

Dynamic Event-Triggered State Estimation for Markov Jump Neural Networks With Partially Unknown Probabilities

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Abstract—This article focuses on the investigation of finite-time dissipative state estimation for Markov jump neural networks. First, in view of the subsistent phenomenon that the state estimator cannot capture the system modes synchronously, the hidden Markov model with partly unknown probabilities is introduced in this article to describe such asynchronization constraint. For the upper limit of network bandwidth and computing resources, a novel dynamic event-triggered transmission mechanism, whose threshold parameter is constructed as an adjustable diagonal matrix, is set between the estimator and the original system to avoid data collision and save energy. Then, with the assistance of Lyapunov techniques, an event-based asynchronous state estimator is designed to ensure that the resulting system is finite-time bounded with a prescribed dissipation performance index. Ultimately, the effectiveness of the proposed estimator design approach combining with a dynamic event-triggered transmission mechanism is demonstrated by a numerical example.

Index Terms—Asynchronous state estimation, dissipativity, dynamic event-triggered mechanism, Markov jump neural networks.

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I. INTRODUCTION

AS A potent information processing modeling pattern, neural networks are widely applied in various fields, including, but not limited to pattern recognition, solving optimization problems, associative memory, and image processing [1]–[3]. In these applications, an interesting problem has been detected, that is, recurrent neural networks cannot reliably catch long-term dependencies, which implies that the network output is less affected by the early input when the temporal sequences increases. Noting that the parameters of neural networks can jump in a set of stochastic finite modes, Tino *et al.* [4] have investigated and proposed an effective network framework named Markov jump neural networks to deal with the above problem. Because of the excellent ability to model various complex systems, Markov jump neural networks have attracted considerable attention from researchers, and a wealth of significant results have been reported in [5]–[8]. The issue of extended dissipativity analysis has been studied in [5], and a novel Lyapunov functional has been further proposed to deal with time delay. To handle the impact of an uncertain factor on Markov jump neural networks, the piecewise time-varying transition probabilities have been studied in [8].

On the other hand, most existing investigations of Markov jump neural networks are based on an ideal assumption that system modes can be fully accessed by filter/state estimator/controller such that the filter/state estimator/controller modes can keep working synchronously with system modes [9]–[11]. As a matter of fact, practical systems are inevitably affected by some disturbances, such as communication delays, data dropouts, and missing measurements, which may lead to losing the mode information of systems. Thus, it is significant to study filtering/state estimation/control for Markov jump systems with asynchronous activities [12]–[14]. In [12], the design method of the asynchronous controller has been extended to nonlinear Markov jump systems in the continuous-time domain. Shen *et al.* [13] have considered more practical circumstances that modes of the filter and quantizer are both asynchronous with that of Markov jump neural networks. However, the cases of asynchronous activities with unknown probabilities are rarely considered in the state estimator design for Markov jump neural networks.

All the above literature deals with the control/state estimation problem in the infinite-time interval. From the perspective

of practical engineering, the states of control systems are always expected to be convergent over a fixed time interval, which signifies that achieving the control effect after a long time is almost meaningless [15], [16]. Therefore, the finite-time analyses of the system performance have been actively investigated recently, such as finite-time synchronization [17], command filtered backstepping control [18], and finite-time stability [19]. With the help of finite-time control technique, a new error compensation mechanism has been proposed in [18] such that the stability of the closed-loop systems could be guaranteed with higher tracking precision and faster response. These significant results also motivate us to research the asynchronous state estimation problem in the finite-time interval.

In the above state estimation problems, the common communication strategy is that all the system measurements are transmitted by the time-triggered mechanism, which means that the measurements must be sent to the estimator within a fixed period, whether necessary or not. However, in practice, both network bandwidth and the energy of battery-based sensors are usually limited [20]–[22]; therefore, transferring data frequently will inevitably occupy more network resources and result in a waste of finite electricity. Under this circumstance, the event-triggered mechanism, which can effectively reduce the frequency of data transmission by an event-triggered condition, has gained considerable attention in recent years [23]–[28]. In [25], the tracking controller with an event-triggered scheme is able to effectively reduce the communication burden while guaranteeing the property of finite-time convergence. By utilizing the impact of transmission delay, a novel distribution-based event-triggered scheme has been established in [28], which is helpful to reduce the energy cost. However, the above event-triggered mechanism cannot dynamically adapt to the operation of the system because its triggered threshold is a prior predetermined constant. In contrast to a usual event-triggered strategy with a fixed threshold, the self-triggered strategies used in [29] and [30] are able to adjust task periods as the variation of the system by precomputing the next update time. However, self-triggered conditions are more conservative than the event-triggered ones, and more constraints on the system structure are needed for the design or implementation of controllers in the self-triggered form [31]. As a result, the dynamic event-triggered strategy has received increasing attention recently. Based on a conventional event-triggered mechanism, the dynamic event-triggered strategy has been investigated in [32], which is able to further save the communication resources with an additional dynamical variable. Because of some unavoidable imperfections, such as communication delays and sporadic measurements, two customized dynamic event-triggered strategies have been designed in [33] for updating the controller such that the desired event-triggered control system is stable. In [34], the state estimators under dynamic event-triggered schemes have been presented for artificial neural networks. However, for the Markov jump neural networks, the dynamic event-based state estimation issues have not been fully discussed.

Based on the aforementioned observations, this article is interested in the asynchronous state estimation problem for

Markov jump neural networks, where the measurements from the original system to the estimator are monitored by a dynamic event-triggered transmission mechanism. With the help of the Lyapunov analysis approaches, an asynchronous event-based state estimator is designed to guarantee that the researched system reaches the expected dissipation performance in the finite-time interval. To demonstrate the feasibility and advantage of the proposed estimation method, an illustrative example is presented at the end. One thing worth noticing is that considering the asynchronous phenomenon and finite-time constraint may increase the complexity of designing desired state estimator. The primary contributions are listed as follows.

- 1) By refactoring the triggered threshold, a novel dynamic event-triggered strategy for Markov neural networks is proposed, which has the potential to strike a tradeoff between reducing transmission burden and improving system performance.
- 2) A hidden Markov model with partially inaccessible probabilities is introduced in this article to deal with the asynchronization constraint between the systems and the estimator.
- 3) For Markov jump neural networks, the state estimation problem in finite time is handled by combining with both asynchronous behavior and event-triggered mechanism in the view of practical utilization.

Notations: $l_2[0, \infty)$ is the space of square-summable vector functions over $[0, \infty)$. “*” denotes the term that is induced by symmetry, and $\text{diag}\{\dots\}$ stands for a block-diagonal matrix. $\Pr\{\cdot\}$ stands for the probability. Besides, $\mathcal{E}\{x\}$ and $\mathcal{E}\{x|y\}$ will, respectively, mean the expectation of x and the expectation of x condition on y .

II. PRELIMINARY ANALYSIS

A. Plant Description

Let $\mathcal{G} \triangleq \{1, 2, \dots, G\}$ be a finite set; the Markov process can be described as the parameter $\{\phi(k) \in \mathcal{G}, k \geq 0\}$. For $\forall \mu, v \in \mathcal{G}$, the related transition probability matrix $\Omega_1 = \{\varphi_{\mu v}\}$ subjects to

$$\Pr\{\phi(k+1) = v | \phi(k) = \mu\} = \varphi_{\mu v} \quad (1)$$

where $\varphi_{\mu v} \in [0, 1]$ and $\sum_{v=1}^G \varphi_{\mu v} = 1$.

Then, we consider the Markov jump neural networks with G modes defined by

$$\begin{cases} x(k+1) = A_{\phi(k)}x(k) + D_{\phi(k)}f(x(k)) + M_{1\phi(k)}o(k) \\ y(k) = B_{\phi(k)}x(k) + M_{2\phi(k)}o(k) \\ z(k) = C_{\phi(k)}x(k) \end{cases} \quad (2)$$

where the vectors $x(k) \in \mathbb{R}^{n_x}$, $y(k) \in \mathbb{R}^{n_y}$, and $z(k) \in \mathbb{R}^{n_z}$ are the neural state, the measurement output, and the output signal to be estimated. The external disturbance $o(k) \in \mathbb{R}^{n_o}$ belongs to $l_2[0, \infty)$. The matrices $A_{\phi(k)} \in \mathbb{R}^{n_x \times n_x}$, $D_{\phi(k)} \in \mathbb{R}^{n_x \times n_o}$, $M_{1\phi(k)} \in \mathbb{R}^{n_x \times n_o}$, $B_{\phi(k)} \in \mathbb{R}^{n_y \times n_x}$, $M_{2\phi(k)} \in \mathbb{R}^{n_y \times n_o}$, and $C_{\phi(k)} \in \mathbb{R}^{n_z \times n_x}$ are known for every $\phi(k)$. Especially, $A_{\phi(k)} \triangleq \text{diag}\{a_{1,\phi(k)}, \dots, a_{n_x,\phi(k)}\}$ and $D_{\phi(k)}$ denotes the connection weight matrix. The functions

$f(x(k)) = [f_1(x_1(k)) \dots f_{n_x}(x_{n_x}(k))]^T$ represent neuron activation functions of the neural networks.

Furthermore, to describe the asynchronous activities between the estimated system and the estimator, a hidden Markov chain $\psi(k)$ is introduced, which belongs to $\mathcal{H} = \{1, 2, 3, \dots, H\}$. $\forall \mu \in \mathcal{G}$ and $\forall i \in \mathcal{H}$, the related conditional probability matrix is $\Omega_2 = \{\tau_{\mu i}\}$, and $\psi(k)$ subjects to

$$\Pr\{\psi(k) = i | \phi(k) = \mu\} = \tau_{\mu i} \quad (3)$$

where $\tau_{\mu i} \in [0, 1]$ and $\sum_{i=1}^H \tau_{\mu i} = 1$. It is worth pointing out that, in this article, we allow some conditional probabilities $\tau_{\mu i}$ to be unknown. In other words, the conditional probability matrix Ω_2 is partly accessible, which can be described as following form:

$$\Omega_2 = \begin{bmatrix} \tau_{11} & ? & ? & \tau_{14} \\ ? & \tau_{22} & ? & \tau_{24} \\ \tau_{31} & ? & \tau_{34} & ? \\ \tau_{41} & \tau_{42} & ? & \tau_{44} \end{bmatrix} \quad (4)$$

where “?” represents that corresponding $\tau_{\mu i}$ is unknown. For the sake of analysis later, we denote $\mathcal{H} = \mathcal{H}_{uk}^\mu \cup \mathcal{H}_k^\mu \forall \mu \in \mathcal{G}$ with

$$\begin{cases} \mathcal{H}_{uk}^\mu \triangleq \{i | \tau_{\mu i} \text{ is unknown} \} \\ \mathcal{H}_k^\mu \triangleq \{i | \tau_{\mu i} \text{ is known} \} \end{cases} \quad (5)$$

and $\bar{\varrho}_\mu^\kappa = \sum_{i \in \mathcal{H}_k^\mu} \tau_{\mu i} \varrho_{\mu i}$, $\tau_\mu^\kappa = \sum_{i \in \mathcal{H}_k^\mu} \tau_{\mu i}$.

Remark 1: Due to the difficulty to obtain the information of the hidden Markov chain completely, modeling the asynchronous behavior by a hidden Markov chain with partially inaccessible probabilities is significant. In this article, three cases of partially inaccessible conditional probability matrices are considered: fully accessible, partly accessible, and almost inaccessible. The impact of different probability accessibility on the resulting system will be analyzed in Section IV.

B. Dynamic Event-Triggered Transmission Mechanism

Reducing the processing of unnecessary data can effectively save the computing resources. For this purpose, a dynamic event-triggered transmission mechanism with a special threshold parameter is employed to decide whether the system output $y(k)$ needs to be transmitted to the state estimator. Denote the current triggered instant as $h_m (m \geq 0)$; the dynamic event-triggered condition can be established as follows:

$$\Gamma(\zeta(k), \rho(k), \sigma_{\psi(k)}) \triangleq \rho^T(k)\rho(k) - y^T(k)\sigma_{\psi(k)}y(k) - \frac{1}{\delta_{\psi(k)}}\zeta(k) \quad (6)$$

where $\rho(k) \triangleq y(k) - y(h_m)$ with the current triggered output $y(h_m)$ and the latest system outputs $y(k)$, $\delta_{\psi(k)}$, and $\zeta(k)$ are known positive scalars and internal dynamic variables, respectively. The threshold $\sigma_{\psi(k)}$ is designed as a diagonal matrix satisfying

$$\begin{cases} \sigma_{\psi(k)} = \text{diag}\{\sigma_{1\psi(k)}, \sigma_{2\psi(k)}, \dots, \sigma_{n_y\psi(k)}\} \\ \sigma_{t\psi(k)} \in [0, 1], \quad t \in \{1, 2, \dots, n_y\}. \end{cases}$$

When the condition $\Gamma(\zeta(k), \rho(k), \sigma_{\psi(k)}) \geq 0$, the triggered output $y(h_m)$ will be updated. Accordingly, the next triggered instant h_{m+1} subjects to

$$h_{m+1} = \inf\{k > h_m | \Gamma(\zeta(k), \rho(k), \sigma_{\psi(k)}) \geq 0\}. \quad (7)$$

Then, the internal dynamic variable $\zeta(k)$ will change with

$$\begin{cases} \zeta(k+1) = \eta_{\psi(k)}\zeta(k) + y^T(k)\sigma_{\psi(k)}y(k) - \rho^T(k)\rho(k) \\ \zeta(0) = \zeta_0 \geq 0 \end{cases} \quad (8)$$

where $\eta_{\psi(k)}$ is a given scalar. It is worth noting that the given parameters $\eta_{\psi(k)}$ and $\delta_{\psi(k)}$ satisfy

$$\eta_{\psi(k)} \in (0, 1), \quad \eta_{\psi(k)}\delta_{\psi(k)} \geq 1. \quad (9)$$

Remark 2: Note that dynamic event-triggered condition (6) is a uniform framework, including two special cases. Setting $\zeta(k) = 0$ or $\delta_{\psi(k)} = \infty$, the dynamic event-triggered condition will reduce to the static one in [35]. Further adjusting the threshold matrix $\sigma_{\psi(k)}$ to approach zero, then all the measurements will be transmitted, which implies that the event-triggered mechanism is converted to the time-triggered mechanism.

Remark 3: In fact, for a complex system, the fluctuations of its subsystems are usually different, and some subsystems may diverge too fast to estimate. For conventional event-triggered mechanism, the scalar triggered threshold, such as the form $\sigma_i \in [0, 1]$ in [22], is determined according to the worst case operating scenario. In this case, the transmission will be triggered more frequently to insure the estimation performance. Thus, the triggered threshold in this article is reconstructed as a diagonal matrix in this article instead of the usual scalar form. By flexibly adjusting the element $\sigma_{t\psi(k)}$ in the threshold matrix $\sigma_{\psi(k)}$ according to the performance requirements of each subsystem, the proposed dynamic triggering mechanism has greater potential to strike a balance between reducing communication burden and improving system performance.

Remark 4: The systems studied in this article are discrete-time systems, and as can be seen from (7), the event-triggered interval is greater than zero [36]. It means that the proposed event-triggered transmission mechanism will not consider Zeno behavior.

C. Event-Based Asynchronous State Estimator

Consider the invalidity and inaccuracy of state measurement, we bring in the following asynchronous state estimator, which can reconstruct the state by using the measurement output:

$$\begin{cases} \tilde{x}(k+1) = \tilde{A}_{\psi(k)}\tilde{x}(k) + \tilde{K}_{\psi(k)}(y(h_m) - B_{\phi(k)}\tilde{x}(k)), \\ \tilde{z}(k) = \tilde{C}_{\psi(k)}\tilde{x}(k) \end{cases} \quad (10)$$

where $k \in [h_m, h_{m+1})$, $\tilde{x}(k) \in \mathbb{R}^{n_x}$ is the state estimation, and $\tilde{z}(k) \in \mathbb{R}^{n_z}$ is the estimation of $z(k)$. The state estimator matrices $\tilde{A}_{\psi(k)}$, $\tilde{K}_{\psi(k)}$, and $\tilde{C}_{\psi(k)}$ are to be confirmed later.

D. State Estimation Error System

For convenience, the subscripts μ , ν , and ι are used in place of the parameters $\phi(k)$, $\phi(k+1)$, and $\psi(k)$, respectively. By integrating (2) and (10), an error system for the state estimator is derived as

$$\begin{cases} \bar{x}(k+1) = \bar{A}_{\mu i} \bar{x}(k) + \bar{D}_{\mu} f(x(k)) \\ \quad - \bar{K}_i \rho(k) + \bar{M}_{\mu i} o(k) \\ \bar{z}(k) = \bar{C}_{\mu i} \bar{x}(k) \end{cases} \quad (11)$$

where

$$\begin{aligned} \bar{x}(k) &= [x^T(k) \quad \bar{x}^T(k)]^T, \quad \bar{z}(k) = z(k) - \tilde{z}(k), \\ \bar{A}_{\mu i} &= \begin{bmatrix} A_{\mu} & 0 \\ \bar{K}_i B_{\mu} & \bar{A}_i - \bar{K}_i B_{\mu} \end{bmatrix}, \quad \bar{M}_{\mu i} = \begin{bmatrix} M_{1\mu} \\ \bar{K}_i M_{2\mu} \end{bmatrix} \\ \bar{D}_{\mu} &= [D_{\mu}^T \quad 0]^T, \quad \bar{K}_i = [0 \quad \bar{K}_i^T]^T, \quad \bar{C}_{\mu i} = [C_{\mu} \quad -\bar{C}_i]. \end{aligned}$$

Up until now, the preliminary analyses of the researched neural networks and the correlative state estimator have been completed. To facilitate the derivation of theorems later, some critical definitions and lemmas are needed.

Definition 1 [16]: For a positive definite matrix Υ and some scalars $0 < e_1 < e_3$, $0 < e_2$, and $\mathcal{T} \in N^+$, system (11) is stochastically finite-time bounded regarding $(e_1, e_2, e_3, \Upsilon, \mathcal{T})$ if the following relation holds:

$$\begin{cases} \sum_{k=0}^{\mathcal{T}} o^T(k) o(k) \leq e_2^2 \\ \bar{x}^T(0) \Upsilon \bar{x}(0) \leq e_1^2 \Rightarrow \mathcal{E}\{\bar{x}^T(k) \Upsilon \bar{x}(k)\} \leq e_3^2. \end{cases} \quad (12)$$

To further derive the definition of dissipativity, the energy supply function is introduced, which is described as

$$\begin{cases} J(U, \bar{z}(k), o(k)) = \mathcal{E}\left\{\sum_{k=0}^U \beta^{-k} r(\bar{z}(k), o(k))\right\}, \\ r(\bar{z}(k), o(k)) = \bar{z}^T(k) \Lambda_1 \bar{z}(k) + 2\bar{z}^T(k) \Lambda_2 o(k) \\ \quad + o^T(k) \Lambda_3 o(k) \end{cases} \quad (13)$$

where $\beta > 1$ and $U \in N^+$. The real matrices Λ_1 , Λ_2 , and Λ_3 are with appropriate dimensions and satisfy $\Lambda_1 = \Lambda_1^T \leq 0$ and $\Lambda_3^T = \Lambda_3$. To facilitate later derivation, let Λ_{1+} be a positive semidefinite matrix and $-(\Lambda_{1+})^2 = \Lambda_1$.

Definition 2 [37]: System (11) is stochastically finite-time exponential dissipative regarding $(e_1, e_2, e_3, \Upsilon, \mathcal{T}, \Lambda_1, \Lambda_2, \Lambda_3)$ if the following requirements are achieved.

C1: System (11) is finite-time bounded regarding $(e_1, e_2, e_3, \Upsilon, \mathcal{T})$.

C2: For every $U > 0$ and a given scalar $\bar{\gamma} > 0$, the estimation error $\bar{z}(k)$ in system (11) satisfies

$$J(U, \bar{z}(k), o(k)) \geq \bar{\gamma} \sum_{k=0}^U o^T(k) o(k) \quad (14)$$

under the zero initial condition.

Lemma 1 [38]: For a scalar ε and nonsingular matrix W , the positive definite matrix Q satisfies

$$\begin{aligned} (W - \varepsilon Q)Q^{-1}(W - \varepsilon Q)^T &\geq 0 \\ \Leftrightarrow \varepsilon^2 Q - \varepsilon W^T - \varepsilon W &\geq -WQ^{-1}W^T. \end{aligned} \quad (15)$$

Lemma 2 [39]: For the dynamic law (8), the internal dynamic variable $\zeta(k) \geq 0$ holds for all nonnegative integers k if the given scalars $\delta_{\psi(k)}$ and $\eta_{\psi(k)}$ satisfy the precondition (9).

Lemma 3 [40]: For the neuron activation functions $f(x(k))$ in (2), if the following two assumptions are satisfied, the following holds.

Assumption 1: $f(x(k))$ is continuous and $f(0) = 0$.

Assumption 2: There exist constant diagonal matrices S_1 and S_2 with appropriate dimensions such that

$$S_1 \leq \frac{f(x_1) - f(x_2)}{x_1 - x_2} \leq S_2 \quad \forall x_1 \neq x_2, \quad x_1, x_2 \in \mathbb{R}^{n_x}.$$

Then, it holds that

$$\begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix}^T \begin{bmatrix} \tilde{S}_1 \Theta_{\mu i} & \tilde{S}_2 \Theta_{\mu i} \\ * & \Theta_{\mu i} \end{bmatrix} \begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix} \leq 0 \quad (16)$$

where $\Theta_{\mu i} = \text{diag}\{\Theta_{1\mu i}, \dots, \Theta_{n_x \mu i}\}$ is positive definite, and

$$\tilde{S}_1 = S_1 S_2, \quad \tilde{S}_2 = -\frac{S_1 + S_2}{2}.$$

III. MAIN RESULTS

Considering the asynchronous constraint, a dynamic event-triggered state estimator will be designed with the form (10) for system (2). Meanwhile, a sufficient condition, which can ensure the expected property for system (11), will be deduced with the aid of the Lyapunov function.

Theorem 1: Given a scalar β ($\beta > 1$) and a diagonal matrix σ_i , system (11) is stochastically finite-time exponential dissipative regarding $(e_1, e_2, e_3, \Upsilon, \mathcal{T}, \Lambda_1, \Lambda_2, \Lambda_3)$ if there exist positive numbers α and γ and positive definite matrices P_{μ} , $Q_{\mu i}$, and O_{μ} such that, for any $\mu \in \mathcal{G}$, the following inequalities hold.

$$\bar{Q}_{\mu}^{\mathcal{K}} + (1 - \tau_{\mu}^{\mathcal{K}}) Q_{\mu i} < P_{\mu}, \quad \iota \in \mathcal{H}_{uk}^{\mu} \quad (17)$$

$$\Xi \triangleq \begin{bmatrix} \Xi_1 & \Xi_2 \\ * & \Xi_3 \end{bmatrix} < 0, \quad \iota \in \mathcal{H} \quad (18)$$

$$\begin{aligned} \max_{\mu \in \mathcal{G}} \{\lambda_{\max}(\tilde{P}_{\mu})\} e_1^2 + \frac{\zeta_0}{\delta} + \max_{\mu \in \mathcal{G}} \{\lambda_{\max}(O_{\mu})\} e_2^2 \\ < \frac{e_3^2 \min_{\mu \in \mathcal{G}} \{\lambda_{\min}(\tilde{P}_{\mu})\}}{\beta^{\mathcal{T}}} \end{aligned} \quad (19)$$

where $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ represent the maximum and minimum eigenvalues of a real matrix, respectively. Furthermore

$$\begin{aligned} \Xi_1 &= \begin{bmatrix} -\bar{P}_{\mu}^{-1} & 0 \\ * & -I \end{bmatrix}, \quad \tilde{P}_{\mu} \triangleq \Upsilon^{-\frac{1}{2}} P_{\mu} \Upsilon^{-\frac{1}{2}}, \\ \Xi_2 &= \begin{bmatrix} \bar{A}_{\mu i} & \bar{D}_{\mu} & -\bar{K}_i & 0 & \bar{M}_{\mu i} & \bar{M}_{\mu i} \\ \Lambda_{1+} \bar{C}_{\mu i} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \Xi_3 &\triangleq \begin{bmatrix} \Xi_3^{11} & \Xi_3^{12} & 0 & 0 & \Xi_3^{15} \\ * & -\Theta_{\mu i} & 0 & 0 & 0 \\ * & * & \Xi_3^{33} & 0 & 0 \\ * & * & * & \Xi_3^{44} & 0 \\ * & * & * & * & \Xi_3^{55} \end{bmatrix} \end{aligned}$$

$$\Xi_3^{11} = -\beta Q_{\mu i} - L^T \tilde{S}_1 \Theta_{\mu i} L + \left(\frac{1}{\delta_i} + \alpha\right) \bar{B}_{\mu}^T \sigma_i \bar{B}_{\mu},$$

$$L = [I \quad 0], \quad \bar{B}_{\mu} = [B_{\mu} \quad 0], \quad \bar{P}_{\mu} = \sum_{v=1}^G \varphi_{\mu v} P_v$$

$$\begin{aligned}
\Xi_3^{12} &= -L^T \tilde{S}_2 \Theta_{\mu i}, \quad \Xi_3^{33} = -\left(\frac{1}{\delta_i} + \alpha\right), \quad \Xi_3^{44} = \frac{\eta_i - \beta + \alpha}{\delta_i} \\
\Xi_3^{15} &= \left[\left(\frac{1}{\delta_i} + \alpha\right) \bar{B}_\mu^T \sigma_i M_{2\mu} - \bar{C}_{\mu i}^T + \left(\frac{1}{\delta_i} + \alpha\right) \bar{B}_\mu^T \sigma_i M_{2\mu} \right] \\
\Xi_3^{55} &= \begin{bmatrix} \Xi_3^{55(1)} & 0 \\ * & \Xi_3^{55(2)} \end{bmatrix} \\
\Xi_3^{55(1)} &= -O_\mu + \left(\frac{1}{\delta_i} + \alpha\right) M_{2\mu}^T \sigma_i M_{2\mu} \\
\Xi_3^{55(2)} &= -\Lambda_3 + \gamma I + \left(\frac{1}{\delta_i} + \alpha\right) M_{2\mu}^T \sigma_i M_{2\mu}.
\end{aligned}$$

Proof: In the first place, we assume that the matrices $\varrho_{\mu i}$ satisfy the following inequality:

$$\sum_{i=1}^H \tau_{\mu i} \varrho_{\mu i} < P_\mu. \quad (20)$$

Noting that the conditional probabilities $\tau_{\mu i}$ are not completely known, one has

$$\begin{aligned}
&\sum_{i=1}^H \tau_{\mu i} \varrho_{\mu i} - P_\mu \\
&= \sum_{i \in \mathcal{H}_k^\mu} \tau_{\mu i} \varrho_{\mu i} + \sum_{i \in \mathcal{H}_{uk}^\mu} \tau_{\mu i} \varrho_{\mu i} - P_\mu \\
&= \bar{\varrho}_\mu^\kappa + (1 - \tau_\mu^\kappa) \sum_{i \in \mathcal{H}_{uk}^\mu} \frac{\tau_{\mu i}}{(1 - \tau_\mu^\kappa)} \varrho_{\mu i} - P_\mu \\
&= \sum_{i \in \mathcal{H}_{uk}^\mu} \frac{\tau_{\mu i}}{(1 - \tau_\mu^\kappa)} [\bar{\varrho}_\mu^\kappa + (1 - \tau_\mu^\kappa) \varrho_{\mu i} - P_\mu] \quad (21)
\end{aligned}$$

with $\sum_{i \in \mathcal{H}_{uk}^\mu} (\tau_{\mu i} / (1 - \tau_\mu^\kappa)) = 1$. In the aforementioned derivation, an equivalence between (20) and (17) is established such that the assumption (20) holds, which will be used later.

In what follows, the finite-time boundedness of system (11) will be analyzed based on the following Lyapunov function:

$$V(k) = \bar{x}^T(k) P_\mu \bar{x}(k) + \frac{1}{\delta_i} \zeta(k). \quad (22)$$

Consider the condition (16) in Lemma 3 and the event-triggered scheme with $\Gamma(\zeta(k), \rho(k), \sigma_\psi(k)) < 0$, the following derivation can be obtained by S-Procedure [41]:

$$\begin{aligned}
&\mathcal{E}\{V(k+1) - \beta V(k) - o^T(k) O_\mu o(k)\} \\
&< \mathcal{E}\{V(k+1) - \beta V(k) - o^T(k) O_\mu o(k)\} \\
&\quad - \alpha \mathcal{E}\{\Gamma(\zeta(k), \rho(k), \sigma_\psi(k))\} \\
&\quad - \mathcal{E}\left\{ \begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix}^T \begin{bmatrix} \tilde{S}_1 \Theta_{\mu i} & \tilde{S}_2 \Theta_{\mu i} \\ * & \Theta_{\mu i} \end{bmatrix} \begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix} \right\} \\
&= \sum_{i=1}^H \tau_{\mu i} (\bar{x}^T(k+1) \bar{P}_\mu \bar{x}(k+1) - \beta \bar{x}^T(k) P_\mu \bar{x}(k) \\
&\quad + \frac{1}{\delta_i} (\zeta(k+1) - \beta \zeta(k)) - o^T(k) O_\mu o(k) \\
&\quad - \alpha \left(\rho^T(k) \rho(k) - y^T(k) \sigma_i y(k) - \frac{1}{\delta_i} \zeta(k) \right) \\
&\quad - \left[\begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix}^T \begin{bmatrix} \tilde{S}_1 \Theta_{\mu i} & \tilde{S}_2 \Theta_{\mu i} \\ * & \Theta_{\mu i} \end{bmatrix} \begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix} \right]). \quad (23)
\end{aligned}$$

Recalling the inequality (20), it yields

$$\begin{aligned}
&\mathcal{E}\{V(k+1) - \beta V(k) - o^T(k) O_\mu o(k)\} \\
&< \sum_{i=1}^H \tau_{\mu i} (\bar{x}^T(k+1) \bar{P}_\mu \bar{x}(k+1) - \beta \bar{x}^T(k) \varrho_{\mu i} \bar{x}(k) \\
&\quad + \frac{1}{\delta_i} (\zeta(k+1) - \beta \zeta(k)) - o^T(k) O_\mu o(k) \\
&\quad - \alpha \left(\rho^T(k) \rho(k) - y^T(k) \sigma_i y(k) - \frac{1}{\delta_i} \zeta(k) \right) \\
&\quad - \left[\begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix}^T \begin{bmatrix} \tilde{S}_1 \Theta_{\mu i} & \tilde{S}_2 \Theta_{\mu i} \\ * & \Theta_{\mu i} \end{bmatrix} \begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix} \right]) \\
&= X^T(k) \sum_{i=1}^H \tau_{\mu i} (\bar{\Xi}_2^T \bar{P}_\mu \bar{\Xi}_2 + \bar{\Xi}_3) X(k) \quad (24)
\end{aligned}$$

where

$$\begin{aligned}
X(k) &= \begin{bmatrix} \bar{x}^T(k) & f^T(x(k)) & \rho^T(k) & \zeta^{\frac{1}{2}}(k) & o^T(k) \end{bmatrix}^T \\
\bar{\Xi}_2 &= \begin{bmatrix} \bar{A}_{\mu i} & \bar{D}_\mu & -\bar{K}_i & 0 & \bar{M}_{\mu i} \end{bmatrix} \\
\bar{\Xi}_3 &\triangleq \begin{bmatrix} \Xi_3^{11} & \Xi_3^{12} & 0 & 0 & \Xi_3^{15} \\ * & -\Theta_{\mu i} & 0 & 0 & 0 \\ * & * & \Xi_3^{33} & 0 & 0 \\ * & * & * & \Xi_3^{44} & 0 \\ * & * & * & * & \Xi_3^{55} \end{bmatrix} \\
\bar{\Xi}_3^{15} &= \left(\frac{1}{\delta_i} + \alpha\right) \bar{B}_\mu^T \sigma_i M_{2\mu} \\
\bar{\Xi}_3^{55} &= -O_\mu + \left(\frac{1}{\delta_i} + \alpha\right) M_{2\mu}^T \sigma_i M_{2\mu}.
\end{aligned}$$

Using the Schur complement, it is obvious that the inequality (18) is equivalent to $\bar{\Xi}_2^T \bar{P}_\mu \bar{\Xi}_2 + \bar{\Xi}_3 < 0$, which leads to

$$\mathcal{E}\{V(k+1)\} < \beta V(k) + o^T(k) O_\mu o(k). \quad (25)$$

Subsequently, through employing successive substitutions of (25), it can be obtained that

$$\begin{aligned}
&\mathcal{E}\{V(k)\} \\
&< \beta V(k-1) + \max_{\mu \in G} \{\lambda_{\max}(O_\mu)\} o^T(k-1) o(k-1) \\
&< \beta^k V(0) + \max_{\mu \in G} \{\lambda_{\max}(O_\mu)\} \sum_{t=0}^{k-1} \beta^{k-t-1} o^T(t) o(t) \\
&< \beta^T \left[V(0) + \max_{\mu \in G} \{\lambda_{\max}(O_\mu)\} \sum_{t=0}^T o^T(t) o(t) \right] \\
&< \beta^T \left[\max_{\mu \in G} \{\lambda_{\max}(\tilde{P}_\mu)\} e_1^2 + \frac{\zeta_0}{\delta} + \max_{\mu \in G} \{\lambda_{\max}(O_\mu)\} e_2^2 \right] \quad (26)
\end{aligned}$$

where $\tilde{P}_\mu \triangleq \Upsilon^{-1/2} P_\mu \Upsilon^{-1/2}$ and $k \leq T$. Recalling the conditions $\zeta(k) \geq 0$ and $\delta_i > 0$ in Lemma 2, one gets

$$\begin{aligned}
\mathcal{E}\{V(k)\} &= \mathcal{E}\left\{ \bar{x}^T(k) P_\mu \bar{x}(k) + \frac{1}{\delta_i} \zeta(k) \right\} \\
&> \mathcal{E}\{ \bar{x}^T(k) P_\mu \bar{x}(k) \} \\
&> \min_{\mu \in G} \{\lambda_{\min}(\tilde{P}_\mu)\} \mathcal{E}\{ \bar{x}^T(k) \Upsilon \bar{x}(k) \}. \quad (27)
\end{aligned}$$

Combining with inequalities (26) and (27), one can derive $\mathcal{E}\{ \bar{x}^T(k) \Upsilon \bar{x}(k) \}$

$$< \frac{\beta^T [\max_{\mu \in G} \{\lambda_{\max}(\tilde{P}_\mu)\} e_1^2 + \frac{\zeta_0}{\delta} + \max_{\mu \in G} \{\lambda_{\max}(O_\mu)\} e_2^2]}{\min_{\mu \in G} \{\lambda_{\min}(\tilde{P}_\mu)\}}. \quad (28)$$

From the inequality (1), it can be straightforward concluded that $\mathcal{E}\{\bar{x}^T(k) \Upsilon \bar{x}(k)\} < e_2^2$, which signifies that system (11) is stochastically finite-time bounded regarding $(e_1, e_2, e_3, \Upsilon, T)$.

Now, one is in a position to analyze the dissipativity of system (11) with zero initial conditions. Considering the supply rate $r(\bar{z}(k), o(k))$ and inequality (24), the following function can be established:

$$\mathcal{F} \triangleq \mathcal{E}\{V(k+1) - \beta V(k) - \bar{z}^T(k) \Lambda_1 \bar{z}(k) - 2\bar{z}^T(k) \Lambda_2 o(k) - o^T(k) (\Lambda_3 - \gamma I) o(k)\}. \quad (29)$$

Via the similar derivation as that for (23) and (24), it is not difficult to get

$$\begin{aligned} \mathcal{F} &< \sum_{i=1}^H \tau_{\mu i} (\bar{x}^T(k+1) \tilde{P}_\mu \bar{x}(k+1) - \beta \bar{x}^T(k) \varrho_{\mu i} \bar{x}(k) \\ &\quad + \frac{1}{\delta_i} (\zeta(k+1) - \beta \zeta(k)) - \bar{z}^T(k) \Lambda_1 \bar{z}(k) \\ &\quad - 2\bar{z}^T(k) \Lambda_2 o(k) - o^T(k) (\Lambda_3 - \gamma I) o(k) \\ &\quad - \alpha \left(\rho^T(k) \rho(k) - y^T(k) \sigma_i y(k) - \frac{1}{\delta_i} \zeta(k) \right)) \\ &\quad - \begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix}^T \begin{bmatrix} \tilde{S}_1 \Theta_{\mu i} & \tilde{S}_2 \Theta_{\mu i} \\ * & \Theta_{\mu i} \end{bmatrix} \begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix} \\ &= X^T(k) \sum_{i=1}^H \tau_{\mu i} (\tilde{\Xi}_2^T \tilde{P}_\mu \tilde{\Xi}_2 - F_2^T \Lambda_1 F_2 + F_3) X(k) \end{aligned} \quad (30)$$

where

$$\begin{aligned} F_2 &= [\bar{C}_{\mu i} \quad 0 \quad 0 \quad 0 \quad 0] \\ F_3 &\triangleq \begin{bmatrix} \Xi_3^{11} & \Xi_3^{12} & 0 & 0 & F_3^{15} \\ * & -\Theta_{\mu i} & 0 & 0 & 0 \\ * & * & \Xi_3^{33} & 0 & 0 \\ * & * & * & \Xi_3^{44} & 0 \\ * & * & * & * & F_3^{55} \end{bmatrix} \\ F_3^{15} &= -\bar{C}_{\mu i}^T + \left(\frac{1}{\delta_i} + \alpha \right) \bar{B}_\mu^T \sigma_i M_{2\mu} \\ F_3^{55} &= -\Lambda_3 + \gamma I + \left(\frac{1}{\delta_i} + \alpha \right) M_{2\mu}^T \sigma_i M_{2\mu}. \end{aligned}$$

By employing the Schur complement to (18), $\tilde{\Xi}_2^T \tilde{P}_\mu \tilde{\Xi}_2 - F_2^T \Lambda_1 F_2 + F_3 < 0$ holds, which means that

$$\mathcal{E}\{V(k+1)\} < \mathcal{E}\{\beta V(k) + r(\bar{z}(k), o(k)) - \gamma o^T(k) o(k)\}. \quad (31)$$

Under zero initial conditions, namely, $\bar{x}(0) = 0$ and $\zeta_0 = 0$, one further has

$$\mathcal{E}\{V(U+1)\} < \mathcal{E}\left\{ \sum_{k=0}^U \beta^{U-k} [r(\bar{z}(k), o(k)) - \gamma o^T(k) o(k)] \right\}. \quad (32)$$

Noting that $V(U+1) > 0$, it follows that

$$\begin{aligned} &\gamma \sum_{k=0}^U \beta^{U-k} o^T(k) o(k) \\ &< \mathcal{E}\left\{ \sum_{k=0}^U \beta^{U-k} r(\bar{z}(k), o(k)) \right\} \\ &\Rightarrow \gamma \sum_{k=0}^U \beta^{-k} o^T(k) o(k) < \mathcal{E}\left\{ \sum_{k=0}^U \beta^{-k} r(\bar{z}(k), o(k)) \right\} \\ &\Rightarrow \gamma \beta^{-U} \sum_{k=0}^U o^T(k) o(k) < \mathcal{E}\left\{ \sum_{k=0}^U \beta^{-k} r(\bar{z}(k), o(k)) \right\}. \end{aligned} \quad (33)$$

Letting $\bar{\gamma} = \gamma \beta^{-U}$, it is intuitive to find that the inequality (33) satisfies the condition (C2) in Definition 2. In conclusion, system (11) is of stochastically finite-time exponential dissipativity regarding $(e_1, e_2, e_3, \Upsilon, T, \Lambda_1, \Lambda_2, \Lambda_3)$, which finishes the proof. ■

Now, based on the discussion of Theorem 1, the design approach of the event-based asynchronous state estimator will be provided by introducing the following slack matrix:

$$W_{\mu i} = \begin{bmatrix} W_{1\mu i} & W_{3i} \\ W_{2\mu i} & W_{3i} \end{bmatrix}. \quad (34)$$

Theorem 2: Given scalars ε and β ($\beta > 1$) and a diagonal matrices σ_i , system (11) is stochastically finite-time exponential dissipative regarding $(e_1, e_2, e_3, \Upsilon, T, \Lambda_1, \Lambda_2, \Lambda_3)$ if there exist positive numbers $\alpha, \theta_1, \theta_2, \theta_3$, and γ and matrices $P_\mu = \begin{bmatrix} P_{1\mu} & P_{2\mu} \\ * & P_{3\mu} \end{bmatrix} > 0$, $\varrho_{\mu i} = \begin{bmatrix} \varrho_{1\mu i} & \varrho_{2\mu i} \\ * & \varrho_{3\mu i} \end{bmatrix} > 0$, $O_\mu > 0$, $\tilde{\mathcal{A}}_i$, $\tilde{\mathcal{K}}_i$, $\tilde{\mathcal{C}}_i$, and $W_{\mu i}$ such that, for any $\mu \in \mathcal{G}$, the inequality (17) and the following conditions hold:

$$\tilde{\Xi} \triangleq \begin{bmatrix} \tilde{\Xi}_1 & \tilde{\Xi}_2 \\ * & \tilde{\Xi}_3 \end{bmatrix} < 0, \quad i \in \mathcal{H} \quad (35)$$

$$\theta_1 \Upsilon < P_\mu < \theta_2 \Upsilon, \quad O_\mu < \theta_3 I \quad (36a)$$

$$\theta_2 e_1^2 + \frac{\zeta_0}{\delta_i} + \theta_3 e_2^2 < \frac{\theta_1 e_3^2}{\beta^T} \quad (36b)$$

where

$$\begin{aligned} \tilde{\Xi}_1 &= \begin{bmatrix} \varepsilon^2 \tilde{P}_\mu - \varepsilon W_{\mu i}^T - \varepsilon W_{\mu i} & 0 \\ * & -I \end{bmatrix}, \quad \tilde{\Xi}_2 = \begin{bmatrix} \tilde{\Xi}_2^{11} & \tilde{\Xi}_2^{12} \\ \tilde{\Xi}_2^{21} & \tilde{\Xi}_2^{22} \end{bmatrix} \\ \tilde{\Xi}_2^{11} &= \begin{bmatrix} W_{1\mu i} A_\mu + \tilde{\mathcal{K}}_i B_\mu & \tilde{\mathcal{A}}_i - \tilde{\mathcal{K}}_i B_\mu & W_{1\mu i} D_\mu & -\tilde{\mathcal{K}}_i \\ W_{2\mu i} A_\mu + \tilde{\mathcal{K}}_i B_\mu & \tilde{\mathcal{A}}_i - \tilde{\mathcal{K}}_i B_\mu & W_{2\mu i} D_\mu & -\tilde{\mathcal{K}}_i \end{bmatrix} \\ \tilde{\Xi}_2^{12} &= \begin{bmatrix} 0 & W_{1\mu i} M_{1\mu} + \tilde{\mathcal{K}}_i M_{2\mu} & W_{1\mu i} M_{1\mu} + \tilde{\mathcal{K}}_i M_{2\mu} \\ 0 & W_{2\mu i} M_{1\mu} + \tilde{\mathcal{K}}_i M_{2\mu} & W_{2\mu i} M_{1\mu} + \tilde{\mathcal{K}}_i M_{2\mu} \end{bmatrix} \\ \tilde{\Xi}_2^{21} &= [\Lambda_{1+} C_\mu \quad -\Lambda_{1+} \tilde{\mathcal{C}}_i \quad 0 \quad 0]. \end{aligned}$$

In addition, the parameters in state estimator (10) can be obtained by

$$\tilde{\mathcal{A}}_i = W_{3i}^{-1} \tilde{\mathcal{A}}_i, \quad \tilde{\mathcal{K}}_i = W_{3i}^{-1} \tilde{\mathcal{K}}_i, \quad \tilde{\mathcal{C}} = \tilde{\mathcal{C}}_i. \quad (37)$$

Proof: According to Lemma 1, the positive definite matrix $\tilde{P}_\mu$ satisfies $-W_{\mu i} \tilde{P}_\mu^{-1} W_{\mu i}^T \leq \varepsilon^2 \tilde{P}_\mu - \varepsilon W_{\mu i}^T - \varepsilon W_{\mu i}$, which implies that

$$\hat{\Xi} \triangleq \begin{bmatrix} \hat{\Xi}_1 & \hat{\Xi}_2 \\ * & \hat{\Xi}_3 \end{bmatrix} < 0, \quad i \in \mathcal{H} \quad (38)$$

TABLE I
PARAMETERS OF SYSTEM (2)

	$\phi(k) = 1$	$\phi(k) = 2$	$\phi(k) = 3$
$A_{\phi(k)}$	$\begin{bmatrix} 0.3 & 0 \\ 0 & 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}$	$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.6 \end{bmatrix}$
$B_{\phi(k)}$	$\begin{bmatrix} 0.2 & 0 \\ 0 & 0.7 \end{bmatrix}$	$\begin{bmatrix} 0.6 & 0 \\ 0 & 0.9 \end{bmatrix}$	$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.8 \end{bmatrix}$
$C_{\phi(k)}$	$[0.2 \ 0.35]$	$[0.1 \ 0.25]$	$[0.3 \ 0.35]$
$D_{\phi(k)}$	$\begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}$	$\begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.5 \end{bmatrix}$
$M_{1\phi(k)}$	$[0.1 \ 0.3]^T$	$[0.7 \ 0.3]^T$	$[0.2 \ 0.8]^T$
$M_{2\phi(k)}$	$[0.2 \ 0.2]^T$	$[0.5 \ 0.9]^T$	$[0.9 \ 0.9]^T$

where

$$\hat{\Xi}_1 = \begin{bmatrix} -W_{\mu} \bar{P}_{\mu}^{-1} W_{\mu}^T & 0 \\ * & -I \end{bmatrix}. \quad (39)$$

To facilitate the following derivation, one defines the following matrix with appropriate dimensions:

$$\mathcal{W} = \text{diag} \left\{ \begin{bmatrix} W_{\mu}^{-1} & 0 \\ * & I \end{bmatrix}, I \right\}. \quad (40)$$

Then, it is straightforward to obtain the inequality (18) by performing a congruence transformation to (38) with $\mathcal{W}$.

Furthermore, in the case that the condition (36a) holds, it can be deduced that

$$\lambda_{\max}(\tilde{P}_{\mu}) < \theta_2, \quad \lambda_{\min}(\tilde{P}_{\mu}) > \theta_1, \quad \lambda_{\max}(O_{\mu}) < \theta_3$$

which implies that (1) can be guaranteed by (36b). To sum up, the expected property for system (11) is guaranteed, and the desired state estimator parameters can be solved at the same time. This completes the proof. ■

Remark 5: As an algorithm based on linear matrix inequality, the resulting conditions in Theorem 2 have a polynomial-time complexity. Looking back on the Markov jump neural networks (2) and the state estimator (10), we can get the total number of decision variables $N = (6n_x^2 + n_x)GH + (2n_x^2 + n_x n_y + n_z n_x + n_o^2)H + 4n_x^2 G$ and the number of LMIs $M = n_x(2 \sum_{\mu} |\mathcal{H}_{uk}^{\mu}| + 5GH + 4G) + n_o(2GH + G) + n_z GH + 2GH + H$, where $|\mathcal{H}_{uk}^{\mu}|$ represents the capacity of the set $\mathcal{H}_{uk}^{\mu}$. It is obvious that the computational burden depends polynomially on the variable dimensions, the network size, and the amount of estimator modes.

IV. EXAMPLES

The purpose here is to show the effectiveness of combining the state estimator design methods and dynamic event-triggered transmission mechanism through considering the Markov jump neural networks (2) with three modes and the parameters in Table I.

In addition, the external disturbance is chosen as $o(k) = e^{-0.3k} \cos(0.8k)$, and by defining the activation functions as

$$\begin{cases} f_1(x_1(k)) = \tanh(0.6x_1(k)) - 0.2 \sin(x_1(k)) \\ f_2(x_2(k)) = \tanh(-0.4x_2(k)) \end{cases}$$

TABLE II
FOUR DIFFERENT CONDITIONAL PROBABILITY MATRICES Ω_2

Fully Accessible (Case I)	Partly Accessible (Case II)
$\begin{bmatrix} 0.3 & 0.5 & 0.2 \\ 0.3 & 0.6 & 0.1 \\ 0.3 & 0.5 & 0.2 \end{bmatrix}$	$\begin{bmatrix} ? & ? & 0.2 \\ 0.3 & 0.6 & 0.1 \\ ? & 0.5 & ? \end{bmatrix}$
Partly Accessible (Case III)	Almost Inaccessible (Case IV)
$\begin{bmatrix} ? & ? & 0.2 \\ 0.3 & ? & ? \\ ? & 0.5 & ? \end{bmatrix}$	$\begin{bmatrix} ? & ? & ? \\ 0.3 & ? & ? \\ ? & ? & 0.2 \end{bmatrix}$

it can be found that

$$\tilde{S}_1 = \text{diag}\{-0.16, 0\}, \quad \tilde{S}_2 = \text{diag}\{-0.3, 0.2\}.$$

The transition probability matrix Ω_1 , which is introduced to describe the switching of the system mode $\phi(k)$, is given by

$$\Omega_1 = \begin{bmatrix} 0.25 & 0.6 & 0.15 \\ 0.2 & 0.65 & 0.15 \\ 0.1 & 0.7 & 0.2 \end{bmatrix}.$$

On the other hand, the hidden Markov process $\psi(k)$ satisfies $\psi(k) \in \{1, 2, 3\}$, and four conditional probability matrices are defined in Table II to take into account the different asynchronous cases where the probabilities $\tau_{\mu i}$ are partly accessible.

With respect to the event-triggered scheme (6) and (8), we choose the threshold matrices as

$$\sigma_{\psi(k)} = \begin{cases} \text{diag}\{0.6, 0.8\}, & \psi(k) = 1 \\ \text{diag}\{0.3, 0.1\}, & \psi(k) = 2 \\ \text{diag}\{0.6, 0.5\}, & \psi(k) = 3 \end{cases} \quad (41)$$

and the initial value $\zeta_0 = 0$. According to Lemma 2, we set $\delta_1 = 10$, $\delta_2 = 2$, $\delta_3 = 10$, $\eta_1 = 0.1$, $\eta_2 = 0.8$, and $\eta_3 = 0.1$.

To facilitate the solution of the state estimator through using the linear matrix inequality, several parameters in Theorem 2 need to be ascertained and set. Inspired by the methods of optimizing parameters in the reference [16], the finite-time parameters are selected as $e_1 = 1$, $e_2 = 0.707$, $e_3 = 15$, $\Upsilon = \text{diag}\{1, 1, 1, 1\}$, $\mathcal{T} = 30$, and $\beta = 1.04$. Then, recalling Lemma 1, we give $\epsilon = 1.5$. The dissipativity parameters are chosen as $\Lambda_1 = -0.36$, $\Lambda_2 = -1.1$, and $\Lambda_3 = 5$.

It is significant that different combinations of asynchronous cases and boundary e_3 will result in different dissipation indices. The corresponding comparison results are listed in Table III, and two evolution principles can be found obviously in this table. First, for the fixed boundary e_3 , the optimal dissipation index $\bar{\gamma}$ is decreasing when changing the conditional probability matrix Ω_2 from Case I to Case IV, which reflects that the performance of system degrades with the loss of the conditional probability information. Furthermore, there exists monotonicity in the augment of the dissipative performance when the boundary e_3 is increasing.

Based on the aforementioned parameter settings, the gains of estimator (10) can be obtained straightway by solving a set of linear matrix inequalities in Theorem 2. Subsequently, for the partly accessible case (Case II), the optimal dissipation

TABLE III
OPTIMAL DISSIPATION INDEX $\bar{\gamma}$ FOR DIFFERENT e_3 'S

$\bar{\gamma}$	$e_3 = 10$	$e_3 = 15$	$e_3 = 20$	$e_3 = 25$
Case I	1.2764	1.2949	1.3011	1.3042
Case II	1.1809	1.2117	1.2148	1.2179
Case III	1.0668	1.0914	1.0976	1.1038
Case IV	0.8880	0.9311	0.9404	0.9465

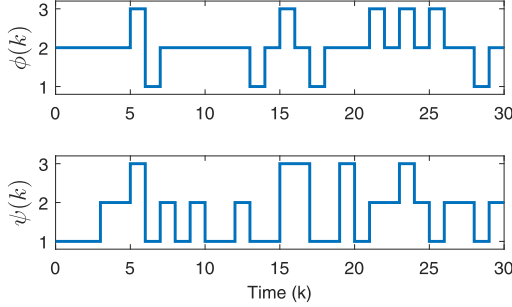


Fig. 1. Switching modes in the system and the estimator.

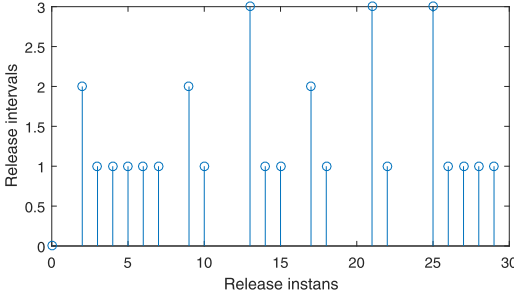


Fig. 2. Release intervals for the event-triggered condition (6).

index $\bar{\gamma}$ for the finite-time exponential dissipative performance is 1.2117, and the desired gains are acquired as

$$\begin{aligned} \tilde{A}_1 &= \begin{bmatrix} 0.2680 & -0.3150 \\ -0.0415 & 0.0382 \end{bmatrix}, & \tilde{A}_2 &= \begin{bmatrix} 0.1059 & -0.4906 \\ -0.1703 & -0.1867 \end{bmatrix} \\ \tilde{A}_3 &= \begin{bmatrix} 0.3089 & -0.3149 \\ -0.1210 & 0.1112 \end{bmatrix}, & \tilde{K}_1 &= \begin{bmatrix} -0.0056 & -0.2827 \\ -0.2187 & -0.1387 \end{bmatrix} \\ \tilde{K}_2 &= \begin{bmatrix} -0.1318 & -0.3824 \\ -0.2691 & -0.2901 \end{bmatrix}, & \tilde{K}_3 &= \begin{bmatrix} 0.0156 & -0.2798 \\ -0.2608 & -0.1378 \end{bmatrix} \\ \tilde{C}_1 &= [-0.2262 \quad -0.3560], & \tilde{C}_2 &= [-0.2703 \quad -0.2807] \\ \tilde{C}_3 &= [-0.1799 \quad -0.4861]. \end{aligned}$$

Meanwhile, two sequences depicted in Fig. 1 illustrate that the modes of system (2) and estimator (10) are switched asynchronously. Under initial condition $x(0) = [0.2 \quad -0.3]^T$, the event-triggered behavior within the finite interval $[0, 30]$ is represented in Fig. 2 by, respectively, defining release intervals and release time as the vertical axis and the horizontal axis. The output $z(k)$ and its estimations $\tilde{z}(k)$ are shown in Fig. 3, which means that, with the dispatch of the dynamic event-triggered transmission scheme, the estimator outputs can closely track the original output. In other words, the proposed dynamic event-triggered mechanism is able to effectively reduce data transmission while maintaining the tracking effect of the estimator. The estimation error $\tilde{z}(k)$ described in the

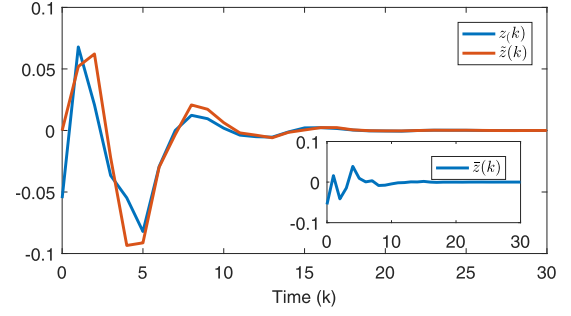


Fig. 3. Output $z(k)$ and its estimations $\tilde{z}(k)$ under the dispatch of the event-triggered scheme.

TABLE IV
OPTIMAL DISSIPATION INDEX $\bar{\gamma}$ FOR THE TRIGGERED THRESHOLD σ_i WITH DIFFERENT STRUCTURES

$\bar{\gamma}$	$\sigma_i = 0.001I$	σ_i in Eq.(41)	$\sigma_i = 0.27I$	$\sigma_i = 0.54I$
Case I	1.4183	1.2949	1.2919	1.2024
Case II	1.3610	1.2117	1.2117	1.1007
Case III	1.3061	1.0914	1.1099	0.9589
Case IV	1.2024	0.9311	0.9465	0.7858

TABLE V
TE FOR THE TRIGGERED THRESHOLD σ_i WITH DIFFERENT STRUCTURES

TE	$\sigma_i = 0.001I$	σ_i in Eq.(41)	$\sigma_i = 0.27I$	$\sigma_i = 0.54I$
Case I	100%	83.9%	96.7%	84.1%
Case II	100%	81.7%	95.6%	81.4%
Case III	?	?	?	?
Case IV	?	?	?	?

subgraph of Fig. 3 also clearly reflects the tracking effect of the estimator.

Up until now, we have just analyzed the tracking result of the event-based estimator under Case II. In what follows, a comparative verification is established to further illustrate the feasibility of the proposed event-triggered transmission scheme according to the exponential dissipative performance and the transmission efficiency (TE), which is defined as

$$TE = \frac{\text{The amount of triggered output}}{\text{The total of system output}} \times 100\%.$$

For a fixed number of system output, in order to reduce the communication burden, we hope that the amount of triggered output is as small as possible, which also means that the TE is as low as possible.

The effects of the triggered threshold σ_i with different structures on both of the index $\bar{\gamma}$ and TE are reported in Tables IV and V, respectively. The symbol “?” in Table V signifies that the TE is indeterminate because most of the probability information of the conditional probability matrix Ω_2 is lost in Cases III and IV. Under this circumstance, the following discussion only considers Cases I and II.

Setting the triggered threshold $\sigma_i = 0.001I$, it can be seen that all the measurements are transmitted. As discussed in Remark 2, it implies that the event-triggered mechanism is converted to the time-triggered one. However, implementing the event-triggered mechanism with σ_i in (41), the TE is 83.9% in Case I and 81.7% in Case II. Thus, the event-triggered

mechanism may be a more appropriate strategy in the cases of communication resources that need to be saved. Then, let us focus on the event triggering mechanism with a scalar threshold (such as $\theta > 0$ in [23]). As shown in Tables IV and V, using the scalar threshold 0.27, namely, $\sigma_i = 0.27I$, needs to occupy more transmission resources to maintain the similar dissipation performance as the diagonal matrix (41). On the contrary, with the similar TE as the diagonal matrix (41), the dissipation performance of using the threshold $\sigma_i = 0.54I$ is obviously worse than that of employing the diagonal matrix (41). The above observation concludes that the proposed event-triggered transmission mechanism has greater potential to strike a balance between reducing transmission burden and improving system performance.

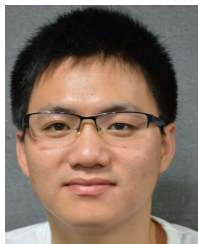
V. CONCLUSION

In this article, the finite-time exponential dissipative state estimation issue for Markov jump neural networks has been discussed with fully considering the asynchronization constraint. An asynchronous state estimator combining with an event-triggered scheme has been presented to ensure that the resulting system satisfies a prescribed dissipation performance index within the finite time, and the estimator gains can be acquired through solving a set of linear matrix inequalities. In Section IV, the impact of different asynchronous situations on the resulting system has been analyzed. In addition, the simulation data have proved that the proposed event-triggered transmission mechanism can effectively reduce the amount of data transmission while ensuring the expected estimation performance. In practice, noting that there are many undesired factors in networked control systems [42], such as packet-dropout [43] and time delay [44], it is significant to develop the resulting method for Markov jump neural networks with communication constraints, which is one of our future studies.

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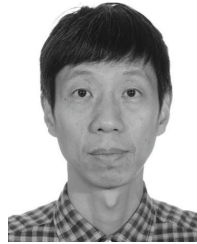
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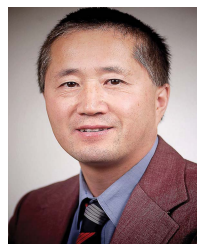
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