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Design of linear residual generators for fault detection and isolation in nonlinear systems

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ABSTRACT

A systematic method for the design of disturbance-decoupled linear residual generators for fault detection and isolation (FDI) in nonlinear systems is developed. Necessary and sufficient conditions for the existence of linear residual generators for nonlinear systems are derived. As long as these conditions are satisfied, we obtain explicit design formulas for the residual generator, with eigenvalue assignment. The proposed formulation and results provide a nonlinear generalisation of standard FDI methods for linear systems. The method is applied to three case studies: a bio reactor, a continuous stirred tanked reactor (CSTR) and a process network.

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1. Introduction

Higher demand for safety and reliability has made fault diagnosis a major topic of research over the past three decades (Ding, 2008; Frank et al., 2000; Frank & Ding, 1997). A fault is an unexpected/unpermitted major deviation in the process variables from normal conditions (Ding, 2008). Faults could arise due to several reasons, including mechanical failures, power failures, human errors, etc. Faults could lead to consequences ranging from off- spec product resulting in loss of profit, to potentially catastrophic explosions causing fatalities. These considerations provide a strong motivation for the development of methods and strategies for quick fault diagnosis that would guide the operator to bring the system back to normal operation (Ding, 2008).

Fault diagnosis techniques can be broadly grouped into two categories: hardware-redundancy-based fault diagnosis and analytical-redundancy-based fault diagnosis (Ding, 2008). Hardware-redundancy-based techniques consist of a reconstruction of the system using identical hardware components parallel to the process (Ding, 2008). This has been used in some safety-critical systems including aircrafts and nuclear power plants. However, while this technique certainly has its advantages in terms of reliability, it is limited by high costs, as constructing an identical redundant system for the sole purpose of fault diagnosis may not make economic sense in capital intensive industries (Ding, 2008). Analytical redundancy on the other hand comprises of a virtual reconstruction of the system using a process model which is implemented in software form on a computer (Ding, 2008; Frank, 1987a, 1987b; Frank et al., 2000; Frank & Ding, 1997; Gertler, 1991). Analytical redundancy is achieved through the known interdependence among the process variables provided by the model (Ding, 2008; Frank, 1987a, 1987b, 1990; Frank et al., 2000; Frank & Ding, 1997). The evolution of process variables of the virtual system will follow the outputs of the real system in the absence of faults and will show a measurable deviation in the presence of faults. The essence of analytical redundancy in fault diagnosis is checking consistency of the actual system behaviour against the system model. Any inconsistency is measured in terms of residuals that deviate from zero only in the presence of a specific fault. Moreover, since accurate modelling of a real system is difficult and the effect of unknown disturbances or uncertainties could be corrupt the residual signal, it is important to carefully define the residual in a way that makes it unaffected by those disturbances. The central objective in model-based fault diagnosis is to develop a residual generator for each of the possible faults, in a way that the residual is unaffected by the other faults and unknown disturbances.

One of the most widely studied approaches in the area of model-based fault detection and isolation (FDI) is the observer-based fault diagnosis approach. The first observer-based FDI method for linear systems was proposed by Beard and Jones in the early 1970s (Beard, 1971; Jones, 1973) which was a historic milestone in the area of fault diagnosis. Following this, many authors approached the fault diagnosis using a single or multiple Luenberger observers or Kalman filters (Clark, 1978a, 1978b; Clark et al., 1975; Ding, 2008; Frank, 1987a, 1987b; Frank & Ding, 1997; Frank & Keller, 1980; Mehra & Peschon, 1971). In the late 70s the question of sensitivity of fault diagnosis schemes to modelling errors and unknown disturbances was raised which led to the development of FDI schemes that included disturbance decoupling conditions(Frank, 1987a, 1987b, 1990; Patton et al., 1987; Watanabe & Himmelblau, 1982; Wüunnenberg & Frank, 1987). In general, observer-based FDI methods for linear systems can be grouped into the following

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four categories (Ding, 2008; Frank, 1990; Frank et al., 2000) (i) Fault Detection Filter (ii) Diagnostic Observer (iii) Parity Space Approach (iv) Frequency Domain Approach. In the 90s interconnections between the amongst these methods were studied and equivalence between these methods has been established(Ding, 2008; Ding et al., 1999; Frank et al., 2000; Gertler, 1991; Magni & Mouyon, 1994). Thus parameters of the residual generator obtained using one approach can be transformed to derive the parameters of the residual generator for any other approach (Ding, 2008; Ding et al., 1999; Frank et al., 2000; Magni & Mouyon, 1994). For a review of fault diagnosis for linear systems the reader is referred to excellent surveys by Frank and Ding (Frank, 1990; Frank & Ding, 1997) and for more details on linear methods including the interconnections amongst different implementations the reader is referred to (Ding, 2008).

Many industrial systems, like chemical processes, exhibit strong nonlinearities which may render the application of linear methods ineffective. To design a reliable FDI system, explicit consideration of the nonlinear dynamics is needed for residual generation. Some fundamental results on the feasibility of disturbance decoupled fault detection and isolation have been derived in (De Persis & Isidori, 2001) using a differential geometric perspective, where the problem of fault detection was formulated in terms of the existence of an unobservability subspace and a quotient observable subsystem solely affected by the fault of interest. Following this, there have been studies dedicated to actuator fault detection and subsequent fault tolerant control in nonlinear systems including detection of a single fault (Mhaskar et al., 2006, 2012) using a replica of the process model, and isolation amongst multiple faults (Mhaskar et al., 2008, 2012) based on the assumption that each input in the system can directly affect only one state equation. There have also been approaches based on banks of high gain observers for generating residuals, with rigorously established convergence properties via Lyapunov methods, that have been shown to be applicable to the detection of a single sensor fault at a time (Du & Mhaskar, 2014) and at most two faults (sensor and/or actuator) in which case a potentially large number of observers are required to distinguish between the two faults (Du et al., 2013; Shahnazari & Mhaskar, 2018). In another work, linear matrix inequalities were used to prove convergence properties of a class of nonlinear state observers in Lipschitz nonlinear systems under full state observability from each one of the measurements, that was subsequently used for diagnosis of sensor faults occurring one at a time (Rajamani & Ganguli, 2004).

In another direction, there have been efforts seeking extensions of observer-based FDI methods to nonlinear systems in the spirit of linear systems methods (Frank et al., 2000; Seliger & Frank, 1991), and the challenges of building observer-based disturbance-decoupled residual generators became evident (Gertler, 2000). So far, concrete results have been restricted to special classes of nonlinear systems, including bilinear systems (Kinnaert, 1999) and globally Lipschitz systems with triangular structure (De Persis & Isidori, 2001; Hammouri et al., 1999).

The present work will approach observer-based FDI for nonlinear systems from the point of view of exact observer error linearisation (Kazantzis & Kravaris, 1998). The residual generator will be constructed to be an observer which, in the absence of faults, has linear disturbance-decoupled error dynamics, with the residual function identically vanishing on the observer invariant manifold. It will be shown that, with the proposed formulation, easy-to-check necessary and sufficient conditions for the existence of such a residual generator can be derived, leading to simple formulas for observer design with eigenvalue assignment. Moreover, fault isolation can be accomplished via multiple residual generators, one for each fault, decoupled from the other faults and the system disturbances. The proposed formulation and results provide a direct nonlinear generalisation of standard linear FDI methods (Ding, 2008).

The outline of the paper is as follows. In the next section, the disturbance decoupled fault detection problem for a nonlinear system using a linear residual generator is formulated, and the pertinent design equations are derived. This is followed by necessary and sufficient conditions for the existence of disturbance decoupled linear residual generators, including a design formula for the residual generator. Then, the problem of fault isolation in the presence of multiple faults is handled via multiple residual generators. Finally, the applicability of the method is demonstrated through chemical engineering examples.

2. Disturbance-decoupled detection of a single fault using a linear residual generator

Consider a nonlinear process described by:

$$\dot{x} = F(x) + G(x)f + \sum_{i=1}^{m} E_i(x)w_i$$
$$y = H(x) + J(x)f + \sum_{i=1}^{m} K_i(x)w_i \tag{2.1}$$

where $x \in \mathbb{R}^n$ denotes the vector of states, $y \in \mathbb{R}^p$ denotes the vector of measured outputs. $f \in \mathbb{R}$ and $w_i \in \mathbb{R}$, $i = 1, 2, \dots, m$ are the fault and the disturbances/uncertainties respectively (system inputs) and $F(x), G(x), E_i(x), H(x), J(x), K_i(x)$ are smooth functions. Under normal operation of the process, the input f (fault) is identically equal to zero, however in an abnormal situation (equipment failure), f becomes nonzero, and this is what needs to be detected on the basis of the measurements. The inputs w_i describe normal variability of process conditions (disturbances) and/or model uncertainty. It is in the presence of this variability that the fault must be detected, and the conclusion (normal or faulty operation) must be unaffected by the presence of w_i (disturbance-decoupled detection).

In this work, we will study the problem of disturbance-decoupled fault detection on the basis of calculating a quantity r called the residual, which is identically zero under normal operation (i.e. when $f(t) = 0$) and nonzero under an abnormal situation (i.e. when $f(t) \neq 0$), and is unaffected by the disturbances w_i. More specifically, this work will study the design of a **linear** filter, called the residual generator, of the form

$$\dot{z} = Az + By$$
$$r = Cz + Dy \tag{2.2}$$

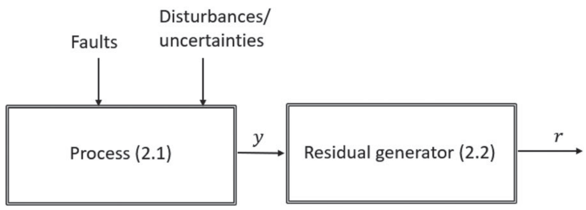

Figure 1. Fault detection scheme.

with state $z \in \mathbb{R}^s$, output $r \in \mathbb{R}$ (the residual), and parameters A, B,C, D being $s \times s, s \times p, 1 \times s$ and $1 \times p$ matrices respectively with (C, A) observable pair, so that the response of the residual r in the series connection of (2.1) followed by (2.2) (see Figure 1)

$$\frac{d}{dt}\begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} F(x) \\ Az + BH(x) \end{bmatrix} + \begin{bmatrix} G(x) \\ BJ(x) \end{bmatrix} f + \sum_{i=1}^{m} \begin{bmatrix} E_i(x) \\ BK_i(x) \end{bmatrix} w_i$$

$$r = [Cz + DH(x)] + [DJ(x)]f + \sum_{i=1}^{m} [DK_i(x)]w_i \quad (2.3)$$

has the following properties:

(i) $r(t)$ asymptotically approaches zero when f is identically zero
(ii) $r(t)$ is unaffected by the disturbances w_i
(iii) $r(t)$ is affected by the fault f

In other words, for any initial conditions $\begin{bmatrix} x(0) \\ z(0) \end{bmatrix}$ and any disturbances $w_i(t)$,

$$\lim_{t\to\infty} r(t) = 0 \ if \ f(t) = 0$$

$$\lim_{t\to\infty} r(t) \neq 0 \ if \ f(t) \neq 0$$

The responsiveness of r to faults and insensitivity to disturbances ensures fault detection while precluding the possibility of false alarms.

The present study focuses on designing *linear* residual generators for nonlinear systems given by (2.1) because of the practicality of linear filters in design and implementation. We will derive necessary and sufficient conditions for the existence of a linear residual generator based on a disturbance-decoupled linear functional observer. As long as these conditions are satisfied, we will derive simple design formulas for the residual generator, with eigenvalue assignment capability.

2.1 Design conditions for the residual generator for disturbance decoupled fault detection

In this subsection we derive specific design conditions that the residual generator must satisfy to meet the requirements (i)-(iii). The first and foremost requirement of the residual generator is that the residual must vanish in the absence of faults or disturbances (asymptotically converge to zero in the presence of initialisation errors). In other words, the residual generator should act as a functional observer that tracks an output identically equal to zero. Consequently (Ding, 2008; Kravaris, 2011, 2016; Luenberger, 1971), there must exist a differentiable map $T(x)$ from $\mathbb{R}^n$ to $\mathbb{R}^s$ such that:

$$\frac{\partial T(x)}{\partial x}F(x) - AT(x) - BH(x) = 0 \quad (2.4)$$

$$CT(x) + DH(x) = 0 \quad (2.5)$$

Conditions (2.4) and (2.5) state that $z = T(x)$ is an invariant manifold of the zero-input dynamics of system (2.3), on which the residual r is identically equal to zero. It will be seen in Proposition 3.1 in the next section that the necessary and sufficient conditions for existence of such an invariant manifold depend on $F(x)$ and $H(x)$ only, and not on the (A, B, C, D) matrices of the residual generator.

If conditions (2.4) and (2.5) are satisfied, the observer error dynamics (expressed in terms of the off-the-manifold coordinate $e = z - T(x)$) and the residual are given by:

$$\frac{d(z - T(x))}{dt} = A(z - T(x)) + \left(BJ(x) - \frac{\partial T(x)}{\partial x}G(x)\right)f + \sum_{i=1}^{m}\left(BK_i(x) - \frac{\partial T(x)}{\partial x}E_i(x)\right) w_i \quad (2.6)$$

$$r = C(z - T(x)) + DJ(x)f + \sum_{i=1}^{m} DK_i(x)w_i \quad (2.7)$$

It should be noted here that the zero-input dynamics of (2.6)–(2.7) is exactly linear and moreover, if the matrix A is Hurwitz, the zero-input response is

$$z(t) - T(x(t)) = e^{At}(z(0) - T(x(0))) \to 0$$

$$r(t) = Ce^{At}(z(0) - T(x(0))) \to 0$$

which means that the manifold $z = T(x)$ is attractive and the residual $r(t)$ asymptotically approaches zero.

The second requirement for the residual generator is that the residual must remain completely unaffected by any disturbances $w_i(t)$ present in the system. Disturbance decoupling can be achieved if the coefficients of w_i in (2.6) and (2.7) vanish, i.e, for all $i = 1, \ldots, m$,

$$\frac{\partial T(x)}{\partial x}E_i(x) - BK_i(x) = 0 \quad (2.8)$$

$$DK_i(x) = 0 \quad (2.9)$$

The third requirement for the residual generator is that the residual r must be affected by the input f, so that the fault can be detected by monitoring the residual. Therefore, the coefficients of the input f in (2.6) and (2.7) must not be all zero:

$$\begin{bmatrix} \frac{\partial T}{\partial x}(x)G(x) - BJ(x) \\ DJ(x) \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (2.10)$$

In summary, the residual generator should satisfy the following design conditions:

(i) The functional observer conditions (2.4) and (2.5)

(ii) The disturbance decoupling conditions (2.8) and (2.9) for all disturbances
(iii) The fault detectability condition (2.10)

Remark 2.1: Some parallels can be drawn between the observer approach formulated here and the differential geometric perspective in (De Persis & Isidori, 2001). Specifically, the disturbance decoupling and fault detectability conditions can be expressed in geometric terms as $\begin{bmatrix} E_i(x) \\ K_i(x) \end{bmatrix} \in \Omega^{\perp} \forall i = 1, .., m$ and $\begin{bmatrix} G(x) \\ J(x) \end{bmatrix} \notin \Omega^{\perp}$, where $\Omega^{\perp}$ is the annihilator of the codistribution Ω spanned by the rows of the matrix $\begin{bmatrix} \frac{\partial T(x)}{\partial x} & -\mathrm{B} \\ 0 & \mathrm{D} \end{bmatrix}$.

3. Solution of the design conditions

For the design of the residual generator (2.2), one must be able to find the matrices A, B, C and D and a differentiable map $T(x)$ so that the design conditions (2.4), (2.5), (2.8) and (2.9) are satisfied. In addition, it is desired that the matrix A is Hurwitz with prescribed eigenvalues for stability and fast response of the error dynamics. The following proposition provides necessary and sufficient conditions for the residual generator (2.2) to satisfy (2.4) and (2.5).

Proposition 3.1: *There exists a residual generator of the form* (2.2) *satisfying the functional observer design conditions* (2.4) *and* (2.5) *if and only if there exist constant row vectors* $v_0, v_1, \ldots, v_{s-1}, v_s$ $\mathbb{R}^p$ *that satisfy:*

$$v_0 H(x) + L_F(v_1 H(x)) + \ldots + L_F^{s-1}(v_{s-1}H(x)) + L_F^s(v_s H(x)) = 0 \tag{3.1}$$

where L_F *denotes the Lie derivative operator* $L_F = \sum_{j=1}^{n} F_j(x) \frac{\partial}{\partial x_j}$.

Proof: (i) *Necessity*: Suppose that there exists $T(x) = \begin{bmatrix} T_1(x) \\ T_2(x) \\ \vdots \\ T_s(x) \end{bmatrix}$ such that (2.4) is satisfied, i.e.

$$\begin{bmatrix} L_F T_1(x) \\ L_F T_2(x) \\ \vdots \\ L_F T_s(x) \end{bmatrix} = \mathrm{A} \begin{bmatrix} T_1(x) \\ T_2(x) \\ \vdots \\ T_s(x) \end{bmatrix} + \begin{bmatrix} \mathrm{B}_1 H(x) \\ \mathrm{B}_2 H(x) \\ \vdots \\ \mathrm{B}_s H(x) \end{bmatrix}$$

where $\mathrm{B}_1, \ldots, \mathrm{B}_s$ denote the rows of the matrix B. Applying the Lie derivative operator L_F to each component of the above equation (k–1) times, we find that for $k = 1,2,3 \ldots$

$$\begin{bmatrix} L_F^k T_1(x) \\ L_F^k T_2(x) \\ \vdots \\ L_F^k T_s(x) \end{bmatrix} = \mathrm{A}^k \begin{bmatrix} T_1(x) \\ T_2(x) \\ \vdots \\ T_s(x) \end{bmatrix} + \begin{bmatrix} (\mathrm{A}^{k-1}\mathrm{B})_1 H(x) + L_F((\mathrm{A}^{k-2}\mathrm{B})_1 H(x)) + \ldots + L_F^{k-1}(\mathrm{B}_1 H(x)) \\ (\mathrm{A}^{k-1}\mathrm{B})_2 H(x) + L_F((\mathrm{A}^{k-2}\mathrm{B})_2 H(x)) + \ldots + L_F^{k-1}(\mathrm{B}_2 H(x)) \\ \vdots \\ (\mathrm{A}^{k-1}\mathrm{B})_s H(x) + L_F((\mathrm{A}^{k-2}\mathrm{B})_s H(x)) + \ldots + L_F^{k-1}(\mathrm{B}_s H(x)) \end{bmatrix}$$

and we can calculate

$$\begin{aligned} &(L_F^s + \alpha_1 L_F^{s-1} + \ldots + \alpha_s I) T_\iota(x) \\ &= ((\mathrm{A}^{s-1}\mathrm{B})_i + \alpha_1(\mathrm{A}^{s-2}\mathrm{B})_i + \ldots + \alpha_{s-1}\mathrm{B}_i) H(x) \\ &\quad + L_F(((\mathrm{A}^{s-2}\mathrm{B})_i + \ldots + \alpha_{s-2}B_i) H(x))) \\ &\quad + \ldots + L_F^{s-1}(\mathrm{B}_i H(x)) \end{aligned} \tag{3.2}$$

where $\alpha_1, \ldots, \alpha_s$ are the coefficients of the characteristic polynomial of the matrix A.

At the same time, the mapping $T(x)$ must satisfy (2.5), hence applying $(L_F^s + \alpha_1 L_F^{s-1} + \ldots + \alpha_s I)$ on each component of equation (2.5) and using (3.2) gives:

$$\begin{aligned} 0 &= (\mathrm{CA}^{s-1}\mathrm{B} + \alpha_1 \mathrm{CA}^{s-2}\mathrm{B} + \ldots + \alpha_{s-1}\mathrm{CB} + \alpha_s \mathrm{D}) H(x) \\ &\quad + L_F((\mathrm{CA}^{s-2}\mathrm{B} + \ldots + \alpha_{s-2}\mathrm{CB} + \alpha_{s-1}\mathrm{D}) H(x))) \\ &\quad + \ldots + L_F^{s-1}((\mathrm{CB} + \alpha_1 \mathrm{D}) H(x)) + L_F^s(\mathrm{D}H(x)) \end{aligned}$$

which proves that (3.1) is satisfied.

(ii) *Sufficiency*: Suppose that there exist constant row vectors $v_0, v_1, \ldots, v_{s-1}, v_s$ that satisfy (3.1). Consider the following choices of (A, B, C, D) matrices:

$$A = \begin{bmatrix} 0 & 0 & \cdots & 0 & -\alpha_s \\ 1 & 0 & \cdots & 0 & -\alpha_{s-1} \\ 0 & 1 & \cdots & 0 & -\alpha_{s-2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -\alpha_1 \end{bmatrix},$$

$$B = \begin{bmatrix} \alpha_s \\ \alpha_{s-1} \\ \vdots \\ \alpha_2 \\ \alpha_1 \end{bmatrix} v_s - \begin{bmatrix} v_0 \\ v_1 \\ \vdots \\ v_{s-2} \\ v_{s-1} \end{bmatrix},$$

$$C = [0\ 0\ \cdots\ 0\ 1], D = -v_s \tag{3.3}$$

For the above A and C matrices (in observer canonical form), the design conditions (2.4) and (2.5) can be written componentwise as follows:

$$\frac{\partial T_1(x)}{\partial x} F(x) + \alpha_s T_s(x) - \mathrm{B}_1 H(x) = 0 \tag{3.4}$$

$$\frac{\partial T_2(x)}{\partial x} F(x) - T_1(x) + \alpha_{s-1} T_s(x) - \mathrm{B}_2 H(x) = 0 \tag{3.5}$$

$$\vdots$$

$$\frac{\partial T_s(x)}{\partial x}F(x) - T_{s-1}(x) + \alpha_1 T_s(x) - B_s H(x) = 0 \quad (3.6)$$

$$T_s(x) + DH(x) = 0 \quad (3.7)$$

We observe that the above equations are easily solvable sequentially for $T_s(x), T_{s-1}(x), \ldots, T_1(x)$, starting from the last equation and going up. In particular, for the chosen B and D matrices, we find from (3.7), (3.6), ..., (3.5):

$$T_s(x) = v_s H(x)$$

$$T_{s-1}(x) = L_F(v_s H(x)) + v_{s-1}H(x)$$

$$\vdots$$

$$T_2(x) = L_F^{s-2}(v_s H(x)) + \ldots + v_2 H(x)$$

$$T_1(x) = L_F^{s-1}(v_s H(x)) + \ldots + L_F(v_2 H(x)) + v_1 H(x)$$

whereas (3.4) gives:

$$L_F^{s}(v_s H(x)) + L_F^{s-1}(v_{s-1}H(x)) + \ldots + L_F(v_1 H(x)) + v_0 H(x) = 0$$

which is exactly (3.1). Thus, we have proved that

$$T(x) = \begin{bmatrix} v_1 H(x) + L_F(v_2 H(x)) + \ldots + L_F^{s-1}(v_s H(x)) \\ v_2 H(x) + \ldots + L_F^{s-2}(v_s H(x)) \\ \vdots \\ v_{s-1}H(x) + L_F(v_s H(x)) \\ v_s H(x) \end{bmatrix} \quad (3.8)$$

satisfies the design conditions (2.4) and (2.5) when $v_0, v_1, \ldots, v_{s-1}, v_s$ satisfy (3.1) and the A, B, C, D matrices are chosen according to (3.3).

It is important to observe that the sufficiency part of the proof is constructive: it gives an explicit solution of the design equations (2.4) and (2.5) in terms of the vectors $v_0, v_1, \ldots, v_{s-1}, v_s$ that satisfy (3.1). Moreover, the eigenvalues of the A-matrix of the derived residual generator are the roots of the polynomial $\lambda^s + \alpha_1\lambda^{s-1} + \ldots + \alpha_{s-1}\lambda + \alpha_s$, therefore they are assignable. The following Proposition provides necessary and sufficient conditions for the derived residual generator to meet the disturbance decoupling specifications (2.8) and (2.9). ■

Proposition 3.2: *Suppose that there exist constant row vectors $v_0, v_1, \ldots, v_{s-1}, v_s$ $\mathbb{R}^p$ that satisfy* (3.1) *and that the residual generator matrices (A, B, C, D) have been chosen according to* (3.3), *so that* (2.4) *and* (2.5) *hold with T(x) given by* (3.8). *The residual generator will satisfy the disturbance decoupling conditions* (2.8) *and* (2.9) *if and only if for all i = 1,2 ... ,m:*

$$\begin{aligned} & v_0 K_i(x) + L_{E_i}(v_1 H(x)) + L_{E_i}L_F(v_2 H(x)) + \ldots + L_{E_i}L_F^{s-1}(v_s H(x)) = 0 \\ & v_1 K_i(x) + L_{E_i}(v_2 H(x)) + \ldots + L_{E_i}L_F^{s-2}(v_s H(x)) = 0 \\ & \vdots \\ & v_{s-2}K_i(x) + L_{E_i}(v_{s-1}H(x)) + \ldots + L_{E_i}L_F(v_s H(x)) = 0 \\ & v_{s-1}K_i(x) + L_{E_i}(v_s H(x)) = 0 \\ & v_s K_i(x) = 0 \end{aligned} \quad (3.9)$$

Proof: The disturbance decoupling conditions (2.8) and (2.9) can be written in component form, for $i = 1,2, \ldots m$, as follows:

$$\frac{\partial T_1(x)}{\partial x}E_i(x) - B_1 K_i(x) = 0$$

$$\frac{\partial T_2(x)}{\partial x}E_i(x) - B_2 K_i(x) = 0$$

$$\vdots$$

$$\frac{\partial T_{s-1}(x)}{\partial x}E_i(x) - B_{s-1}K_i(x) = 0$$

$$\frac{\partial T_s(x)}{\partial x}E_i(x) - B_s K_i(x) = 0$$

$$DK_i(x) = 0$$

Substituting the expressions for B, D and $T(x)$from (3.3) and (3.8) to the above equations lead to the following conditions:

$$L_{E_i}L_F^{s-1}(v_s H(x)) + \ldots + L_{E_i}L_F(v_2 H(x)) + L_{E_i}(v_1 H(x)) - \alpha_s v_s K_i(x) + v_0 K_i(x) = 0$$

$$L_{E_i}L_F^{s-2}(v_s H(x)) + \ldots + L_{E_i}L_F(v_3 H(x)) + L_{E_i}(v_2 H(x)) - \alpha_{s-1}v_s K_i(x) + v_1 K_i(x) = 0$$

$$\vdots$$

$$L_{E_I}L_F^{2}(v_s H(x)) + L_{E_i}L_F(v_{s-1}H(x)) + L_{E_i}(v_{s-2}H(x)) - \alpha_3 v_s K_i(x) + v_{s-3}K_i(x) = 0$$

$$L_{E_i}L_F(v_s H(x)) + L_{E_i}(v_{s-1}H(x)) - \alpha_2 v_s K_i(x) + v_{s-2}K_i(x) = 0$$

$$L_{E_i}(v_s H(x)) - \alpha_1 v_s K_i(x) + v_{s-1}K_i(x) = 0$$

$$-v_s K_i(x) = 0$$

which can be written equivalently as

$$L_{E_i}L_F^{s-1}(v_s H(x)) + \ldots + L_{E_i}L_F(v_2 H(x)) + L_{E_i}(v_1 H(x)) + v_0 K_i(x) = 0$$

$$L_{E_i}L_F^{s-2}(v_s H(x)) + \ldots + L_{E_i}L_F(v_3 H(x)) + L_{E_i}(v_2 H(x)) + v_1 K_i(x) = 0$$

$$\vdots$$

$$L_{E_I}L_F^2(v_sH(x)) + L_{E_i}L_F(v_{s-1}H(x)) + L_{Ei}(v_{s-2}H(x)) + v_{s-3}K_i(x) = 0$$

$$L_{E_i}L_F(v_sH(x)) + L_{E_i}(v_{s-1}H(x)) + v_{s-2}K_i(x) = 0$$

$$L_{E_i}(v_sH(x)) + v_{s-1}K_i(x) = 0$$

$$v_sK_i(x) = 0$$

This completes the proof.

The following Proposition provides necessary and sufficient conditions for the derived residual generator to meet the fault detectability condition (2.12). ■

Proposition 3.3: *Suppose that there exist constant row vectors $v_0, v_1, \ldots, v_{s-1}, v_s$ $\mathbb{R}^p$ that satisfy* (3.1) *and that the residual generator matrices (A, B, C, D) have been chosen according to* (3.3), *so that* (2.4) *and* (2.5) *hold with T(x) given by* (3.8). *The residual generator will satisfy the fault detectability condition* (2.10) *if and only if*

$$\begin{bmatrix} v_0J(x) + L_G(v_1H(x)) + L_GL_F(v_2H(x)) + \ldots + L_GL_F^{s-1}(v_sH(x)) \\ v_1J(x) + L_G(v_2H(x)) + \ldots + L_GL_F^{s-2}(v_sH(x)) \\ \vdots \\ v_{s-2}J(x) + L_G(v_{s-1}H(x)) + L_GL_F(v_sH(x)) \\ v_{s-1}J(x) + L_G(v_sH(x)) \\ v_sJ(x) \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3.10)$$

Proof: For B and D defined via (3.3) and $T(x)$ given by (3.8), condition (2.10) is equivalent to: is 'either $v_sJ(x) \neq 0$' or

$$\begin{bmatrix} \alpha_s \\ \alpha_{s-1} \\ \vdots \\ \alpha_2 \\ \alpha_1 \end{bmatrix} v_sJ(x) - \begin{bmatrix} v_0J(x) + L_G(v_1H(x)) + L_GL_F(v_2H(x)) + \ldots + L_GL_F^{s-1}(v_sH(x)) \\ v_1J(x) + L_G(v_2H(x)) + \ldots + L_GL_F^{s-2}(v_sH(x)) \\ \vdots \\ v_{s-2}J(x) + L_G(v_{s-1}H(x)) + L_GL_F(v_sH(x)) \\ v_{s-1}J(x) + L_G(v_sH(x)) \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

and further to

$$\begin{bmatrix} v_0J(x) + L_G(v_1H(x)) + L_GL_F(v_2H(x)) + \ldots + L_GL_F^{s-1}(v_sH(x)) \\ v_1J(x) + L_G(v_2H(x)) + \ldots + L_GL_F^{s-2}(v_sH(x)) \\ \vdots \\ v_{s-2}J(x) + L_G(v_{s-1}H(x)) + L_GL_F(v_sH(x)) \\ v_{s-1}J(x) + L_G(v_sH(x)) \\ v_sJ(x) \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

Summarising the results of Propositions 3.1–3.3, we conclude that the design of a an s-th order linear residual generator is feasible if and only if there exist constant row vectors $v_0, v_1, \ldots, v_{s-1}, v_s$ $\mathbb{R}^p$ that satisfy

(1) $L_F^s(v_sH(x)) + L_F^{s-1}(v_{s-1}H(x)) + \ldots + L_F(v_1H(x)) + v_0H(x) = 0$

(2) $\Omega \begin{bmatrix} E_i(x) \\ K_i(x) \end{bmatrix} = 0 \ \forall\, i = 1, .., m$ and

(3) $\Omega \begin{bmatrix} G(x) \\ J(x) \end{bmatrix} \neq 0$

where

$$\Omega = \begin{bmatrix} \frac{\partial}{\partial x}[L_F^{s-1}(v_s H(x))] + \ldots + \frac{\partial}{\partial x}[L_F(v_2 H(x))] + \frac{\partial}{\partial x}(v_1 H(x)) & v_0 \\ \frac{\partial}{\partial x}[L_F^{s-2}(v_s H(x))] + \ldots + \frac{\partial}{\partial x}(v_2 H(x)) & v_1 \\ \vdots & \vdots \\ \frac{\partial}{\partial x}[L_F(v_s H(x))] + \frac{\partial}{\partial x}(v_{s-1} H(x)) & v_{s-2} \\ \frac{\partial}{\partial x}(v_s H(x)) & v_{s-1} \\ 0 & v_s \end{bmatrix}$$

The last two conditions state that the vectors $\begin{bmatrix} E_i(x) \\ K_i(x) \end{bmatrix}$, $i = 1, .., m$ belong to the annihilator of the codistribution spanned by the rows of the matrix Ω, whereas $\begin{bmatrix} G(x) \\ J(x) \end{bmatrix}$ does not (cf. Remark 2.1). Also, it is important to note that all three conditions are independent of the choice of eigenvalues for the residual generator; if constant row vectors $v_0, v_1, \ldots, v_{s-1}, v_s$ can be found to satisfy them, any arbitrary eigenvalues can be assigned.

Remark 3.1: In case $\begin{bmatrix} G(x) \\ J(x) \end{bmatrix} \in span\left(\begin{bmatrix} E_1(x) \\ K_1(x) \end{bmatrix}, \begin{bmatrix} E_2(x) \\ K_2(x) \end{bmatrix}, \ldots, \begin{bmatrix} E_m(x) \\ K_m(x) \end{bmatrix}\right)$ for all x, the disturbance decoupling conditions become incompatible with the fault detectability condition, hence fault detection is infeasible in the presence of disturbances.

Remark 3.2: Using the Lie derivative notation on a vector function as the vector of the Lie derivatives of its components, e.g. $L_F \begin{bmatrix} H_1(x) \\ \vdots \\ H_p(x) \end{bmatrix} = \begin{bmatrix} L_F H_1(x) \\ \vdots \\ L_F H_p(x) \end{bmatrix}$, and accordingly notation for higher-order Lie derivatives of vector functions, the conditions of Propositions 3.1–3.3 may be written in a compact form as

$$[v_0 \ v_1 \ \ldots \ v_{s-1} \ v_s][\Gamma_o(x)\Gamma_{w_1}(x)\ldots\Gamma_{w_m}(x)\Gamma_f(x)] = [0 \ 0 \ *] \tag{3.11}$$

where:

$$\Gamma_o(x) = \begin{bmatrix} H(x) \\ L_F H(x) \\ \vdots \\ L_F^{s-1} H(x) \\ L_F^s H(x) \end{bmatrix}$$

$$\Gamma_{w_i}(x) = \begin{bmatrix} K_i(x) & 0 & \cdots & 0 & 0 \\ L_{E_i} H(x) & K_i(x) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ L_{E_i} L_F^{s-2} H(x) & L_{E_i} L_F^{s-3} H(x) & \cdots & K_i(x) & 0 \\ L_{E_i} L_F^{s-1} H(x) & L_{E_i} L_F^{s-2} H(x) & \cdots & L_{E_i} H(x) & K_i(x) \end{bmatrix}$$

$$\Gamma_f(x) = \begin{bmatrix} J(x) & 0 & \cdots & 0 & 0 \\ L_G H(x) & J(x) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ L_G L_F^{s-2} H(x) & L_G L_F^{s-3} H(x) & \cdots & J(x) & 0 \\ L_G L_F^{s-1} H(x) & L_G L_F^{s-2} H(x) & \cdots & L_G H(x) & J(x) \end{bmatrix}$$

and the symbol * indicates a nonzero matrix block. In this form, the linear dependence of the conditions on the unknown vectors $v_0, v_1, \ldots, v_{s-1}, v_s$ becomes explicit.

For the special case when the system (2.1) is linear, i.e.

$$\dot{x} = Fx + Gf + \sum_{i=1}^{m} E_i w_i \tag{3.12}$$

$$y = Hx + Jf + \sum_{i=1}^{m} K_i w_i$$

the design conditions (2.4), (2.5), (2.8) and (2.9) become the standard design conditions for linear residual generators for linear system (Ding, 2008)

$$TF - AT - BH = 0 \tag{3.13}$$

$$CT + DH = 0 \tag{3.14}$$

$$TE - BK = 0 \tag{3.15}$$

$$DK = 0 \tag{3.16}$$

where $E = [E_1 \ldots E_m]$ and $K = [K_1 \ldots K_m]$, whereas the fault detectability condition (2.10) becomes $\begin{bmatrix} TG - BJ \\ DJ \end{bmatrix} \neq 0$.

For the choices of A,B,C,D matrices given by (3.3),

$$T = \begin{bmatrix} v_1 H + v_2 HF + \ldots + v_s HF^{s-1} \\ v_2 H + \ldots + v_s HF^{s-2} \\ \vdots \\ v_{s-1} H + v_s HF \\ v_s H \end{bmatrix}$$

and the conditions on the residual generator can be combined in a compact form as

$$[v_0\ v_1\ \ldots\ v_{s-1}\ v_s][\tilde{\Gamma}_o \tilde{\Gamma}_w \tilde{\Gamma}_f] = [0\ 0\ *] \tag{3.17}$$

where

$$\tilde{\Gamma}_o = \begin{bmatrix} H \\ HF \\ \vdots \\ HF^{s-1} \\ HF^s \end{bmatrix}$$

$$\tilde{\Gamma}_w = \begin{bmatrix} K & 0 & \cdots & 0 & 0 \\ HE & K & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ HF^{s-2}E & HF^{s-3}E & \cdots & K & 0 \\ HF^{s-1}E & HF^{s-2}E & \cdots & HE & K \end{bmatrix}$$

$$\tilde{\Gamma}_f = \begin{bmatrix} J & 0 & \cdots & 0 & 0 \\ HG & J & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ HF^{s-2}G & HF^{s-3}G & \cdots & J & 0 \\ HF^{s-1}G & HF^{s-2}G & \cdots & HG & J \end{bmatrix}$$

and the symbol * indicates a nonzero matrix block. Equation (3.17) is exactly the condition given by Ding (Ding, 2008) for linear systems of the form (3.12).

Thus, the results of Propositions 3.1, 3.2 and 3.3 provide a direct generalisation of standard results on linear systems to nonlinear systems.

Remark 3.3: In the linear systems literature (Ding, 2008), the vectors $v_0, v_1, \ldots, v_{s-1}, v_s$ that satisfy (3.17) are called *parity vectors*, and the set of parity vectors, when nonempty, defines a linear space which is called the parity space(Chow & Willsky, 1984). The nonlinear generalisation developed in this section offers a nonlinear analog of parity vectors, defined as the ones satisfying (3.1), (3.9) and (3.10) or equivalently (3.11).

Remark 3.4: The parity vectors $v_0, v_1, \ldots, v_{s-1}, v_s$ provide information about the measurements that are being used in the residual generator. If the j-th element of all of these vectors happens to be 0, this means that the measurement y_j is not used for fault detection since both B and D will have their j-th column identically zero. This situation may arise in applications and will be discussed in the applications section. In the event of multiple solutions for the set of parity vectors $v_0, v_1, \ldots, v_{s-1}, v_s$, this feature might be used minimise the total number of sensors that are used.

Remark 3.5: In the majority of applications, process disturbances do not affect sensors and sensor disturbances do not affect the process. This motivates considering the following special case:

$$\dot{x} = F(x) + G(x)f + \sum_i E_i(x) w_i^p \tag{3.18}$$

$$y = H(x) + J(x)f + \sum_i K_i(x) w_i^s$$

where w_i^p denotes a process disturbance and w_i^s a sensor disturbance. For this special class of systems, the disturbance decoupling conditions (3.9) get simplified since for every disturbance, either $E_i(x)$ or $K_i(x)$ vanishes, depending on whether it is a process or sensor disturbance. A sensor disturbance generally places more restrictions than a process disturbance. In particular, we see from (3.9) that

(a) A process disturbance w_i^p places no restriction on v_0 since the corresponding $K_i(x) = 0$.
(b) A sensor disturbance w_i^s imposes the restriction that $[v_0\ v_1\ \ldots\ v_{s-1}\ v_s]K_i(x) = 0$. In case a disturbance affects only a specific sensor measuring y_j, this implies that the j-th element of $v_0, v_1, \ldots, v_{s-1}, v_s$ must equal to 0, hence the measurement y_j must not be used in the residual generator.

Remark 3.6: For the special case of a scalar residual generator ($s = 1$), the design conditions become

$$v_0 H(x) + L_F(v_1 H(x)) = 0$$

$$v_0 K_i(x) + L_{E_i}(v_1 H(x)) = 0, i = 1, \ldots, m$$

$$v_1 K_i(x) = 0, i = 1, \ldots, m$$

$$\begin{bmatrix} v_0 J(x) + L_G(v_1 H(x)) \\ v_1 J(x) \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The above conditions take an even simpler form in case all states are measurable, i.e. $H(x) = x$:

$$v_0 x + v_1 F(x) = 0$$

$$v_0 K_i(x) + v_1 E_i(x) = 0, i = 1, \ldots, m$$

$$v_1 K_i(x) = 0, i = 1, \ldots, m$$

$$\begin{bmatrix} v_0 J(x) + v_1 G(x) \\ v_1 J(x) \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Remark 3.7: The linear residual generator formulation developed in the previous and the present section is amenable to a slight generalisation. Instead of using residual generator (2.2), it is possible to use a residual generator involving additive nonlinear output injection terms:

$$\dot{z} = Az + \beta(y) \tag{3.19}$$

$$r = Cz + \delta(y)$$

where $\beta(\cdot)$ and $\delta(\cdot)$ are smooth functions $\mathbb{R}^p \to \mathbb{R}^s$ and $\mathbb{R}^p \to$ respectively. Then, the mapping $T(x)$ from $\mathbb{R}^n$ to $\mathbb{R}^s$ must satisfy the functional observer conditions:

$$\frac{\partial T(x)}{\partial x}F(x) - AT(x) - \beta(H(x)) = 0 \tag{3.20}$$

$$CT(x) + \delta(H(x)) = 0 \tag{3.21}$$

and the observer error dynamics, in the absence of faults and disturbances, will still be linear:

$$\frac{d(z - T(x))}{dt} = A(z - T(x))$$
$$r = C(z - T(x))$$

therefore it will have the same convergence properties when A is Hurwitz. Proposition 3.1 gets modified as follows:

Proposition 3.4: *There exists a residual generator of the form* (3.19) *satisfying the functional observer design conditions* (3.20) *and* (3.21) *if and only if there exist functions* $v_0(y), v_1(y), \ldots, v_{s-1}(y), v_s(y)$ *from* $\mathbb{R}^p$ *to* $\mathbb{R}$ *that satisfy:*

$$v_0(H(x)) + L_F(v_1(H(x))) + \ldots + L_F^{s-1}(v_{s-1}(H(x))) + L_F^s(v_s(H(x))) = 0 \tag{3.22}$$

If such functions can be found, the residual generator can be built by using

$$A = \begin{bmatrix} 0 & 0 & \cdots & 0 & -\alpha_s \\ 1 & 0 & \cdots & 0 & -\alpha_{s-1} \\ 0 & 1 & \cdots & 0 & -\alpha_{s-2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -\alpha_1 \end{bmatrix},$$
$$\beta(y) = \begin{bmatrix} \alpha_s \\ \alpha_{s-1} \\ \vdots \\ \alpha_2 \\ \alpha_1 \end{bmatrix} v_s(y) - \begin{bmatrix} v_0(y) \\ v_1(y) \\ \vdots \\ v_{s-2}(y) \\ v_{s-1}(y) \end{bmatrix}, \tag{3.23}$$
$$C = [0\ 0\ \cdots\ 0\ 1], \delta(y) = -v_s(y)$$

and it is possible to verify that

$$T(x) = \begin{bmatrix} v_1(H(x)) + L_F(v_2(H(x))) + \ldots + L_F^{s-1}(v_s(H(x))) \\ v_2(H(x)) + \ldots + L_F^{s-2}(v_s(H(x))) \\ \vdots \\ v_{s-1}(H(x)) + L_F(v_s(H(x))) \\ v_s(H(x)) \end{bmatrix} \tag{3.24}$$

satisfies (3.20) *and* (3.21).

One can accordingly define disturbance decoupling and fault detectability design conditions for the residual generator, which will impose extra conditions on the functions $v_0(y), v_1(y), \ldots, v_{s-1}(y), v_s(y)$ *mutatis mutandis.*

■

4. Fault isolation

Till now we considered the problem of detecting a single scalar fault in the presence of disturbances. However, for systems with multiple faults, in addition to detecting the occurrence of faults it is necessary to correctly identify which fault/faults have occurred. To this end, consider the following system involving n_f possible faults:

$$\dot{x} = F(x) + \sum_{i=1}^{n_f} G_i(x) f_i \tag{4.1}$$

$$y = H(x) + \sum_{i=1}^{n_f} J_i(x) f_i$$

with state $x \in \mathbb{R}^n$, output $y \in \mathbb{R}^p$ and inputs $f_i \in \mathbb{R}$, $i = 1, 2, \ldots, n_f$, and with $F(x), H(x), G_i(x), J_i(x)$ smooth functions, and assume that

(i) $n_f \leq p$ i.e. that the number of faults does not exceed the number of measurements.
(ii) the vectors $\begin{bmatrix} G_i(x) \\ J_i(x) \end{bmatrix}$, $i = 1, \ldots, n_f$ are linearly independent for every x. In other words, that no fault can enter the model equations the same way as a linear combination of some other faults.

The above are clearly necessary conditions fault distinguishability.

Remark 4.1: In general, fault distinguishability may be defined as injectivity or left invertibility of the input/output map $(f_1, \ldots, f_{nf}) \rightarrowtail y$. Sufficient conditions may be derived by taking derivatives of each output of order up to the relative orders, and checking the left invertibility of the matrix of the coefficients of the input vector. Specifically, denoting by ρ_j the relative order of output y_j with respect to the input vector and by $\mathcal{C}(x)$ the $p \times n_f$ characteristic matrix, with entries

$$\mathcal{C}_{ji}(x) = \begin{cases} L_{G_i} L_F^{\rho_j - 1} H_j(x), & \text{if } \rho_j > 0 \\ J_{ji}(x), & \text{if } \rho_j = 0 \end{cases}$$

a sufficient condition for left invertibility of the input/output map is Rank $\mathcal{C}(x) = n_f$.

The residual generator formulated in Section 3 can be applied to build a fault isolation scheme in a straightforward manner. To isolate a specific fault f_k, one can try to construct a residual generator of the form:

$$\dot{z}_k = Az_k + B_k y \tag{4.2}$$

$$r_k = Cz_k + D_k y$$

which satisfies the fault detectability condition (3.10) for fault f_k and the disturbance decoupling conditions (3.9) for $w_i = f_i, i \neq k$, along with the functional observer condition (3.1).

If this is feasible for every fault, then one can build an overall system of residual generators, working in parallel, and each one

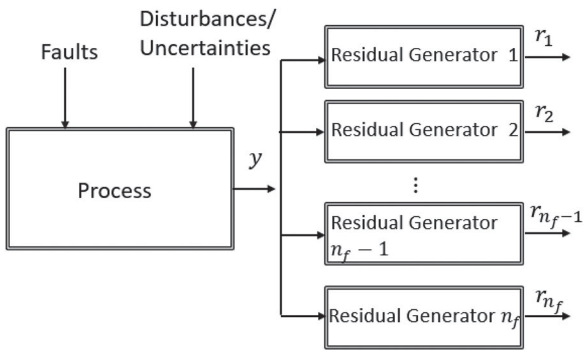

Figure 2. Fault isolation scheme.

detecting a specific fault (see also Figure 2):

$$\begin{aligned} \dot{z}_1 &= Az_1 + B_1 y \\ &\vdots \\ \dot{z}_{n_f} &= Az_{n_f} + B_{n_f} y \\ r_1 &= Cz_1 + D_1 y \\ &\vdots \\ r_{n_f} &= Cz_{n_f} + D_{n_f} y \end{aligned} \tag{4.3}$$

where for any initial condition $\begin{bmatrix} x(0) \\ z_1(0) \\ \vdots \\ z_{n_f}(0) \end{bmatrix}$, $\lim_{t\to\infty} r_i(t) = 0 \ if \ f_i(t) = 0$, $\lim_{t\to\infty} r_i(t) \neq 0 \ if \ f_i(t) \neq 0$

This will solve the fault isolation problem.

Remark 4.1: In the foregoing formulation, the same pair of (C, A) matrices are used in all residual generators, leading to the same assigned eigenvalues for all residual generators. More generally, different pairs of (C, A) matrices could be used.

Remark 4.2: System (4.1) involves faults but no disturbances. More generally, one could consider

$$\dot{x} = F(x) + \sum_{i=1}^{n_f} G_i(x) f_i + \sum_{i=1}^{m} E_i(x) w_i \tag{4.7}$$

$$y = H(x) + \sum_{i=1}^{n_f} J_i(x) f_i + \sum_{i=1}^{m} K_i(x) w_i$$

Every residual generator in this case, must satisfy disturbance decoupling conditions for all w_i in addition to the disturbance decoupling conditions for the other faults. In general, the disturbance decoupling conditions for w_i may impose an increase in the number of necessary measurements p, beyond the number of faults n_f.

5. Representative applications to chemical processes

In this section, case studies are presented to demonstrate the use of linear residual generators for fault diagnosis in nonlinear chemical process systems. In chemical processes, dynamic models are generally composed of conservation equations and inventory rate equations of the form: (Accumulation) = (In) – (Out) + (Generation), with the nonlinearities often appearing only in the generation terms, associated with kinetic rate expressions. This makes them amenable to the design conditions, with parity vectors that are independent of the reaction rates, which are often uncertain. Three application examples are studied in this section, specifically an anaerobic digester (bio-reactor), a continuous stirred tank reactor (CSTR) and a process network consisting of a CSTR and flash separator with a recycle stream.

5.1 Bio-reactor

Anearobic digestion is a complex biochemical system, in which organic compounds are converted to biogas, consisting primarily of methane and carbon dioxide. Anerobic digestion of a soluble susbtrate can be modelled as a two-step process: The acidogenic bacteria first convert the organic soluble substrate to a volatile fatty acid mixture and then the acids are utilised by methanogenic bacteria to produce the biogas. It is assumed that the digestion occurs in a CSTR (see Figure 3) and the feed only consists of soluble substrates and no biomass and no volatile fatty acids. The mathematical model is as follows:

$$\begin{aligned} \frac{dX_1}{dt} &= -(D_r + f(t))X_1 + \frac{(\mu_{\max1} + w(t))S_1}{K_{s1} + S_1} X_1 \\ \frac{dS_1}{dt} &= (D_r + f(t))(S_0 - S_1) - \frac{1}{Y_1} \frac{(\mu_{\max1} + w(t))S_1}{K_{s1} + S_1} X_1 \\ \frac{dX_2}{dt} &= -(D_r + f(t))X_2 + \frac{\mu_{\max2} S_2}{K_{s2} + S_2} X_2 \\ \frac{dS_2}{dt} &= -(D_r + f(t))S_2 + \frac{c_{12}}{Y_1} \frac{(\mu_{\max1} + w(t))S_1}{K_{s1} + S_1} X_1 \\ &\quad - \frac{\mu_{\max2} S_2}{K_{s2} + S_2} \frac{X_2}{Y_2} \\ y_1 &= X_1 \\ y_2 &= S_1 \\ y_3 &= X_2 \\ y_4 &= S_2 \end{aligned} \tag{5.1}$$

where S_1 and S_2 are the concentration of the soluble organic substrate and volatile fatty acids respectively, X_1 and X_2 are the concentration of acidogenic and methanogenic biomass respectively, $\mu_1(S_1) = \frac{\mu_{\max1} S_1}{K_{s1} + S_1}$ and $\mu_2(S_2) = \frac{\mu_{\max2} S_2}{K_{s2} + S_2}$ are the specific growth rates of acidogenic and methanogenic bacteria respectively, with $\mu_{\max1}, \mu_{\max2}$, the corresponding maximum specific growth rates and K_{S1}, K_{S2} the saturation constants,

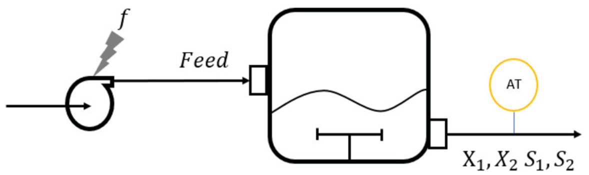

Figure 3. Bio-reactor schematic.

Table 1. Bio-reactor parameters.

Parameter	Value	Parameter	Value
D_r	0.2d^{-1}	$\mu_{\max 2}$	0.36d^{-1}
$\mu_{\max 1}$	4d^{-1}	Y_1	0.11 g/g
K_{s1}	0.023g/L	Y_2	0.003 g/mmol
S_0	10 g/L	c_{12}	16.95 mmol/g
K_{s2}	2.3mmol/L		

Y_1, Y_2 are the biomass yield coefficients, c_{12} is the stoichiometric coefficient of conversion of S_1 to S_2, S_0 is the concentration of organic substrate in the feed and D_r is the dilution rate. $f(t)$ represents a fault in the dilution rate and $w(t)$ represents the uncertainty in the maximum growth rate of acidogenic bacteria.

The bio-reactor parameters are listed in Table 1 and the initial conditions are $X_1(0) = 0.1$ g/L, $X_2(0) = 40$ g/L, $S_1(0) = 10$ g/L, $S_2(0) = 0.1$ mmol/L.

The model (5.1) can be converted to deviation form, relative to reference steady state conditions corresponding to absence of faults or uncertainties: $X_1' = X_1 - X_{1,ref}, S_1' = S_1 - S_{1,ref}, X_2' = X_2 - X_{2,ref}, S_2' = S_2 - S_{2,ref}$ and $y_1' = X_1', y_2' = S_1', y_3' = X_2', y_4' = S_2'$, where the subscript *ref* denotes reference steady state value. The goal is to build a residual generator to detect the dilution rate fault $f(t)$ in the presence of the disturbance $w(t)$ in the acidogenic reaction rate expression.

To this end, a scalar residual generator is built ($s = 1$), with the design conditions (see Remark 3.6) satisfied for following choice of parity vectors

$$v_0 = \left[D_r, D_r, D_r\left(-1 + \frac{1}{Y_1}\right)\frac{Y_1}{Y_2 c_{12}}, D_r\left(-1 + \frac{1}{Y_1}\right)\frac{Y_1}{c_{12}}\right] \tag{5.2}$$

$$v_1 = \left[1, 1, \left(-1 + \frac{1}{Y_1}\right)\frac{Y_1}{Y_2 c_{12}}, \left(-1 + \frac{1}{Y_1}\right)\frac{Y_1}{c_{12}}\right]$$

Using the parity vectors (5.2) and the design parameters $A = -\alpha_1 = -1, B = \alpha_1 v_1 - v_0, C = 1, D = -v_1$, leads to the following first order residual generator:

$$\frac{dz}{dt} = -z + (1 - D_r)\left(y_1' + y_2' + \left(-1 + \frac{1}{Y_1}\right)\frac{Y_1}{Y_2 c_{12}} y_3' + \left(-1 + \frac{1}{Y_1}\right)\frac{Y_1}{c_{12}} y_4'\right)$$

$$r = z - \left(y'_1 + y'_2 + \left(-1 + \frac{1}{Y_1}\right)\frac{Y_1}{Y_2 c_{12}} y_3' + \left(-1 + \frac{1}{Y_1}\right)\frac{Y_1}{c_{12}} y_4'\right)$$

From (5.3), we see that at steady state, the residual is given by

$$r_s = -D_r\left(X'_{1,s} + S'_{1,s} + \left(-1 + \frac{1}{Y_1}\right)\frac{Y_1}{Y_2 c_{12}} X'_{2,s} + \left(-1 + \frac{1}{Y_1}\right)\frac{Y_1}{c_{12}} S'_{2,s}\right) \tag{5.4}$$

and it is zero in the absence of fault and disturbances (when system is at reference steady state). One can also observe, from the steady state equations of the system (5.1) in deviation form, that the new steady state obtained in the presence of only disturbances and no fault, satisfies (5.4) with $r_s = 0$, as a result of the disturbance-decoupling property of the residual generator. On the other hand, again from (5.1) in deviation form, in the presence of a constant fault of size f_s,

$$D_r\left(X'_{1,s} + S'_{1,s} + \left(-1 + \frac{1}{Y_1}\right)\frac{Y_1}{Y_2 c_{12}} X'_{2,s} + \left(-1 + \frac{1}{Y_1}\right)\frac{Y_1}{c_{12}} S'_{2,s}\right) + f_s\left(X_{1,s} + S_{1,s} + \left(-1 + \frac{1}{Y_1}\right)\frac{Y_1}{Y_2 c_{12}} X_{2,s} + \left(-1 + \frac{1}{Y_1}\right)\frac{Y_1}{c_{12}} S_{2,s}\right) = 0 \tag{5.5}$$

where the subscript s denotes the new steady state of the bioreactor.

Combining (5.4) and (5.5), the conclusion is that

$$r_s = f_s\left(X_{1,s} + S_{1,s} + \left(-1 + \frac{1}{Y_1}\right)\frac{Y_1}{Y_2 c_{12}} X_{2,s} + \left(-1 + \frac{1}{Y_1}\right)\frac{Y_1}{c_{12}} S_{2,s}\right) \tag{5.6}$$

From (5.6) it is clear that a constant fault of size $f_s \neq 0$, leads to a residual $r_s \neq 0$.

The residual generator is simulated for two cases: (i) No fault in the dilution rate but under uncertainty in the maximum growth rate of acidogenic bacteria of size $w(t) = 0.5\mu_{\max 1}$. (ii) A fault in the dilution rate which is a step of size 0.5 applied at $t = 2$ and $w(t) = 0.5\mu_{\max 1}$.

The residuals for cases (i) and (ii) are plotted in Figure 4. For the fault-free case the residual shows no deviation for all times whereas in case (ii), there is a deviation that starts at time $t = 2$ d (when the fault occurs) and settles at $r_s = 1.431$.

Remark 5.1: As noted in Remark 3.4, parity vectors are not uniquely defined; multiple solutions could exist for v_0, v_1. For example, in the present problem,

$$v_0 = \left[\frac{1}{Y_1},\ 1,\ 0,\ 0\right] \tag{5.7}$$

$$v_1 = \left[\frac{D}{Y_1},\ D,\ 0,\ 0\right]$$

is an alternative pair of parity vectors. With this choice, only two measurements are required namely, the acidogenic biomass $y_1 = X_1$ and the soluble organic substrate $y_2 = S_1$.

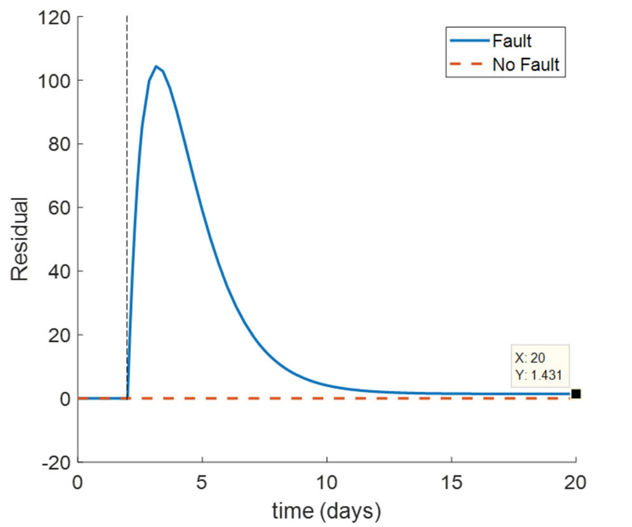

Figure 4. Residual as a function of time. Fault $f(t) = 0.5$ occurs at time $t = 2$ d. The final value of the residual is 1.431.

5.2 Non-isothermal Continuous Stirred Tank Reactor (CSTR)

Consider a non-isothermal Continuous Stirred Tank Reactor (see Figure 5) where a chemical reaction A $\rightarrow$ B takes place. It is assumed the reactor is well-mixed and has constant volume and the feed does not contain species B. The dynamics of the reactor are as follows:

$$\frac{dC_A}{dt} = \frac{F}{V}(C_{A,in} - C_A) - (k_0 + w(t)) \cdot R(C_A, \theta)$$

$$\frac{d\theta}{dt} = \frac{F}{V}(\theta_{in} - \theta) - \frac{UA}{\rho c_p V}(\theta - \theta_j) + \frac{-\Delta H_R}{\rho c_p}(k_0 + w(t)) \cdot R(C_A, \theta) \quad (5.8)$$

$$\frac{d\theta_j}{dt} = \frac{F_j}{V_j}(\theta_{j,in} + f_2(t) - \theta_j) + \frac{UA}{\rho_j c_{p_j} V_j}(\theta - \theta_j)$$

$$y_1 = C_A + f_1(t)$$

$$y_2 = \theta$$

$$y_3 = \theta_j$$

where C_A, θ, θ_j and $C_{A,in}, \theta_{in}, \theta_{j,in}$ represent the concentration, reactor temperature and coolant temperature of the outlet and inlet streams respectively. F and F_j are the feed and coolant flowrates respectively. V and V_j are the reactor volume and cooling jacket volume respectively. $k_0 R(C_A, \theta)$ is the reaction rate, with $R(C_A, \theta) = e^{\frac{-E}{R\theta}} C_A^{1.2}$. $\Delta H_R$is the enthalpy of the reaction. ρ, c_p and $\rho_j, c_{p_j}$are the densities and heat capacities of the reactor contents and cooling fluid respectively. All three states are assumed to be measurable. There is an uncertainty in the pre-exponential factor $w(t)$ of the reaction rate. Two faults are considered namely a sensor fault in the concentration measurement ($f_1(t)$) and a process fault in the inlet jacket temperature ($f_2(t)$).

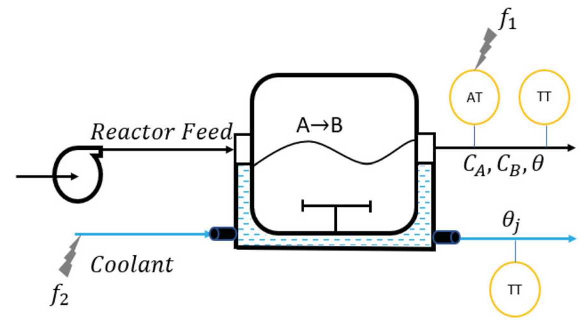

Figure 5. Continuous stirred tank reactor schematic.

The model (5.8) can be converted to deviation form relative to reference conditions corresponding to absence of faults and uncertainties: $C_A' = C_A - C_{A,ref}, \theta' = \theta - \theta_{j,ref}, \theta_j' = \theta_j - \theta_{j,ref}$, where the subscript *ref* denotes reference steady state value.

Our goal is to design a fault diagnosis scheme that can detect and isolate faults f_1 and f_2 in the presence of uncertainties in the reaction rate. To this end, two scalar residual generators of the form (6.2) are built (i) to estimate the analytical sensor fault (f_1) while considering f_2 as an additional disturbance. (ii) to estimate inlet jacket temperature fault f_2 considering f_1 as an additional disturbance.

Residual generator 1: Detection of the analytical sensor fault f_1 while considering $f_2$as an additional disturbance.

A scalar ($s = 1$) residual generator can be designed with the following parity vectors:

$$v_0 = -\left[1, \frac{F\rho c_p + UA}{F(-\Delta H_R)}, \frac{-UA}{F(-\Delta H_R)}\right] \quad (5.9)$$

$$v_1 = -\left[\frac{V}{F}, \frac{V\rho c_p}{F(-\Delta H_R)}, 0\right]$$

and design parameters $\mathrm{A} = -\alpha_1 = -1, \mathrm{B} = \alpha_1 v_1 - v_0$, $\mathrm{C} = 1, \mathrm{D} = -v_1$:

$$\frac{dz_1}{dt} = -z_1 + \left(-\frac{V}{F} + 1\right)y_1' + \left(-\frac{V\rho c_p}{F(-\Delta H_R)} + \frac{F\rho c_p + UA}{F(-\Delta H_R)}\right)y_2' - \frac{UA}{F(-\Delta H_R)}y_3'$$

$$r_1 = z_1 + \frac{V}{F}y_1' + \frac{V\rho c_p}{F(-\Delta H_R)}y_2' \quad (5.10)$$

From (5.10), we see that at steady state, the residual is given by

$$r_{1,s} = (C'_{A,s} + f_{1,s}) + \frac{\rho c_p}{(-\Delta H_R)}\theta'_s + \frac{UA}{F(-\Delta H_R)}(\theta'_s - \theta'_{j,s}) \quad (5.11)$$

On the other hand, from the first two steady state equations of the system (5.8) in deviation form,

$$C'_{A,s} + \frac{\rho c_p}{(-\Delta H_R)}\theta'_s + \frac{UA}{F(-\Delta H_R)}(\theta'_s - \theta'_{j,s}) = 0 \quad (5.12)$$

irrespective of the presence or absence of disturbance w or fault f_2. Therefore,

$$r_{1,s} = f_{1,s} \tag{5.13}$$

The steady state of the residual is nonzero when fault f_1 is nonzero.

Residual Generator 2: Detection of the inlet cooling jacket temperature fault f_2 considering f_1 as an additional disturbance.

A scalar ($s = 1$) residual generator can be designed with the following parity vectors:

$$v_0 = \left[0, +\frac{UA}{\rho_j C_{pj} F_j}, -1 - \frac{UA}{\rho_j C_{pj} F_j}\right] \tag{5.14}$$

$$v_1 = -\left[0, 0, \frac{V_j}{F_j}\right]$$

and design parameters $\mathrm{A} = -\alpha_1 = -1, \mathrm{B} = \alpha_1 v_1 - v_0, \mathrm{C} = 1, \mathrm{D} = -v_1$:

$$\frac{dz_2}{dt} = -z_2 - \left(\frac{UA}{\rho_j c_{p_j} F_j}\right) y_2' - \left(\frac{V_j}{F_j} - 1 - \frac{UA}{\rho_j c_{pj} F_j}\right) y_3'$$

$$r_2 = z_2 + \frac{V_j}{F_j} y_3' \tag{5.15}$$

From (5.16), we see that at steady state, the residual is given by

$$r_{2,s} = -\left(\frac{UA}{\rho_j c_{p_j} F_j}\right) \theta'_s + \left(1 + \frac{UA}{\rho_j c_{pj} F_j}\right) \theta'_{j,s} \tag{5.16}$$

On the other hand, from the third steady state equation of the system (5.8) in deviation form,

$$(f_{2,s} - \theta'_{j,s}) + \frac{UA}{\rho_j c_{p_j} V_j} (\theta'_s - \theta'_{j,s}) = 0 \tag{5.17}$$

From (5.16) and (5.17), it is evident that the residual tracks the closure of the jacket energy balance with or without the fault and we have

$$r_{2,s} = f_{2,s} \tag{5.18}$$

The two residual generators are tested on the following scenario: $f_1(t) = \begin{cases} 0, t < 1 \\ 0.1, t \geq 1 \end{cases}, f_2(t) = \begin{cases} 0, t < 2 \\ 10, t \geq 2 \end{cases}$. $w(t)$ is uniformly distributed in the interval $[-0.05k_0, 0.05k_0]$. The data used for simulations are in Table 2 and the initial conditions of the state variables are $C_A(0) = 0, \theta(0) = 300, \theta_j(0) = 278.15$. The residuals are plotted in Figure 6.

Both residuals from time $t = 0$–1 hr are identically 0. When the sensor fault occurs at time $t = 1$ hr a deviation is seen in r_1 whereas r_2 is identically zero. At time $t = 2$ hr a deviation is observed in r_2 indicating the presence of a fault in the inlet coolant temperature.

Remark 5.2: We see from (5.13) and (5.18) that in this particular application, the residuals at steady state are equal to the values of the respective faults. This can also be seen in Figure 6 where the residuals r_1 and r_2 tend to f_1 and f_2 asymptotically. Thus, in addition to disturbance decoupled fault detection and isolation, the residuals provide estimates of the sizes of the faults.

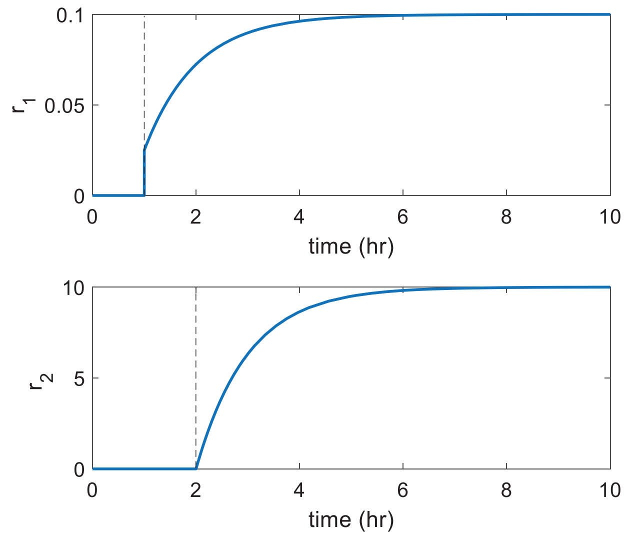

Figure 6. Residuals vs time for non-isothermal CSTR. Fault f_1 occurs at $t = 1$ hr and f_2 at $t = 2$ hr.

Table 2. CSTR parameters.

Parameter	Value	Parameter	Value
F	4 $\mathrm{m^3/hr}$	ρ	1000 $\mathrm{kg/m^3}$
V	1 $\mathrm{m^3}$	c_p	0.23 kJ/(kg K)
V_j	0.03 $\mathrm{m^3}$	ρ_j	1000 $\mathrm{kg/m^3}$
$C_{A,in}$	$\mathrm{kmol/m^3}$	c_{pj}	4 kJ/(kg K)
θ_{in}	300 K	U	500 W/($\mathrm{m^2}$ K)
θ_{jin}	278.15 K	A	10 $\mathrm{m^2}$
k_0	$3 \times 10^8 \mathrm{hr^{-1} m^{0.6} kmol^{-0.2}}$	A_j	1 $\mathrm{m^2}$
E	5×10^4 kJ/kmol	R	8.314 kJ/(kmol K)
ΔH_R	-5×10^4 kJ/kmol		

5.3 Process network

As our final example, we consider a process network consisting of a CSTR and a flash separator (see Figure 7). This process is considerably more complex than the previous two case studies due to the presence of parallel reactions and a recycle stream. In this plant, two parallel exothermic chemical reactions $A \to B, A + A \to C$ with B being the desired product. The outlet stream of the reactor goes to the separator and a part of it is recycled back to the reactor. The mathematical model of the process takes the following form:

Reactor Mass Balance

$$\frac{dC_A}{dt} = \frac{F}{V}(C_{A,in} - C_A) + \frac{F_r}{V}(C_{Ar} - C_A) - (k_1 + w_1(t)) \cdot e^{\frac{-E_1}{R\theta}} C_A - (k_2 + w_2(t)) \cdot e^{\frac{-E_2}{R\theta}} C_A^2$$

$$\frac{dC_B}{dt} = \frac{F_r}{V}(C_{Br} - C_B) - \frac{F}{V} C_B + (k_1 + w_1(t)) \cdot e^{\frac{-E_1}{R\theta}} C_A$$

Reactor Energy Balance

$$\frac{d\theta}{dt} = \frac{F}{V}(\theta_{in} - \theta) + \frac{F_r}{V}(\theta_r - \theta) - \frac{(U + f_2(t))A}{\rho c_p V}(\theta - \theta_J) + \frac{-\Delta H_{R1}}{\rho c_p}(k_1 + w_1(t)) \cdot e^{\frac{-E_1}{R\theta}} C_A + \frac{-\Delta H_{R2}}{\rho c_p}(k_2 + w_2(t)) \cdot e^{\frac{-E_2}{R\theta}} C_A^2$$

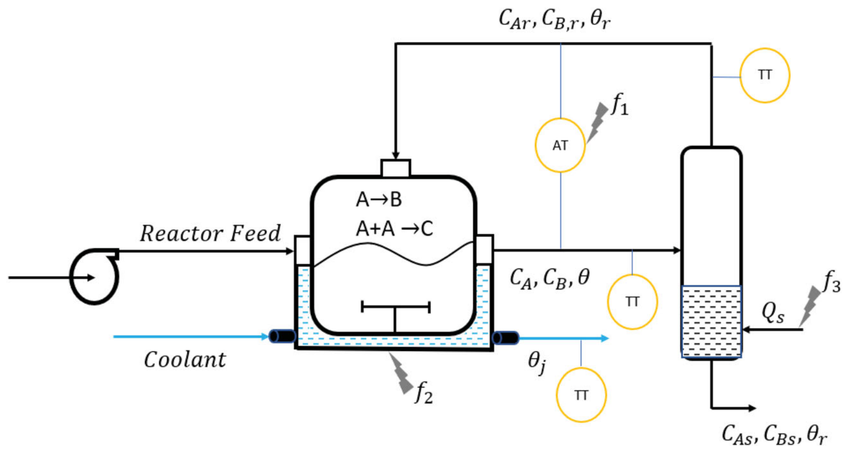

Figure 7. Reactor separator network.

Cooling Jacket Energy Balance

$$\frac{d\theta_j}{dt} = \frac{F_j}{V_j}(\theta_{j,in} - \theta_j) + \frac{(U + f_2(t))A}{\rho_j c_{p_j} V_j}(\theta - \theta_j) \tag{5.15}$$

Flash Separator Mass Balance

$$\frac{dC_{Af}}{dt} = \frac{F_b}{V_f}(C_A - C_{Af}) + \frac{F_r}{V_f}(C_A - C_{Ar}) + w_3$$

$$\frac{dC_{Bf}}{dt} = \frac{F_b}{V_f}(C_B - C_{Bf}) + \frac{F_r}{V_f}(C_B - C_{Br}) + w_4$$

$$C_{Ar} = \frac{\alpha_A C_{Bf}\rho}{\rho + (\alpha_A - 1)C_{Bf}MW_A}$$

$$C_{Br} = \frac{\alpha_B C_{Bf}\rho}{\rho + (\alpha_B - 1)C_{Bf}MW_B}$$

Flash Separator Energy Balance

$$\frac{d\theta_r}{dt} = \frac{F_b + F_r}{V_f}(\theta - \theta_r) + \frac{Q_f + f_3(t)}{\rho C p V_f}$$

Outputs

$$y_1 = C_A + f_1(t)$$

$$y_2 = C_B - f_1(t)$$

$$y_3 = \theta$$

$$y_4 = \theta_j$$

$$y_5 = C_{Ar} + f_1(t)$$

$$y_6 = C_{Br} - f_1(t)$$

$$y_7 = \theta_r$$

The reactor contains an inlet feed F consisting of only species A with concentration $C_{A,in}$ and a recycle feed F_r consisting of both A and B (C_{Ar} and C_{Br}). C_A and C_B are the concentrations of A and B in the reactor and the reactor temperature is θ, the inlet feed temperature is θ_{in} and heat is removed from the reactor via a coolant jacket with inlet temperature $\theta_{j,in}$ and outlet temperature θ_j. $V$and V_j are the reactor and cooling jacket volumes respectively. The desired reaction $A \rightarrow B$ has a rate given by $k_1 e^{-\frac{E_1}{R\theta}} C_A$ and the undesired parallel reaction has a rate given by $k_2 e^{-\frac{E_2}{R\theta}} C_A^2$ where E_1, E_2 and k_1, k_2 are the activation energies and pre-exponential factors of the two reactions respectively and R in the exponential term of the reaction rate is the universal gas constant. ΔH_{R1} and ΔH_{R2} are the enthalpies of the two reactions respectively. ρ, c_p and ρ_j, c_{p_j} are the densities and heat capacities of the reactant and cooling fluid respectively. The outlet of the reactor feeds into a separator with volume V_f, operated at a temperature θ_r and has a heat input Q_f. The concentrations of A and B at the bottom of the flash separator are given by C_{Af} and C_{Bf} respectively with a flow rate F_b. The relative volatilities and molecular weights of the two compounds are given by α_A, α_b and MW_A, MW_B respectively. The parameters used for the simulations are listed in Table 3 and the initial conditions are $C_A(0) = 0, C_B(0) = 0, \theta(0) = 300, \theta_j(0) = 300, C_{As}(0) = 0, C_{Bs}(0) = 0, \theta_r(0) = 300$.

It is assumed that there are uncertainties, given by w_1 and w_2, in the pre-exponential factors of both the reaction rates. In addition, there are modelling uncertainties in the concentration equations for the flash separator characterised by w_3 and w_4. Three different faults are considered namely, (i) a sensor fault $f_1$affecting the measurements of C_A, C_B, C_{Ar} and C_{Br}. (ii) a fault in the cooling jacket given by f_2, (iii) a fault in the heat input to the separator given by f_3. Our goal is to detect and isolate the presence of the three faults decoupled from the four uncertainties in the system. To this end, three residual generators are built, one for each fault of interest.

Like in the previous applications, the model is converted to deviation form relative to reference conditions corresponding to absence of faults and uncertainties: $C'_A = C_A - C_{A,ref}, C'_B = C_B - C_{B,ref}, \theta' = \theta - \theta_{j,ref}, \theta'_j = \theta_j - \theta_{j,ref}, C'_{Af} = C_{Af} - C_{Af,ref}, C'_{Bf} = C_{Bf} - C_{Bf,ref}, \theta'_r = \theta_r - \theta_{r.ref}$, where the subscript *ref* denotes reference steady state value.

Residual generator 1: Detection of sensor fault f_1 with all the other faults as disturbances.

$$v_0 = \left[\frac{-\Delta H_{R2}\left(\frac{F}{V}+\frac{F_r}{V}\right)}{\rho c_p}, \left(\frac{-\Delta H_{R2}}{\rho c_p} - \frac{-\Delta H_{R1}}{\rho c_p}\right)\left(\frac{F_r}{V}+\frac{F}{V}\right), \left(\frac{F_r}{V}+\frac{F}{V}\right), \frac{F_j\rho_j c_{pj}}{\rho c_p V}, -\frac{-\Delta H_{R2}F_r}{\rho c_p V}, \frac{\left(\frac{-\Delta H_{R1}}{\rho c_p} - \frac{-\Delta H_{R2}}{\rho c_p}\right)F_r}{V}, -\frac{F_r}{V} \right]$$

$$v_1 = \left[\frac{-\Delta H_{R2}}{\rho c_p}, \left(\frac{-\Delta H_{R2}}{\rho c_p} - \frac{-\Delta H_{R1}}{\rho c_p}\right), 1, \frac{\rho_j c_{pj} V_j}{\rho c_p V}, 0, 0, 0\right] \tag{5.16}$$

and design parameters $A = -\alpha_1 = -1, B = \alpha_1 v_1 - v_0, C = 1, D = -v_1$:

$$\begin{aligned}\frac{dZ'}{dt} &= -z' + \left(\frac{-\Delta H_{R2}}{\rho c_p} - \frac{-\Delta H_{R2}\left(\frac{F}{V}+\frac{F_r}{V}\right)}{\rho c_p}\right)y_1' \\ &+ \left(\frac{-\Delta H_{R2}}{\rho c_p} - \frac{-\Delta H_{R1}}{\rho c_p}\right)\left(1-\left(\frac{F_r}{V}+\frac{F}{V}\right)\right)y_2' \\ &+ \left(1-\left(\frac{F_r}{V}+\frac{F}{V}\right)\right)y_3' + \left(\frac{\rho_j c_{pj} V_j}{\rho c_p V} - \frac{F_j\rho_j c_{pj}}{\rho c_p V}\right)y_4' \\ &+ \frac{-\Delta H_{R2}F_r}{\rho c_p V}y_5' - \frac{\left(\frac{-\Delta H_{R1}}{\rho c_p} - \frac{-\Delta H_{R2}}{\rho c_p}\right)F_r}{V}y_6' + \frac{F_r}{V}y_7' \\ r_1 &= z' - \frac{-\Delta H_{R2}}{\rho c_p}y_1' - \left(\frac{-\Delta H_{R2}}{\rho c_p} - \frac{-\Delta H_{R1}}{\rho c_p}\right)y_2' \\ &- y_3' - \frac{\rho_j c_{pj} V_j}{\rho c_p V}y_4'\end{aligned} \tag{5.17}$$

Following the same steps as before, we have the following expression of the residual in terms of the fault of interest at steady state:

$$\begin{aligned} r_{1,s} = \Bigg(&-\left(\frac{-\Delta H_{R2}\left(\frac{F}{V}+\frac{F_r}{V}\right)}{\rho c_p}\right) \\ &+ \left(\frac{-\Delta H_{R2}}{\rho c_p} - \frac{-\Delta H_{R1}}{\rho c_p}\right)\left(\left(\frac{F_r}{V}+\frac{F}{V}\right)\right) \\ &+ \frac{-\Delta H_{R2}F_r}{\rho c_p V} + \frac{\left(\frac{-\Delta H_{R1}}{\rho c_p} - \frac{-\Delta H_{R2}}{\rho c_p}\right)F_r}{V}\Bigg)f_{1,s}\end{aligned} \tag{5.18}$$

Table 3. CSTR and separator parameters.

Parameter	Value	Parameter	Value
F	4 m^3/hr	ρ	1000 kg/m^3
F_r	0.2 m^3/hr	c_p	0.23 kJ/(kg K)
V	1 m^3	ρ_j	1000kg/m^3
V_j	0.03 m^3	c_{pj}	4 kJ/(kg K)
C_{Ain}	kmol/m^3	U	500 W/(m^2 K)
θ_{in}	300 K	A	10 m^2
θ_{jin}	278.15 K	A_j	1 m^2
k_1	$3 \times 10^8 \text{hr}^{-1}$	V_f	1m^2
k_2	$3 \times 10^{10} \text{hr}^{-1}\text{m}^3\text{mol}^{-1}$	R	8.314 kJ/(kmol K)
E_1	5×10^4 kJ/kmol	MW_A	50 kg/kmol
E_2	5.1×10^4 kJ/kmol	MW_B	100 kg/kmol
ΔH_{R1}	-5×10^4 kJ/kmol	α_A	10
ΔH_{R2}	-5.5×10^4 kJ/kmol	α_B	1
Q_f	10^4kJ/ hr		

Residual generator 2: Detection of cooling jacket fault f_2 with all the other faults as disturbances. A scalar ($s = 1$) residual generator can be designed with the following parity vectors:

$$v_0 = \left[0, 0, -\frac{UA}{\rho_j c_{pj} V_j}, \frac{UA}{\rho_j c_{pj} V_j} + \frac{F_j}{V_j}, 0, 0, 0\right]$$

$$v_1 = [0, 0, 0, 1, 0, 0, 0]$$

and design parameters $A = -\alpha_1 = -1, B = \alpha_1 v_1 - v_0$, $C = 1, D = -v_1$:

$$\frac{dZ'}{dt} = -z' + \left(\frac{UA}{\rho_j c_{p_j} F_j}\right)y_3' + \left(1 - \frac{UA}{\rho_j c_{pj} V_j} - \frac{F_j}{V_j}\right)y_4'$$

$$r_2 = z' - y_4' \tag{5.19}$$

Following the same steps as before, we have the following expression of the residual in terms of the fault of interest at steady state.

$$r_{2,s} = -\frac{f_{2,s}A}{\rho_j c_{p_j} F_j}(\theta_s - \theta_{j,s}) \tag{5.20}$$

Residual generator 3: Detecting fault in flash separator heat input f_3 with all the other faults as disturbances.

A scalar ($s = 1$) residual generator can be designed with the following parity vectors:

$$v_0 = \left[0, 0, -\frac{F_b + F_r}{V_f}, 0, 0, 0, \frac{F_b + F_r}{V_f}\right]$$

$$v_1 = [0, 0, 0, 0, 0, 0, 1]$$

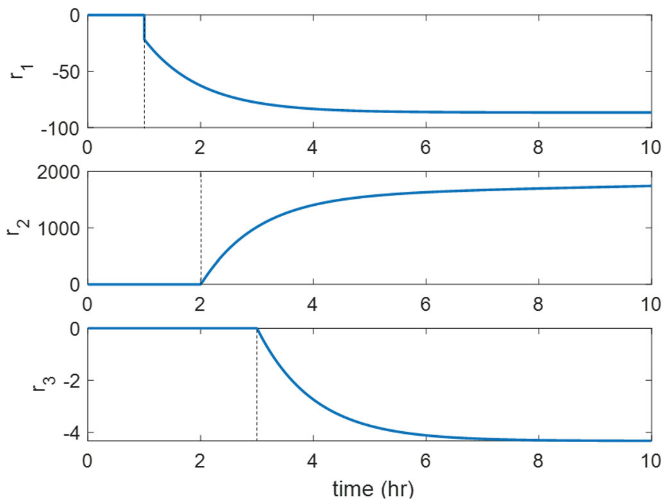

Figure 8. Residuals vs time for Process Network. Faults f_1, f_2, f_3 occur at $t = 1,2,3$ hr respectively.

and design parameters $\mathrm{A} = -\alpha_1 = -1, \mathrm{B} = \alpha_1 \nu_1 - \nu_0$, $\mathrm{C} = 1, \mathrm{D} = -\nu_1$:

$$\frac{dZ'}{dt} = -z' + \left(\frac{F_b + F_r}{V_f}\right) y_3' + \left(1 - \frac{F_b + F_r}{V_f}\right) y_8'$$

$$r_3 = z' - y_8' \tag{5.21}$$

Following the same steps as before, we have the following expression of the residual in terms of the fault of interest at steady state.

$$r_{3,s} = \frac{-f_{3,s}}{\rho C p V_f} \tag{5.22}$$

The three residual generators are tested in a scenario with faults occurring in the following way: $f_1(t) = \begin{cases} 0, t < 1 \\ 0.1, t \geq 1 \end{cases}$, $f_2(t) = \begin{cases} 0, t < 2 \\ -100e^{0.01t}, t \geq 2 \end{cases}$, $f_3(t) = \begin{cases} 0, t < 3 \\ 1000, t \geq 3 \end{cases}$. Uncertainties w_1 and w_2 are uniformly distributed in the intervals $[-0.05k_{10}, 0.05k_{10}]$ and $[-0.05k_{20}, 0.05k_{20}]$ respectively. w_3 and w_4 are Gaussian distributions $N(0, 1)$ and $N(0, 2)$ respectively. The plots of the three residuals are shown in Figure 8. In the interval $t = [0,1]$ all residuals are identically zero. When the sensor fault occurs at time $t = 1$ hr there is a deviation in r_1 from sensor fault f_1 whereas residuals r_2 and r_3 are unaffected. After the onset of cooling jacket fault f_2 at time $t = 2$ hr, r_2 shows a deviation but r_3 remains identically equal to zero, until the fault f_3 in the heat input to the flash separator occurs at time $t = 3$ hr.

Remark 5.3: It is to be noted that the requirement of full state measurements can be done away with if f_1 is absent or is assumed to be an additional disturbance. Isolation of faults f_2 and f_3 requires only the 3 temperature measurements, namely reactor, cooling jacket, and separator temperature.

6. Conclusions

This work derived necessary and sufficient conditions of existence of a **linear** residual generator for disturbance-decoupled fault detection in a nonlinear system. As long as these conditions are satisfied, we have shown that the design of residual generators with eigenvalue assignment is straightforward. Using a linear residual generator for every fault, decoupled from the other faults and the system disturbances, immediately gives rise to a linear fault diagnoser for the nonlinear system.

Not every nonlinear system satisfies the feasibility conditions for a linear residual generator. However, a large class of chemical processes involve 'localised' nonlinearities in a way that they permit the design of linear residual generators. Therefore, the results of this work are expected to enable future industrial fault diagnosis applications.

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