|y_{it}^{r\omega}|$ cannot start from $t = 0$. One approach to solving this issue is to extend to time horizon to $T' = T + \max_{i,r,\omega} T_{ir}^{\omega}$ and let $w_{it}^{r\omega} = 0$ for $t > T$, so that $Y_i^{r\omega} = \left(\sum_{t=T_{ir}^{\omega}}^{T+T_{ir}^{\omega}}|y_{it}^{r\omega}| + \sum_{t=0}^{T_{ir}^{\omega}-1} w_{it}^{r\omega}\right)/2$ (constraint (1l)) can be used to indicate the maintenance type. Figure 2 illustrates this issue.

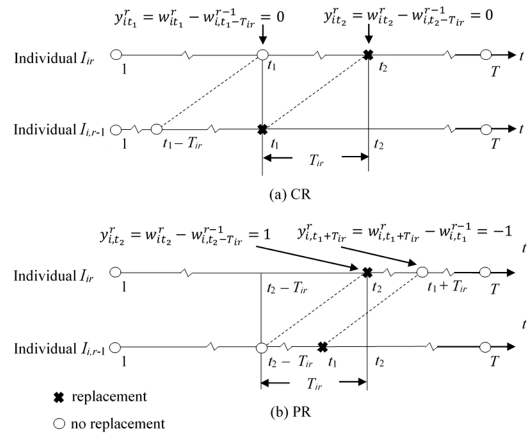

Figure 1. Illustration of distinguishing PR and CR

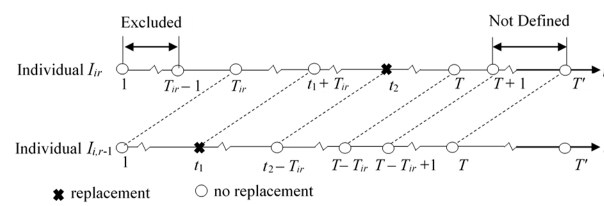

Figure 2. Illustration of boundary issue

The absolute function, $|y_{it}^{r\omega}|$, can be linearized by a pair of deviation variables $u_{it}^{r\omega}$ and $v_{it}^{r\omega}$ [11]. We replace $|y_{it}^{r\omega}|$ with Equation (2) in the constraint (1l), and add constraint (1r) to (1t) in the DEF model. Notice that constraint (1s) is unnecessary for the linearization but can lead to a stronger formulation.

$$|y_{it}^{r\omega}| = u_{it}^{r\omega} + v_{it}^{r\omega}, i \in N, r \in R, t \in \{0, \dots, T'\}, \omega \in \Omega \quad (2)$$

$$y_{it}^{r\omega} = u_{it}^{r\omega} - v_{it}^{r\omega}, i \in N, r \in R, t \in \{0, \dots, T'\}, \omega \in \Omega \quad (1r)$$

$$u_{it}^{r\omega} + v_{it}^{r\omega} \le 1,\ i \in N, r \in R, t \in \{0, \dots, T'\}, \omega \in \Omega \quad (1s)$$

$$u_{it}^{r\omega}, u_{it}^{r\omega} \in \{0,1\}, i \in N, r \in R, t \in \{0, \dots, T'\}, \omega \in \Omega \quad (1t)$$

2.2 *Constraints*

Constraint (1b) is the definition of $\tilde{x}_{it}^{r\omega}$, which ensures that individual $I_{i,r}$ is replaced at or before $t + 1$ when it is replaced at or before t. Constraint (1c) implies that individual $I_{i,r+1}$ can only be replaced after $I_{i,r}$ is replaced. Constraints (1d) and (1e) ensures that the maintenance cost d incurs when any component is replaced at time t. Constraints (1f) and (1g) ensure that individual $I_{i,r}$ has to be replaced before or at the end of its lifetime T_{ir}^{ω}. Constraint (1h) implies that only individual 1 could be replaced at time 0. In stochastic programming, it is required that the decision at $t = 0$ is the same as x_i for all scenarios, known as the non-anticipativity constraint, and this constraint is imposed by constraint (1i). The constraint (1j) forces all failed components at time $t = 0$ to be replaced. Constraints (1k) and (1l) define the auxiliary variable $Y_i^{r\omega}$, which is critical to identify the type of maintenance. Constraint

(1m) provides the full definition of variable $y_{it}^{r\omega}$. Constraints (1n) – (1p) are the definitions of variable $w_{it}^{r\omega}$. Constraint (1q) is the binary constraint for all decision variables. The linearization of $|y_{it}^{r\omega}|$ can be found in Equation (2) and constraints (1r) – (1t).

2.3 *Progressive hedging algorithm (PHA)*

The proposed DEF model is an integer program with pure binary decision variables. The lack of structural property prohibits us to use efficient algorithm to handle this problem, such as Benders decomposition. Therefore, we use progressive hedging algorithm (PHA) to solve the proposed model. PHA is a scenario-by-scenario decomposition method. The optimal solution of each scenario is first obtained without considering non-anticipativity constraint, then PHA penalizes the deviation of first-stage solution from the average solution of all scenarios to force non-anticipativity constraint holds. The detail of PHA is described as follows, where $\mathbf{cx} + E(Q(\mathbf{x}, \omega))$ is the concise presentation of a two-stage stochastic programming model with $\mathbf{x}$ representing the first-stage decision variables and $Q(\mathbf{x}, \omega)$ representing the subproblem in scenario ω. The penalty factor is denoted by ρ [12].

Progressive hedging algorithm (PHA)
1. **Initialization**: Let $v \leftarrow 0$, $\varepsilon \leftarrow 10^{-2}$; $\mathbf{x}_\omega^{(v)} \leftarrow \arg\min_{\mathbf{x}} (\mathbf{cx} + Q(\mathbf{x}, \omega))$, $\forall \omega \in \Omega$; $\bar{\mathbf{x}}^v \leftarrow \sum_{\omega \in \Omega} p(\omega) \mathbf{x}_\omega^v$; $\mathbf{w}_\omega^v \leftarrow \rho(\mathbf{x}_\omega^v - \bar{\mathbf{x}}^v)$, $\forall \omega \in \Omega$. 2. **Update iteration variable**: $v \leftarrow v + 1$. 3. **Decomposition**: $\mathbf{x}_\omega^{(v)} \leftarrow \arg\min_{\mathbf{x}} (\mathbf{cx} + \mathbf{w}_\omega^{v-1} + \frac{\rho}{2} \|\mathbf{x} - \bar{\mathbf{x}}^{v-1}\| + Q(\mathbf{x}, \omega))$ $\forall \omega \in \Omega$. 4. **Aggregation**: $\bar{\mathbf{x}}^v \leftarrow \sum_{\omega \in \Omega} p(\omega) \mathbf{x}_\omega^v$. 5. **Update price**: $\mathbf{w}_\omega^v \leftarrow \mathbf{w}_\omega^{v-1} + \rho(\mathbf{x}_\omega^v - \bar{\mathbf{x}}^v)$, $\forall \omega \in \Omega$. 6. **Calculate converge distance**: $g^v \leftarrow \sum_{\omega \in \Omega} p(\omega) \|\mathbf{x}_\omega^v - \bar{\mathbf{x}}^v\|$, $\forall \omega \in \Omega$. 7. **Termination**: **If** $g^v < \varepsilon$, stop and **return** optimal solution $\bar{\mathbf{x}}^v$. **Else**, go to step 2.

3 NUMERICAL EXAMPLE

In this section, we analyze the changes of optimal PM interval when considering economic dependence for a road maintenance case. In practical road maintenance, joint maintenance of multiple road sections can share the setup cost. The setup cost is usually induced by crew travelling, downtime loss etc.. Road condition is evaluated by the international roughness index (IRI) as in units of inches per mile. A larger IRI stands for a worse road condition. According to the Federal Highway Administration, reconstruction (CM) is considered when IRI exceeds 170in./mile. Therefore, 170in./mile is the failure threshold in this example.

3.1 Parameter estimation

We use real-world pavement IRI data over years from the state of Florida [13] to fit the road degradation process distribution. We select 4 section data as plotted in Figure 3. In each section data, each path represents an independent measurement of the same section.

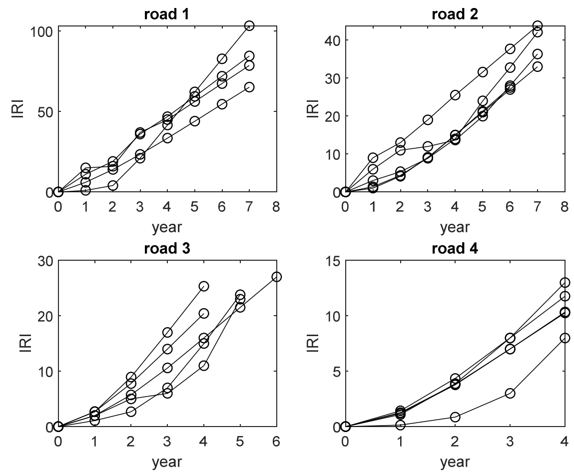

Figure 3. IRI data sets

EM algorithm for gamma process with missing data
$X_i = \{x_i(t_0), x_i(t_1), \cdots, x_i(t_{J_i})\}$: i^{th} data set. $t_m = \max_i t_{J_i}$: maximum time that has an observed data point among all data sets. $\Delta\eta_j = \eta(t_j) - \eta(t_{j-1})$: increment of shape function between two consecutive time points. $\Delta X_{ij} = x_i(t_j) - x_i(t_{j-1})$: increment between two consecutive time points of i^{th} data set. **At the k^{th} iteration:** **(1) *E*-step:** *Estimate* $x_i(t_m)$ *for all i*: $w_i^k = E[X_i(t_m) \| D_{obs}, \theta^k]$ $= \dfrac{\eta^k(t_m) - \eta^k(t_{i,J_i})}{\gamma^k} + X_i(t_{i,J_i})$ *Estimate* $\ln \Delta X_{i,j}$ *for* i *and* $j > t_{J_i}$: $E[\ln \Delta X_{i,j} \| D_{obs}, \theta^k] = \psi(\Delta\eta_j^k) - \ln \gamma^k$ **(2) *M*-step:** *Calculate*: $\gamma^{\text{k+1}} = \frac{n\hat{\eta}_m}{\sum_{i=1}^n w_i^N}$ and $\Delta\eta_i^{k+1} = \psi^{-1}\left[\frac{1}{n}\sum_{i=1}^n \omega_{i,j}^k + \ln \gamma^{k+1}\right]$ where $\hat{\eta}_m = \sum_{j=1}^m \psi^{-1}\left[\frac{1}{n}\sum_{i=1}^n \omega_{i,j}^k + \ln(n\eta_m) - \ln\left(\sum_{i=1}^n w_i^k\right)\right]$ and $\psi(\cdot)$ is the digamma function. **Repeat step (1) and (2) until following condition is met (ε is pre-defined) :** $L(\theta^{N+1}; D_{obs}) - L(\theta^N; D_{obs}) < \varepsilon.$

We assume the deterioration processes of all sections follow a gamma process with different unknown shape parameters $\eta(t)$ and different rate parameters γ. The shape parameter $\eta(t)$ is a function of time t but the function form is also unknown. In some section data, several observations in the last several years are missing, e.g., section 2 and section 3. Thus, we use EM algorithm to estimate the parameters.

The algorithm runs iteratively over two steps – the expectation step (E-step) and the maximization step (M-step). Denote the parameter vector to estimate by θ, the observed data by D_{obs}, the complete data by $D = D_{obs} \cup D_{miss}$ and the log-likelihood function based on D by $L(\theta; D)$. Let θ^k be the estimated parameter vector at the k^{th} EM algorithm. At the expectation step of the $(k+1)^{\text{th}}$ iteration, the missing data is considered as unknown variables and we obtain the expectations of D_{miss} in terms of θ^k. At the maximization step of the $(k+1)^{\text{th}}$ iteration, the expectations of D_{miss} are substituted in the log-likelihood function $Q(\theta|\theta^k)$, which is subsequently maximized over θ and then we obtain θ^{k+1}. For a gamma process, EM algorithm with n data sets can be described as shown above.

The output of the EM algorithm is the estimation of $\hat{\theta} = (\hat{\eta}(t_j), \hat{\gamma})$ for $j = \{1, 2, \ldots, t_m\}$. By using the section data in Figure 3, we have $\hat{\gamma} = (0.5, 1.27, 1.36, 3.26)$ for section 1 to section 4 respectively. We compare different forms of functions to fit $\hat{\eta}$, such as linear, power and exponential. The result of estimated $\hat{\eta}$ values and best fitting function with corresponding parameters are shown in Figure 4.

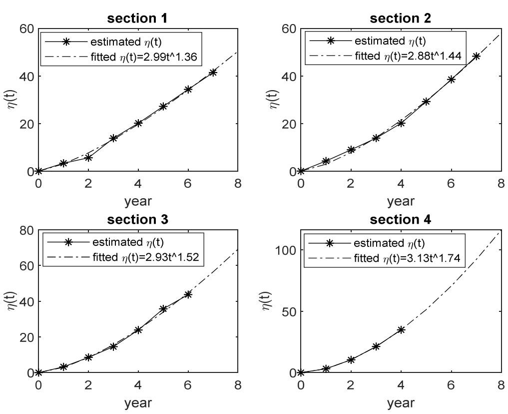

Figure 4. $\hat{\eta}(t)$ estimation

3.2 Experiment results

We consider a planning horizon of $T = 100$ years. The number of sections $n = 4$. The PR cost is 1, CR cost is 10 for all sections. When economic dependence is not considered, the optimal PM intervals can be calculated under age-based policy, which are $\tau = (9, 17, 15, 18)$ years for section 1 to section 4 respectively. When economic dependence is taken into account, the optimal solution is obtained by solving proposed model with PHA. Since we only take first-stage solutions in PHA, we use rolling horizon technique to obtain the solutions of all decision epochs.

We want to figure out how are age-based PM intervals τ changing if we consider economic dependence under different setup cost. Table 1 summarizes the average PM intervals with different setup cost d. For all sections, the average PM interval increases as setup cost increases. The reason is that PM becomes more expensive as setup cost increases. For section 2 and section 4, the age-based PM intervals are close enough so that they have a high chance to group together. Therefore, for both setup costs, section 2 and section 4 have the same PM intervals which increases the possibility that they can be maintained at the same time.

Table 1: average PM interval with different setup cost d

section	age-based	$d = 5$	$d = 100$
1	9	10.9	11.6
2	17	18.8	19
3	15	16.8	17
4	18	18.8	19

4 CONCLUSION

In this paper, we propose a two-stage stochastic program to analytically model the maintenance optimization problem for multi-component systems over the finite planning horizon subject to the uncertainty of component lifetime. The impacts of economic dependence and setup cost are investigated through a road maintenance case. The results show that the optimal PM interval increases when considering economic dependence. In the future research, we will further explore the model structure and analyze the optimal grouping pattern in a more rigorous and comprehensive way.

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