




Leveraging Math Cognition to Combat Health Innumeracy

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Abstract

Rational numbers (i.e., fractions, percentages, decimals, and whole-number frequencies) are notoriously difficult mathematical constructs. Yet correctly interpreting rational numbers is imperative for understanding health statistics, such as gauging the likelihood of side effects from a medication. Several pernicious biases affect health decision-making involving rational numbers. In our novel developmental framework, the natural-number bias—a tendency to misapply knowledge about natural numbers to all numbers—is the mechanism underlying other biases that shape health decision-making. Natural-number bias occurs when people automatically process natural-number magnitudes and disregard ratio magnitudes. Math-cognition researchers have identified individual differences and environmental factors underlying natural-number bias and devised ways to teach people how to avoid these biases. Although effective interventions from other areas of research can help adults evaluate numerical health information, they circumvent the core issue:

We describe the origins of natural-number bias and how researchers may harness the bias to improve rational-number understanding and ameliorate innumeracy in real-world contexts, including health. We recommend modifications to formal math education to help children learn the connections among natural and rational numbers. We also call on researchers to consider individual differences people bring to health decision-making contexts and how measures from math cognition might identify those who would benefit most from support when interpreting health statistics. Investigating innumeracy with an interdisciplinary lens could advance understanding of innumeracy in theoretically meaningful and practical ways.

Keywords

math cognition, decision-making, natural-number bias, ratio bias, denominator neglect, 1-in- x phenomenon, interdisciplinary collaborations

Adults are frequently inundated with statistics in the form of rational numbers, such as fractions (e.g., $1/4$), percentages (e.g., 25%), decimals (e.g., 0.25), and whole-number frequencies (e.g., 25 out of 100). For instance, a woman may be presented with the statistic that one in eight women experience breast cancer in their lifetime and may rely on this information when deciding whether to schedule diagnostic screenings. How does she interpret this statistic? What numerical factors might influence her understanding of this breast-cancer risk? If she were to compare this risk to that of

heart disease, which affects one in four people in the United States, would she consider this to be a lower or higher health risk? Perhaps she, like many other people, would overestimate her risk of cancer relative to heart disease (Scheideler et al., 2017).

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A myriad of factors influence the decisions people make about their own or others' health. In this article, we focus on one critical aspect of many health decisions—

This is an important focus because this (mis)interpretation is malleable and can be improved via intervention. We outline the pernicious biases that affect people's ability to reason with numerical health information. For example, people often neglect denominators and attend only to numerators when comparing health statistics such as 4 in 10,000 versus 3 in 1,000, and, in doing so, reach the mistaken conclusion that 4 in 10,000 is the larger of the two risks, although it is smaller in magnitude than 3 in 1,000. In one real-world example, consumers believed they were getting less value for their money for A&W's third-pound burger than McDonald's Quarter Pounder because 3 is smaller than 4 (Conradt, 2016). This phenomenon, in which people attend only to one of two quantities in a ratio ($a:b$), or only to the numerator or denominator in a fraction in isolation, is known by different terms: *ratio bias* (Bourdin et al., 2022), the *1-in-x phenomenon*, or *denominator neglect* in the health-cognition literature and *natural-number bias* or *whole-number bias* in the domain of math cognition (see Table 1).

Researchers in the field of decision-making have developed communication strategies to help adults reason with numerical health information (e.g., Bonner et al., 2021; Lipkus, 2007; Nelson et al., 2008; Waters et al., 2016; Waters, Foust, et al., 2021; Waters, Maki, et al., 2021; Zikmund-Fisher, 2011). However, these strategies do not necessarily address the root of the numerical biases that impede informed decision-making. Moreover, these biases are not limited to the health domain; they also occur in scholastic (e.g., Ni & Zhou, 2005), financial (Peters et al., 2019; Sobkow et al., 2019), and presumably other contexts in which rational numbers are involved. These biases are all instances of a more general cognitive bias that results from the tendency to misapply knowledge about natural numbers to all other numbers (e.g., although $4 > 3$, $3/1,000 > 4/10,000$; Siegler et al., 2011). As mentioned above, this tendency is known as the natural-number bias in the domain of math cognition, and it arises from constraints on the human perceptual system, statistical regularities in the environment (i.e., prevalence of natural numbers in everyday life), and difficulties with rational-number understanding that stem from formal classroom lessons (Moss & Case, 1999). Figure 1 illustrates our novel developmental framework, which we describe in subsequent sections, including the antecedents of natural-number bias and its downstream

consequences. The two thick gray arrows in Figure 1 are expanded in Figure 2 and illustrate the inflection points at which interventions can occur (i.e., in childhood or during adulthood) to dampen the effects of the natural-number bias.

In the current article, we explicate the benefits of integrating theoretical and methodological insights from the fields of both math cognition and health decision-making. Prior reviews of numeracy in the domain of health decision-making have emphasized different theoretical perspectives that may account for innumeracy, including some from cognitive perspectives (Lipkus & Peters, 2010; Peters, 2020; Peters & Bjälkebring, 2015; Reyna, 2008; Reyna & Brainerd, 2007). In her book *Innumeracy in the Wild*, Ellen Peters (2020) described various mathematical and nonmathematical factors that affect health and financial decision-making. She asserted:

Our grasp of numbers and uncertainty is one of humankind's most distinctive and important traits. It is pivotal to our exceptional ability to control the world around us as we make short-term choices and forecast far into the future. However, even very smart people can struggle with numbers in ways that pose negative consequences for their decision-making. Numeric ability equips individuals with vital tools that allow them to take charge of various aspects of their life. The more numerate enjoy superior health, wealth, and employment outcomes, whereas the innumerate remain more vulnerable (inside cover).

One overarching goal within the domain of numeracy and decision-making is to find ways to make the innumerate less vulnerable to adverse health outcomes. We propose that this can be done, at least in part, through developing and implementing interventions that help individuals think more deeply about the magnitude of rational numbers, which should in turn decrease the likelihood of automatically processing ratios in a biased way. This article is intended to inform efforts to advance this overarching goal of promoting positive health outcomes.

In addition, we propose that our own recent cross-disciplinary efforts have shown promise in integrating theoretical and methodological insights from both math cognition and health decision-making (Choi et al., 2020; Fitzsimmons, Woodbury, et al., in press; Mielicki, Fitzsimmons, Schiller et al., 2022; Scheibe et al., 2022; Thompson et al., 2021; Thompson, Taber, Sidney, et al., 2020). We believe that interdisciplinary, collaborative science that bridges domains has the potential to make

Table 1. Glossary of Terms

Domain-general process	
Relational reasoning	Domain-general reasoning about the relations among concepts instead of the concepts in isolation. For example, a dog running behind a cat represents the “chasing” relation, and 3/7 represents the relation between components of the ratio: the whole (7 of 7) and the parts (3 of 7).
Approximate number system	Cross-species number systems and capacity with numbers Early emerging system that allows people and animals to perceive the approximate numeric value of sets without counting (e.g., 16 dots is more than 8 dots). Some researchers suggest that it is a system that allows people and animals to perceive magnitude instead of number because number covaries with other continuous aspects of stimuli, such as surface area and density, which might instead be what people and animals are perceiving.
Magnitude processing system	The ability to process numerical magnitudes may be made possible by a general system that facilitates processing of magnitudes of any kind (e.g., length, brightness, weight, space).
Ratio processing system	Early emerging system that allows people and animals to perceive approximate ratios without calculating (e.g., 16 blue dots for every 8 yellow dots is the same ratio as 2 blue dots for every 1 yellow dot). The extent to which this system is distinct from the approximate number system is debated.
Number sense	Basic ability that allows people and animals to reason with nonsymbolic numerical magnitudes
Numeric ability (or lack thereof) and preferences	
Numeracy	Math ability or “understanding and use of mathematical concepts” (Peters, 2020); can be assessed with subjective (i.e., perceptions) or objective (i.e., calculations) measures. OECD (2016) defines it as “the ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life” (p. 48).
Innumeracy	Lack of, or deficit in, numeracy
Types of numbers	
Natural numbers	Positive integers beginning with 1
Whole numbers	Positive integers and 0
Rational numbers	Fractions, percentages, decimals, and whole-number frequencies. In symbolic form, a rational number can take the fraction form a/b , where a and b are typically integers, and b is not allowed to be equal to 0. This symbolic form of a rational number explains why integers and decimals are instances of rational numbers (e.g., the integer 2 can be expressed as 2/1 and the decimal .75 can be expressed as 3/4).
Ratio	Relationship between two quantities (e.g., three roses:five daisies) or a part related to a whole (e.g., three roses:eight flowers). The rational-number magnitude associated with a ratio $a:b$ is a/b , where b is not equal to 0 (e.g., the ratio of 3 cups of flour to 4 cups of sugar means there is 3/4 cup of flour for each cup of sugar, or \$50 for 5 sandwiches means a rate of \$10 per sandwich. In this article, a ratio $a:b$ and its fraction or magnitude representation a/b are used interchangeably, which also means that the two quantities a and b are sometimes referred to as the numerator and the denominator.
Unit fraction	Fraction with a numerator of one (e.g., 1/3 or 1/12). From a measurement or number-line perspective, a unit fraction takes the form $1/b$, where b is a positive integer. They are called unit fractions because they represent the units that are iterated or copied in the fraction a/b ; i.e., a/b means copies or iterations of a $1/b$ unit (e.g., 3/5 means 3 copies or iterations of a 1/5 unit).
Symbolic numbers	Refers to presenting numeric information with Arabic or Roman numerals (e.g., 1, 1/2, III)
Nonsymbolic numbers or ratios	Refers to presenting number or ratio information with continuous line segments (e.g., one line segment is twice as long as the other), or discrete, countable objects, sounds, or events (e.g., ↑↑↑↑ to represent 4).
Small-component equivalent fractions	Fractions that often have single-digit numerators and denominators yet are defined relative to a large-component equivalent fraction that has the same magnitude but larger numerator and denominator components (e.g., 1/2, a small-component fraction, is equivalent to 15/30, a large-component fraction).

(continued)

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Large-component equivalent fractions	Fractions that often have double-digit numerators and denominators yet are defined relative to a small-component equivalent fraction that has the same magnitude but smaller numerator and denominator components (e.g., 15/30, a large-component fraction, is equivalent to 1/2, a small-component fraction).
Types of biases	
Natural-number bias	Also known as the whole-number bias. An overgeneralization of natural-number knowledge to all rational numbers. One example is attending to only the numerators or denominators of a fraction in isolation rather than considering the magnitude, or size, of the fraction. People may also judge fractions with larger numerators and denominators (e.g., 15/30) as larger than an equivalent fraction with smaller components (e.g., 1/2).
Ratio bias/denominator neglect	Only considering the numerator, and not the denominator, when reasoning with a fraction or ratio.
1-in- x phenomenon	Refers to presenting risk information as 1 over a denominator, which often causes people to overestimate personal risk compared with presenting risk information as an equivalent ratio with a larger numerator and denominator.

the greatest real-world impact (Klein et al., 2017). Psychology is a “hub science” with strong connections to many other disciplines (Cacioppo, 2007). Similar to other “calls to action” that have urged researchers to clarify terminology and measurement tools to move fields forward (Norton & Nurnberger-Haag, 2018;

Rittle-Johnson, 2017), a collaborative approach among scientists who identify as cognitive, developmental, social, and health psychologists; math-education researchers; and judgment and decision-making researchers is critical to meeting the goal of helping adults appropriately interpret the numerical information

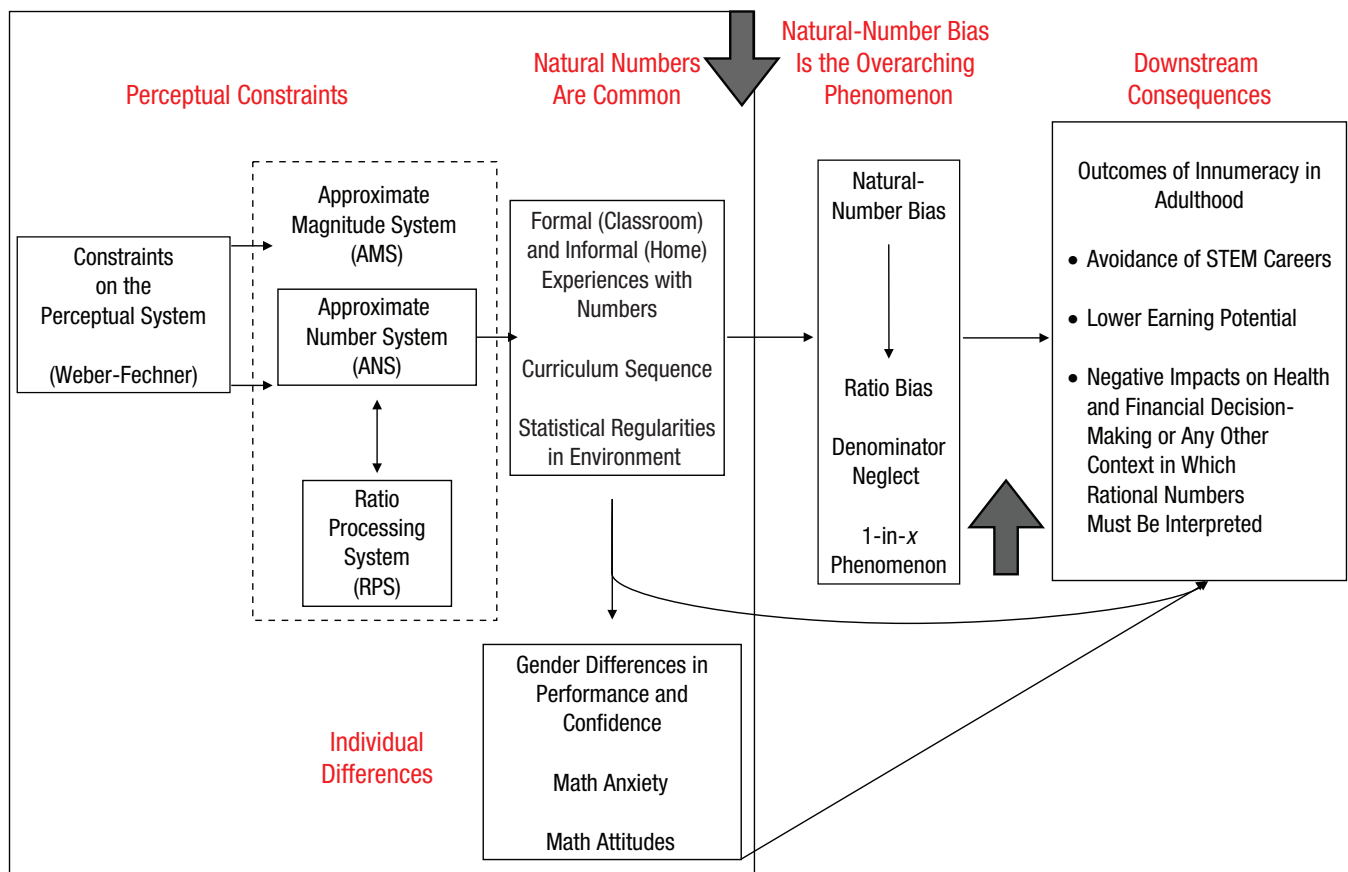


Fig. 1. A novel developmental framework for identifying the antecedents and downstream consequences of the natural-number bias. Each section of the figure is discussed in detail in the text. The thick gray arrows are inflection points at which interventions can occur (i.e., in childhood or during adulthood) to dampen the effects of the natural-number bias. Note that there are downstream consequences of the natural-number bias in any situation in which people must interpret rational numbers.

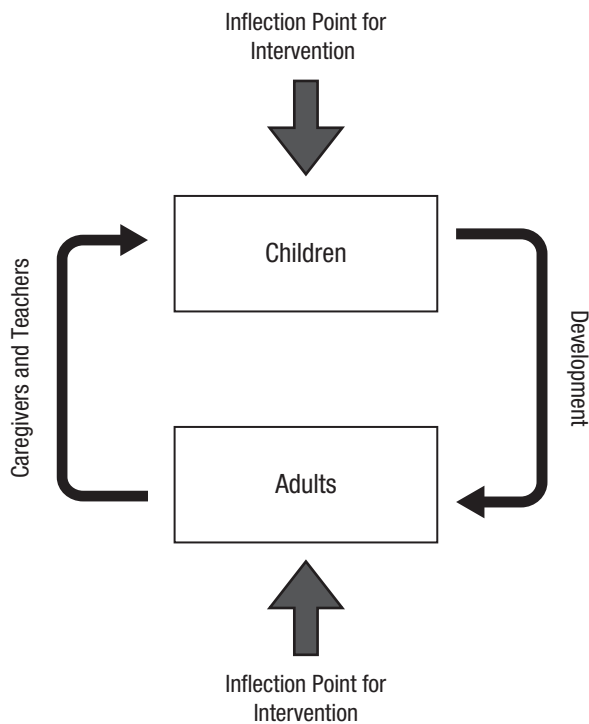


Fig. 2. Inflection points for interventions to override the natural-number bias. If we intervene on adults, they could help children combat natural-number bias as caregivers and teachers. If we intervene on children, they will become adults who are less likely to fall prey to the natural-number bias.

inherent in health statistics. It is very difficult for any one researcher to stay completely abreast of nuances in multiple fields of research, especially when the same constructs are referred to with different terminology (see Table 1). By taking a “team-science” approach (Hall et al., 2018; National Research Council, 2015) in which we examine the research problem through various research lenses, we can create and test multifaceted solutions that may be more impactful than those produced from any one discipline alone.

In the next sections, we provide an overview of (a) the far-reaching negative consequences of innumeracy, or a lack of math literacy, in real-world contexts; (b) a mechanistic explanation of the origins of natural-number bias; (c) differing goals across disciplines for circumventing numerical biases via interventions; and (d) recommendations for ways to improve math education in the service of battling innumeracy.

Innumeracy’s Far Reach

Many people struggle with numeracy, “the ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in

adult life” (Organisation for Economic Cooperation and Development [OECD], 2016, p. 48). Strikingly, results from the Programme for International Student Assessment (OECD, 2019) show that U.S. 15-year-olds perform below the international average on math literacy, which emphasizes the ability to use math in real-world contexts. This is troubling because low numeracy earlier in life relates to poorer math performance in later childhood (Siegler et al., 2012) and has health and financial implications for outcomes in adulthood (National Mathematics Advisory Panel [NMAP], 2008; Peters et al., 2019).¹

Misunderstanding the ways in which numerical magnitudes relate to one another is one source of innumeracy. Difficulties with ratios are due in part to ratios being relational constructs (Siegler et al., 2011; Sidney & Thompson, 2019)—the first quantity must be interpreted *in relation* to the other quantity (e.g., three of seven parts for the ratio 3:7, or its fraction form $3/7$; Paik & Mix, 2003), and both of these units must be interpreted relative to the whole unit (e.g., seven of seven parts is the whole). It is for this reason that ratios can be thought of as analogous to Piaget’s class-inclusion problem used to assess children’s relational reasoning (see also Reyna, 1991; Wolfe & Reyna, 2009). For example, when presented with an array of three red roses and four daisies, young children often respond that there are more daisies than flowers. This is likely because children focus on the two distinct subsets (three red roses vs. four daisies) rather than the subset of four daisies relative to the whole set of seven flowers.

Failure to engage in relational reasoning underlies people’s difficulty understanding rational numbers and contributes to natural-number bias (Alibali & Sidney, 2015; Braithwaite & Siegler, 2018b; Fitzsimmons et al., 2020b; Lamon, 2020; Ni & Zhou, 2005; Sidney & Thompson, 2019; Siegler et al., 2013). This bias likely arises because of people’s extensive experience with, and confidence in, dealing with natural numbers (Dehaene & Mehler, 1992; Fitzsimmons et al., 2020a). From a young age, people have extensive experience reasoning with natural numbers. However, these experiences are narrowly constrained to reasoning about absolute magnitudes (e.g., three red roses). Experiences that focus on reasoning about *relative* magnitudes (e.g., how the number of roses relates to the total number of flowers) are less common and tend to occur later in development. Natural-number bias arises when the parts are dissociated from the whole for quantities that mathematically exist in relation to one another. Furthermore, natural-number bias is not relegated just to those individuals who are low in numeracy. Rather, it is a pervasive problem that affects people of all ages (Alibali & Sidney, 2015; Braithwaite & Siegler, 2018b; Fazio et al., 2017; Ni & Zhou, 2005; Opfer & DeVries, 2008),

cultures (Alonso-Díaz et al., 2019; DeWolf & Vosniadou, 2015; Gómez et al., 2015; Van Hoof et al., 2020), and levels of expertise (Obersteiner et al., 2013; Reyna, 2004). This suggests that decision-making biases do not suddenly occur in adulthood and thus may be amenable to earlier intervention.

Innumeracy exerts downstream consequences in educational, health, and financial contexts (Nelson et al., 2008; NMAP, 2008; Peters et al., 2019; Trevena et al., 2021). Natural-number bias can lead directly to innumeracy in the health domain (Fig. 1, rightmost box). Innumeracy, in turn, is also associated with avoidance of higher-level math courses and STEM careers (Ashcraft & Kirk, 2001), which can affect access to higher education and earning potential (NMAP, 2008). Innumeracy is also related to negative outcomes in financial contexts (Peters et al., 2019) and arguably any context in which rational numbers must be accurately interpreted.

Identifying a Common Driver of Innumeracy: Natural-Number Bias

Natural-number bias is a broad phenomenon that accounts for a myriad of findings, including the overgeneralization of whole-number arithmetic strategies when solving fraction arithmetic problems (Siegler et al., 2011), consideration of only natural numbers when evaluating possible values of variables (Van Hoof et al., 2015), and failure to recognize that between any two fractions there are an infinite number of other possible fractions (Vamvakoussi & Vosniadou, 2010). For example, when presented with rational numbers such as $1/8$ and $1/4$, people may fail to attend to the numerators because they are the same (Bonato et al., 2007; Opfer & DeVries, 2008; Schneider & Siegler, 2010; Thompson & Opfer, 2008). Said another way, in the breast-cancer versus heart-disease example above (one in eight vs. one in four), people are more likely to only attend to the denominator rather than considering the fraction's magnitude because the denominators differ from one another and the numerators do not. Knowing that eight is greater than four, people may mistakenly decide that $1/8 > 1/4$. This is an example of natural-number bias because this error demonstrates an overgeneralization of knowledge about natural numbers (that eight is greater than four) and a failure to comprehend the magnitude of the ratio formed by the relation between the numerator and denominator: $1/8 = 12.5\%$, which is less than $1/4 = 25\%$. Not only do children and adults sometimes fail to understand which fraction is larger, but natural-number knowledge also interferes in estimating just how large (or small) fractions are, for example, considering $1/60$ to be closer to

$1/1$ than $1/1,440$ because 60 is closer to 1 than 1,440 (Opfer & DeVries, 2008; Thompson & Opfer, 2008).

In Figure 1, we propose a novel developmental trajectory, drawing on research specifically from math cognition and education, that likely contributes to innumeracy. In this trajectory, we describe (a) the limitations of the perceptual system; (b) the processing systems capable of representing large approximate numbers, ratios, and magnitudes more generally that are shared across ages, cultures, expertise levels, and species; (c) the statistical regularities in the environment and formal classroom instruction on rational numbers that could predispose people to the natural-number bias; (d) critical individual differences that affect performance on pure numerical tasks and health decision-making tasks; and (e) the real-world consequences of natural-number bias on innumeracy. The developmental trajectory identifies two critical inflection points for intervention—one in childhood and one in adulthood—that are represented with the large gray arrows in the figure. First, we briefly describe the subcomponents of our novel developmental framework. We then provide more details about our claim that natural-number bias is the overarching phenomenon that encompasses ratio bias, denominator neglect, and the 1-in- x phenomenon. Last, we describe how natural-number bias may be adaptive in some contexts, how individual differences influence decision-making involving rational numbers, and some educational interventions that improve natural-number bias.

What Underlies the Natural-Number Bias?

Perceptual constraints²

From a historical perspective, research from as early as the 1800s has shown that the human perceptual system is limited in systematic ways. Consider the Weber-Fechner law: For a person to notice a change in the intensity of a stimulus (e.g., the number of additional candles beyond just one that must be lit for one to notice a difference in the brightness of a large, dark room), the difference must reach a specific threshold proportional to the intensity of the original stimulus. In the case of the visual perception of nonsymbolic numbers (Fig. 3), a person decides that four objects is greater in magnitude than two more quickly and accurately than deciding that 104 objects is greater than 102 (i.e., size effect) because the ratio of the first pair is considerably larger than the ratio of the second. Furthermore, the more distant two numbers, the easier they are to discriminate from one another (i.e., distance effect; Dehaene, 2011; Dehaene et al., 1998) and remember (Thompson & Opfer, 2016; Thompson & Siegler, 2010). Humans of all ages (McCrink & Wynn, 2007; Xu & Spelke, 2000), even

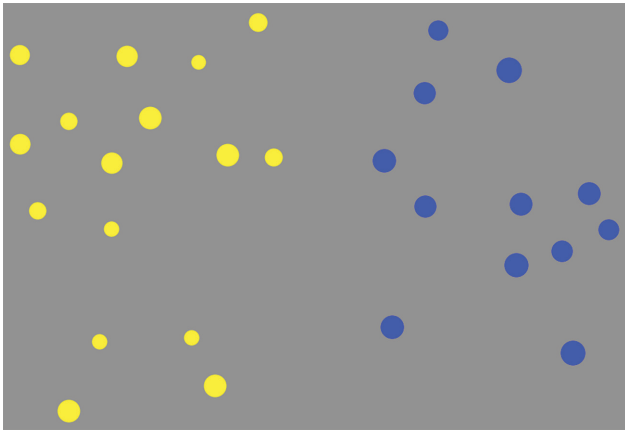


Fig. 3. A nonsymbolic array of dots in which overall surface area is controlled. Dot arrays such as this one are accessible for research via <https://panamath.org>. These arrays can be interpreted in many ways. For example, people may be asked to decide which color is more numerous than the other. The correct answer is that there are more yellow than blue dots ($16 > 12$). Or, people may also be given a cover story that asks them to think about the dots like pieces of candy in a jar and that they should try to maximize their likelihood of choosing a yellow piece of candy because they taste the best (Fazio et al., 2014). In this jar, the ratio of yellow to blue is 16:12 or 4:3. This also means that the person has 16 out of 28 or a 57% chance of picking out a yellow piece of candy from this jar. If there were eight yellow and six blue, the ratio would be the same but expressed with different natural-number components. There are an infinite number of ways to symbolically express this ratio. As indicated in the text, every example of a symbolic or a nonsymbolic ratio is also an additional experience with natural numbers. Thus, natural numbers are necessarily more prevalent in environmental input than rational numbers. Note that the surface area is controlled for the dots in this figure; there are more yellow than blue dots, but some of the yellow dots are substantially smaller than some of the blue dots. This type of experimental control is crucial to show that infants and nonhuman animals have a specific, early emerging capacity for numbers, as opposed to other stimuli characteristics that covary with number (e.g., surface area, density).

those who lack a complex formal counting system (Pica et al., 2004), as well as nonhumans (e.g., Dehaene, 2011; Dehaene et al., 1998; Rugani et al., 2016), can differentiate approximate magnitudes when they differ by a just-noticeable difference (i.e., that individual's threshold). Specifically, humans possess an early emerging system (or systems) to process numerical and/or spatial information. However, the exact nature and developmental trajectory of this system (or systems) and what exactly is processed have been debated. Some have proposed that the approximate number system is used to process large approximate numbers³ (Feigenson et al., 2004; Halberda et al., 2012), whereas others assert that separate core knowledge systems process number and space (Spelke & Kinzler, 2007). Some argue that a dedicated ratio-processing system processes rational numbers (Jacob et al., 2012; Matthews et al., 2015), whereas others argue that the approximate magnitude system processes magnitudes of any kind (Leibovich

et al., 2017; Newcombe et al., 2015). Constraints of the human perceptual system have implications for how humans reason about natural and other rational numbers.

Natural numbers are extremely prevalent in the environment

Although humans are equipped with an early emerging system that allows them to automatically process rational-number information, the environment plays a key role in how humans process symbolic numbers (e.g., Braithwaite & Siegler, 2018b; Ni & Zhou, 2005). Across many cultures, humans have substantially more experience with symbolic (i.e., Arabic numerals) natural numbers than other symbolic rational numbers. Given the structure of symbolic fractions, any experience with symbolic ratios, fractions, decimals, percentages, or whole-number frequencies is necessarily also an experience with symbolic natural numbers that compose the ratio. For instance, every time that a person encounters the fraction $1/2$, they also encounter the natural numbers 1 and 2. This is one likely reason⁴ that the fraction $1/2$ is rated as highly familiar—and more familiar than the equivalent fraction $15/30$ (Fitzsimmons et al., 2020a; Fitzsimmons & Thompson, in press). Because $1/2$ is composed of the numerator 1 and the denominator 2, natural numbers more familiar than 15 and 30 (Dehaene & Mehler, 1992), this may contribute to the overall impression of greater familiarity of $1/2$ relative to $15/30$.

Symbolic and nonsymbolic natural numbers (magnitudes, such as a collection of objects or partitioned lengths; Begolli et al., 2020; Boyer et al., 2008) are prevalent in formal classroom environments as well as informal out-of-school environments (Bustamante et al., 2020; Carraher et al., 1985). People are acutely attuned to this numeric information, possibly because of the early emerging numerical knowledge described above as well as humans' sensitivity to patterns in the environment. In fact, even young children have a propensity to spontaneously focus on numeric information in their environments (Chan & Mazzocco, 2016; Hannula et al., 2010), and before their first birthday, infants learn to recognize new patterns (e.g., ABB) that diverge from previously encountered statistical regularities in their environment (Marcus et al., 1999; Saffran et al., 1996).

The type and quantity of linguistic input about numbers that children receive at home can support their later math development (Eason & Ramani, 2020; Levine et al., 2010; Pruden et al., 2011). However, children's environmental input is biased because small natural numbers (e.g., 1, 2, 3) are more prevalent in the early linguistic environment relative to large natural numbers (e.g., 231, 232, 233; Dehaene & Mehler, 1992).

The prevalence of natural numbers in the environment likely contributes to natural-number bias. For example, children's experiences that illustrate how natural numbers such as 15 and 30 are larger than 1 and 2 may contribute to their tendency to overestimate large-component (e.g., 15/30) relative to small-component (e.g., 1/2) equivalent fractions (Braithwaite & Siegler, 2018b). In addition, not only are children and adults more familiar with fractions containing small relative to large natural-number components, but they are also more accurate when estimating equivalent fractions with smaller relative to larger natural-number components (Fitzsimmons et al., 2020a; Fitzsimmons & Thompson, in press). Furthermore, environmental input can lead to the types of math errors that children make. When the various fraction arithmetic problems found in popular elementary math textbooks were entered into a computational model as input, the model's errors resembled those made by children (Braithwaite & Siegler, 2018a). One common error involved the overapplication of whole-number arithmetic procedures to fraction arithmetic problems. For example, adding across numerators *and* denominators leads to an incorrect answer on a fraction addition problem. Natural-number bias errors are not limited to children in educational contexts but are also observed for adults in a prisoner's-dilemma decision-making scenario. Adults were more likely to cooperate rather than defect when they were offered 300 cents compared with \$3 (Furlong & Opfer, 2009) because $300 > 3$, although the monetary incentives were equivalent (i.e., 300 cents = \$3).

Modifying the environment can have dramatic educational implications. Moss and Case (1999) tested an experimental curriculum that changed the order in which rational-number types were introduced to leverage children's understanding of percentages toward understanding decimals and fractions. This intervention improved children's understanding of rational numbers and reduced natural-number bias relative to a control group of children who learned about rational-number types in the typical order (fractions first, followed by decimals). Moss and Case's intervention was effective but was implemented over the course of several months. Importantly, however, even small modifications can minimize common mathematical misconceptions, as has been shown with a different misconception relating to children's understanding of the equal sign (McNeil, 2008; McNeil et al., 2015). Young children have extensive practice with "right-blank" addition problems (i.e., $2 + 3 = \underline{\quad}$) and subsequently develop a misconception that the equals sign means "find the answer" rather than an indication that the quantities on both sides should be the same. By simply practicing problems in a workbook with the blanks on the left (i.e., $\underline{\quad} = 2 + 3$), children

developed a more conceptual understanding of equivalence (i.e., what is on the left of the equation must be equal to what is on the right) that persisted across 5 to 6 months. These studies suggest that the nature of children's experiences in classrooms affects their reasoning. Given that people have substantially more experience with natural numbers compared with other rational numbers, these common experiences may influence their reasoning when they encounter numerical information in everyday life.

Individual differences affect rational-number understanding

There are important individual differences (e.g., objective and subjective numeracy, magnitude understanding, math attitudes, math anxiety, confidence) that may affect how accurately people interpret numerical risk information. As we describe later, it is imperative to consider a variety of individual differences when evaluating how people reason about rational numbers during health decision-making.

Natural and rational numbers are not coherently integrated in formal instruction

In the United States, the Common Core State Standards in Math (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) recommends that symbolic fractions (e.g., $3/7$) should not be formally introduced in math classrooms until the third grade. However, by this point in the curriculum, children have had nearly a decade of formal and informal experiences with natural numbers that can help or hinder them as they reason about fractions in formal and informal math contexts.

The disconnect between natural numbers and fractions is exacerbated because U.S. teachers receive little guidance on how to emphasize the shared mathematical structure of natural numbers and fractions to their students (for suggestions on using number lines to illustrate connections between natural numbers and fractions, however, see Siegler et al., 2010). In fact, the Common Core State Standards exclusively focus on children's ability to understand absolute magnitudes of whole numbers prior to third grade. The standards focus on fractions as relational magnitudes over a protracted period, from third through fifth grade, and then address ratios and proportions in sixth grade. Thus, it is likely that children are learning about different aspects of rational numbers from different teachers across several years of formal schooling. Even after

years of fraction instruction, middle school students (Siegler et al., 2011), community college students (Schneider & Siegler, 2010), and even college students at selective universities (Fazio et al., 2017) succumb to fraction errors that arise from their misapplication of natural-number knowledge to all other numbers (Siegler, 2016).

Natural-Number Bias is the Overarching Phenomenon

Several phenomena identified in the field of decision-making—ratio bias, denominator neglect, and the 1-in- x phenomenon (described below)—are the result of natural-number bias (i.e., the tendency to focus on natural numbers and natural-number strategies when reasoning about ratios). Our innovative interpretation is that natural-number bias is the overarching construct that subsumes these phenomena identified in the decision-making literature.

At its core, the natural-number bias is a failure to engage in relational reasoning (Gentner, 1983) by considering concepts in isolation rather than in relation to one another.⁵ Rational numbers and natural numbers share many common features that can be adaptively leveraged when people attempt to apply their knowledge about natural numbers when reasoning with rational numbers (Sidney, Thalluri, et al., 2019; Yu et al., 2020, 2022). However, as our many examples have illustrated, overgeneralizing and misapplying the properties of natural numbers to rational numbers can lead to inaccurate estimates of rational-number magnitude. In general, this is problematic and can lead to several biases in decision-making, including ratio bias, denominator neglect, and the 1-in- x phenomenon (described below). These biases, which are just different instances of failure to engage in relational reasoning to process rational-number magnitudes, can hinder accurate estimates of health statistics presented as rational numbers, which may have adverse consequences for health decision-making.

Ratio bias and denominator neglect

Ratio bias, also known as denominator neglect, occurs when people focus on the natural-number components of a ratio. This bias has been shown in probability scenarios, such as giving people a choice between two bowls of jellybeans, one containing 1 red and 9 white and another containing 9 red and 90 white and asking them to choose which bowl would optimize their likelihood of choosing a red jellybean. In these scenarios, people will often choose from the bowl with a larger *absolute* number of red jellybeans (9 vs. 1), even when

the proportion of red to white jellybeans is lower in that container ($9/100 = 9\%$ vs. $1/10 = 10\%$; Denes-Raj & Epstein, 1994; Peters et al., 2006).

Critically, it is also an example of natural-number bias, a failure to attend to the relation between two numbers while instead focusing on each quantity in isolation. Studies in math cognition have conducted analyses of the open-ended strategy reports in which children and adults describe how they estimated and compared fraction magnitudes. It is common for people to explicitly mention in their strategy reports that they focused on numerators in isolation (Fazio et al., 2017; Sidney, Thalluri, et al., 2019; Siegler et al., 2011; Siegler & Thompson, 2014; Thompson et al., 2021). People also focused on just numerators early in the COVID-19 pandemic when the news media and politicians claimed that the seasonal flu was more common than COVID-19 because the absolute number of flu deaths was greater than the absolute number of COVID-19 deaths (Mielicki, Fitzsimmons, Schiller, et al., 2022; Netburn, 2020; Scheibe et al., 2022; Thompson, Taber, Sidney, & Coifman, 2020; Thompson et al. 2021). This misunderstanding of case-fatality rates likely influenced people's beliefs that COVID-19 was less of a threat than the seasonal flu. Therefore, given that natural-number bias involves either a focus on numerators or denominators in isolation, we argue that it is the overarching phenomenon that encompasses ratio bias and denominator neglect.

1-in- x phenomenon

According to research on the 1-in- x phenomenon, people overestimate risks represented with one as the numerator (e.g., 1 in 5) relative to equivalent risks with a larger numerator (e.g., 15 in 75; Pighin et al., 2011; Sirota et al., 2018; Zikmund-Fisher, 2011). It is important to note that the phenomenon in which people overestimate the magnitude of ratios with smaller relative to larger numerator and denominator components is not specific to ratios with a numerator of one. For example, some adults demonstrated a *reverse bias* when they estimated the location of equivalent ratios on number lines (Fitzsimmons et al., 2020b), or when they were more accurate at comparing fractions that were incongruent relative to congruent with the order of natural numbers (e.g., $11/19 > 23/52$, although 11 and 19 are smaller natural numbers than 23 and 52; Obersteiner et al., 2020; Reinhold et al., 2020). Their responses suggested that they used the heuristic that smaller components = larger magnitudes. This heuristic may develop based on their experiences that, in general, ratios with larger denominators have smaller magnitudes. Thus, the 1-in- x phenomenon, in which 1 in 5 is overestimated relative to 15 in 75, is similar to biases for ratios with numerators other than one.

Children and adults are influenced by the size of the independent natural-number components of ratios in complex ways. It is important for future research to investigate the conditions under which people demonstrate the larger components = larger magnitude or the smaller components = larger magnitude bias.

Adaptive Strategy Choice: Is Natural-Number Bias Always Bad?

Is natural-number bias always a bad thing, or in some contexts is the automatic processing of the natural-number components of a ratio a more efficient strategy than computing the ratio's magnitude? According to the adaptive strategy choice account (Alibali & Sidney, 2015; Fazio et al., 2017; Fitzsimmons et al., 2020b; Siegler, 1996), people adaptively choose which strategy to deploy depending on interactions among characteristics of the task at hand, the specific natural numbers involved in the stimuli, recent experiences, and individual differences, such as objective math ability (Alibali & Sidney, 2015; Fazio et al., 2017). Even when people encounter the same problem a second time, they may use a different strategy to solve it (cf. strategy variability: Siegler, 1994). That is, strategy variability can be viewed as advantageous, adaptive, and predictable under some circumstances (Fazio et al., 2017; Fitzsimmons et al., 2020b; Sidney, Thalluri, et al., 2019).

One case in which attending to only the numerators or denominators in isolation may result in efficient strategies is when people are asked to compare two unit fractions in a pure numerical context ($1/2$, $1/3$, $1/4$, $1/5$, $1/6$, $1/7$, $1/8$, and $1/9$) to choose the larger magnitude. In this case, it is more adaptive and efficient to use natural-number strategies (e.g., 8 is less than 9, so based on the smaller components = larger magnitudes heuristic, 8 is a bigger share of the whole) than to precisely calculate the ratio's magnitude (Bonato et al., 2007; Schneider & Siegler, 2010). By relying on the heuristic that larger denominators typically result in unit fractions that have smaller magnitudes relative to the same whole unit, people can ignore the numerators altogether in their unit-fraction comparisons. Based on the adaptive strategy choice account, strategy variability in people's approach to rational numbers is not only meaningful (i.e., not random) and predictable (Fazio et al., 2017; Sidney, Thalluri, et al., 2019) but also provides opportunities to shape people's thinking by subtle manipulation of the problem-solving context (e.g., asking people to compare unit fractions to each other instead of other types of fractions).

It is an open empirical question whether cognitive biases, such as natural-number bias, have the potential to lead to positive health intentions and behaviors. For

example, if the woman who is trying to make sense of the 1-in-8 risk of breast cancer in her lifetime mistakenly thinks about just the denominator and reasons that 8 is a pretty big number, at least relative to 4 (i.e., heart-disease denominator), she might perceive her lifetime risk for breast cancer as greater. Even if her risk estimate is not objectively accurate, her heightened risk estimate, in part resulting from the risk being presented in 1-in- x format, might still prompt her to adhere to recommended health behaviors, such as yearly mammograms after the age of 50 and monthly breast self-exams.

Some health decision-making researchers have suggested that the 1-in- x format be adopted to communicate health risks in a more personally meaningful way, on the premise that one possible reason why small-probability risks might be overestimated is that the format encourages people to think more about themselves or another individual as the "1" in the numerator (Pighin et al., 2011). However, others (Cuite et al., 2008; Waters et al., 2016; Zikmund-Fisher, 2011, 2013a) have warned that the 1-in- x format should be avoided when communicating health risks because this format is likely to result in misunderstanding—typically, overestimation—of personal health risks. The 1-in- x format does persist in health communications, such as the breast-cancer and heart-disease statistics the woman was considering above (see <https://www.cancer.org/cancer/cancer-basics/lifetime-probability-of-developing-or-dying-from-cancer.html>).

Instead, we suggest that percentages might be considered easier to comprehend than the 1-in- x format. Mathematically, percentages involve the use of a common unit—"by the 100," from the definition of percent as per (by) cent (100). This facilitates the process of comparing relative quantities. Although percentages are a relational construct, they perceptually look like whole numbers (Sidney et al., 2021; Thompson et al., 2021) because they convey "multiples" of a common unit (i.e., 100 iterations of the unit $1/100$). This feature of percentages enables individuals to effectively apply their knowledge about natural numbers when processing ratios. In fact, people can just disregard the percentage sign in 75% and think about it like 75 because they are so familiar with the fact that 75 is a big number relative to 100 (Siegler & Opfer, 2003).

There are several lines of work that support communicating health risks as percentages.⁶ Math educators (Moss & Case, 1999) have proposed that percentages may function as a conceptual bridge between more familiar natural numbers and less familiar fractions. In addition, people endorse more positive attitudes toward percentages than fractions (Mielicki, Fitzsimmons, Schiller, et al., 2021; Sidney et al., 2021). Thinking about percentages can engage knowledge about natural-number

magnitudes. Reasoning about percentages between 0 and 100 as natural numbers may cause natural-number bias to exert fewer negative effects, but only in some cases (e.g., when comparing magnitudes but not in multiplicative contexts). For example, it is not mathematically accurate to add percentages in the same way that one adds together natural numbers. That is, 20% off an original price plus another 10% off is not the same as 30% off the original price (see Cuite et al., 2008 for empirical evidence in the health decision-making domain).

In summary, the natural-number bias encompasses other cognitive biases studied in the field of decision-making. According to the adaptive strategy choice approach, natural-number bias is maladaptive in most contexts but can sometimes be adaptive. Future work could explore ways for health communicators to adapt materials to leverage the natural-number bias such as using percentages to communicate about ratio information in some contexts.

Consistency in Measurement and Accounting for Individual Differences

Not only do math-cognition and decision-making researchers use different terminology, they also rely on different validated and reliable measures to assess people's objective and subjective math skills. In the decision-making literature, measures of objective numeracy, such as the Rasch-based numeracy scale (Weller et al., 2013) and the Berlin numeracy scale (Cokely et al., 2012), require people to calculate to solve math problems. As it turns out, many people have more negative attitudes about math involving fractions and percentages than about math involving natural numbers (Sidney et al., 2021) and prefer to avoid doing math. Furthermore, objective numeracy scales are often presented as word problems, which are notoriously difficult to parse (Koedinger & Nathan, 2004) and may result in impulsive calculation (Schiller, 2020). Impulsive calculation (i.e., "plug and chug") occurs when people read a math problem and immediately begin solving it before trying to make sense of the underlying structure of the problem (Chi et al., 1981).

Health decision-making researchers also rely on the subjective numeracy scale (Fagerlin et al., 2007) to measure self-reported math ability without the need for participants to calculate their answers, which avoids the issues mentioned above pertaining to objective measures. The subjective numeracy scale is correlated with objective measures of numeracy but is considered a distinct construct (Peters et al., 2019; Waters et al., 2018). The scale is preferable to objective measures because it is quicker to administer.

Our team has observed that objective and subjective numeracy scales are predictive of health decision-making *precisely because* these scales tap into understanding of ratios: Objective measures are composed of items that assess the ability to calculate with ratios, and the subjective scale entails self-assessments about one's ability to work with fractions and percentages. Given that health statistics inherently involve rational numbers and require people to judge the magnitude of their own or others' risks, it is crucial that measures of math skills incorporate knowledge of ratios.

There are several objective measures of math skills validated in the field of math cognition that could be administered by decision-making researchers. One promising candidate is the number-line estimation task, in which participants are asked to estimate the location of numbers on a number line. Decision-making researchers have used number-line estimation tasks to measure adults' symbolic number mappings (Peters & Bjälkebring, 2015), but they have used a number-line estimation task in the 0 to 1,000 range. By second grade, children make precise estimates in this numerical range (Siegler & Opfer, 2003; Siegler et al., 2009); therefore, this measure may not be sensitive to individual differences in adults' rational-number understanding. Instead, researchers should specifically consider number-line estimation tasks with fractions because rational-number understanding is crucial for interpreting health statistics. In research on numerical cognition, performance on fraction number-line estimation tasks has been shown to be correlated with (Fazio et al., 2014; Siegler et al., 2011; Siegler & Thompson, 2014), and predictive of (Siegler et al., 2012), future overall math achievement. Further, fraction number-line estimation tasks are quick to administer and avoid the shortcomings of word problems (Thompson et al., 2021). Recent evidence suggests that math cognition measures, including fraction number-line estimation tasks, predict health decision-making performance when entered simultaneously into models with objective and subjective measures commonly used in the decision-making literature (Thompson et al., 2021). That is, measures from math cognition are not redundant with measures from decision-making but rather explain unique variance in decision-making performance.

Another important individual difference that may affect people's ability to reason with health statistics is math anxiety, or an apprehension about math that occurs in the mere presence or anticipation of numbers and numerical information. Decision-making researchers should always consider the effects of math anxiety on health decision-making performance that involves participants attempting to interpret numerical health

statistics. Math anxiety can be reliably assessed with one item (see Ashcraft, 2002; Núñez-Peña et al., 2014). Recent research has indicated that adults are more anxious about fractions and very large natural numbers than percentages and small natural numbers (Mielicki, Wilkey, et al., 2021). If higher levels of math anxiety are linked to failure to reason deeply about numerical risk information or avoidance of numbers altogether (Thompson et al., 2021), then this could have implications for health decision-making scenarios, which often involve ratios. Choi et al. (2020) indicated that math anxiety, but not induced stress, was associated with performance on objective and subjective numeracy measures. Crucially, girls and women of different ages are less precise than boys and men when estimating the magnitude of numbers in pure numerical contexts as well as health contexts (Fitzsimmons, Woodbury, et al., in press; Geary et al., 2021; Hutchison et al., 2019; Rivers et al., 2021; Thompson & Opfer, 2008). Girls and women are also less confident in their estimation performance even when controlling for overall estimation skills (Rivers et al., 2021; Thompson & Opfer, 2008). In summary, individual differences, such as preexisting math ability, confidence (Fitzsimmons et al., 2020a; Rivers et al., 2021), math anxiety, and gender should always be taken into consideration to fully understand how people (mis)interpret numerical information as they make decisions about their health.

Educational Interventions to Improve Risk Interpretation

Decision-making researchers have devised ways to help people effectively evaluate health statistics in the moment (e.g., Wolfe & Reyna, 2009). In a common approach, researchers present the numerical health information via visualizations, such as icon arrays and risk ladders (Waters et al., 2021). A primary goal of these decision-making approaches is to help people extract the gist of the numerical information (Reyna, 2008) and better evaluate the presented statistics (Bonner et al., 2021; Trevena et al., 2021). In this way, the visuals remove the burden of computing ratios by presenting statistics as percentages, which can be easily “read off” the visual (e.g., 8 out of 10 rows of an icon array containing 10 ovals per row are shaded in, and the y-axis indicates 80%).

Although these types of approaches can be effective at helping people evaluate numerical health information in isolated instances, they typically do not provide people with the tools to effectively evaluate numerical health information in the absence of visuals or other such cognitive supports (e.g., activating relevant prior knowledge: Sidney et al., 2022; Sidney, Thompson, & Rivera, 2019). In contrast, we consider two inflection

points (see thick gray arrows in Fig. 1, which are expanded in Fig. 2) during which the natural-number bias can be inhibited or bypassed in childhood and adulthood. That is, we can teach children or adults how to understand ratios and subsequently transfer this learning to new contexts via the use of instructional interventions grounded in cognitive-science principles, such as the use of analogies to familiar contexts and worked examples.

To summarize, there is a philosophical difference in how decision-making and math-cognition researchers approach the problem of helping people make more mathematically accurate risk assessments. The health decision-making literature recognizes that clinicians do not have time to teach patients about math. Instead, health decision-making researchers rely on either structuring health communications in ways to (a) prevent activation of numerical biases, such as the natural-number bias, or (b) promote the use of noncognitive processes, such as visual perception, that bypass computations entirely (e.g., an icon array in which a percentage risk can be read off without counting). Teaching people how to do the math is a long-term strategy for addressing problems such as the natural-number bias; it is worth the extra effort because people might then be able to translate what they learned in one situation to future situations rather than having to depend on always encountering a well-designed visual display (cf. Zikmund-Fisher, 2013b).

Before discussing our recent work implementing interventions with adults and addressing educational interventions with children, we describe some research from math cognition that informs natural-number bias interventions.

Magnitudes are the gist of rational numbers

Numerical magnitudes are the building blocks of all of mathematics (Siegler, 2016) and a common feature of all rational numbers (Siegler et al., 2011). This perspective aligns with the fuzzy-trace theory (Brainerd & Gordon, 1994; Reyna, 2008). According to this theory, people can rely on the gist of numerical information (approximate or estimated magnitudes) or verbatim (exact) numerical information when making decisions. There is a developmental shift in relying more on gist with age (Reyna, 2012); “intuitive gist-based processing is considered more advanced in fuzzy trace theory” (Reyna & Brainerd, 2008, p. 97) than relying on verbatim information. In accounting for gist and verbatim recall of numerical information, fuzzy-trace theory is a dual-processing model, and such models have been proposed by math-cognition researchers to explain mechanisms underlying natural-number bias. For instance, when reasoning with

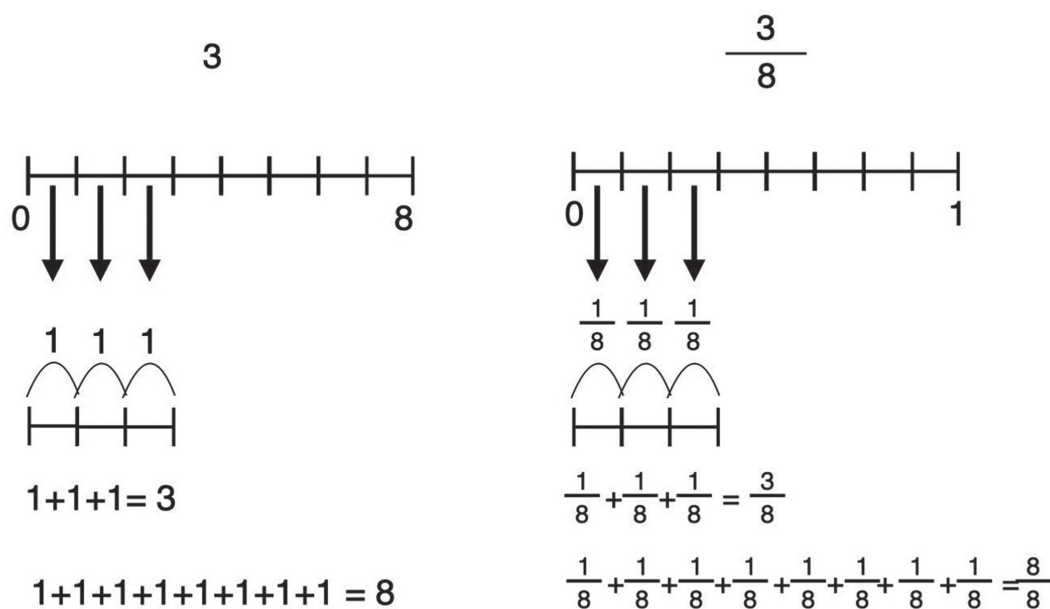


Fig. 4. Bootstrapping whole-number knowledge to improve fraction estimation performance. Iterating the unit of 1 three times shows the location of 3 on the 0 to 8 number line on the left. Iterating the unit of 1/8 three times shows the location of 3/8 on the 0 to 1 number line on the right. In this way, knowledge about natural numbers can be bootstrapped to better understand fractions.

fractions, people must first inhibit their prepotent, automatic whole-number response to later engage in more effortful, strategic reasoning and processing of the ratio's magnitude (Vamvakoussi & Vosniadou, 2004; Van Hoof et al., 2020). The development of rational-number understanding may follow a trajectory like that described by the fuzzy-trace theory. If we consider the gist, or essence, of rational numbers to be their approximate magnitudes, then learners typically move from reasoning with exact information (i.e., the size of the natural-number components of a fraction) to applying more intuitive gist-based reasoning by considering approximate rational-number magnitudes. People may be more likely to rely on the gist, or magnitude, of numbers when they have more robustly activated, or precise, mental representations of the magnitudes of those numbers (Alibali & Sidney, 2015). If we can prompt or teach people to consider the gist of rational-number magnitudes, for instance when estimating the magnitude of a health risk, this may help them avoid the pitfalls of the natural-number bias and optimize decision-making.

Number lines clearly illustrate numerical magnitudes

Teaching about the magnitudes of ratios using number lines—horizontal visual displays in which smaller numbers are located on the left side of the line and larger numbers are located on the right side of the line (see

Fig. 4)—capitalizes on prior natural-number knowledge to build accurate fraction understanding in the middle grades of elementary school (Sidney et al., 2017; Yu et al., 2020, 2022) and in adulthood (Sidney et al., 2022). Number lines are often present in classrooms because they are a critical part of training children how to reason about the magnitude of numbers and compare those magnitudes to one another (see Common Core State Standards Writing Team, 2011; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Number lines have helped people of all ages more accurately interpret the magnitude of numbers relative to one another and to the endpoints of the line (Fitzsimmons, Woodbury, et al., in press; Mielicki, Fitzsimmons, Schiller, et al. 2022; Siegler, 2016; Thompson et al., 2021).⁷ According to the integrated theory of whole number and fractions development (Siegler et al., 2011), number lines are a useful tool to show what whole numbers and fractions have in common: magnitudes whose values can be placed on number lines.

Children then marked the location for 3 on the 0 to 8 number line and were then more accurate when placing 3/8 on the 0 to 1 number line because this fraction is located at the very same point on the number line: 38% of the way across the number line from left to right.

Children can observe how the partitions for 1, 2, and 3 out of 8 units are the same as $1/8$, $2/8$, and $3/8$ out of $8/8$ units (Steffe & Olive, 2009). In Figure 4, we visualize the way in which $3/8$ equals 3 copies of a length of $1/8$ from 0 and how this line segment is analogous to creating 3 equal copies of a length of 1 whole unit from 0.

Number lines also capitalize on spatial-numeric relations to help people easily visualize numerical magnitudes (Dehaene, 2011; Opfer et al., 2010). In cultures with left-to-right orthographies, the mental number line is oriented with smaller numbers located on the left and larger numbers located on the right just like the number-line visual display. Number-line visual displays are thought to be a central conceptual structure (Griffin et al., 1994) to help children and adults map their mental magnitudes onto physical space.

Finally, number lines may make it much easier to identify and relationally reason about the part versus the whole to help people successfully solve class-inclusion problems (Reyna, 1991). The red line that stretches from the left of the number line to the vertical arrow in Figure 5b indicates the magnitude, or the part; the end of the number line represents the whole. Conversely, because the icon arrays commonly used to present health statistics include discrete, compared with continuous, magnitudes, it is possible that these icon arrays may bias people to count and compare subsets of icons (red vs. gray ovals) rather than compare the magnitudes of each subset relative to the whole (red vs. all ovals), as has been found in prior research comparing discrete and continuous formats of ratios (Begolli et al., 2020; Boyer et al., 2008). The extent to which discrete icon array formats encourage counting strategies in health contexts is an empirical question for future research (e.g., Stone et al., 2003).

Interventions to address decision-making biases in adults

Our team has successfully implemented interventions aimed at reducing natural-number bias in adults. One such intervention, used by Thompson and colleagues (2021), is described in more detail in Figure 5a. This intervention featured several components informed by cognitive-science research, including worked examples, analogies to familiar contexts, and using number lines to represent rational numbers.

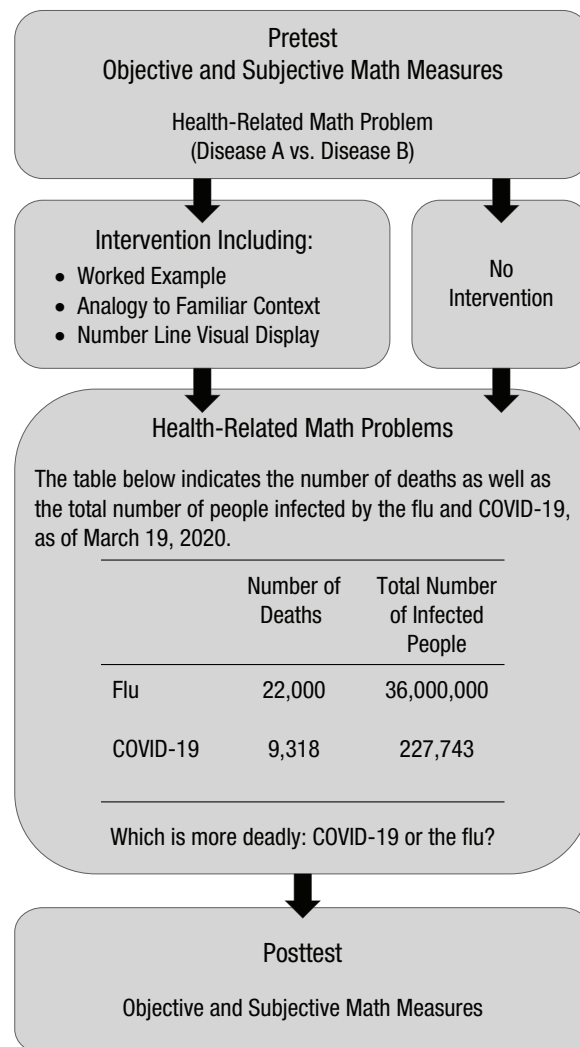
number less than 1% and much closer to 0% than 5%). Procedural and conceptual knowledge develop in tandem with one another (Rittle-Johnson, 2017). We trained participants on *how* to carry out the correct calculation procedures in case some adults lacked the relevant prior knowledge to reason about rates. We also included conceptual training to explain *why* the procedures work the way that they do. Conveying the conceptual information allowed us to highlight the numerical magnitudes of the death-to-infection ratios to help participants override their bias to consider the numerators or denominators in isolation. Of course, it is an open question whether these “light-touch” interventions help adults continue to bypass their cognitive biases as they attempt to interpret health statistics in future contexts. Research inside (Thompson & Siegler, 2010) and outside (Shtulman & Valcarcel, 2012) the domain of mathematics has shown that naive understanding involving biases can be suppressed, but not necessarily eliminated, through instruction.

Helping adults translate among equivalent rational-number types more effectively may also help them to bypass the natural-number bias. Adults may be unaware that there are an infinite number of numbers between any two fractions, and any given fraction can be represented in an infinite number of ways as equivalent fractions, decimals, and percentages (e.g., $1/4$, 0.25, 25%, and 25 of 100 all represent the same magnitude). Alerting adults that the natural-number bias exists may prompt them to slow down and “make sense” of a fraction-related math problem (i.e., rely on relational reasoning). This may encourage people to use more effective strategies to translate rational-number information into an easier-to-interpret format (Alibali & Sidney, 2015; Fazio et al., 2017; Fitzsimmons et al., 2020b; Mielicki, Fitzsimmons, Schiller, et al., 2022; Moss & Case, 1999; Sidney, Thalluri, et al., 2019; Siegler et al., 2011; Siegler & Thompson, 2014; Thompson et al., 2021) rather than impulsively calculating (Schiller, 2020) and overapplying natural-number knowledge to fraction problems. Such interventions would help adults with low numeracy overcome their misunderstanding of rational numbers to engage strategically with rational-number magnitudes (see Alibali & Sidney, 2015; Fitzsimmons et al., 2020b; Schiller et al., 2022; see Fig. 7). This is important because adult caregivers and teachers can pass their correct or incorrect understanding of rational numbers along to children.

Interventions to head off health decision-making biases in childhood

If children learn how to avoid natural-number bias—by being taught about the integrated nature of natural

a



b

Universities Problem

As of early September, University A in the United States had tested 1,154 total students for COVID-19. At University A, 125 students had lab-confirmed cases of COVID-19 or tests that came back “positive.” University B in another part of the United States had tested 22,470 total students for COVID-19. At University B, 249 students had lab confirmed cases of COVID-19 or tests that came back “positive.”

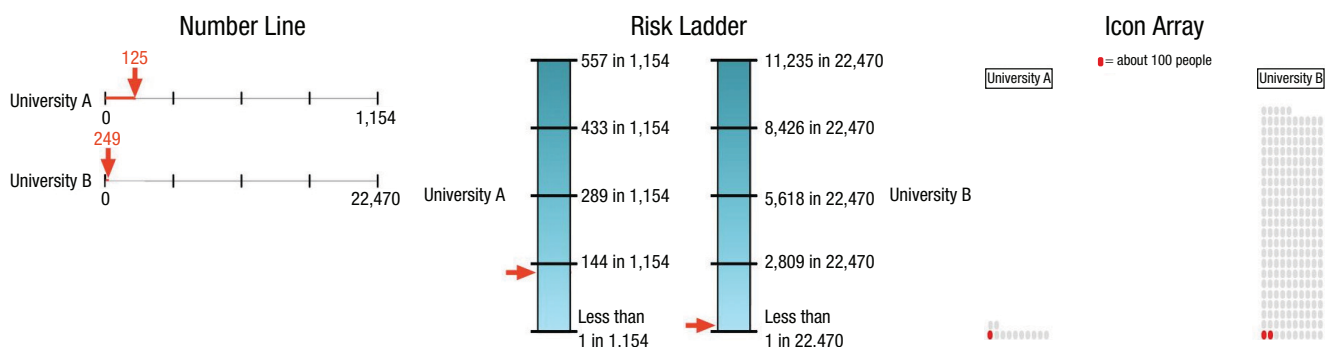


Fig. 5. (continued on next page)

Fig. 5. (a) Survey flow for Thompson et al. (2021). At pretest, a national panel of adults completed objective and subjective math measures and a health-related math problem in which they compared the number of people who died from unnamed Disease A versus Disease B and the total number of people who were infected with Disease A versus Disease B. Half of the participants were then randomly assigned to complete a brief, online educational intervention that included an analogy to a familiar, nonhealth context (i.e., decay rate of fruit in two orchards), a step-by-step worked example to calculate and compare case-fatality rates for COVID-19 and the flu, and number-line visual displays that compared the case-fatality rates for COVID-19 relative to the flu. The other half of the participants did not receive the educational intervention. Both groups completed problems that included 2×2 contingency tables such as the one shown here (see also Wolfe & Reyna, 2009). Those participants who engaged with the educational intervention were more accurate on postintervention health-related math problems pertaining to COVID-19 case-fatality rates than were participants who did not engage with the intervention. Furthermore, in a daily diary follow up across 10 days after the intervention, those participants who engaged with the educational intervention reported higher COVID-19 worry and risk perceptions for themselves and others across portions of the diary relative to participants who did not engage with the intervention. This suggests that the intervention taught participants about the magnitude of the COVID-19 severity relative to the flu. (b) Example visual displays from Mielicki, Fitzsimmons, Schiller, et al. (2022). Number lines, risk ladders, and icon arrays are shown from left to right. A sample of college undergraduates were randomly assigned to solve health-related math problems accompanied by one of three visual displays or no visual display at all. Number lines are not commonly used to convey risk information. However, participants who were randomly assigned to the number-line visual display condition were more accurate when solving COVID-19-related math problems than were participants in the risk-ladder or icon-array visual-display conditions, which did not differ from the no-visual-display-at-all condition. In addition, higher performance on the health-related math problems was associated with higher COVID-19 worry for oneself and others, higher perceptions of COVID-19 severity, and higher endorsement of intentions to engage in preventive health behaviors, even when controlling for baseline math skills. The number lines emphasized the size of the components of the ratio and allowed people to visualize the proportion of positive COVID-19 cases in University A versus University B and directly compare these proportions—as a distance from the left endpoint of the number line—to one another to decide which university had the highest positivity rate. Future research should also assess whether number lines help people make mathematically accurate health decisions in real-world, personalized risk contexts.

numbers, fractions, ratios, decimals, and percentages early in their formal math curriculum—then, according to our developmental framework (see Fig. 2), they should be more likely to develop into adults who do not easily fall prey to numerical biases, thus ameliorating innumeracy in adulthood. Thus, to address natural-number bias at this inflection point, math curricula can be restructured such that more productive connections are made between natural-number knowledge and knowledge about rational numbers (e.g., Yu et al., 2020, 2022). Evidence from an international assessment of students' math understanding (OECD, 2018) consistently indicates that students in the United States are below average in their mathematical literacy. These deficits in mathematical literacy have the potential for far-reaching implications for adults in daily life. There are also differences in teachers' pedagogical content knowledge pertaining to fractions in the United States relative to other countries with higher math achievement (e.g., Ma, 1999), and these differences may contribute to U.S. students' struggles with fractions. Addressing natural-number bias during the childhood inflection point would be beneficial for young learners and could obviate the need to intervene with adults.

Implementing effective interventions at the childhood inflection point requires drawing on research findings about how rational-number understanding progresses across development.⁸ Even 6-month-old infants have a rudimentary understanding of ratios (McCrink & Wynn, 2007), and by preschool, children can reason about continuous magnitudes in match-to-sample scaling tasks (i.e., a stuffed animal likes only drinks that have a certain juice-to-water ratio, and

children have to identify a scaled-down match for this ratio; Boyer et al., 2008). Building on this early emerging understanding of nonsymbolic ratios, fractions are introduced in the first grade in the context of partitioning shapes (see the Common Core State Standards in Mathematics (CCSSM) Geometry standard: 1.G.A.3). However, fractions are not introduced in their symbolic form (i.e., n/m) until the third grade and are not defined in terms of division of relative quantities (i.e., $n/m = n$ divided by m ; CCSSM 5.NF.B.3. The standard is shown at this website: <http://www.corestandards.org/Math/Content/5/NF/B/3/>) until the fifth grade. Interestingly, despite this protracted instructional sequence for rational numbers, sixth and eighth graders with years of formal fraction instruction still make notable errors as they estimate the location of fractions on number lines (Siegler et al., 2011). Even college students make predictable mistakes as they reason about rational-number magnitudes (Fazio et al., 2017; Schneider & Siegler, 2010; Sidney, Thalluri, et al., 2019).

Addressing the natural-number bias in math education requires a different solution than is currently used in U.S. math curricula. Note that the sequence in which the Common Core State Standards in Mathematics (CCSSM) was initially ordered was not informed by rigorous, empirical research in all cases (Common Core Standards Writing Team, 2011; cf. Confrey and colleagues' (2014) concerns pertaining to mathematical progressions). In addition, sometimes the standards were underspecified, and educators received little guidance on how to implement the standards in classrooms (Nurnberger-Haag & Thompson, 2022; Sidney, Thompson, & Rivera, 2019).

a

	Number of deaths	Total number of people infected
Flu	22,000	36,000,000
COVID-19	9,318	227,743

In order to determine which disease is the most fatal, you must consider the number of deaths **RELATIVE** to the total number of people who were infected.

Here's a step-by-step tutorial on how you can figure out the fatality rate for each disease.

First, find the Flu fatality RATE.

Divide the number of **Flu** deaths (22,000) by the total number of people infected by the **Flu** (36,000,000). If you plug these numbers into a calculator, you will see that $22,000 / 36,000,000 = 0.00061$.

Transform this into a percentage by multiplying by 100: $0.00061 * 100 = .06\%$. **This is the fatality rate** for the **Flu**. That's a pretty small number that is less than 1% and closer to 0% than to 5%.

	Number of deaths	Total number of people infected	Fatality Rate
Flu	22,000	36,000,000	$22,000 / 36,000,000 = 0.00061 = 0.06\%$
COVID-19	9,318	227,743	

b

Finally, compare the Flu's fatality rate (0.06%) to COVID-19's fatality rate (4.1%).

You can see that the fatality rate is higher for **COVID-19**. In fact, as of March 19th, 2020 our best estimate is that it was 41 times more fatal.

One important thing to remember is that if you want to figure out a fatality rate, you have to take into consideration both the number of deaths as well as the total number of people infected for each disease.

Here is a graph to help you visualize the **COVID-19** fatality rate (red in bottom bar) **RELATIVE** to the **flu** fatality rate (blue in top bar).

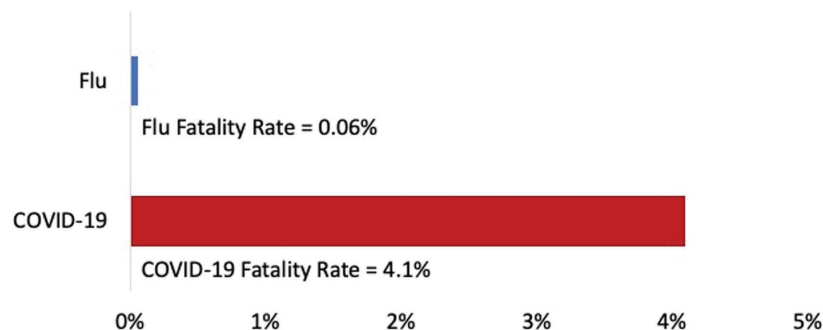


Fig. 6. Excerpt from the worked example used in Thompson et al. (2021). Procedural knowledge about how to calculate and compare case-fatality rates was conveyed in a step-by-step worked example (a) by Thompson and colleagues (2021). Conceptual knowledge was conveyed by describing how the magnitude of the case-fatality rate for the flu was less than 1% and much smaller than 5%. Thompson and colleagues' (2021) worked example also included number lines (b) to help people easily compare case-fatality rates relative to one another.

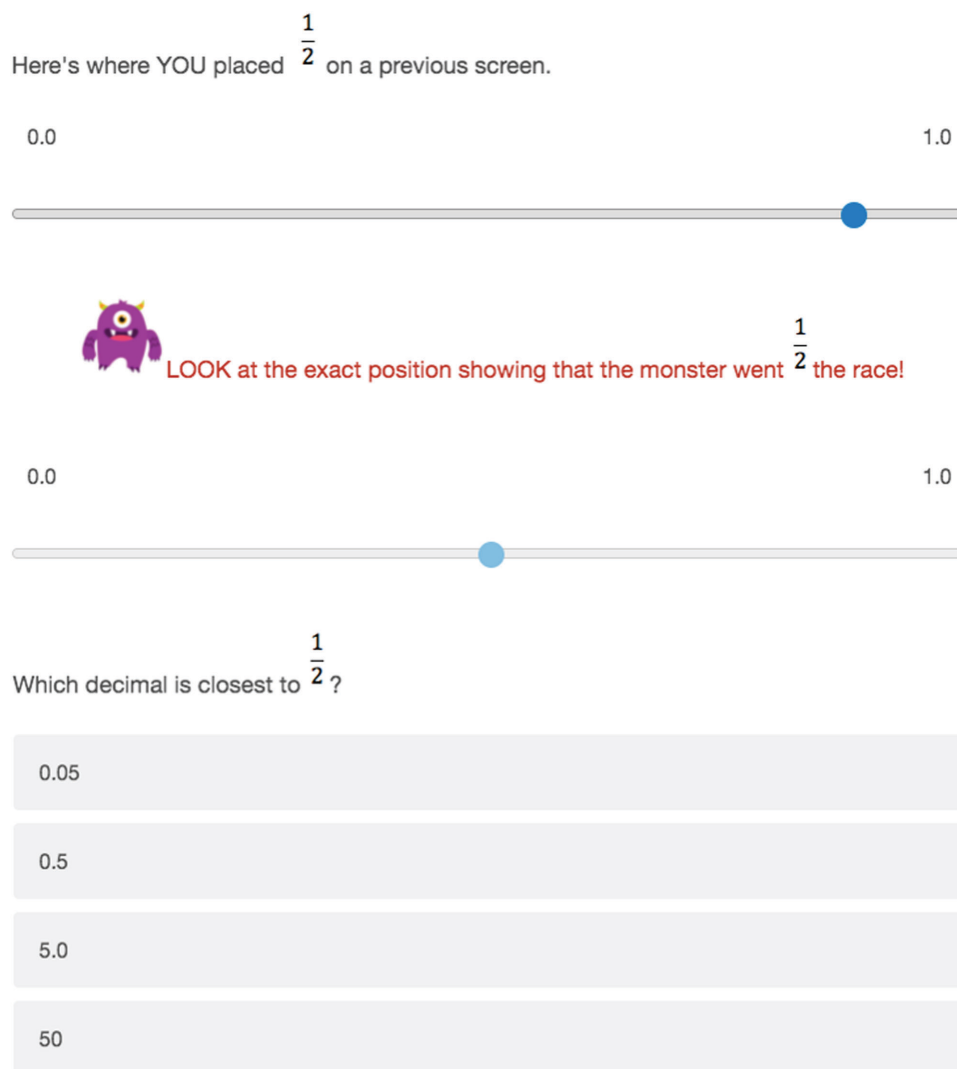


Fig. 7. Translating among fractions, decimals, and percentages can help people accurately interpret the magnitude of rational numbers. College students, who engaged with a playful educational intervention about monsters racing along a number line, demonstrated improvements in their ability to translate among fractions, percentages, and decimals (Schiller et al., 2022). Translation skills are helpful to adults as they attempt to reason about the magnitude of health risks or statistics in difficult-to-handle formats, such as the 1-in- x format.

An unintended consequence of delaying the introduction of fractions as relational concepts and emphasizing natural-number operations in early instruction is the tendency for students to think about quantities in absolute terms instead of relative to one another. One proposed solution is to emphasize ratios as relational concepts much earlier in instruction (see Ni & Zhou, 2005). To help learners make the connection between natural numbers and fractions, teachers could emphasize the multiplicative, or measurement (Steffe & Olive, 2009), aspect of fractions (see Fig. 4). Another promising solution is scaffolding learners' understanding of

rational numbers by helping them make connections across percentages, decimals, and fractions (Moss & Case, 1999; Schiller, 2020; Schiller et al., 2022; see Fig. 5) rather than introducing these concepts separately across several grade levels. For example, students are not expected to translate a fraction into a percentage until the sixth grade, and this translation is only expected for fractions with 100 as the denominator (CCSSI, 2010). More research is needed to assess whether introducing all rational-number notations together in earlier grades would improve rational-number understanding.

Conclusions

Our novel developmental framework provides an approach for thinking about the underlying mechanisms of innumeracy in adulthood. Innumeracy is not just a problem for some adults. People of various ages and expertise levels experience numerical biases, which likely stem from perceptual constraints, statistical regularities in the environment, and missed opportunities for drawing connections across natural-number and ratio instruction in classrooms.

Further, our novel developmental framework takes into consideration that people know and use various strategies as they attempt to solve rational-number problems. Some of these strategies lead to correct responses, whereas others do not. We have devised interventions that prompt people to stop and consider other strategies beyond the prepotent natural-number bias response in which they focus on numerators or denominators in isolation rather than on how numerators relative to denominators express the magnitude of a ratio. Using number lines in our educational interventions makes the underlying magnitudes of the ratios more transparent (i.e., as a percentage of the total number line from left to right) to bypass the natural-number bias. Number lines capitalize on what is similar across natural numbers and rational numbers to integrate numerical understanding across these number types.

An interdisciplinary approach to identifying and intervening on the precursors that contribute to the natural-number bias can have unprecedented influence on decision-making outcomes in health, finance, and any other context involving rational numbers. Unbeknownst to researchers across diverse fields of inquiry, they may be attempting to answer similar research questions through the lenses of their own disciplines. However, collaboration can result in scientific breakthroughs with broader impact. In this call to action, we see significant merit in fostering collaborations across psychology, math education, health decision-making, and health communication. Bringing together relevant expertise from these disparate domains can provide novel insights into adults' misconceptions pertaining to numerical health information.

Transparency

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Notes

1. On the one hand, some research has shown that socioeconomic status and low numeracy are linked (Ramani & Siegler, 2009). Low-income children have different numerical experiences before formal schooling than do their middle-income age mates, and numerical board-game interventions have been shown to close the gap in numerical magnitude understanding between these two groups of children (Ramani & Siegler, 2008, 2009, 2008; Siegler & Ramani, 2008). However, the same researchers have reached different conclusions when analyzing other nationally representative data sets. That is, a host of demographic factors, including elementary-aged children's general intellectual ability and parental income and education level, did *not* predict algebra performance 5 or 6 years later (Siegler et al., 2012). The causes and effects of low numeracy are important topics for continued research.
2. A detailed overview of the Weber-Fechner law, approximate-magnitude system, approximate-number system, ratio-processing system, and perceptual constraints more generally are beyond the scope of this article. However, for overviews, see Clarke and Beck (2021), Feigenson et al. (2004), Leibovich et al. (2017), Matthews et al. (2015), and Newcombe et al. (2015).
3. Some researchers have recently proposed that the approximate-number system processes both natural numbers and rational numbers, subsuming the ratio-processing system (Clarke & Beck, 2021).
4. Half is likely a special case that is a very familiar fraction in its own right. This fraction occurs often in math textbooks (Braithwaite & Siegler, 2018a), children are asked to partition shapes into halves in the first-grade geometry strand of the Common Core State Standards in Mathematics, and half is a benchmark fraction that children in the fourth grade are encouraged to compare to other fractions to assess their magnitudes (CCSSI, 2010; Siegler & Thompson, 2014; Siegler et al., 2011).
5. Some researchers would argue that to reason about similarity one would need to notice what is distinctive, or different (Hunt, 2013). In this way, people may be considering alignable differences across stimuli as they compare them (Markman & Gentner, 2000). More research is needed to assess the "structure-seeking" tendencies (Sidney & Thompson, 2019; cf. Wall et al., 2015) of people as they reason about rational numbers in pure numerical and applied contexts.
6. According to some decision-making researchers (Gigerenzer & Hoffrage, 1995), natural-number frequencies (e.g., 12 in 100) are preferred over percentages because percentages can lead to inaccurate calculations in some contexts. For an alternative view, see Sloman et al. (2003).

7. The Common Core State Standards Writing Team (2011) created some draft resources to help teachers use number lines as they represent ratio relationships in their lessons (see https://commoncoretools.files.wordpress.com/2012/02/ccss_progression_rp_67_2011_11_12_corrected.pdf).

8. It is beyond the scope of this article to provide a comprehensive overview of this literature; however, interested readers can consult Campbell (2005) and Sidney, Thompson, and Opfer (2019).

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