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CONVECTION VELOCITY IN DRAG-REDUCED OSCILLATING PIPE FLOW

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ABSTRACT

Direct Numerical Simulations (DNS) of turbulent pipe flow with periodic inflow/outflow boundary conditions at the ends of a 12 diameter long pipe domain are performed at Reynolds numbers=170, 360 and 720. We simulate standard pipe flow with motionless wall and pipe flow in which drag is reduced by oscillating the wall in the azimuthal direction. We present space-time correlations of the streamwise velocity with separation in the streamwise direction and examine the effect of periodic in-flow/out-flow. It is found that the turbulent fluctuations maintain significant correlation well beyond the time it takes to flow through the simulation domain, implying that streamwise periodicity is not a benign condition, at least for the 12 diameter domain length. We compute the convection velocity from the space-time correlation using the method of Wills (1964) and a new method obtained from the autocorrelations with pure spatial separation and pure time separation, quantities which are much less computationally demanding than the full space-time correlation. The new method uses the the values of the separation in space and time at which the respective correlations first cross zero. Fair agreement is found between the two methods, although the method of zeroes is sensitive to noise in the correlation functions. We also find that the temporal coherence of large-scale motions increases markedly in space and time in the frame of the convection velocity when the drag is reduced by wall oscillation.

Introduction

Space-time correlation provides interesting insights not found in space or time correlations, especially in flows having strong mean velocities. In this paper we consider flow in an oscillating pipe. The space-time auto-correlation coefficient of streamwise velocity fluctuation U with streamwise separation, ΔX, and time-delay, Δt, is given by

$$\rho_{uu}(r,\Delta x,\Delta t) = \frac{\langle u(r,\theta,x,t)u(r,\theta,x+\Delta x,t+\Delta t)\rangle_{x,\theta,t}}{\langle u^2\rangle} \qquad (1)$$

where averaging is done by integrating over the stream-wise (or axial) direction, x, the azimuth angle, θ, and time,t. The zero mean turbulent velocity fluctuation is a periodic and statistically homogeneous function of $0 < x < L$ and $0 < \theta < 2*pi$ and a statistically stationary function of t. r is the radial distance from the center-line, R is the radius of the pipe, and L is the length of the simulated flow domain. Later we shall use $0 < y = R - r < R$ to denote the distance from the pipe wall. Periodicity in x is a consequence of using the periodic inflow/outflow boundary condition, $u_i(x,r,\theta) = u_i(x+L,r,\theta), i = 1,2,3$, a condition that is thought to emulate flow that is statistically homogeneous in x provided L/R is sufficiently large. The sufficiency condition is not well understood, nor is the approach to statistically homogeneous flow in an infinitely long pipe. One idea is that L is sufficiently large if there is a range $L_0 << L - L_0$ in which the spatial correlation coefficient $\rho(\Delta x, \Delta t = 0) = 0$, and that range L_0 is a substantial fraction of L. Conceptually, structures in this range would be uncorrelated with structures at the inlet or the outlet, suggesting that that the inlet/outlet conditions have no effect on structure in this range. However, uncorrelatedness does not imply statistical independence, so the inference must be considered weak.

The convection velocity of turbulence describes the average speed at which turbulent structures (eddies) move downstream. Its principal use occurs in applications of Taylor's Frozen Field Hypothesis (Taylor (1938)), an approximation that makes it possible to estimate the two-point correlation function with spatial separation in terms of the two-point correlation function with time-delay. The latter correlation is found readily from experimental time-series data which are the natural outcome of physical experiments using a velocity probe at a fixed point. If the probe has high frequency response, the inferred two-point spatial correlation has high resolution in space. Conversely the Frozen Field Hypothesis can also be used to convert highly resolved spatial data computed by numerical simulation into temporal data. The Hypothesis assumes that the velocity structure of the turbulence does not change during the time it convects past the probe (Townsend (1980)). It is variously assumed to be equal to the local mean velocity or the bulk mean velocity, but neither value is necessarily correct a priori, and this ambiguity is a major source of error in the correlation functions inferred using Taylor's Hypothesis Del Alamo & Jiménez (2009)),Zaman & Hussain (1981)).

The classical definition of convection velocity given by Wills(1964) (Wills (1964)) is based on the full two-dimensional space-time correlation . This has resulted in convective velocities being approximated using comparatively short time-correlations. Under these conditions the short time-scales of turbulence may appear to behave as though they obey Taylor's frozen flow hypothesis and exhibit a distinct non-decaying periodicity with time delay due to an insufficient time record or streamwise lengths (Wang *et al.* (2020)). The modified method of Del Alamo & Jiménez (2009) is also numerically expensive as it requires the co-spectra of non-linear terms with velocity. In this paper we consider several issues bearing upon the definition and interpretation of the convection velocity. Firstly, what is the effect of scale. Do large-scales of motion travel faster or slower than small scales? Secondly, how is the convection velocity affected by Reynolds number and pipewall oscillation? Lastly, how are the length-scale of the spatial correlation and the time-scale of the temporal correlation related to the convection velocity? This last question leads to a new method of approximating the convective velocity by the ratio of the spatial separation at which the spatial correlation function first crossed zero, Δx_0, to the temporal delay at which the time correlation first crosses zero, Δt_0.

Method

The results presented are gathered from Direct Numerical Simulations of a turbulent pipe flow at three Reynolds numbers: $Re_\tau = 170, 360$ and 720. The simulation is carried out using a highly scalable code Nek5000 (Deville & Mund (2002); Fischer (1997)). The domain of the flow is a locally structured, globally unstructured Cartesian grid. The wall shear stress τ_w is kept fixed using a constant forcing term in the streamwise direction $f_x = 2\tau_w/\rho R$ (R is the radius of the pipe). Spanwise wall oscillations are introduced to achieve drag reduction. Wall oscillations are modeled as a velocity boundary condition, which is sinusoidal in time and invariable in space.

The following terms scale the primitive variables. U_{bulk} is the bulk velocity, which, coupled with the pipe diameter D and viscosity ν describes the bulk Reynolds number of the flow, $Re_{bulk} = \frac{U_{bulk}D}{\nu}$. The pipe length is fixed at $L = 24R$ to encourage structural decorrelation of very large scales in the streamwise direction (Kim & Adrian (1999)). The selected oscillation parameters are kept fixed in the inner units as $w^+_{wall} = 10$ and $T^+_{wall} = 100$. These values were selected from the available literature (Quadrio & Ricco (2004)). The result of drag reduction under the constant forcing term allows the mean flow rate to increase.

Simulations were run for 10000 ν/τ_w, and data is collected every $3.125\nu/\tau_w$ to allow for well resolved temporally averaged statistics and correlations. Results are interpolated using spectrally accurate interpolation routines (Merrill *et al.* (2016)) to a grid uniformly distributed in streamwise and azimuthal directions and that uses Gauss-Legendre-Lobatto nodes in the radial direction. High performance MPI libraries for Python (Otten & Min (2016)) are used to post-process and plot the data. A Hanning window was applied to the time record of velocity fluctuations. Record lengths were varied to achieve a total record length of $T_{rec} \approx \frac{5L}{U_{bulk}}$. A sliding window average of width $\frac{\Delta T U_{bulk}}{R} = 4$ was applied to zero spatial separation time delay correlation function and width $\frac{\Delta X}{R} = 6$ was applied to the zero time delay correlation function to filter out high frequency oscillation which resulted in spurious early time/space zero crossing.

Results

Streamwise-temporal correlations

Contours of the space-time auto-correlation coefficients of the streamwise velocity, u_i are presented in Figure 1 for for a single, representative wall-normal location $y^+ = 110$. This location is chosen to represent the region of the outer-layer that is above the logarithmic layer. The solid black lines indicate zero correlation, and the gray regions correspond to correlation coefficients between 0-7.5% (nominally 'low' correlation). Dark red denotes the region of nominally 'high' correlation (90-100%). Streamwise periodicity of the velocity field causes the correlation coefficients to be periodic in Δx, and the use of Fourier transforms to represent the velocity in time causes the correlation coefficients to also be periodic in Δt. We see that the temporal coherence of the velocity in a frame moving with the convective velocity is greater than the flow through-time, $24R/U_{bulk}$, leading to regions of weak, but non-zero correlation longer than the pipe length. The effect becomes greater with increasing Reynolds number producing weak correlations as long as 48R at Re=720 in the non-oscillating pipe. The coherence length and time are more than doubled when the wall is oscillated to achieve drag reduction.(Compare Figure 1(a) and 1(b).) Clearly, L=24R is insufficient to render the periodic inflow/outflow condition benign. hen running simulations of turbulent flows with drag reductions, researchers should take care to understand that what is considered a sufficiently long domain for standard turbulence may be too short for drag reduction. The space-time auto-correlations for radial velocity are significantly less converged than the corresponding auto-correlations for the streamwise velocity. Wall oscillations impart a resonance in the temporal separation curves most easly seen in Figure 1. The same is apparent for the radial fluctuation correlation. For comparison, the streamwise-temporal auto-correlations of radial velocity are presented in Figure 3. The correlations of radial velocity are significantly more noisey than those of the streamwise velocity.

Convective velocities

The black dotted line passing through the high correlation region shows a line with the slope equal to the convective velocity given by the method of Wills (1964), defined below in the Equation (2). Since the method of Wills involves the behavior of the correlation function at small separations, it is dominated by the contributions of the small scales. On the other hand, the low correlation region corresponds to large-scale structures, which are known to influence the log-layer turbulence (Hutchins & Marusic (2007); Guala *et al.* (2006)). One sees that the slope of the dashed line is greater than the slope of the zero contour line, indicating small scales moving faster than large scales. In the current paper, two methods for calculating convective velocity are presented and compared.

Method of Wills. The first method is the classical method of calculating convective velocity (Wills (1964)), which defines it as the slope of the line that passes through the location of the maximum correlation of the streamwise-temporal correlation function. This is found by finding Δt through a process of a linear regression of the points $(\Delta x, \Delta t, r)$ for each value of streamwise separation Δx and wall-normal location r as

$$\frac{\partial \rho_{u_i u_i}(\Delta x, \Delta t, r)}{\partial \Delta t}|_{\Delta x_i} = 0. \tag{2}$$

From the equation 2 a set of $(\Delta x_i, \Delta t_i)$ where the correlation is maximized is gathered. Assuming a linear fit a linear least squares algorithm can be applied with the slope $c_{u_i u_i} = a_1$.

$$[\Delta x_i] = [a_0, a_1]^\intercal [1, \Delta t_i] \tag{3}$$

Method of Zeroes. In the second method, which is a new method that we propose (referred to as the method of "zero matching"), we approximate the convection velocity through the ratio of the length scale (I_x) in the streamwise direction to the temporal scale (I_t), where the convective velocity $\langle c_{u_i u_i} \rangle_\theta$ is defined as

$$\langle c_{u_i u_i} \rangle_\theta (r) = \frac{I_x(r)}{I_t(r)} = \frac{\Delta x \big| \rho_{u_i u_i}(\Delta x, \Delta t = 0) = 0}{\Delta t \big| \rho_{u_i u_i}(\Delta x = 0, \Delta t) = 0} \tag{4}$$

In Equation (4), the streamwise length scale $I_x(r)$ is defined as the distance to the first zero crossing of the streamwise correlation function at zero time delay, and the temporal scale $I_t(r)$ is defined as the distance to the first zero crossing of the temporal auto-correlation function at zero spatial separation. A cubic spline interpolation with bisection is used to determine the numerical approximation of the first zero crossing. Figure 2 presents convective velocities obtained from Equation (2) and Equation (4) for streamwise fluctuations, and Figure 4 presents the corresponding data for radial fluctuations. Observations regarding drag reduction show that for the lower Reynolds numbers, convective velocities are significantly increased with respect to the bulk mean velocity.

Conclusions

The convective velocity profile calculated with the method of zero matching (Equation (4)) shows a reasonable agreement with the classical method. Near the centerline of the pipe the convective velocity tends towards the centerline velocity. Drag reduction shows a tendency to increase the convective velocity with respect to the bulk mean velocity for streamwise fluctuations. This is shown both using Method 1 and Method 2. This shows that the method presented is consistent with the classical method. Some spurious spikes in Method 2, especially for $Re_\tau = 360$, are attributed to a difficulty to reliably converge on the first zero crossing for a correlation function, which is oscillatory and crosses an abscissa multiple times.

The Method of zeroes gives a reasonably good agreement throughout the domain for the convective velocity related to streamwise fluctuations. Radial fluctuations, however, exhibit a distinct retardation. This is hypothesized to be due to what is observed in isotropic turbulence where the velocity correlation function of a fluctuation transverse to the axis of correlation quickly goes negative and positive again (De Karman & Howarth (1938)) due to continuity requirements. Thus the streamwise length scale based on a first zero crossing is shorter.

Acknowledgements

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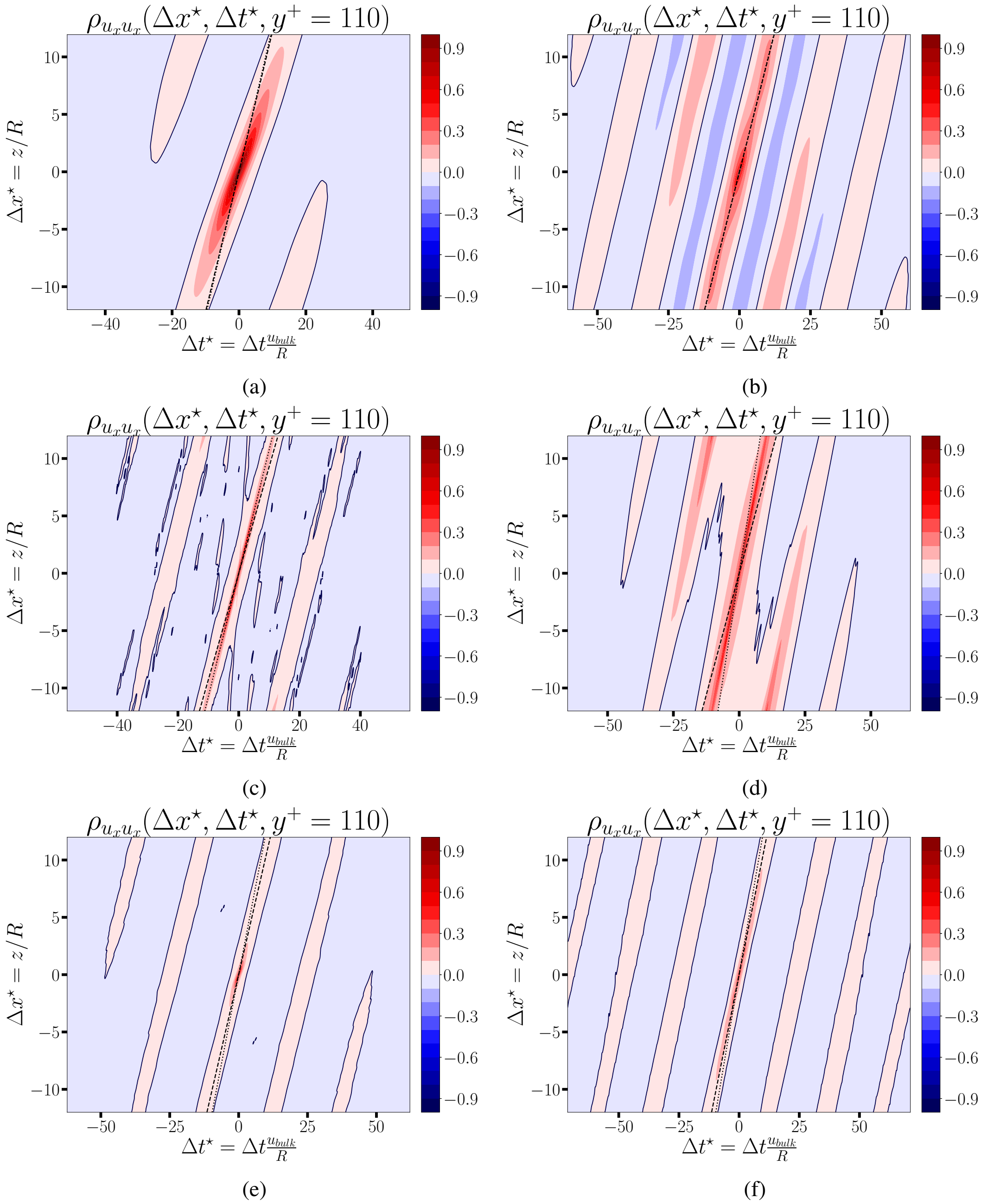
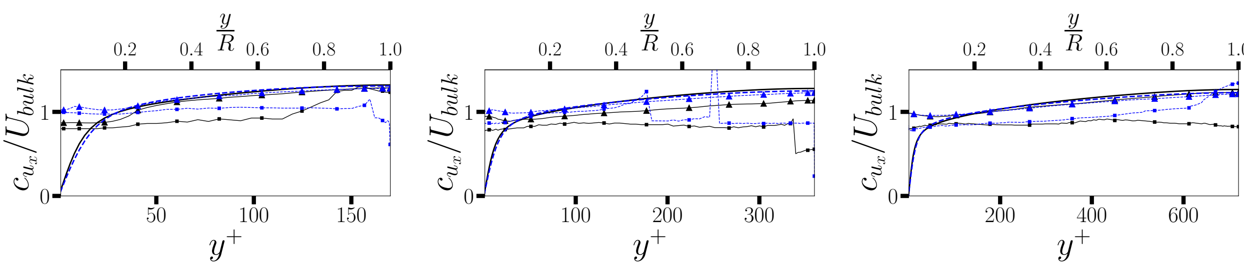
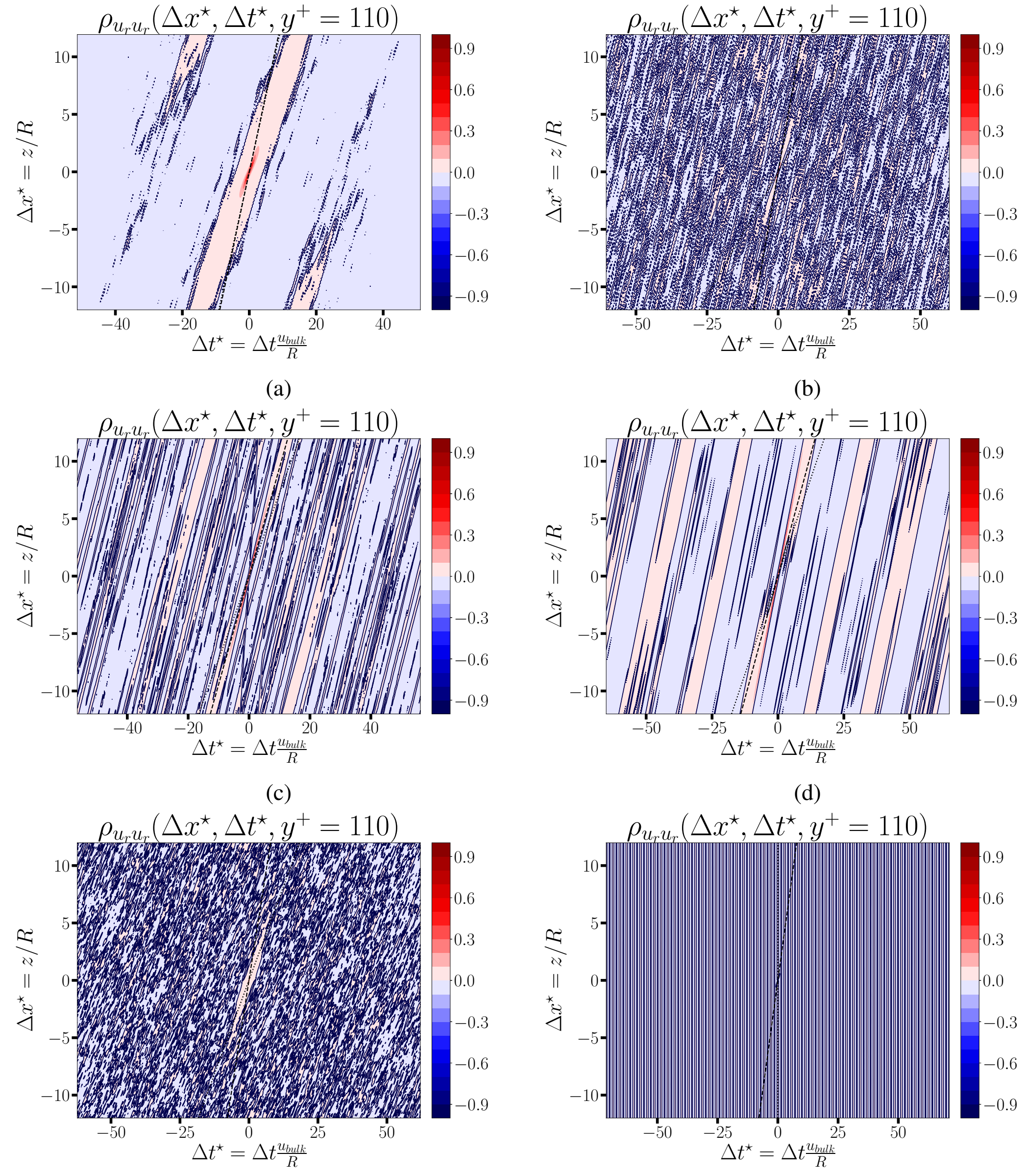


Figure 1: Space-time auto-correlation of streamwise velocity at y+ = 110 above the log-layer for $Re_\tau = 170$ (a),(b); $Re_\tau = 360$ (c),(d); $Re_\tau = 720$ (e),(f). The left-hand plots give the results for flow with no pipe oscillation (zero drag-reduction), and the right-hand plots present the results with oscillation (non-zero drag reduction). Black dashed lines indicate the line of best fit associated with the convection velocity of Wills (2), and the black dotted line is from the method of zeroes. (4). The solid black line corresponds to the zero-contour level.



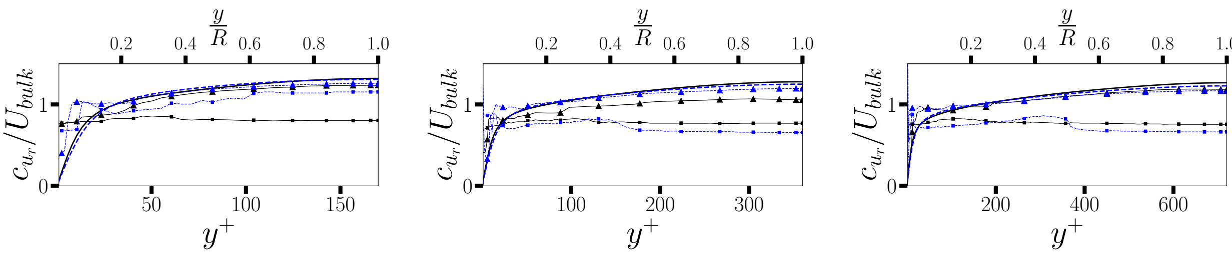
(a) $Re_\tau = 170$ (b) $Re_\tau = 360$ (c) $Re_\tau = 720$

Figure 2: These figures show the approximation of mean convective velocities of the streamwise fluctuations. Black solid lines are standard pipe flow and blue dashed are drag reduced. We compare the mean velocity profile (lines with no markers), Method 1 (Method of Wills, triangles) and Method 2 (zero matching, squares).



(a) (b) (c) (d) (e) (f)

Figure 3: Space-time auto-correlation coefficient of radial velocity at y+ = 110 for $Re_\tau = 170$ (a),(b), $Re_\tau = 360$ (c),(d), $Re_\tau = 720$ (e),(f). The left-hand plots give the results for flow with no pipe oscillation (zero drag-reduction), and the right-hand plots present the results with oscillation (non-zero drag reduction) Black dashed lines indicate the line of best fit associated with the convection velocity of Wills (2), and the black dotted line is from the method of zeroes (4). The solid black line corresponds to the zero-contour level. The noise in the correlations indicates the difficulty in resolving a true convective velocity.



(a) $Re_\tau = 170$ (b) $Re_\tau = 360$ (c) $Re_\tau = 720$

Figure 4: These figures show the approximation of mean convective velocities of the radial fluctuations. Black solid lines are standard pipe flow and blue dashed are drag reduced. We compare the mean velocity profile (lines with no markers), Method 1 (Method of Wills, triangles) and Method 2 (zero matching, squares).