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Pricing and remunerating electricity storage flexibility using virtual links

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ABSTRACT

Ambitious renewable portfolio standards motivate the mass deployment of energy storage resources (ESR) as sources of flexibility. As such, the design of electricity markets that properly remunerate the provision of flexibility services by ESRs is under active development. In this paper, we propose a new market clearing framework that incorporates ESR systems. Compared to existing market designs, our approach models ESR systems using the concept of virtual links (VLs), which capture the transfer/shift of power across time. The VL representation reveals economic incentives available for ESR operations and sheds light into how electricity markets should remunerate ESRs. Our framework also allows us to explore the role of ESR physical parameters on market behavior; specifically, we show that, while energy and power capacity defines the amount of flexibility each ESR can provide, charge/discharge efficiencies play a fundamental role in ESR remuneration and in the ability of the power grid to mitigate market price volatility. The new market design is also computationally attractive in that it is a linear program and thus avoids mixed-integer formulations and formulations with complementarity constraints (used in current designs to capture binary charge/discharge logic). We use our market framework to analyze the interplay between ESRs and market operators and to provide insights into optimal deployment strategies for ESRs in power grids.

1. Introduction

The power grid in the United States is undergoing significant changes driven by increasing adoption of renewable energy. Multiple states such as California, Colorado, and Virginia have set ambitious renewable portfolio standards of 100% by 2050 or earlier (Anon, 2022). Shioshansi et al. and Lund have shown that electricity storage resources (ESR) offer significant flexibility potential (Sioshansi et al., 2009; Lund, 2020) that can facilitate the adoption of renewables; however, current electricity markets managed by independent system operators (ISOs) are not explicitly designed to enable participation of ESRs. In 2018, the Federal Energy Regulatory Commission (FERC) released Order 841 that aimed to remove barriers limiting ESR participation in wholesale electricity markets (Commission, 2018); a detailed analysis of FERC Order 841 provides more background on the order (Smith, 2019). Since this FERC order was issued, ISOs have proposed various participation models and market rules for ESRs in different energy markets (Chandra, 2020).

Incorporating ESRs into market clearing procedures has been the subject of recent interest in the power systems literature. This interest is driven by the potential for large-scale ESR incorporation into energy infrastructures in the near future, and the need to understand how this will influence operational feasibility and electricity market prices. Parvar et al. presented a modeling framework for coordinated (centralized) markets where ESRs act as price takers; this framework forms the basis for CAISO electricity market regulations (Parvar et al., 2019). Chen and Jing proposed a market clearing formulation that captures ESRs with flexible terminal charge conditions that enables higher flexibility potential (Chen and Jing, 2020). Taylor (2014) and Muñoz-Álvarez and Bitar (2017) introduced the concept of financial storage rights (FSR) in coordinated market settings, with the former using the dual prices of ESR physical constraints and the latter locational marginal prices (LMPs). The concept of physical storage rights (PSRs) has also been proposed as an alternative market product for ESR systems where, instead of being managed by the market operator, ESRs sell their energy, power capacities, and dispatch actions to other market participants (He et al., 2011; Brijs et al., 2016; Thomas et al., 2020). Lüth et al. explore the incorporation of ESR systems in smart grids operated on a peer-to-peer basis (Lüth et al., 2018).

One major computational challenge associated with incorporating ESR models in optimization models (e.g. energy clearing models) is the need to capture charge/discharge logic. Specifically, this logic involves binary logic that prevents simultaneous charging and discharging (which is economically inefficient and thus do not make practical sense). Existing market clearing models incorporate this logic by using complementarity constraints or mixed-integer formulations, which are

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computationally challenging to handle (Choi et al., 2017; Parisio et al., 2014). To bypass this problem, one usually has to restrict the ESR to be ideal (no loss in charging and discharging) (Dall'Anese et al., 2016), or apply relaxations. For instance, Garifi et al. propose a convex relaxation that removes the complementarity constraints and apply a penalty term, which enforces suboptimality of solutions that violate the complementarity constraints (Garifi et al., 2020). However, the value of the penalty parameter is dependent on the problem parameters, which can be challenging to determine in market settings where the bidding parameters are always subject to change by participants.

A significant body of research explores bi-level programs describing the price-making behavior of ESR systems. Pandžić et al. use a bi-level programming model where the ESR and ISO each have strategies to achieve their objectives, with the ESR (upper level) selecting a storage strategy, subject to the (lower lever) market clearing problem in which the ESR strategy is treated as a fixed decision (Pandžić and Kuzle, 2015). Their results demonstrate that ESR systems stand to profit the most when acting as price takers, selecting locations and bidding strategies that do not incur large changes in market prices. Hartwig et al. apply a similar bi-level framework to study the effect of ESR ownership arrangements (Hartwig and Kockar, 2015). Mohsenian-Rad proposes a bi-level programming framework for large-scale geo-distributed ESRs operating under coordination (Mohsenian-Rad, 2015). Huang et al. conduct a comparative analysis of market mechanisms for ESR systems operating under different levels of coordination (Huang et al., 2017), showing that coordinated ESRSs do benefit from economic bidding when their operator faces electricity market uncertainty.

In this work, we propose a new energy market design that aims to incentivize flexibility provided by ESR systems. The market design applies a robust bound on ESR operations that bypasses the need to capture charge/discharge logic (our formulation is a linear program that indirectly enforces this logic). We discuss conditions under which charge/discharge logic is satisfied in traditional market formulations for ESRs; moreover we demonstrate that our new market formulation ensures that the logic is satisfied but at the expense of reducing operational space for ESRs. Our framework is scalable in that it enables the incorporation of a large number of ESRs in market clearing procedures (which would be challenging to do with mixed-integer formulations or formulations with complementarity constraints). In addition, a key feature of our market design is that decomposes ESR charging and discharging operations into temporal energy transfer and net-charging/discharging components. The charging operation is treated as a paid service instead of utility gained by the ESR (as is done in load modeling). Energy transfer is captured using *virtual links* (VLs), which are pathways that capture transfer/shifting of power across time. Our framework reveals that VLs exploit *temporal price differences* (volatility) by storing and discharging power at strategic times in order to maximize profit. We also show that our framework ensures that ESRs are remunerated consistently as a part of the market clearing process. We also use our framework to explore the effect of different physical parameters on the flexibility provision of each ESR and on market outcomes. Specifically, we show that ESR roundtrip efficiencies play a fundamental role in the ability of the ISO to mitigate market volatility. Using numerical experiments, we explore optimal investment and operational strategies for ESRs and their interplay with ISO goals.

The paper is structured as follows. In Section 2 we provide the operational ESR model that is incorporated into the market clearing formulation. In Section 3 we provide the basic market clearing model, and show how ESRs can be incorporated into this model with different features, building complexity with successive formulations. We present illustrative numerical experiments in Section 4, and provide analysis to interpret the results. Section 5 concludes the paper.

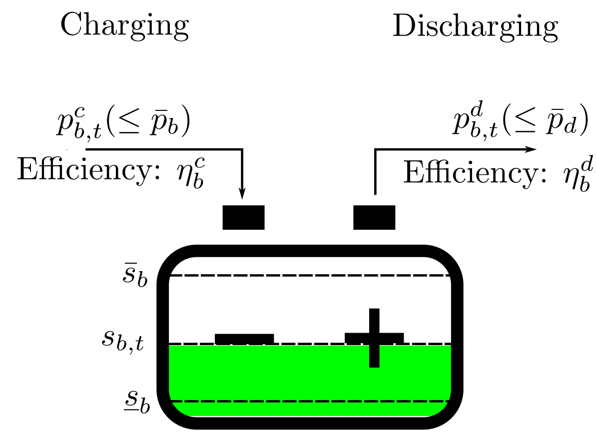


Fig. 1. Schematic of the ESR operational model at a given time point t.

2. ESR operational model

We first lay out the operational ESR system model that we use in this paper. The operational model is based on work by Parvar et al. (2019) and captures key constraints relevant to the modeling of ESRs in market settings. Fig. 1 summarizes our model notation. We also refer readers to Section B in the Appendix for a complete nomenclature.

The ESR operational model keeps track of the charge states and operations of an ESR system indexed $b \in \mathcal{B}$ over a discretized time horizon $\mathcal{T} := \{1, 2, \dots, T\}$; the model aims to ensure that it obeys physical constraints including charge/discharge status, capacity limits, and terminal charge requirements. Specifically, we define $s_{b,t} \in \mathbb{R}$ to be the state-of-charge (SOC) at the end of time interval $t \in \mathcal{T}$, $p^c_{b,t} \in \mathbb{R}_+$ be the charging power of ESR b during time interval $t \in \mathcal{T}$, and $p^d_{b,t} \in \mathbb{R}_+$ be the discharging power of ESR b during time interval $t \in \mathcal{T}$. Each ESR system $b \in \mathcal{B}$ is characterized by a set of key physical parameters.

- $\eta^c_b \in [0, 1]$: charging efficiency
- $\eta^d_b \in [0, 1]$: discharging efficiency
- $\underline{s}_b \in \mathbb{R}_+$: minimum SOC
- $\bar{s}_b \in \mathbb{R}_+$: maximum SOC
- $s_{b,0} \in [\underline{s}_b, \bar{s}_b]$: SOC at time 0
- $\bar{p}_b \in \mathbb{R}_+$: power capacity for charging/discharging

The storage model is formulated by the following set of constraints:

$$s_{b,t} = s_{b,t-1} + \eta^c_b p^c_{b,t} - \frac{1}{\eta^d_b} p^d_{b,t}, \ (b,t) \in \mathcal{B} \times \mathcal{T} \tag{2.1a}$$

$$\underline{s}_b \leq s_{b,t} \leq \bar{s}_b, \ (b,t) \in \mathcal{B} \times \mathcal{T} \tag{2.1b}$$

$$p^c_{b,t} + p^d_{b,t} \leq \bar{p}_b, \ (b,t) \in \mathcal{B} \times \mathcal{T} \tag{2.1c}$$

$$0 \leq p^c_{b,t} \perp p^d_{b,t} \geq 0, \ (b,t) \in \mathcal{B} \times \mathcal{T} \tag{2.1d}$$

$$s_{b,T} \geq s_{b,0}, \ b \in \mathcal{B} \tag{2.1e}$$

Eq. (2.1a) captures dynamics of SOC over time and associated energy losses. We implicitly assume that the charging and discharging time is 1 h (so the time increment Δt is not included in the model). Constraint (2.1b) captures the SOC capacity constraints. Constraint (2.1c) captures power capacity constraints. Constraint (2.1d) is the complementarity constraint that captures charging/discharging logic (at each time interval, an ESR must be either charging or discharging, but not both). We note that, while physically possible to charge and discharge an ESR simultaneously, the complementarity constraint is enforced in ESR modeling for economic and efficiency reasons. Specifically, it does not make economic sense to charge and discharge simultaneously (due to energy losses). Constraint (2.1e) captures the terminal condition for SOC, which is not physically necessary but rather a market operation requirement.

One can capture the charging/discharging complementarity by using binary variables or directly as complementarity constraints, which will make the model mixed-integer or nonconvex in nature. We thus see that the presence of complementarity makes the storage operational model computationally challenging in nature, which limits the ability to capture large numbers of ESRs in market clearing models.

Eq. (2.1a) can be written in the following equivalent form:

$$s_{b,t} = s_{b,0} + \eta^c_b \sum_{t'=1}^{t} p^c_{b,t'} - \frac{1}{\eta^d_b} \sum_{t'=1}^{t} p^d_{b,t'} \tag{2.2}$$

This allows us to write the storage model (2.1) purely in terms of p^c and p^d:

$$\Delta \underline{s}_{b,t} \le \eta^c_b \sum_{t'=1}^{t} p^c_{b,t'} - \frac{1}{\eta^d_b} \sum_{t'=1}^{t} p^d_{b,t'} \le \Delta \bar{s}_b,\ \ t \in \mathcal{T} \tag{2.3a}$$

$$p^c_{b,t} + p^d_{b,t} \le \bar{p}_b,\ \ t \in \mathcal{T} \tag{2.3b}$$

$$0 \le p^c_{b,t} \perp p^d_{b,t} \ge 0,\ \ t \in \mathcal{T} \tag{2.3c}$$

where $\Delta \underline{s}_{b,t} := \underline{s}_b - s_{b,0}$ for $t < T$, $\Delta \underline{s}_{b,T} := 0$, and $\Delta \bar{s}_b := \bar{s}_b - s_{b,0}$. Inequality (2.1e) is merged into the left-hand side of (2.3a) via the definition of $\Delta \underline{s}_{b,T}$. Note that from a computational perspective, the aggregated formulation (2.3) has a dense Jacobian and thus might slow down linear solvers. However, as we will show later, this formulation allows us to reveal how energy capacity becomes a key resource that defines the amount of flexibility an ESR can provide. In other words, the aggregated formulation is introduced simply for theoretical analysis; if one desires to avoid dense Jacobians, one can simply write down the dynamic constraints explicitly.

3. Market formulation with storage

The market clearing formulations that we propose apply the concept of VLs to model ESR systems. Virtual links were first proposed to capture load-shifting flexibility from data centers (Zhang et al., 2020; Zhang and Zavala, 2021) and are not standard in the power systems literature; as such, we introduce a family of formulations with increasing complexity.

3.1. Notation and terminology

We begin our discussion by introducing basic notation for a space–time network without considering ESR systems. The detailed setup and derivation for the space–time clearing formulation is discussed by Zhang and Zavala (2021). Here, we provide a brief review of the setup. The market considers, over time horizon $\mathcal{T} := \{1, 2, \dots, T\}$, a set of suppliers (owners of power plants) $\mathcal{S}$ and consumers (owners of loads) $\mathcal{D}$. Each participant is connected to a transmission network comprised of geographical nodes $\mathcal{N}$ and transmission lines $\mathcal{L}$ (owned by transmission service providers).

Each supplier $i \in \mathcal{S}$ is connected to node $n(i) \in \mathcal{N}$ in the network. The supplier bids into the market by offering power at bid price (often simply referred to as "bid") $\alpha^p_{i,t} \in \mathbb{R}_+$ and offers available capacity $\bar{p}_{i,t} \in [0, \infty)$ for each $t \in \mathcal{T}$. We define $\mathcal{S}_n := \{i \in \mathcal{S} \mid n(i) = n\} \subseteq \mathcal{S}$ (set of suppliers connected to node n). The cleared allocation for supplier $i \in \mathcal{S}$ (load injected) are denoted as $p_{i,t}$ and must satisfy $p_{i,t} \in [0, \bar{p}_{i,t}]$. We use p to denote the collection of all cleared allocations.

Each consumer $j \in \mathcal{D}$ is connected to node $n(j) \in \mathcal{N}$. The consumer bids into the market by requesting power at bid price $\alpha^d_{j,t} \in \mathbb{R}_+$ and requests a maximum capacity $\bar{d}_{j,t} \in [0, \infty)$ for each $t \in \mathcal{T}$. We define $\mathcal{D}_n := \{j \in \mathcal{D} \mid n(j) = n\} \subseteq \mathcal{D}$ (set of consumers connected to node n). For simplicity, we assume that there is only one consumer at a given node ($\mathcal{D}_n$ are singletons). The cleared allocation for consumer $j \in \mathcal{D}$ (load withdrawn) is denoted as $d_{j,t}$ and must satisfy $d_{j,t} \in [0, \bar{d}_{j,t}]$. We use d to denote the collection of all cleared allocations.

The transmission owner owns the whole network, where each (undirected) line $l \in \mathcal{L}$ is defined by its sending node $\mathrm{snd}(l) \in \mathcal{N}$ and receiving node $\mathrm{rec}(l) \in \mathcal{N}$. For the purpose of linearizing the market clearing problem (the details of which are provided by Zhang and Zavala (2021)), we decompose each line l into a couple of directed edges: $l^+ := (\mathrm{snd}(l), \mathrm{rec}(l))$ and $l^- := (\mathrm{rec}(l), \mathrm{snd}(l))$. The set of all directed edges from such decomposition is $\mathcal{K} := \cup_{l \in \mathcal{L}} \{l^+, l^-\}$. For each node $n \in \mathcal{N}$, we define its set of receiving lines $\mathcal{K}^{\mathrm{rec}}_n := \{k \in \mathcal{K} \mid n = \mathrm{rec}(k)\} \subseteq \mathcal{K}$ and its set of sending lines $\mathcal{K}^{\mathrm{snd}}_n := \{k \in \mathcal{K} \mid n = \mathrm{snd}(k)\} \subseteq \mathcal{K}$. Each line offers a bid price $\alpha^f_{k,t} \in \mathcal{R}_+$ and capacity $\bar{f}_{k,t} \in [0, \infty)$. Each cleared flow $f_{k,t}$ must satisfy the bounds $f_{k,t} \in [-\bar{f}_{k,t}, \bar{f}_{k,t}]$ and the collection f must obey the direct-current (DC) power flow Eqs. (3.4)

$$f_{l^+,t} - f_{l^-,t} = B_l(\theta_{\mathrm{snd}(l),t} - \theta_{\mathrm{rec}(l),t}), \tag{3.4}$$

where $B_l \in \mathbb{R}_+$ is the line susceptance and $\theta_n \in \mathbb{R}$ is the phase angle at node $n \in \mathcal{N}$. The DC power flow model is a linear model and requires small phase angle differences across transmission lines $\theta_{\mathrm{snd}(l),t} - \theta_{\mathrm{rec}(l),t} \in [-\Delta\bar{\theta}'_{l,t}, \Delta\bar{\theta}'_{l,t}]$. The limits on phase angle differences and the capacity constraints for flows are captured by Eq. (3.5)

$$-\Delta\bar{\theta}_{l,t} \le \theta_{\mathrm{snd}(l),t} - \theta_{\mathrm{rec}(l),t} \le \Delta\bar{\theta}_{l,t} \tag{3.5}$$

where $\Delta\bar{\theta}_{l,t} := \min\{\bar{f}_{l,t} B^{-1}_{l,t}, \Delta\bar{\theta}'_{l,t}\}$.

We use $\pi_{n,t} \in \mathbb{R}_+$ to represent the clearing price at the space–time node $(n, t) \in \mathcal{N} \times \mathcal{T}$. The collection of clearing prices is denoted as π; these are also known as nodal prices or locational marginal prices (LMPs) and are used to remunerate market stakeholders. We observe that in a typical market, suppliers and transmission owners *offer a service* to the grid, while consumers *request a service* from the grid. Making this distinction is important because we will see later that there are multiple ways to capture ESRs in a market clearing framework, which differ in whether ESR is treated as a pure service provider or as a *prosumer* (a simultaneous service provider and consumer). This standard market clearing process is illustrated in Fig. 2.

3.2. Base electricity market formulation

We begin our discussion with a space–time market clearing formulation with no ESR presence formulated in Eq. (3.6)

$$\min_{d,p,f,\theta} \quad \sum_{t \in \mathcal{T}} \Big(\sum_{i \in \mathcal{S}} \alpha^p_{i,t} p_{i,t} + \sum_{k \in \mathcal{K}} \alpha^f_{k,t} f_{k,t} - \sum_{j \in D} \alpha^d_{j,t} d_{j,t} \Big) \tag{3.6a}$$

$$\text{s.t.} \quad \sum_{k \in \mathcal{K}^{\mathrm{rec}}_n} f_{k,t} + \sum_{i \in \mathcal{S}_n} p_{i,t} = \sum_{k \in \mathcal{K}^{\mathrm{snd}}_n} f_{k,t} + \sum_{j \in D_n} d_{j,t}, \quad (\pi_{n,t}) \quad n \in \mathcal{N}, t \in \mathcal{T} \tag{3.6b}$$

$$f_{l^+,t} - f_{l^-,t} = B_l(\theta_{\mathrm{snd}(l),t} - \theta_{\mathrm{rec}(l),t}), \quad l \in \mathcal{L}, t \in \mathcal{T} \tag{3.6c}$$

$$(d, p, \theta) \in C \tag{3.6d}$$

where $C := C^d \times C^p \times C^\theta$ captures the capacity constraints for all variables:

$$C_d := \{d \mid d_{j,t} \in [0, \bar{d}_{j,t}]\ \forall\ j \in \mathcal{D}, t \in \mathcal{T}\} \tag{3.7a}$$

$$C_p := \{p \mid p_{i,t} \in [0, \bar{p}_{i,t}]\ \forall\ i \in \mathcal{S}, t \in \mathcal{T}\} \tag{3.7b}$$

$$C_\theta := \{\theta \mid \theta_{\mathrm{rec}(k),t} - \theta_{\mathrm{snd}(k),t} \in [-\Delta\bar{\theta}_{k,t}, \Delta\bar{\theta}_{k,t}]\ \forall\ k \in \mathcal{K}, t \in \mathcal{T}\} \tag{3.7c}$$

The objective (3.6a) is the negative *social surplus* or *total welfare*, which captures the value of the demand served (to be maximized) and the cost of total supply and transmission services (to be minimized). The transmission cost is typically not included in the market clearing literature; this cost is included here to highlight an important analogy between transmission costs and power-shifting costs of ESR systems (to be discussed later). Constraint (3.6b) is the power balance constraint at each node n (Kirchhoff's current law). Constraint (3.6c) is the DC power flow equation for each line l.

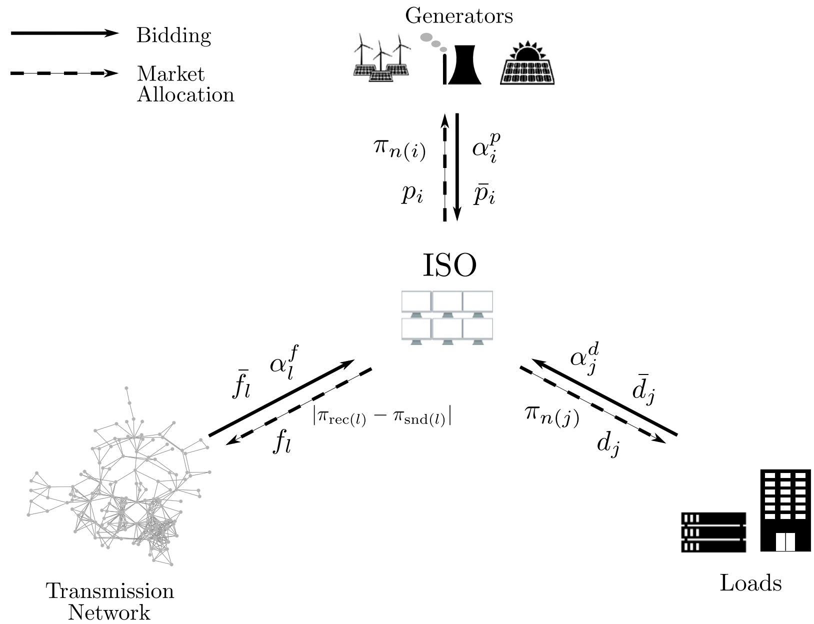


Fig. 2. Sketch of base (standard) market clearing framework with no ESRs.

The solution of (3.6) gives the *primal* allocations (p, d, f) and the *dual* allocations (prices) π. The dual allocations are the optimal solutions of the dual variables associated with the power balance constraints (3.6b). These dual allocations are electricity LMPs that clear the market. This market clearing process is illustrated in Fig. 2. Most often, prices will vary across locations and times (thus giving rise to space–time volatility).

We use (p, d, f, π) to denote the primal–dual allocation obtained from the solution of the clearing formulation. Under this market clearing mechanism, each supplier i is cleared at the price $\pi_{n(i),t}$ for each unit of power provided at time t, and each consumer j pays at the price $\pi_{n(j),t}$ for each unit of power consumed at time t. Each transmission line l is remunerated by the price difference $|\pi_{\mathrm{snd}(l),t} - \pi_{\mathrm{rec}(l),t}|$ at time t. The profit function for each participant (and at each time) is defined as a function of the LMPs and the primal allocations:

$$\phi^p_{i,t} := (\pi_{n(i),t} - \alpha^p_{i,t})p_{i,t} \tag{3.8a}$$

$$\phi^d_{j,t} := (\alpha^d_{j,t} - \pi_{n(j),t})d_{j,t} \tag{3.8b}$$

$$\phi^f_{k,t} := (\pi_{\mathrm{rec}(l),t} - \pi_{\mathrm{snd}(l),t} - \alpha^f_k)f_k \tag{3.8c}$$

3.3. Base market formulation with ESR

We now extend the space–time market formulation to capture ESR systems as a new type of market participant denoted by the set $\mathcal{B}$. Each ESR $b \in \mathcal{B}$ is connected to the power grid at node $n(b) \in \mathcal{N}$. The ESR bids into the market by offering charging service at bid price $\alpha^{sc}_{b,t}$ and discharging service at bid price $\alpha^{sd}_{b,t}$ for each time $t \in \mathcal{T}$.

The clearing formulation for markets with ESR participants is presented in (3.9).

$$\min_{d,p,f,\theta,p^c,p^d} \sum_{t\in\mathcal{T}} \Big(\sum_{i\in\mathcal{S}} \alpha^p_{i,t}p_{i,t} + \sum_{k\in\mathcal{K}} \alpha^f_{k,t}f_{k,t} - \sum_{j\in\mathcal{D}} \alpha^d_{j,t}d_{j,t} + \sum_{b\in\mathcal{B}} \alpha^{sc}_{b,t}p^c_{b,t} + \sum_{b\in\mathcal{B}} \alpha^{sd}_{b,t}p^d_{b,t} \Big) \tag{3.9a}$$

$$\text{s.t.} \sum_{k\in\mathcal{K}^{\mathrm{rec}}_n} f_{k,t} + \sum_{i\in\mathcal{S}_n} p_{i,t} + \sum_{b\in\mathcal{B}_n} p^d_{b,t} = \sum_{k\in\mathcal{K}^{\mathrm{snd}}_n} f_{k,t} + \sum_{j\in\mathcal{D}_n} d_{j,t} + \sum_{b\in\mathcal{B}_n} p^c_{b,t},\ (\pi_{n,t})\, n\in\mathcal{N}, t\in\mathcal{T} \tag{3.9b}$$

$$f_{l^+,t} - f_{l^-,t} = B_l(\theta_{\mathrm{snd}(l),t} - \theta_{\mathrm{rec}(l),t}), \quad l\in\mathcal{L}, t\in\mathcal{T} \tag{3.9c}$$

$$\Delta\underline{s}_{b,t} \le \eta^c_b \sum_{t'=1}^{t} p^c_{b,t'} - \frac{1}{\eta^d_b}\sum_{t'=1}^{t} p^d_{b,t'} \le \Delta\bar{s}_b, \quad b\in\mathcal{B}, t\in\mathcal{T} \tag{3.9d}$$

$$p^c_{b,t} + p^d_{b,t} \le \bar{p}_b, \quad b\in\mathcal{B}, t\in\mathcal{T} \tag{3.9e}$$

$$p^c_{b,t}, p^d_{b,t} \ge 0, \quad b\in\mathcal{B}, t\in\mathcal{T} \tag{3.9f}$$

$$(d, p, \theta) \in \mathcal{C} \tag{3.9g}$$

The social surplus (3.9a) captures the charging and discharging costs of ESRs over the entire time horizon. The nodal power balances (3.9b) capture ESR charging and discharging dynamics. Constraints (3.9d)–(3.9f) capture the operational constraints of each ESR participant. Note that (3.9d)–(3.9f) are equivalent to (2.3) except for (3.9f), where the complementarity constraint in (2.3c) is relaxed. This means that it is possible for the market to deliver power allocations and prices that result in infeasible operations for ESRs (simultaneous charge and discharge). However, we will show that the optimal solutions of (3.9) satisfy the complementarity constraints (2.3c) under non-negative prices (which is a key insight obtained from our theoretical analysis).

The objective function (3.9a) is different from most market clearing formulations that capture ESRs in which the charging terms of ESR are not treated as a cost but a consumer surplus (i.e., they are assigned the same sign as the load term). There are a couple of disadvantages with these types of formulations; from the modeling perspective, treating the charging terms as a consumer surplus implies that ESR participants benefit only from the charging actions in the market because they are obtaining electricity. This is not consistent with reality because energy arbitrage requires both charging and discharging actions (so charging without discharging does not add value to ESR). In addition, from the computational perspective, assigning opposite signs for charging and discharging leads to optimal solutions with simultaneous charging and discharging, which makes complementarity constraints necessary.

We now establish the market properties of (3.9) to demonstrate how to remunerate ESR participants. The partial Lagrange function is:

$$L(d,p,f,\theta,p^c,p^d,\pi) = \sum_{t\in\mathcal{T}} \Big(\sum_{i\in\mathcal{S}} \alpha^p_{i,t}p_{i,t} + \sum_{k\in\mathcal{K}} \alpha^f_{k,t}f_{k,t} - \sum_{j\in\mathcal{D}} \alpha^d_{j,t}d_{j,t} + \sum_{b\in\mathcal{B}} \alpha^{sc}_{b,t}p^c_{b,t} + \sum_{b\in\mathcal{B}} \alpha^{sd}_{b,t}p^d_{b,t} \Big) - \sum_{n\in\mathcal{N},t\in\mathcal{T}} \pi_{n,t} \Big(\sum_{k\in\mathcal{K}^{\mathrm{rec}}_n} f_{k,t} + \sum_{i\in\mathcal{S}_n} p_{i,t} + \sum_{b\in\mathcal{B}_n} p^d_{b,t}$$

$$- \sum_{k\in\mathcal{K}_n^{\text{snd}}} f_{k,t} - \sum_{j\in D_n} d_{j,t} - \sum_{b\in\mathcal{B}_n} p^c_{b,t})$$

$$= -\sum_{t\in\mathcal{T}} \Bigg\{ \sum_{j\in\mathcal{N}_d} (\alpha^d_{j,t} - \pi_{n(j)}) d_{j,t} + \sum_{i\in\mathcal{S}} (\pi_{n(i),t} - \alpha^p_i) p_{i,t} + \sum_{k\in\mathcal{K}} (\pi_{\text{rec}(k)} + \pi_{\text{snd}(k)} + \alpha^f_{k,t}) f_{k,t} + \sum_{b\in\mathcal{B}} \left[(\pi_{b,t} - \alpha^{sd}_{b,t}) p^d_{b,t} - (\pi_{b,t} + \alpha^{sc}_{b,t}) p^c_{b,t} \right] \Bigg\} \quad (3.10)$$

and the Lagrange dual problem is:

$$\max_{\pi} \mathcal{D}(\pi) := \min_{d,p,f,\theta,p^c,p^d} L(d,p,f,\theta,p^c,p^d,\pi) \quad (3.11a)$$

$$\text{s.t.} f_{l^+,t} - f_{l^-,t} = B_l(\theta_{\text{snd}(l),t} - \theta_{\text{rec}(l),t}), \quad l\in\mathcal{L}, t\in\mathcal{T} \quad (3.11b)$$

$$\Delta\underline{s}_{b,t} \le \eta^c_b \sum_{t'=1}^{t} p^c_{b,t'} - \frac{1}{\eta^d_b}\sum_{t'=1}^{t} p^d_{b,t'} \le \Delta\bar{s}_b, \quad b\in\mathcal{B}, t\in\mathcal{T} \quad (3.11c)$$

$$0 \le p^c_{b,t} + p^d_{b,t} \le \bar{p}_b, \quad b\in\mathcal{B}, t\in\mathcal{T} \quad (3.11d)$$

$$p^c_{b,t}, p^d_{b,t} \ge 0, \quad b\in\mathcal{B}, t\in\mathcal{T} \quad (3.11e)$$

$$\eta^c_b \sum_{t'=1}^{T} p^c_{b,t'} - \frac{1}{\eta^d_b}\sum_{t'=1}^{T} p^d_{b,t'} \ge 0, \quad b\in\mathcal{B} \quad (3.11f)$$

$$(d,p,\theta)\in C \quad (3.11g)$$

The Lagrange dual function $\mathcal{D}(\pi)$ can be decomposed into individual profit maximization problems. The ESR profit maximization problem for each $b\in\mathcal{B}$ is:

$$\max_{p^c_b, p^d_b} \sum_{t\in\mathcal{T}} (\pi_{b,t} - \alpha^{sd}_{b,t}) p^d_{b,t} - (\pi_{b,t} + \alpha^{sc}_{b,t}) p^c_{b,t} \quad (3.12a)$$

$$\text{s.t.} \Delta\underline{s}_{b,t} \le \eta^c_b \sum_{t'=1}^{t} p^c_{b,t'} - \frac{1}{\eta^d_b}\sum_{t'=1}^{t} p^d_{b,t'} \le \Delta\bar{s}_b, \quad t\in\mathcal{T} \quad (3.12b)$$

$$0 \le p^c_{b,t} + p^d_{b,t} \le \bar{p}_b, \quad t\in\mathcal{T} \quad (3.12c)$$

$$p^c_{b,t}, p^d_{b,t} \ge 0, \quad t\in\mathcal{T} \quad (3.12d)$$

$$\eta^c_b \sum_{t'=1}^{T} p^c_{b,t'} - \frac{1}{\eta^d_b}\sum_{t'=1}^{T} p^d_{b,t'} \ge 0 \quad (3.12e)$$

Inspection of the ESR profit maximization problem (3.12) provides an intuitive reason for the optimal dispatch of ESR to satisfy the complementarity constraint under non-negative prices. We observe that, for any non-negative value of π, the objective will not incentivize charging and discharging at the same time. Specifically, at each time interval $t\in\mathcal{T}$, discharging is incentivized when $\pi_{b,t} \ge \alpha^{sd}_{b,t} > 0$, and charging is incentivized when $\pi_{b,t} \le -\alpha^{sc}_{b,t} < 0$; importantly, under non-negative prices, the latter condition will never occur. However, this does not mean ESR will not charge at all: the lower bound on the SOC(the first inequality of (3.12b)) will prompt the ESR to charge enough power to satisfy the terminal condition for SOC at times with lower prices. This also implies that, with non-negative prices, the ESR will not charge more than necessary for the terminal condition requirement. This is consistent with the notion that charging is part of the service that ESR offers to transport (shift) electricity over time. The structure of the ESR profit problem thus highlights the consistency of our market clearing formulation.

With the profit maximization problem (3.12), we are now able to establish satisfaction of the complementarity condition rigorously. Let $(d^*, p^*, f^*, \theta^*, p^{c*}, p^{d*}), \pi^*$ denote the optimal primal–dual solution of (3.9). A sufficient condition for satisfaction of complementarity is provided in the following theorem.

Theorem 1. *For any $b\in\mathcal{B}, t\in\mathcal{T}$, we have that $p^{c*}_{b,t}\cdot p^{d*}_{b,t} = 0$ if*

$$\pi^*_{b,t} \ge -\frac{1}{1-\eta_b}(\eta_b\alpha^{sd}_{b,t} + \alpha^{sc}_{b,t}) \quad (3.13)$$

Proof. See Appendix. □

Condition (3.13) provides a lower bound for prices so that ESR will not benefit from charging and discharging simultaneously. The bound is violated by negative prices, where there is local excess power and ESR is encouraged to charge and discharge simultaneously in order to consume this excess power without violating the SOC constraints. In practice, various inflexible components of power systems may lead to negative prices, including renewable energy generation (with its stochastic availability) and ramp-constrained generation units (e.g., coal and natural gas).

We also note that, if ESR has ideal performance with no losses ($\eta_b = 1$), the market formulation always recovers an optimal solution that satisfies complementarity (regardless of the prices). This is a key insight that reveals interplay between efficiency and market incentives. We also note that (3.13) is a sufficient but not necessary condition because other constraints (e.g. SOC bounds) may prevent simultaneous charging and discharging despite negative prices. We will show that this result will allow us to design an alternative market formulation that ensures charge/discharge complementarity without enforcing complementarity constraints explicitly (thus facilitating computational tractability).

3.4. Alternative market formulation with robust bound

So far we have shown that simply relaxing the complementarity constraints may lead to allocations for ESRs that are not economically efficient. To tackle this issue, we propose an alternative formulation based on the robust battery dispatch formulation recently proposed by Nazir and Almassalkhi (2021). The idea (in brief) is to approximate the charge and discharge operations of ESR with a series of net-charge decisions $p^c_{b,t} - p^d_{b,t}$ with a net-charge efficiency $\frac{\eta^c_b}{\eta^d_b}$. Instead of including an exact upper bound for SOC as in (3.9d), we replace it with a conservative upper bound function as shown on the right-hand side of constraint (3.14).

$$\eta^c_b \sum_{t'=1}^{t} p^c_{b,t'} - \frac{1}{\eta^d_b}\sum_{t'=1}^{t} p^d_{b,t'} \le \frac{\eta^c_b}{\eta^d_b}\left(\sum_{t'=1}^{t} p^c_{b,t'} - p^d_{b,t'}\right) \quad (3.14)$$

This leads to the market clearing formulation:

$$\min_{d,p,f,\theta,p^c,p^d} \sum_{t\in\mathcal{T}} \Big(\sum_{i\in S}\alpha^p_{i,t}p_{i,t} + \sum_{k\in\mathcal{K}}\alpha^f_{k,t}f_{k,t} - \sum_{j\in D}\alpha^d_{j,t}d_{j,t} + \sum_{b\in\mathcal{B}}\alpha^{sc}_{b,t}p^c_{b,t} + \sum_{b\in\mathcal{B}}\alpha^{sd}_{b,t}p^d_{b,t} \Big) \quad (3.15a)$$

$$\text{s.t.} \sum_{k\in\mathcal{K}_n^{\text{rec}}} f_{k,t} + \sum_{i\in S_n} p_{i,t} + \sum_{b\in\mathcal{B}_n} p^d_{b,t} = \sum_{k\in\mathcal{K}_n^{\text{snd}}} f_{k,t} + \sum_{j\in D_n} d_{j,t} + \sum_{b\in\mathcal{B}_n} p^c_{b,t}, \ (\pi_{n,t}) \quad n\in\mathcal{N}, t\in\mathcal{T} \quad (3.15b)$$

$$f_{l^+,t} - f_{l^-,t} = B_l(\theta_{\text{snd}(l),t} - \theta_{\text{rec}(l),t}), \quad l\in\mathcal{L}, t\in\mathcal{T} \quad (3.15c)$$

$$\eta^c_b \sum_{t'=1}^{t} p^c_{b,t'} - \frac{1}{\eta^d_b}\sum_{t'=1}^{t} p^d_{b,t'} \ge \Delta\underline{s}_{b,t}, \quad b\in\mathcal{B}, t\in\mathcal{T} \quad (3.15d)$$

$$\frac{\eta^c_b}{\eta^d_b}\left(\sum_{t'=1}^{t} p^c_{b,t'} - p^d_{b,t'}\right) \le \Delta\bar{s}_b, \quad b\in\mathcal{B}, t\in\mathcal{T} \quad (3.15e)$$

$$p^c_{b,t} + p^d_{b,t} \le \bar{p}_b, \quad b\in\mathcal{B}, t\in\mathcal{T} \quad (3.15f)$$

$$p^c_{b,t}, p^d_{b,t} \ge 0, \quad b\in\mathcal{B}, t\in\mathcal{T} \quad (3.15g)$$

$$(d,p,\theta)\in C \quad (3.15h)$$

Note that the only difference between our baseline ESR clearing model (3.9) and (3.15) is that the robust SOC upper bound (3.15e) replaces the exact SOC upper bound in (3.9d). Also, the inequality (3.14) implies that the feasible region of (3.9) is a subset of the feasible region of (3.15) (it is a tightening constraint). This means that the clearing formulation (3.15) compromises the optimal total welfare value in exchange for guaranteeing a feasible operation for ESRs. Importantly, the compromise between total welfare and feasible operation is a function of ESR efficiency (the higher the charging and discharging efficiencies, the less the compromise is). Specifically, note that there is no compromise if $\eta_b^c = \eta_b^d = 1$ and the inequality (3.14) becomes an equality. This again highlights the key role of efficiencies.

Since we only changed one of the ESR constraints, we can easily obtain the profit maximization problem for each ESR using the same Lagrangian dual analysis to obtain:

$$\max_{p_b^c, p_b^d} \quad \sum_{t\in\mathcal{T}} (\pi_{b,t} - \alpha_{b,t}^{sd}) p_{b,t}^d - (\pi_{b,t} + \alpha_{b,t}^{sc}) p_{b,t}^c \tag{3.16a}$$

$$\text{s.t.} \quad \eta_b^c \sum_{t'=1}^{t} p_{b,t'}^c - \frac{1}{\eta_b^d} \sum_{t'=1}^{t} p_{b,t'}^d \ge \Delta \underline{s}_{b,t}, \quad t \in \mathcal{T} \tag{3.16b}$$

$$\frac{\eta_b^c}{\eta_b^d} \left(\sum_{t'=1}^{t} p_{b,t'}^c - p_{b,t'}^d \right) \le \Delta \bar{s}_b, \quad t \in \mathcal{T} \tag{3.16c}$$

$$p_{b,t}^c + p_{b,t}^d \le \bar{p}_b, \quad t \in \mathcal{T} \tag{3.16d}$$

$$p_{b,t}^c, p_{b,t}^d \ge 0, \quad t \in \mathcal{T} \tag{3.16e}$$

Let $(d^*, p^*, f^*, \theta^*, p^{c*}, p^{d*}), \pi^*$ denote the optimal primal–dual solution of (3.15). Using the previous definitions, we can establish the following result.

Theorem 2. *Formulation (3.15) delivers allocations that satisfy the complementarity $p_{b,t}^{c*} \cdot p_{b,t}^{d*} = 0$ for any $b \in \mathcal{B}, t \in \mathcal{T}$.*

Proof. See Appendix. □

This theorem is significant because it guarantees that we can satisfy the complementarity charging/charging constraints without explicitly including them in the market clearing formulation. Such constraints are difficult to handle computationally, as they introduce nonconvexity. Moreover, we note that this result allows us to formulate the market clearing problem as a linear program and to incorporate many ESRs.

3.5. Market formulation with virtual links

Intuitively, each ESR participant aims to make profit via energy arbitrage (exploit price differences across time). In addition, ESR flexibility can help electricity markets =mitigate price volatility. However, none of these intuitive advantages is obvious from formulation (3.9). This motivates us to propose a mathematically-equivalent formulation that models flexibility of ESR using VLs ; this approach will show that charging and discharging of power can be seen as a temporal transfer (transport) of power from the charging time to discharging time. This approach will also reveal proper strategies to remunerate ESRs for the provision of their flexibility and will help highlight the key role of efficiencies.

In the formulation, we view ESR operations in a different way. Instead of modeling operations as a series of charging and discharging decisions over time, we model operations using the concept of net charging and discharging, as well as energy transfer, capturing the exchange of electricity between the grid and ESRs. We break down the operations of ESRs into the following three categories for modeling purposes:

1. *Net-charging:* buying an amount of electricity from the market at a time period and storing it for the rest of the period.
2. *Net-discharging:* selling an amount of electricity to the market at a time period that will not be replaced by electricity purchase later.
3. *Energy transfer:* transporting certain amount of energy from one time to another time. This captures charging/discharging certain amount of electricity at one time and discharging/charging it later.

We note that energy transfer operations have no effect on the net change of state of charge for the ESR over the whole time period of market clearing, while net-charging and net-discharging operations do affect the charge state.

The ESR modeling of the alternative formulation is illustrated in Fig. 3. We define decision variables $p_{b,t}^{nc} \in \mathbb{R}_+, p_{b,t}^{nd} \in \mathbb{R}_+$ to capture the net-charging and net-discharging operations of each ESR $b \in \mathcal{B}$ at time $t \in \mathcal{T}$. To model energy transfer, we extend the concept of VLs proposed in the market design work by Zhang and Zavala (2021). In that work, VLs were proposed to capture non-physical (meaning not via the physical transmission network) and lossless shift of electricity loads enabled by geo-distributed computing infrastructure. Here, we extend this concept to capture non-physical and lossy energy transfer in time enabled by ESRs.

Let $\mathcal{V}$ be the set of all VLs ; each VL $v = (b(v), t_c(v), t_d(v)) \in \mathcal{V}$ has an associated ESR $b(v)$, charging time $t_c(v)$, and discharging time $t_d(v)$. We require that $t_c(v) \neq t_d(v)$ for all $v \in \mathcal{V}$. Implicitly, each VL v is also associated with a location $n(v) := n(b(v))$ from the associated ESR. We define $\mathcal{V}_{n,t}^{\text{in}} := \{v \in \mathcal{V} \,|\, t_d(v) = t, n(v) = n\}$, $\mathcal{V}_{n,t}^{\text{out}} := \{v \in \mathcal{V} \,|\, t_c(v) = t, n(v) = n\}$ as the set of discharging and charging VLs at space–time node (n,t), respectively. These sets form two partitions of VLs on the dimension of space–time nodes: $\mathcal{V} = \cup_{(n,t)\in\mathcal{N}\times\mathcal{T}} \mathcal{V}_{n,t}^{\text{in}} = \cup_{(n,t)\in\mathcal{N}\times\mathcal{T}} \mathcal{V}_{n,t}^{\text{out}}$. We also define $\mathcal{V}_{b,t}^{\text{in}} := \{v \in \mathcal{V} \,|\, t_d(v) = t, b(v) = b\}$, $\mathcal{V}_{b,t}^{\text{out}} := \{v \in \mathcal{V} \,|\, t_c(v) = t, b(v) = b\}$ as the set of discharging and charging VLs for ESR b at time t, respectively. These sets further form partitions for the sets $\mathcal{V}_{n,t}^{\text{in}}$ and $\mathcal{V}_{n,t}^{\text{out}}$: $\mathcal{V}_{n,t}^{\text{in}} = \cup_{b\in\mathcal{B}_n} \mathcal{V}_{b,t}^{\text{in}}$, $\mathcal{V}_{n,t}^{\text{out}} = \cup_{b\in\mathcal{B}_n} \mathcal{V}_{b,t}^{\text{out}}$. Each VL v is also associated with a bid price for the power shift.

The cleared power shifts (virtual flows) are defined as $\delta_v \in \mathbb{R}_+$. Note that shifting power via ESR systems incurs loss of power due to charging and/or discharging efficiency (i.e., $\eta_c, \eta_d < 1$). This means the amount of power discharged may be strictly less than the amount of power charged for each VL . For consistency, we define δ_v as the amount of power charged by VL v at time $t_c(v)$; the amount of power discharged at time $t_d(v)$ by v can be calculated as $\eta_{b(v)}\delta_v$, where $\eta_{b(v)} := \eta_{b(v)}^c \cdot \eta_{b(v)}^d$ is the aggregated (round-trip) efficiency of a pair of charging and discharging actions for ESR $b(v)$. We now can compute the charging and discharging power $p_{b,t}^c, p_{b,t}^d$ as:

$$p_{b,t}^c = \sum_{v\in\mathcal{V}_{b,t}^{\text{out}}} \delta_v + p_{b,t}^{nc} \tag{3.17a}$$

$$p_{b,t}^d = \sum_{v\in\mathcal{V}_{b,t}^{\text{in}}} \eta_{b(v)}\delta_v + p_{b,t}^{nd} \tag{3.17b}$$

The total amount of power charged and discharged by all ESRs at space–time node (n,t) can be expressed in terms of δ as:

$$\sum_{b\in\mathcal{B}_n} p_{b,t}^c = \sum_{b\in\mathcal{B}_n} \left[\sum_{v\in\mathcal{V}_{b,t}^{\text{out}}} \delta_v + p_{b,t}^{nc} \right] = \sum_{v\in\mathcal{V}_{n,t}^{\text{out}}} \delta_v + \sum_{b\in\mathcal{B}_n} p_{b,t}^{nc} \tag{3.18a}$$

$$\sum_{b\in\mathcal{B}_n} p_{b,t}^d = \sum_{b\in\mathcal{B}_n} \left[\sum_{v\in\mathcal{V}_{b,t}^{\text{in}}} \eta_{b(v)}\delta_v + p_{b,t}^{nd} \right] = \sum_{v\in\mathcal{V}_{n,t}^{\text{in}}} \eta_{b(v)}\delta_v + \sum_{b\in\mathcal{B}_n} p_{b,t}^{nd} \tag{3.18b}$$

The clearing formulation with VLs is formulated as:

$$\min_{d,p,f,\theta,\delta} \sum_{t\in\mathcal{T}} \left(\sum_{i\in\mathcal{S}} \alpha_{i,t}^p p_{i,t} + \sum_{k\in\mathcal{K}} \alpha_{k,t}^f f_{k,t} - \sum_{j\in\mathcal{D}} \alpha_{j,t}^d d_{j,t} \right) + \sum_{v\in\mathcal{V}} \alpha_v^\delta \delta_v \tag{3.19a}$$

$$\text{s.t.} \sum_{k\in\mathcal{K}_n^{\text{rec}}} f_{k,t} + \sum_{i\in\mathcal{S}_n} p_{i,t} + \sum_{v\in\mathcal{V}_{n,t}^{\text{in}}} \eta_{b(v)}\delta_v + \sum_{b\in\mathcal{B}_n} p_{b,t}^{nd} = \sum_{k\in\mathcal{K}_n^{\text{snd}}} f_{k,t} + \sum_{j\in\mathcal{D}_n} d_{j,t} + \sum_{v\in\mathcal{V}_{n,t}^{\text{out}}} \delta_v + \sum_{b\in\mathcal{B}_n} p_{b,t}^{nc}, \ (\pi_{n,t})\, n \in \mathcal{N}, t \in \mathcal{T} \tag{3.19b}$$

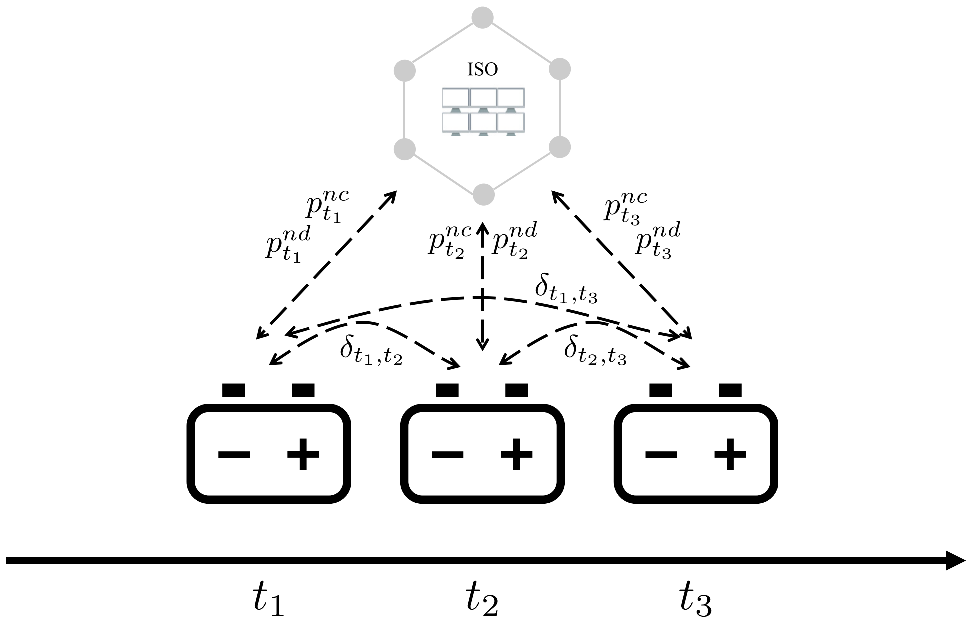


Fig. 3. Alternative formulation for ESR systems using VLs (ESR index b is omitted in notations).

$$f_{l^+,t} - f_{l^-,t} = B_l(\theta_{\mathrm{snd}(l),t} - \theta_{\mathrm{rec}(l),t}), \quad l \in \mathcal{L}, t \in \mathcal{T} \tag{3.19c}$$

$$\eta_b^c \sum_{t'=1}^{t} \left[\sum_{v \in \mathcal{V}^{\mathrm{out}}_{b,t'}} \delta_v - \sum_{v \in \mathcal{V}^{\mathrm{in}}_{b,t'}} \delta_v \right] \geq \Delta \underline{s}_{b,t} + \frac{1}{\eta_b^d} \sum_{t'=1}^{t} p^{nd}_{b,t'}, \quad b \in \mathcal{B}, t \in \mathcal{T} \tag{3.19d}$$

$$\frac{\eta_b^c}{\eta_b^d} \sum_{t'=1}^{t} \left[\sum_{v \in \mathcal{V}^{\mathrm{out}}_{b,t'}} \delta_v - \sum_{v \in \mathcal{V}^{\mathrm{in}}_{b,t'}} \eta_b \delta_v \right] \leq \Delta \bar{s}_b - \eta_b^c \sum_{t'=1}^{t} p^{nc}_{b,t'}, \quad b \in \mathcal{B}, t \in \mathcal{T} \tag{3.19e}$$

$$p^{nc}_{b,t} + p^{nd}_{b,t} + \sum_{v \in \mathcal{V}^{\mathrm{out}}_{b,t}} \delta_v + \sum_{v \in \mathcal{V}^{\mathrm{in}}_{b,t}} \eta_b \delta_v \leq \bar{p}_b, \quad b \in \mathcal{B}, t \in \mathcal{T} \tag{3.19f}$$

$$\delta \geq 0 \tag{3.19g}$$

$$(d, p, \theta) \in C \tag{3.19h}$$

With these formulations laid out we can now establish the following result, which establishes that the clearing formulations with VLs is mathematically-equivalent to the clearing formulation (3.15).

Theorem 3. *Let* $\alpha_v^{\delta} = \alpha^{sc}_{b(v),t_c(v)} + \eta_{b(v)} \alpha^{sd}_{b(v),t_d(v)}$ *for each* $v \in \mathcal{V}$*. Then, formulations* (3.15) *and* (3.19) *are equivalent.*

Proof. See Appendix. □

Theorem 3 shows the equivalence between the market formulations when each VL is assigned a proper bid. The bid assigned for each VL covers the cost for the corresponding charging and discharging operations. Also, the bid is a function of the round-trip efficiency to account for lost power in the transfer process. Fig. 4 illustrates the market clearing process with VLs modeling the flexibility of ESR systems.

A key advantage of the VL-based framework, compared to (3.9) or (3.15), is the ability to allow for more sophisticated bidding procedures by ESRs. In Theorem 3, the equivalence between the formulations require that each VL has the same bid. More generally, however, each ESR is allowed to submit different bids for its VLs (for its shift of power across time). For instance, ESRs can submit a higher bid for VLs that span a longer period of time, as the energy transferred via those VLs will occupy the charge storage space for a longer time, compared to those transferred via other VLs . This bidding behavior cannot be captured in (3.15), as bidding is made for charging and discharging operations as a whole.

Under the market clearing framework with VLs, each ESR makes a payment for every unit of electricity it purchases, and is remunerated for every unit of electricity sold to the market, as well as every unit of energy transfer via its VLs . The total remuneration for each ESR is:

$$\sum_{v \in \mathcal{V}_b} \pi_v^d (\eta_b \delta_v) + \sum_{t \in \mathcal{T}} \pi_{b,t} p^{nd}_{b,t} - \sum_{v \in \mathcal{V}_b} \delta_v \pi_v^c - \sum_{t \in \mathcal{T}} \pi_{b,t} p^{nc}_{b,t}$$
$$= \sum_{v \in \mathcal{V}_b} (\eta_b \pi_v^d - \pi_v^c) \delta_v + \sum_{t \in \mathcal{T}} \pi_{b,t} (p^{nd}_{b,t} - p^{nc}_{b,t}) \tag{3.20}$$

where $\pi_v^d := \pi_{n(b(v)),t_d(v)}$, $\pi_v^c := \pi_{n(b(v)),t_c(v)}$, $\delta_b := \{\delta_v\}_{v \in \mathcal{V}_b}$, $p_b^{nc} := \{p^{nc}_{b,t}\}_{t \in \mathcal{T}}$, $p_b^{nd} := \{p^{nd}_{b,t}\}_{t \in \mathcal{T}}$. This reveals the different elements of the service remuneration.

The profit maximization problem for each ESR b under the VL framework is provided Eq. (3.21). This problem reveals how the market clearing formulation with VLs (3.19) remunerates flexibility of ESRs. The objective function (3.21a) shows that each ESR maximizes the total profit from VLs (shifting) and net-charging/discharging. On the first term, each VL is remunerated by a "discounted" price difference $\eta_b \pi_v^d - \pi_v^c$ across time, where the price at the discharge time is multiplied by the round-trip efficiency of the ESR. This results from the fact that each ESR has less electricity to sell than it buys when the efficiency is strictly less than one (if the originally stored electricity is not considered).

$$\max_{\delta_b, p_b^{nc}, p_b^{nd}} \sum_{v \in \mathcal{V}_b} (\eta_b \pi_v^d - \pi_v^c - \alpha_v^{\delta}) \delta_v + \sum_{t \in \mathcal{T}} \left[(\pi_{b,t} - \alpha^{sd}_{b,t}) p^{nd}_{b,t} - (\pi_{b,t} + \alpha^{sc}_{b,t}) p^{nc}_{b,t} \right] \tag{3.21a}$$

$$\text{s.t.}\, \eta_b^c \sum_{t'=1}^{t} \left[\sum_{v \in \mathcal{V}^{\mathrm{out}}_{b,t'}} \delta_v - \sum_{v \in \mathcal{V}^{\mathrm{in}}_{b,t'}} \delta_v \right] \geq \Delta \underline{s}_{b,t} + \frac{1}{\eta_b^d} \sum_{t'=1}^{t} p^{nd}_{b,t'} \tag{3.21b}$$

$$\frac{\eta_b^c}{\eta_b^d} \sum_{t'=1}^{t} \left[\sum_{v \in \mathcal{V}^{\mathrm{out}}_{b,t'}} \delta_v - \sum_{v \in \mathcal{V}^{\mathrm{in}}_{b,t'}} \eta_b \delta_v \right] \leq \Delta \bar{s}_b - \eta_b^c \sum_{t'=1}^{t} p^{nc}_{b,t'} \tag{3.21c}$$

$$p^{nc}_{b,t} + p^{nd}_{b,t} + \sum_{v \in \mathcal{V}^{\mathrm{out}}_{b,t}} \delta_v + \sum_{v \in \mathcal{V}^{\mathrm{in}}_{b,t}} \eta_b \delta_v \leq \bar{p}_b, \quad t \in \mathcal{T} \tag{3.21d}$$

$$\delta_b, p_b^{nc}, p_b^{nd} \geq 0 \tag{3.21e}$$

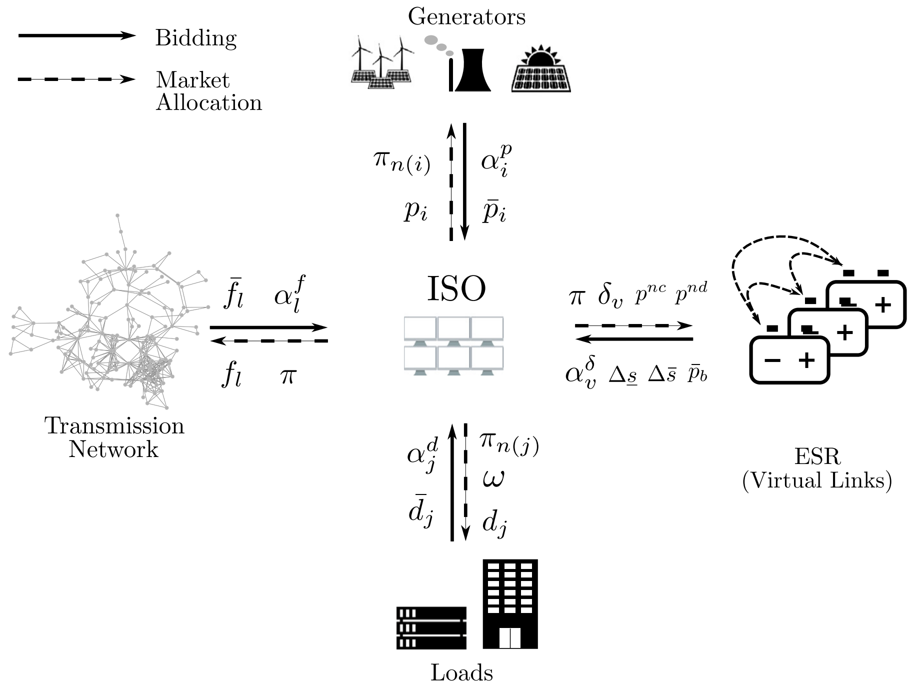


Fig. 4. Market clearing framework with ESR using VLs .

Importantly, in order to make a VL v profitable (thus activating the VL in the market), the price difference needs to satisfy:

$$\eta_b \pi_v^d - \pi_v^c - \alpha_v^\delta \geq 0 \Rightarrow \pi_v^d - \pi_v^c \geq \frac{1-\eta_b}{\eta_b}\pi_v^c + \frac{1}{\eta_b}\alpha_v^\delta \tag{3.22}$$

The bound reveals some interesting properties of the market design and how it incentivizes temporal shifting. The bid plays a key role for defining the minimum price difference; a higher bid means larger price difference is needed to activate the VL . We also observe that a higher round-trip efficiency η_b shrinks the minimum price difference need for activation if the charging price is non-negative, but expands the difference if the charging price is very negative. In other words, *ESRs with higher efficiencies will be more likely to be cleared in the market (have a competitive advantage)*.

To state the previous result more formally, we let $\hat{\pi}_v := \frac{1-\eta_b}{\eta_b}\pi_v^c + \frac{1}{\eta_b}\alpha_v^\delta$ define the minimum price difference. We can compute the derivative of this difference with respect to efficiency as follows:

$$\frac{\partial \hat{\pi}_v}{\partial \eta_b} = -\frac{1}{\eta_b^2}(\pi_v^c + \alpha_v^\delta). \tag{3.23}$$

We can see that, when $\pi_v^c > -\alpha_v^\delta$, more efficient ESR systems provide VLs that can be activated with smaller price differences; when $\pi_v^c < -\alpha_v^\delta$, less efficient ESR systems provide VLs that can be activated with smaller price differences. This also provides the interesting insight that *less efficient ESRs might be preferred in the event of negative market prices*, as they can process the excess energy more easily knowing that more of it will be lost due to their lower storage efficiency.

Finally, we observe that the value of price π_v^c also affects the minimum price difference $\hat{\pi}_v$. When the ESR is not ideal ($\eta_b < 1$), $\hat{\pi}_v$ monotonically increases with π_v^c. This shows that, *in order to activate a VL shift, the price difference must be larger if the clearing prices are higher*. On the other hand, when clearing prices are negative, even a negative price difference could be enough to incentivize the VL . All these insights demonstrate how VLs can be used to understand market conditions that incentivize temporal power shifts (which is difficult to do with formulations in the literature).

The second term in the objective function (3.21a) represents the remuneration rate for net-charging and discharging, and is identical to the remuneration rate shown in Eq. (3.16). At each time interval $t \in \mathcal{T}$, net-discharging is incentivized when $\pi_{b,t} \geq \alpha_{b,t}^{sd} > 0$, and net-charging is incentivized when $\pi_{b,t} \leq -\alpha_{b,t}^{sc} < 0$. The key difference between net-charging/discharging in this VL formulation and charging/discharging in (3.16) is that net-charging/discharging variables ($p_{b,t}^{nc}$ and $p_{b,t}^{nd}$) only account for the amount of power charged/discharged that impact the net change of SOC. Charging/discharging variables ($p_{b,t}^c$ and $p_{b,t}^d$) in (3.16) account for total charging/discharging power at each time, including the contribution from energy transfer over time. Indeed, formulation (3.21) reveals different ways in which ESRs react to the prices. If the prices are volatile within the time frame, an ESR generates profits by transferring power over time using VLs ; if the prices are not volatile, an ESR can choose to buy/sell power if the prices are low/high, or if there is a need to satisfy the lower and upper limits for SOC. These market properties are thus consistent with rational economic behavior.

We now inspect the constraints in the profit maximization formulation (3.21). The constraints show that there are two key resources that each ESR exploits to maximize profit: power capacity and charge capacity. The notion of power capacity is more straightforward to explain and is shown in (3.21d); at each time, the sum of power for all operations must be upper-bounded by the power limit $\bar{p}_b$. The notion of charge capacity is bit complex to explain under the VL formulation. Specifically, the parameters $\Delta\bar{s}_b/\Delta\underline{s}_b$ define the maximum amount of net charge that can be accumulated/depleted from the ESR. The key observation is that net-charging/discharging operations consume these resources permanently within the time period, while VLs only use these resources temporarily. This is reflected on the right-hand side of constraints (3.21b) and (3.21c), where the summation terms of p^{nd} and p^{nc} variables have the effect of tightening the bound of SOC for future times. This implies that *ESRs will be more reluctant to do net-charging/discharging at earlier times in the horizon* as doing so might reduce the marginal profit obtainable from VLs later in the horizon. Again, this highlights that the proposed formulation is consistent with rational economic behavior.

4. Case studies

In this section, we present a couple of studies to demonstrate the properties of the proposed market design. Julia code and data for reproducing the results can be found at https://github.com/zavalab/JuliaBox/tree/master/StorageVirtualLink.

Table 1
Parameter table for single-node case study. Δp: maximum ramp limit; s_0: Initial SOC level.

Scenario	Δp (MW)	s_0 (MWh)
1	25	50
2	15	50
3	15	95
4	5	50

4.1. 3-Time, single-node system

We consider a simple hourly market with a single spatial node (no transmission network) operated over a 3-hour time period. The purpose of this small case is to demonstrate theoretical properties that we have established, and to highlight some empirical observations that will be relevant for our second case (which will be more difficult to isolate in the more complex setting).

The single node is connected to one load, one generator, and one ESR system. The amount of load requested is $\bar{d} = [25, 100, 25]$ MWh over the time period, with bids $\alpha^d = [30, 60, 40]$ \$/MWh. The generator has a capacity of $\bar{p} = [50, 50, 50]$ MWh and bids $\alpha^p = [5, 20, 10]$ \$/MWh. In order to create cases with negative prices, we enforce a ramping constraint for the generator of the form:

$$|p_{t+1} - p_t| \leq \Delta p \tag{4.24}$$

where Δp is the maximum ramp limit. The ESR we consider has charging efficiency $\eta^c = 0.9$, discharging efficiency $\eta^d = 0.8$, SOC bounds $\underline{s}_b = 0$ MWh, $\bar{s}_b = 100$ MWh, and $\bar{p}_b = 10$ MW.

To demonstrate the properties of our market design, we consider four different scenarios with varying ramping capability of the generator and with different initial SOC values. The parameter values of each scenario are tabulated in Table 1. In scenario 1, we assume the generator has ample ramping capability (25 MW) and the ESR starts with half of its charge capacity (50 MWh). In scenarios 2 and 4, we reduce the ramping capability to 15 MW and 5 MW to induce negative prices and observe how the ESR system reacts. In scenario 3, we increase the s_0 value so that the SOC upper bound could potentially be hit at the optimal solution.

We solve formulations (3.9) (base market formulation) and (3.19) (VL-based market formulation) over the four scenarios. The optimal solutions are shown in Table 2. Here, ϕ denotes the total welfare value, π the optimal prices, p^c/p^d the charging/discharging power, and s the SOC level. Note that they do not necessarily correspond to decision variables in the formulations, and might be calculated from optimal solutions; for instance, for the VL formulation (3.19) p^c/p^d are calculated from optimal values of δ and p^{nc}/p^{nd} via (3.17)

We observe that scenario 1 produces a series of positive prices that are commonly seen in real settings. Here, the ramping constraints are not active, and both market formulations produce the same optimal solution that satisfies the complementarity constraints for ESRs. Same observations can be made in scenarios 2 and 4, where the optimal solutions of the models agree and both satisfy the complementarity constraints. As the generator becomes more inflexible (with less ramping capability), more negative prices occur, leading to higher price differences that the ESR can exploit to increase its profits. As a result, the ESR fully uses its power capacity to do price arbitrage and to buy extra electricity.

In scenario 3 we start to observe differences in the primal optimal solutions between the market formulations. Model (3.9) delivers an optimal solution that does not satisfy the complementarity constraint at time $t = 1$, where the price become quite negative (-35 \$/MWh). The reason is that, under market formulation (3.9) (without the complementarity constraint) the ESR will take advantage of the negative price by charging and discharging simultaneously to consume extra electricity (which is not economically efficient). On the other hand, the robust bound developed for model (3.9) rules out this possibility, and therefore the optimal solution satisfies the complementarity constraint. The robust bound is also reflected at the value of s at $t = 1$, which is 100 for (3.9) but 99 for (3.19), showing that the robust bound on SOC is active in this case (it is preventing a solution where $s = 100$ is reached). This case demonstrates that, unlike the base market formulation (3.9), the VL-based formulation (3.19) finds optimal solutions that are economically efficient for ESRs.

A simple comparison with the other scenarios show that the robust bound will only be active when the ESR operates near the SOC upper bound. As a result of the robust bound being active, the total welfare value becomes smaller. These results verify the theoretical limitations of formulation (3.9) identified in Theorem 1, where large and negative prices are likely to lead to infeasible operations for ESRs. Note that scenario 4 shows how Theorem 1 provides a sufficient but not necessary condition for guaranteeing feasible operations, as negative prices that breaks the bound in (3.13) may still co-exist with feasible operations in the primal space.

4.2. 30-Node system with one ESR

We now consider the IEEE 30-node system with Active Power Increase (API) conditions from PGLib-OPF library (Babaeinejadsarookolaee et al., 2019). The network topology is shown in Fig. 5(a). We ran the market over this system for a period of $T = 24$ hours. The original case file contains physical parameters for transmission lines and generators, as well as the cost function of generators. The bid of each generator is selected as the linear term coefficient of the cost function. The file also specifies a load level at a fixed time. To run the market over multiple hours, for each load we sample a load level for each time from a uniform distribution between 0.75 and 1.25 times of the specified load level. Each load j is assigned a constant bid cost of $\alpha^d_{j,t} = 200$ \$/MWh, which resembles the value-of-loss-load (VOLL) penalty for load curtailment (note that in real life the VOLL value could be significantly higher than the 200 \$/MWh).

In this market, we consider three identical ESR systems distributed at nodes 5, 15, and 24 (green nodes in Fig. 5(a)). Each ESR has charging efficiency $\eta^c = 0.95$ and discharging efficiency $\eta^d = 0.85$. To consider the effect of varying energy and power capacity, we define an integer multiplier K for the other parameters ($K = 0$ are base conditions). Thus, each ESR has SOC bounds $\underline{s}_b = 0$ MWh, $\bar{s}_b = 4K$ MWh, initial SOC$s_0 = 2K$ MWh, and power capacity $\bar{p}_b = 1K$ MW. The multiplier K takes values $\{0, 1, 5, 10, 15, 20, 25, 50\}$. The baseline scenario ($K = 0$) corresponds to the case with no ESR installed.

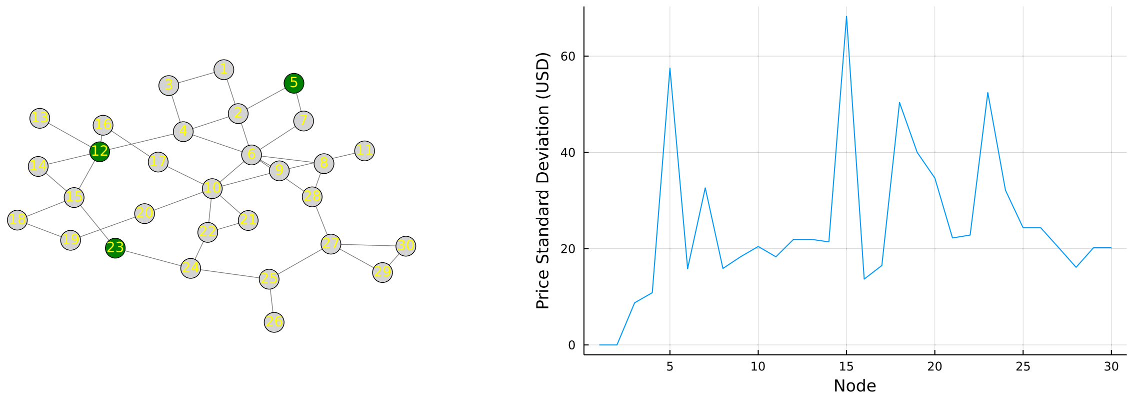
Fig. 5(b) shows the temporal standard deviation of nodal prices at each node under base conditions. We observe that nodes 5 an 15 have the highest temporal price volatility over all nodes, while node 24 only exhibits medium level of volatility. In addition, we also run the case where each ESR participates individually with 3 times the capacity.

Table 3 tabulates the total market welfare values for different K values in the case where all three ESRs participate in the market, using the base market formulation (3.9) and the virtual-link-based formulation (3.19). We note that the two market models attain the same solutions for all cases. The reason is that this case study does not suffer from negative prices as scenario 3 in Section 4.1.

Fig. 6 shows the total remuneration for each ESR for different multiplier values K. First, we observe that the choice of location has a significant effect on the economic benefits of ESR. We observe that the economic benefits of offering flexibility varies by location. Moreover, an interesting observation is that ESR 3 gains higher remuneration than ESR 2 for all cases, despite the fact that the node of ESR 2 has the highest temporal price volatility while the node of ESR 3 has only medium temporal price volatility. This implies that placing an ESR at a node where prices are volatile at the base case does not necessarily lead to high economic benefits. In fact, ESRs 1 and 2 are located at nodes with similar price volatility, but the economic potential of

Table 2
Numerical results of the 3-time, single-node case study. ϕ: total welfare; π: LMP; p^c: charging power; p^d: discharging power; s: SOC. Results other than total welfare ϕ are concatenated over time; (3.9) and (3.19) are base market formulation and virtual-link-based market formulation, respectively.

Model	Scenario	ϕ ($)	π ($/MWh)	p^c (MW)	p^d (MW)	s (MWh)
(3.9)	1	3883.72	[5, 60, 10]	[10, 0, 3.89]	[0, 10, 0]	[59, 46.5, 50]
	2	3822.0	[−0.1, 60, −0.1]	[10, 0, 10]	[0, 10, 0]	[59, 46.5, 55.5]
	3	3708.60	[−35, 60, 10]	[8.14, 0, 8.33]	[1.86, 10, 0]	[100, 87.5, 95]
	4	3422.0	[−24.9, 60, −0.1]	[10, 0, 10]	[0, 10, 0]	[59, 46.5, 55.5]
(3.19)	1	3883.72	[5, 60, 10]	[10, 0, 3.89]	[0, 10, 0]	[59, 46.5, 50]
	2	3822.0	[−0.1, 60, −0.1]	[10, 0, 10]	[0, 10, 0]	[59, 46.5, 55.5]
	3	3633.72	[−35, 60, 10]	[4.44, 0, 9.44]	[0, 10, 0]	[99, 86.5, 95]
	4	3422.0	[−0.1, 60, −24.9]	[10, 0, 10]	[0, 10, 0]	[59, 46.5, 55.5]



(a) (b)

Fig. 5. (a) Network topology of IEEE 30-node system (each green node has an installed ESR) and (b) nodal price standard deviation under base conditions.

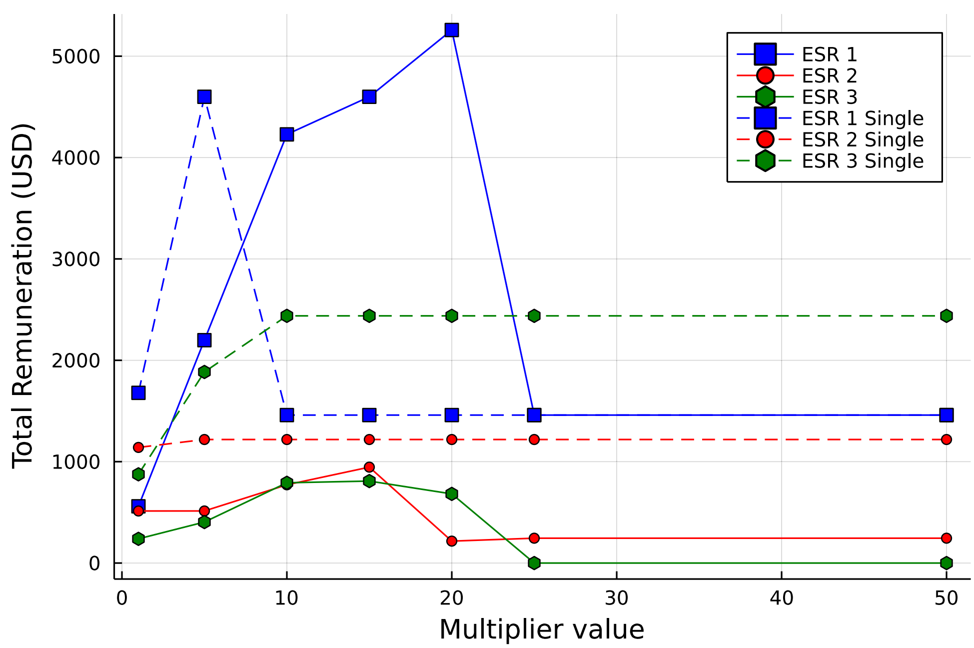


Fig. 6. Total remuneration of each ESR for different multiplier values K. Solid lines are obtained from the solution where all ESRs simultaneously participate in the market, and dashed lines ("Single" in legend) are obtained from solutions where each ESR solely participates in the market with 3 times higher capacity.

ESR 1 is consistently higher than that of ESR 2, in both individual and simultaneous participation cases. This gives rise to the interesting question of optimal placement of flexible technologies from an investor perspective (how to maximize economic returns).

In Fig. 6 we observe that bidding a high capacity value does not necessarily means higher remuneration. In most cases, the total remuneration tends to increase at smaller multiplier values and then decrease at larger multiplier values. To shed more light on this issue, Fig. 7 shows the temporal price volatility at the three buses equipped with an ESR over different multiplier values. We observe that each node has a breaking point where price volatility sharply drops, and the break points for nodes 15 and 24 come at much smaller multiplier value compared to that of node 5. As a result, high multiplier values wipe out the economic incentives for price arbitrage that ESR system can benefit

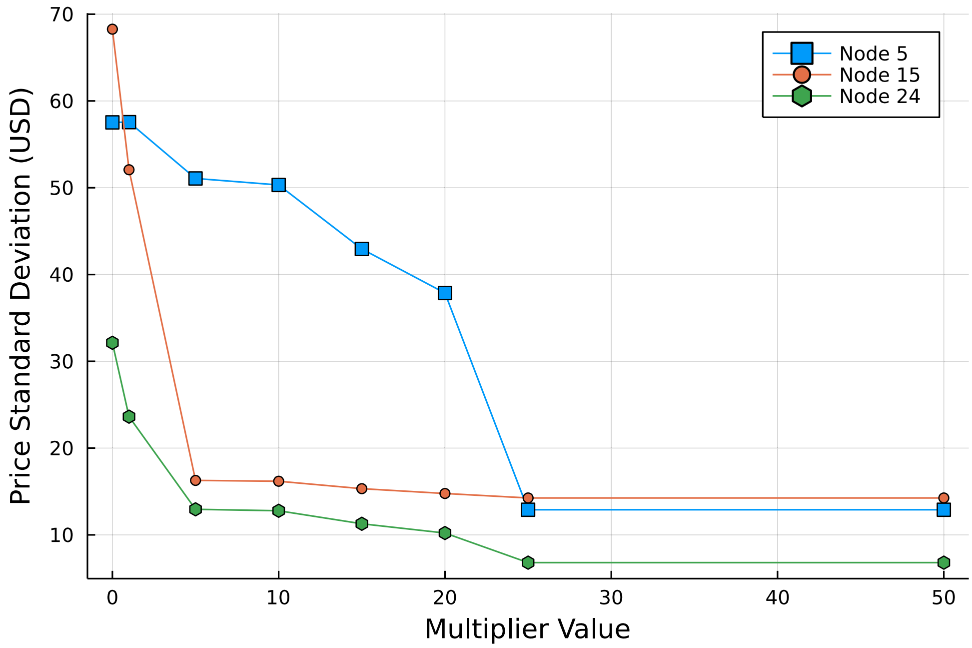


Fig. 7. Temporal standard deviation of nodal prices at nodes 5, 15, and 24 for different values of multiplier K. All ESRs participate in the market simultaneously.

Table 3
Total welfare values (in \$$10^6$) for different values of multiplier K with all three ESRs under base market formulation (3.9) and the virtual-link-based formulation (3.19).

Model	$K = 0$	$K = 5$	$K = 10$	$K = 15$	$K = 20$	$K = 25$	$K = 50$
(3.9)	1.839	1.843	1.845	1.847	1.848	1.848	1.848
(3.19)	1.839	1.843	1.845	1.847	1.848	1.848	1.848

from. This explains why ESR 1 receives much higher remuneration in the simultaneous participation case, as shown in Fig. 6.

All these observations demonstrate the trade-off that ESR stakeholders face when bidding in the market: *bidding a small amount of flexibility might miss the chance to sell more flexibility (and get paid more), while bidding too much flexibility reduces the economic incentives and hurts the received remuneration. Therefore, ESRs need to be strategic in how much flexibility to offer.*

Fig. 8 shows the temporal price volatility at different nodes for the case of $K = 0$, $K = 20$, and $K = 20$ with individual ESR participation. We observe that $K = 20$ case leads to the lowest price standard deviation, while $K = 0$ case leads to the highest price standard deviation. Cases of $K = 20$ with individual ESR participation exhibit volatility between $K = 0$ and $K = 20$. Spatial price volatility also follows the same trend. This demonstrates that from the ISO's perspective, it is more beneficial to have a larger number of smaller ESRs distributed at different locations than to have a few large (centralized) ESRs at a single location, if the goal is to reduce system-level price volatility. Interestingly, our market does not make any assumption about the ownership of ESRs, meaning that the market framework gives the same solution regardless of the ownership of ESRs.

If each ESR belongs to a different entity, we can observe that multiple small ESRs will compete with each other to offer more flexibility, leading to a higher reduction in price volatility with cheaper cost (less remuneration for ESRs). This can be verified in Fig. 6, where at some multiplier values (e.g., $K = 5$), the remuneration for all ESRs combined in the simultaneous participation case is even lower than the remuneration for a single large ESR in the individual participation case. On the other hand, if all the ESRs belong to the same entity, the results imply that ESR investors may be incentivized to reduce distribution of ESR units across different locations, in order to avoid possible reduction in remuneration.

5. Conclusions and future work

Recent trends of renewable energy absorption calls for much higher needs for flexibility resources. New FERC orders and regulations call for ISOs to implement new markets to allow ESR participation, but research work in this area is lacking. In this paper, we propose a new energy market clearing framework that models ESR flexibility using VLs . We discuss the application of a robust bound for ESR operations that helps satisfy charge/discharge complementarity conditions without using mixed-integer formulations. We prove a theoretical lower bound for prices where the robust bound might be needed. In this way, the market clearing model is able to run with a large number of ESRs without delivering clearing solutions that do not satisfy complementarity. Our market framework decomposes the operations of ESRs to reveal the incentives for ESR operations. Specifically, the concept of VLs clearly reveal how ESRs are incentivized by temporal price differences. By applying the concept of VLs and Lagrangian dual analysis, we show that load shifting is remunerated via temporal price volatility, and efficiency of ESR plays a key role in the economic benefits of the load-shifting flexibility. We also demonstrate that the SOC capacity and power capacity are the real resources that determine how much load-shifting flexibility an ESR can provide.

The case studies reveal several interesting directions for future work. From an investor perspective, the capacity sizing and optimal placement of ESR resources can be done via a bilevel programming formulation that embeds our market clearing framework. This is feasible as our market clearing framework avoids mixed-integer formulation from complementarity. From the ESR operator perspective, it is worth exploring the optimal bidding strategy in detail. In this work we assume truthful bidding, i.e. the bid price and capacity perfectly reflect the operating cost and capacity of the ESR system. However, the case studies identify possible incentives for ESRs to bid less capacity than their actual capacity. In addition, bids are a lower bound on the profit of VLs as found by Zhang and Zavala (2021), which provides another motivation for the submission of higher bids. From the ISO perspective, it is worth exploring how to extend our market framework for multi-scale markets. The current market is formulated on a day-ahead basis, and thus VLs only incentivize load shifting in short period of time. The incentives for long-term load-shifting will have to be provided in another layer of market as real-life electricity markets work on

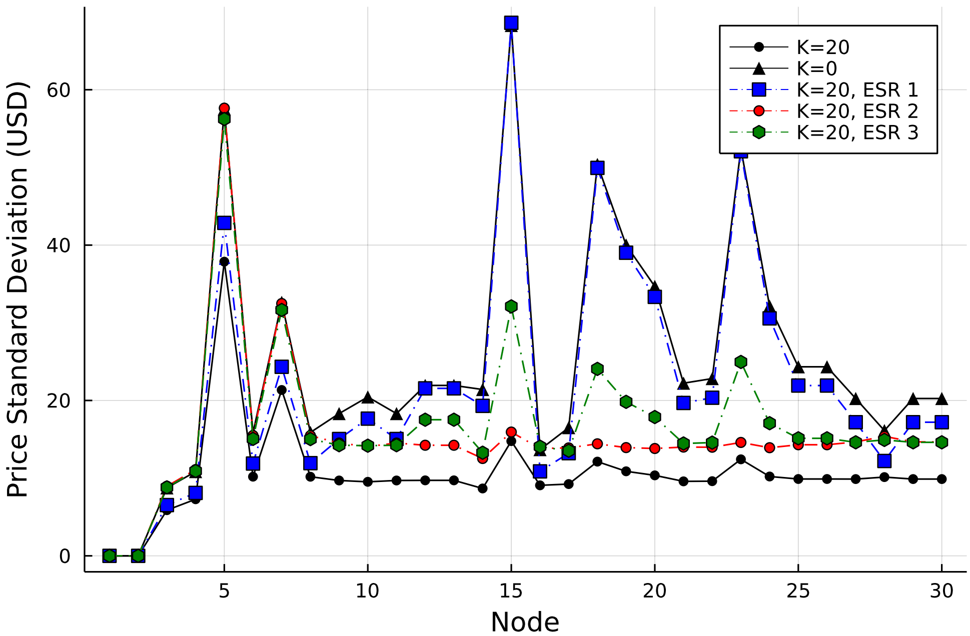


Fig. 8. Temporal standard deviation of nodal prices for $K = 0$, $K = 20$, and $K = 20$ with individual ESR participation. The solid line with $K = 20$ denotes the case where all ESRs participate simultaneously, and the dashed lines denote the case where only one ESR participates (participating ESR is denoted in legend).

multiple time scales. One possible approach is to design another market at a higher scale (e.g., seasonal or monthly scale) that determines the operating SOC level (which is a parameter in the current market framework). The change of SOC levels at different seasons/months can also be capture via the concept of VLs .

CRediT authorship contribution statement

Weiqi Zhang: Methodology, Formal Analysis, Software, Writing – original draft, Visualization. **Victor M. Zavala:** Conceptualization, Supervision, Writing – review & editing, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data and code are provided in link added to manuscript.

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Appendix A. Proofs

Proof of Theorem 1. We inspect the profit maximization problem (3.12) for each ESR $b \in \mathcal{B}$. Given π, let (p^c_b, p^d_b) be a feasible solution of (3.12). For convenience, let $\psi(p^c_b, p^d_b)$ denote the objective function (3.12a). Define $\epsilon_t := \min\{p^c_{b,t}, \frac{1}{\eta_b} p^d_{b,t}\} \geq 0$ for every $t \in \mathcal{T}$, and $\epsilon := \{\epsilon_t\}_{t\in\mathcal{T}}$. First, we observe that $(p^c_b - \epsilon, p^d_b - \eta_b\epsilon)$ is also a feasible solution to (3.12) because

$$
\begin{aligned}
&\eta^c_b \sum_{t'=1}^{t} (p^c_{b,t'} - \epsilon_{t'}) - \frac{1}{\eta^d_b} \sum_{t'=1}^{t} (p^d_{b,t'} - \eta_b \epsilon_{t'}) \\
&= \eta^c_b \sum_{t'=1}^{t} p^c_{b,t'} - \frac{1}{\eta^d_b} \sum_{t'=1}^{t} p^d_{b,t'} + \sum_{t'=1}^{t} \eta^c_b (\epsilon_{t'} - \epsilon_{t'}) \\
&= \eta^c_b \sum_{t'=1}^{t} p^c_{b,t'} - \frac{1}{\eta^d_b} \sum_{t'=1}^{t} p^d_{b,t'}
\end{aligned}
$$

holds or every $t \in \mathcal{T}$. Next, we observe that $(p^c_b - \epsilon, p^d_b - \eta_b\epsilon)$ satisfies the complementarity condition. This is true because, by definition of ϵ, at every time t we have that either $p^c_{b,t} = 0$ or $p^d_{b,t} = 0$. In the end, we show that $(p^c_b - \epsilon, p^d_b - \eta_b\epsilon)$ produces a better objective value than (p^c_b, p^d_b) under the condition of (3.13):

$$
\begin{aligned}
&\psi(p^c_b - \epsilon, p^d_b - \eta_b\epsilon) - \psi(p^c_b, p^d_b) \\
&= \sum_{t\in\mathcal{T}} (\pi_{b,t} - \alpha^{sd}_{b,t})(p^d_{b,t} - \eta_b\epsilon_t) - (\pi_{b,t} + \alpha^{sc}_{b,t})(p^c_{b,t} - \epsilon_t) - (\pi_{b,t} - \alpha^{sd}_{b,t}) p^d_{b,t} \\
&\quad + (\pi_{b,t} + \alpha^{sc}_{b,t}) p^c_{b,t} \\
&= \sum_{t\in\mathcal{T}} \eta_b \alpha^{sd}_{b,t} \epsilon_t + \alpha^{sc}_{b,t} \epsilon_t + \epsilon_t (1 - \eta_b) \pi_{b,t} \\
&\geq \sum_{t\in\mathcal{T}} \epsilon_t \left(\eta_b \alpha^{sd}_{b,t} + \alpha^{sc}_{b,t} + (1 - \eta_b) \frac{-1}{1 - \eta_b} (\eta_b \alpha^{sd}_{b,t} + \alpha^{sc}_{b,t}) \right) \\
&= 0
\end{aligned}
$$

meaning that the optimal solution must satisfy the complementarity condition. □

Proof of Theorem 2. We inspect the profit maximization problem (3.16) for each ESR $b \in \mathcal{B}$. Given π, let (p^c_b, p^d_b) be a feasible solution of (3.16). For convenience, let $\psi(p^c_b, p^d_b)$ denote the objective function (3.16a). Define $\epsilon_t := \min\{p^c_{b,t}, p^d_{b,t}\} \geq 0$ for every $t \in \mathcal{T}$, and $\epsilon := \{\epsilon_t\}_{t\in\mathcal{T}}$. First, we observe that $(p^c_b - \epsilon, p^d_b - \eta_b\epsilon)$ is also a feasible solution to (3.12) because

$$
\begin{aligned}
&\eta^c_b \sum_{t'=1}^{t} (p^c_{b,t'} - \epsilon_{t'}) - \frac{1}{\eta^d_b} \sum_{t'=1}^{t} (p^d_{b,t'} - \epsilon_{t'}) = \eta^c_b \sum_{t'=1}^{t} p^c_{b,t'} - \frac{1}{\eta^d_b} \sum_{t'=1}^{t} p^d_{b,t'} \\
&\quad + \sum_{t'=1}^{t} (\frac{1}{\eta^d_b} - \eta^c_b) \epsilon_{t'} \geq \eta^c_b \sum_{t'=1}^{t} p^c_{b,t'} - \frac{1}{\eta^d_b} \sum_{t'=1}^{t} p^d_{b,t'}
\end{aligned}
$$

holds for every $t \in \mathcal{T}$. Next we observe that $(p^c_b - \epsilon, p^d_b - \epsilon)$ satisfies the complementarity condition. This is true because, by definition of ϵ, at every time t we have that either $p^c_{b,t} = 0$ or $p^d_{b,t} = 0$. In the end we show that $(p_b^c - \epsilon, p_b^d - \epsilon)$ produces a better objective value than (p_b^c, p_b^d):

$$\begin{aligned}
&\psi(p_b^c - \epsilon, p_b^d - \epsilon) - \psi(p_b^c, p_b^d) \\
&= \sum_{t\in\mathcal{T}} (\pi_{b,t} - \alpha_{b,t}^{sd})(p_{b,t}^d - \epsilon_t) - (\pi_{b,t} + \alpha_{b,t}^{sc})(p_{b,t}^c - \epsilon_t) - (\pi_{b,t} - \alpha_{b,t}^{sd})p_{b,t}^d \\
&\quad + (\pi_{b,t} + \alpha_{b,t}^{sc})p_{b,t}^c \\
&= \sum_{t\in\mathcal{T}} (\alpha_{b,t}^{sd} + \alpha_{b,t}^{sc})\,\epsilon_t \\
&\geq 0
\end{aligned}$$

meaning that the optimal solution must satisfy the complementarity condition. □

Proof of Theorem 3. To establish equivalence, we show the solution of one formulation can be obtained from some solution of the other formulation with the same objective value. To do this, we inspect the linear system (3.17). For convenience, we write the system as $y = Ax$, where $y := [p_b^c, p_b^d]$ captures the solution of (3.15), and $x := [\delta, p_b^{nc}, p_b^{nd}]$ captures the solution of (3.19). Trivially, one can obtain y from an optimal x following (3.17). Thus, for the rest of the proof we focus on how to obtain a solution x given y. Let $\mathcal{X}_y$ be the set of solutions to the system (3.17) given y. Note that matrix A is full row rank as the identity matrix $\mathbf{I}_{2T,2T}$ is a submatrix of A. This implies that given an arbitrary y, the solution set $\mathcal{X}_y$ is non-empty (one trivial solution would be to set $p_b^{nc} = p_b^c$, $p_b^{nd} = p_b^d$).

We now show that there exists $x \in \mathcal{X}_y$ that is feasible to constraints (3.19d)–(3.19g) given y from a feasible solution of (3.15). (3.19f) holds naturally for every $x \in \mathcal{X}_y$ as implied by the fact that y satisfies constraint (3.15f). To show how x can be guaranteed to satisfy other constraints, we write down a procedure of how to find the solution. We first compute the net change of SOC given y:

$$\Delta\text{SOC} := \eta_b^c \sum_{t=1}^{t} p_{b,t}^c - \frac{1}{\eta_b^d} \sum_{t=1}^{t} p_{b,t}^d$$

If $\Delta\text{SOC} = 0$ then the ESR is purely shifting energy within the time window, meaning that we can set $p_{b,t}^{nc} = 0$ and $p_{b,t}^{nd} = 0$. If $\Delta\text{SOC} > 0/\Delta\text{SOC} < 0$ then solution y induces a net-charging/discharging behavior, so some of the $p_{b,t}^{nc}/p_{b,t}^{nd}$ are set with strictly positive values. The values are assigned such that $p_{b,t}^{nd} = 0$ and $\eta_b^c \sum_{t\in\mathcal{T}} p_{b,t}^{nc} = \Delta\text{SOC}$ if $\Delta\text{SOC} > 0$, or $p_{b,t}^{nc} = 0$ and $\frac{1}{\eta_b^d} \sum_{t\in\mathcal{T}} p_{b,t}^{nd} = -\Delta\text{SOC}$ if $\Delta\text{SOC} < 0$. Once $p_{b,t}^{nc}, p_{b,t}^{nd}$ are determined, we are left with a time graph, where each time node is either a source node defined by $p_{b,t}^c - p_{b,t}^{nc}$ or a sink node defined by $p_{b,t}^d - p_{b,t}^{nd}$. By construction, we have that

$$\eta_b \cdot \left(\sum_{t\in\mathcal{T}} p_{b,t}^c - p_{b,t}^{nc} \right) = \sum_{t\in\mathcal{T}} p_{b,t}^d - p_{b,t}^{nd}$$

so the charge into and out of the ESR is balanced. Then, a feasible δ can be found by repetitively pairing source and sink nodes until all nodes have zero net energy. With this procedure, one can verify that the resulting solution x satisfies constraints (3.19d)–(3.19g) by the feasibility of y in (3.15).

To conclude, we show that the new solution gives the same objective value. Let (x, y) be a pair of solutions to $y = Ax$. We have that:

$$\begin{aligned}
&\sum_{t\in\mathcal{T}} \alpha_{b,t}^{sc} p_{b,t}^c + \alpha_{b,t}^{sd} p_{b,t}^d \\
&= \sum_{t\in\mathcal{T}} \alpha_{b,t}^{sc} \left(\sum_{v\in\mathcal{V}_{b,t}^{\text{out}}} \delta_v + p_{b,t}^{nc} \right) + \alpha_{b,t}^{sd} \left(\sum_{v\in\mathcal{V}_{b,t}^{\text{in}}} \eta_{b(v)}\delta_v + p_{b,t}^{nd} \right) \\
&= \sum_{t\in\mathcal{T}} \left(\alpha_{b,t}^{sc} p_{b,t}^{nc} + \alpha_{b,t}^{sd} p_{b,t}^{nd} \right) + \sum_{v\in\mathcal{V}} (\alpha_{b,t}^{sc} + \eta_{b(v)}\alpha_{b,t}^{sd})\delta_v \\
&= \sum_{t\in\mathcal{T}} \left(\alpha_{b,t}^{sc} p_{b,t}^{nc} + \alpha_{b,t}^{sd} p_{b,t}^{nd} \right) + \sum_{v\in\mathcal{V}} \alpha_v^{\delta}\delta_v
\end{aligned}$$

This concludes the proof. □

Appendix B. Table of nomenclature

Type	Notation	Description
Sets	$\mathcal{B}$	Set of ESRs
	$\mathcal{T}$	Discretized time horizon, $\{1, 2, \ldots, T\}$
	$\mathcal{N}$	Set of nodes (buses)
	$\mathcal{S}(\mathcal{S}_n)$	Set of suppliers (connected to node n)
	$\mathcal{D}(\mathcal{D}_n)$	Set of consumers (connected to node n)
	$\mathcal{L}$	Set of transmission lines
	$\mathcal{K}(\mathcal{K}_n^{\text{rec/snd}})$	Set of directed (receiving/sending) transmission lines
	$\mathcal{V}(\mathcal{V}_{n,t}^{\text{in/out}})$	Set of VL (going into/out of space–time node (n,t))
Parameters	$\eta_b^{c/d} \in [0,1]$	Charging/discharging efficiency of ESR b
	$\eta_b \in [0,1]$	Aggregated (round-trip) efficiency of ESR b
	$\underline{s}_b/\bar{s}_b \in \mathbb{R}_+$	Minimum/maximum state of charge of ESR b
	$s_{b,0} \in [\underline{s}_b, \bar{s}_b]$	State of charge of ESR b at time 0
	$\bar{p}_b \in \mathbb{R}_+$	Power capacity of ESR b for charging/discharging
	$\alpha_{i,t}^p \in \mathbb{R}_+$	Bid price of supplier i at time t
	$\bar{p}_{i,t} \in \mathbb{R}_+$	Available capacity of supplier i at time t
	$\alpha_{j,t}^d \in \mathbb{R}_+$	Bid price of consumer j at time t
	$\bar{d}_{j,t} \in \mathbb{R}_+$	Available capacity of consumer j at time t
	$\alpha_{k,t}^f \in \mathbb{R}_+$	Bid price of directed transmission line k at time t
	$\bar{f}_{k,t} \in \mathbb{R}_+$	Available capacity of directed transmission line k at time t
	$\bar{\theta}_{k,t} \in \mathbb{R}_+$	Phase angle limit across directed transmission line k at time t
	$\alpha_{b,t}^{sc/sd} \in \mathbb{R}_+$	Charging/discharging bid price of ESR b at time t
	$\alpha_v^{\delta} \in \mathbb{R}_+$	Bid price of VL v

Type	Notation	Description
Variables	$s_{b,t} \in \mathbb{R}_+$	State of charge of ESR b at time t
	$p_{b,t}^{c/d} \in \mathbb{R}_+$	Charging/discharging power of ESR b at time t
	$p_{i,t} \in \mathbb{R}_+$	Cleared allocation for supplier i at time t
	$d_{j,t} \in \mathbb{R}_+$	Cleared allocation for consumer j at time t
	$f_{k,t} \in \mathbb{R}_+$	Cleared allocation for directed transmission line k at time t
	$\theta_{n,t} \in \mathbb{R}$	Phase angle of node n at time t
	$\pi_{n,t} \in \mathbb{R}$	Local marginal price of node n at time t
	$\phi_{i,t}^p \in \mathbb{R}$	Profit function of supplier i at time t
	$\phi_{j,t}^d \in \mathbb{R}$	Profit function of consumer j at time t
	$\phi_{k,t}^f \in \mathbb{R}$	Profit function of directed transmission line k at time t
	$p_{b,t}^{nc/nd} \in \mathbb{R}_+$	Net-charging/net-discharging power of ESR b at time t
	$\delta_v \in \mathbb{R}_+$	Cleared allocation of VL v

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