

# Pulsating hydrogen-deficient white dwarfs and pre-white dwarfs observed with *TESS*

## III. Asteroseismology of the DBV star GD 358

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## ABSTRACT

**Context.** The collection of high-quality photometric data by space telescopes, such as the already completed *Kepler* mission and the ongoing *TESS* program, is revolutionizing the area of white-dwarf asteroseismology. Among the different kinds of pulsating white dwarfs, there are those that have He-rich atmospheres, and they are called DBVs or V777 Her variable stars. The archetype of these pulsating white dwarfs, GD 358, is the focus of the present paper.

**Aims.** We report a thorough asteroseismological analysis of the DBV star GD 358 (TIC 219074038) based on new high-precision photometric data gathered by the *TESS* space mission combined with data taken from the Earth.

**Methods.** We reduced *TESS* observations of the DBV star GD 358 and performed a detailed asteroseismological analysis using fully evolutionary DB white-dwarf models computed accounting the complete prior evolution of their progenitors. We assessed the mass of this star by comparing the measured mean period separation with the theoretical averaged period spacings of the models, and we used the observed individual periods to look for a seismological stellar model. We detected potential frequency multiplets for GD 358, which we use to identify the harmonic degree ( $\ell$ ) of the pulsation modes and rotation period.

**Results.** In total, we detected 26 periodicities from the *TESS* light curve of this DBV star using a standard pre-whitening. The oscillation frequencies are associated with nonradial  $g$ (gravity)-mode pulsations with periods from  $\sim 422$  s to  $\sim 1087$  s. Moreover, we detected 8 combination frequencies between  $\sim 543$  s and  $\sim 295$  s. We combined these data with a huge amount of observations from the ground. We found a constant period spacing of  $39.25 \pm 0.17$  s, which helped us to infer its mass ( $M_\star = 0.588 \pm 0.024 M_\odot$ ) and constrain the harmonic degree  $\ell$  of the modes. We carried out a period-fit analysis on GD 358, and we were successful in finding an asteroseismological model with a stellar mass ( $M_\star = 0.584^{+0.025}_{-0.019} M_\odot$ ), compatible with the stellar mass derived from the period spacing, and in line with the spectroscopic mass ( $M_\star = 0.560 \pm 0.028 M_\odot$ ). In agreement with previous works, we found that the frequency splittings vary according to the radial order of the modes, suggesting differential rotation. Obtaining a seismological made it possible to estimate the seismological distance ( $d_{\text{seis}} = 42.85 \pm 0.73$  pc) of GD 358, which is in very good accordance with the precise astrometric distance measured by *GAIA* EDR3 ( $\pi = 23.244 \pm 0.024$ ,  $d_{\text{GAIA}} = 43.02 \pm 0.04$  pc).

**Conclusions.** The high-quality of data measured with the *TESS* space telescope, used in combination with data taken from ground-based observatories, provides invaluable information for conducting asteroseismological studies of DBV stars, analogously to what happens with other types of pulsating white-dwarf stars. The currently operating *TESS* mission, together with the advent of other similar space missions and new stellar surveys, will give an unprecedented boost to white dwarf asteroseismology.

**Key words.** stars — pulsations — stars: interiors — stars: evolution — stars: white dwarfs



## 1. Introduction

Pulsating white dwarfs (WD) and pre-WDs constitute a long-studied and reliably established class of compact variable stars, both from observational and theoretical grounds. Their brightness variations are multiperiodic, with periods between 100 s and 7 000 s, and amplitudes up to 0.4 mag in typical optical light curves (Winget & Kepler 2008; Fontaine & Brassard 2008; Althaus et al. 2010; Córsico et al. 2019). The variability is associated to low-degree ( $\ell \leq 2$ ) nonradial  $g$ (gravity)-mode pulsations excited by a physical mechanism related to the partial ionization of the dominant chemical species in the zone of driving, located in the external layers. In warm and cool pulsating WDs, the opacity bumps associated to these partial-ionization zones are responsible for the appearance of an outer convection zone, which also strongly contributes to  $g$ -mode pulsation instabilities (Brickhill 1991; Goldreich & Wu 1999; Wu & Goldreich 1999). The first pulsating WD was discovered in 1968 (Landolt 1968), and currently more than 300 objects are known. They are classified in various categories, including DAVs or ZZ Ceti stars (pulsating WDs with H atmospheres), DBV or V777 Her stars (pulsating WDs with helium atmospheres), and pulsating PG 1159 or GW Vir stars, among others (Winget & Kepler 2008; Fontaine & Brassard 2008; Althaus et al. 2010; Córsico et al. 2019).

Since the discovery of the first pulsating WDs, observations of these variable stars have been steadily increasing, thanks to single-site observations and also multi-site campaigns like those of the Whole Earth Telescope (WET; Nather et al. 1990). Subsequently, a dramatic growth in the number of known pulsators was made possible thanks to the identification of candidates from the spectral observations of the Sloan Digital Sky Survey (SDSS, York et al. 2000; Kleinman et al. 2013; Kepler et al. 2015, 2016, 2019a), examples of which are the works of Mukadam et al. (2004a,b, 2006). Finally, in recent years, the area has received a strong boost driven by the uninterrupted observations from space made by the *Kepler* telescope, both main mission (Borucki et al. 2010) and *K2* mode (Howell et al. 2014). Indeed, these efforts paved the way to the analysis of 32 ZZ Ceti stars and three DBV stars (Østensen et al. 2011; Greiss et al. 2014; Bell et al. 2015; Hermes et al. 2017a,b; Bell et al. 2017; Bell 2017; Duan et al. 2021), until the mission was terminated due to lack of fuel in 2018. The successor to *Kepler* is the Transiting Exoplanet Survey Satellite (*TESS*, Ricker et al. 2015). This space mission has observed 200 000 brightest stars in 85% of the whole sky in 2019 and 2020 in the first part of the mission. *TESS* performs extensive time-series photometry that allows to discover pulsating stars, and, in particular, variable hot subdwarfs, WDs, and pre-WDs with  $\text{mag} < 16$ , with short (120 s) cadence. Starting in July 2020, it is now also observing in 20 s cadence.

Relevant to this work are the DBV stars, which are pulsating He atmosphere WDs with effective temperatures in the range  $22\,400 \lesssim T_{\text{eff}} \lesssim 32\,000$  K and pulsate with  $g$ -mode periods between 120 and 1080 s (Winget & Kepler 2008; Córsico et al. 2019). The existence of the DBV class of pulsating WDs was anticipated through theoretical arguments (Winget et al. 1982b) before it was confirmed observationally shortly after (Winget et al. 1982a). Pulsations in DBVs are thought to be excited by a combination of the  $\kappa$  mechanism acting in the He partial ionization zone — and thus setting the blue edge of the DBV instability strip (Winget et al. 1983b), and the “convective driving” mechanism (Brickhill 1991; Dupret et al. 2008; Quirion et al. 2008; Van Grootel et al. 2017).

Due to the high-quality *Kepler* and *K2* observations, three DBV stars, KIC 8626021 (Østensen et al. 2011), PG 0112+104 (Hermes et al. 2017b), and EPIC 228782059 (Duan et al. 2021), were intensively studied with space data. In particular, KIC 8626021 has been repeatedly modelled by several independent research groups (Bischoff-Kim & Østensen 2011; Córsico et al. 2012; Bischoff-Kim et al. 2014; Giannicche et al. 2018; Charpinet et al. 2019), who have explored its internal structure with unprecedented precision. The first DBV pulsator observed extensively with *TESS*, the star EC 0158–160 or WD 0158–160 (TIC 257459955), was analyzed and modelled by Bell et al. (2019), who found nine independent frequencies appropriate for asteroseismology.

In this work, we present new *TESS* observations of the already known DBV star GD 358. It has been scrutinized extensively from the ground for 3 decades. While some time-series photometry of GD 358 has been obtained from space (e.g., Castanheira et al. 2005), in this work we examine this archetypal DBV star using intensive high-precision photometry from space for the first time. We also perform a detailed asteroseismological analysis of this star on the basis of the fully evolutionary models of DB WDs computed by Althaus et al. (2009). The present study is the third part of our series of papers devoted to the study of pulsating H-deficient WDs observed with *TESS*. The first article is focused on six already known GW Vir stars (Córsico et al. 2021), and the second one is devoted to the discovery of two new GW Vir stars, specifically DOVs (Uzundag et al. 2021).

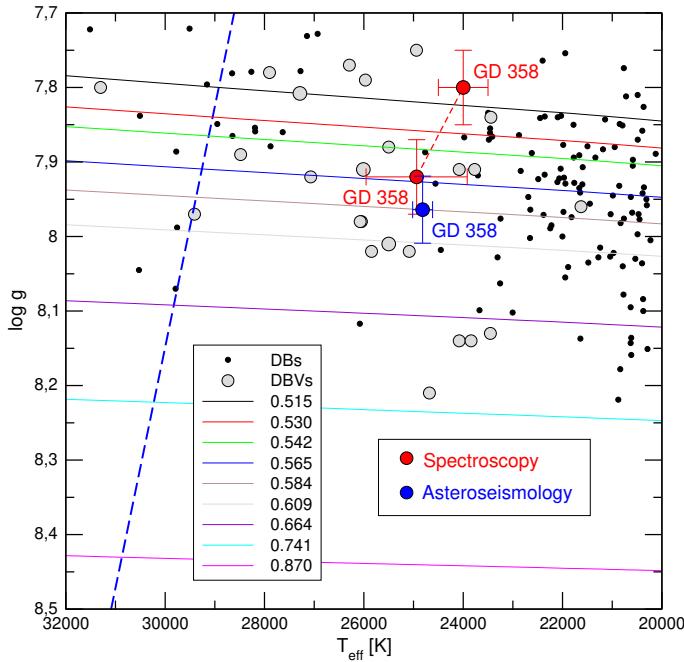
The paper is organized as follows. In Section 2 we provide a brief account of the main characteristics of GD 358. In Sect. 3, we describe the methods we apply to obtain the pulsation periods of the target star. A brief summary of the stellar models of DB WD stars employed for the asteroseismological analysis of GD 358 is provided in Sect. 4. Sect. 5 is devoted to the asteroseismological modelling of the target star, including the look for a possible uniform period spacing in the period spectrum by applying significance tests, the derivation of the stellar mass using the period separation, and by carrying out a period-to-period fit with the goal of finding an asteroseismological model. Finally, in Sect. 6, we summarize our results.

## 2. The target star: GD 358

The location of GD 358 in the  $\log T_{\text{eff}}\text{-}\log g$  diagram is depicted in Fig. 1. We have included the evolutionary tracks of DB WDs computed by Althaus et al. (2009). Independently of the precise location of GD 358 dictated by the spectroscopic parameters (see below), the star appears to be in the middle of the DBV instability strip, with a stellar mass somewhat lower than the average mass of the C/O-core WDs ( $\sim 0.6M_{\odot}$ ). We describe the basic characteristics of GD 358 below and summarize its stellar properties in Table 1. GD 358 (or V777 Her) has a *TESS* Input Catalog (TIC) number TIC 219074038. It is the brightest ( $m_V = 13.7$ ) and most extensively studied DBV star. This prototypical object provides the designation for the class of V777 Her (or DBV) variable stars. The pulsations of GD 358 were discovered in 1982 (Winget et al. 1982b). Its spectroscopic surface parameters are  $T_{\text{eff}} = 24\,937 \pm 1018$  K and  $\log g = 7.92 \pm 0.05$  according to Bédard et al. (2017) from optical data (see Fig. 1), although the previous analysis by Nitta et al. (2012) and Koester et al. (2014) using optical and UV data give  $T_{\text{eff}} = 24\,000 \pm 500$  K and  $\log g = 7.8 \pm 0.05$  (Fig. 1). Recently, Kong & Luo (2021) derived the atmospheric parameters of GD 358 with LAMOST data and found  $T_{\text{eff}} = 24\,075 \pm 124$  K and  $\log g = 7.827 \pm 0.01$  dex. These values are in perfect agreement with the parameters derived by

**Table 1.** Characteristics of GD 358. Columns 1, 2, 3, 4, 5, and 6 correspond to the *TESS* input catalog number, name of the object, effective temperature, surface gravity, Gaia EDR3 parallax, and distance, respectively. There are three spectroscopic determinations of the atmospheric parameters, the first row corresponding to the values from Bédard et al. (2017), the second one from Nitta et al. (2012); Koester et al. (2014), and the third one from Kong & Luo (2021). For details, see the text.

TIC	Name	$T_{\text{eff}}$ [K]	$\log g$ [cgs]	$\pi$ [mas]	$d$ [pc]
219074038	GD 358 (V777 Her)	$24\,937 \pm 1018$	$7.92 \pm 0.05$	$23.244 \pm 0.024$	$43.02 \pm 0.04$
		$24\,000 \pm 500$	$7.80 \pm 0.05$		
		$24\,075 \pm 124$	$7.827 \pm 0.01$		



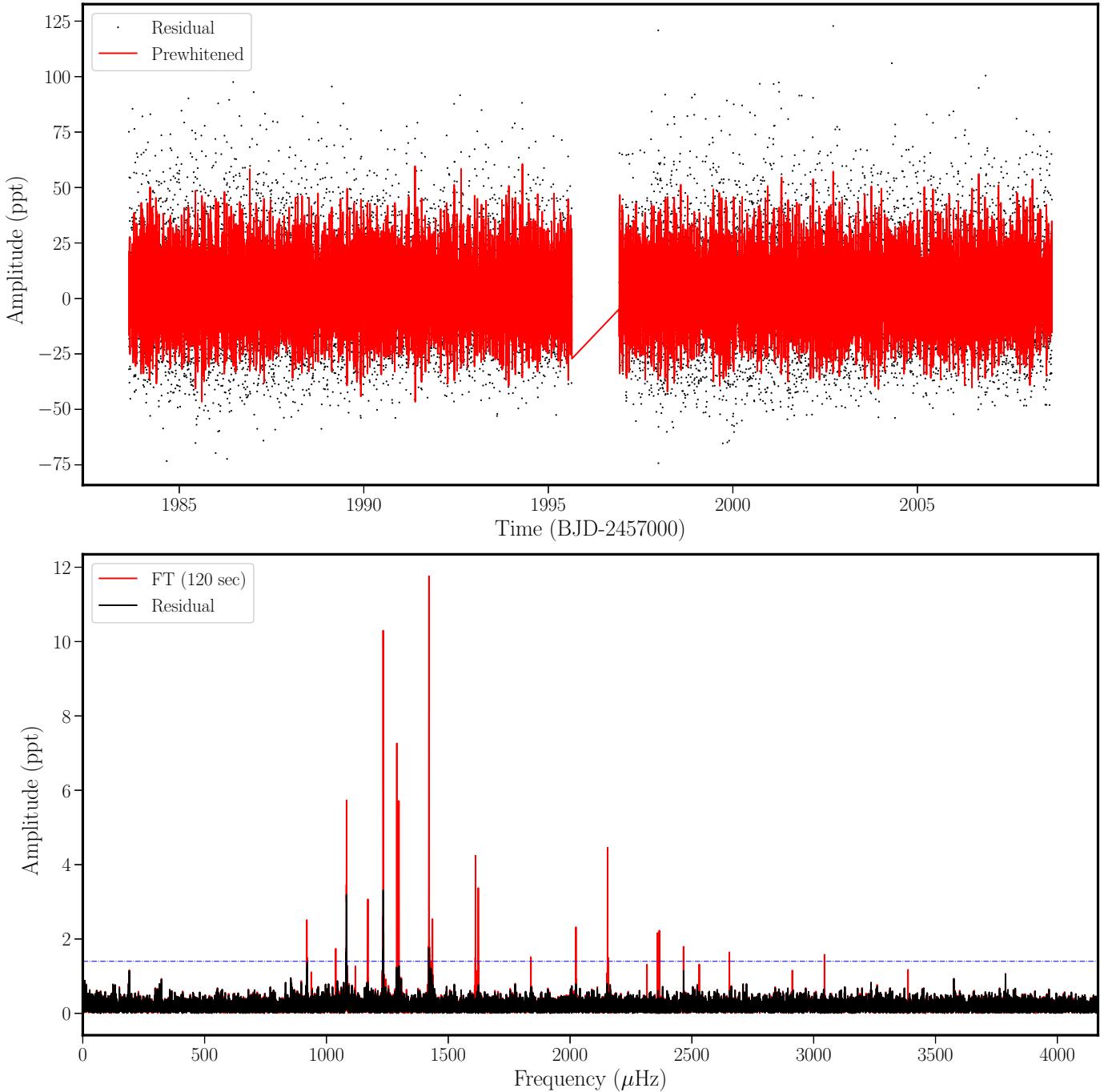
**Fig. 1.** Location of DB WDs on the  $T_{\text{eff}}$  –  $\log g$  diagram (Kepler et al. 2019b), marked with small black circles. Also depicted is the location of the published DBV stars (gray circles), according to the compilation by Cúrsico et al. (2019). The target star of the present paper, GD 358, is highlighted with large red circles according to spectroscopy, being the two locations of this star corresponding to two spectroscopic determinations of  $T_{\text{eff}}$  and  $\log g$  according to Koester et al. (2014) ( $T_{\text{eff}} = 24\,000 \pm 500$  K,  $\log g = 7.80 \pm 0.05$ ), which are very close to the recent derivations by Kong & Luo (2021), and according to Bédard et al. (2017) ( $T_{\text{eff}} = 24\,937 \pm 1018$  K,  $\log g = 7.92 \pm 0.05$ ). The location of the asteroseismological model (see Sect. 5.2) is emphasized with a blue circle. The DB WD evolutionary tracks of Althaus et al. (2009) are displayed with different colors according to the stellar-mass values (in solar units). The blue-dashed line represents the theoretical dipole ( $\ell = 1$ ) blue edge of the DBV instability strip, according to Cúrsico et al. (2009).

Nitta et al. (2012) and Koester et al. (2014). The spectroscopic parameters are summarized in Table 1. GD 358 has been extensively observed by the *WET* collaboration (Winet et al. 1994; Vuille et al. 2000; Kepler et al. 2003; Provencal et al. 2009). The most recent and complete analysis of this star has been carried out by Bischoff-Kim et al. (2019), who collected and reduced data from 34 years of photometric observations, including archival data from 1982 to 2006, and 1195.2 hr of observations from 2007 to 2016. Bischoff-Kim et al. (2019) detected a total of 15 independent periods, of which thirteen belong to a series of  $\ell = 1$  pulsation periods with consecutive radial order, the longest continuous sequence of periods observed in a DBV

star until then. The star has repeatedly been the focus of asteroseismological analyses (Bradley & Winget 1994; Metcalfe et al. 2000, 2001; Fontaine & Brassard 2002; Metcalfe 2003) using evolutionary models of DB WDs with simplified chemical profiles. Bischoff-Kim et al. (2019) analyzed this star using models that include parameterized, complex, core-composition profiles to fit the 15 observed periods. They obtain a seismological model with a thickness of the He layer that is qualitatively consistent with the diffusion-calculation picture that predict that the pure-He envelope will steadily grow thicker as the DB star cools (Dehner & Kawaler 1995; Fontaine & Brassard 2002; Althaus & Cúrsico 2004). The *Gaia* EDR3 parallax and distance for GD 358 are  $\pi = 23.244 \pm 0.024$  mas and  $d = 43.02 \pm 0.04$  pc (Bailer-Jones et al. 2021), respectively.

### 3. Observations and data reduction

GD 358 was observed by *TESS* at 2-min cadence on Sector 25 between 2020-May-13 and 2020-June-08, targeted as TIC 219074038 ( $T = 13.9$  mag). The temporal resolution is  $1/T = 0.451$   $\mu$ Hz ( $T$  being the data span of 25.67 days). The light curve for GD 358 released by the Science Processing Operations Center (SPOC) pipeline had large gaps, which we believe were caused by unnecessarily harsh clipping based on quality flags that resulted in a low (67.4%) duty cycle. We re-reduced the Sector 25 data with the same 5-pixel aperture from the SPOC pipeline but with a looser quality-flag cuts, yielding a significantly higher (92.6%) duty cycle. Our final light curve was flattened of long-term trends by the division of second-order polynomial every 2 days. The final light curve is shown in Fig. 2 including 16781 data points spanning 25 days. The average noise level of the amplitude spectra is 0.24 ppt. We calculated a detection threshold of 0.1% false alarm probability (FAP) following the method described in Kepler (1993) (see also Baran & Koen 2021). If the amplitude of a given peak is above this value, there is a 0.1% chance of resulting from random noise fluctuations. All frequencies above the threshold level of 0.1% FAP of 1.45 ppt have been prewhitened a few exceptions. The frequencies at 1657 and 2157  $\mu$ Hz are below the significance threshold with S/N of 5.6 and 4.6 respectively. These frequencies were reported in the previous work by Bischoff-Kim et al. (2019). Moreover, we prewhitened 8 combination frequencies, which are located at beyond 2300  $\mu$ Hz. A few of them at 2315, 2530, 2912 and 3386  $\mu$ Hz are slightly below 0.1% FAP level with S/N of 5.3, 5.5, 4.7 and 5 respectively. All combination frequencies beyond 2400  $\mu$ Hz are detected for the first time and also extracted from the light curve and reported in Table 2. Overall, we have detected 26 frequencies that are concentrated between 900  $\mu$ Hz and 3400  $\mu$ Hz, out of which we identified 8 combination frequencies. Fig. 2 displays the Fourier transform of GD 358. In Table 2 we show the list of periods of GD 358 detected with *TESS*.



**Fig. 2.** TOP: The light curve of the pulsating DBV star GD 358 observed in sector 25. The black dots show the residual flux while the red lines show all prewhitened variations from the light curve. BOTTOM: Fourier transform of the pulsating DBV star GD 358 observed with 120-sec cadence. The dotted horizontal blue line indicates the 0.1% false-alarm-probability (FAP) significance threshold. The black line is the FT of the prewhitened light curve.

The frequencies emphasized with boldface in Table 2 are components of rotational triplets ( $\ell = 1$ ). Rotational multiplets are depicted in Fig. 3. A well-known property of nonradial stellar pulsations is that the eigenfrequencies of degree  $\ell$  split into  $2\ell + 1$  components differing in azimuthal ( $m$ ) number due to stellar rotation. When the rotation is slow and rigid, the frequency splitting can be obtained as  $\delta\nu_{\ell,k,m} = m(1 - C_{\ell,k})\Omega_R$ ,  $\Omega_R$  being the rotational angular frequency of the pulsating star, and  $m = 0, \pm 1, \pm 2, \dots, \pm \ell$  (Unno et al. 1989). The condition of slow rotation implies that  $\Omega_R \ll \nu_{\ell,k}$ . The  $C_{\ell,k}$  constants are the

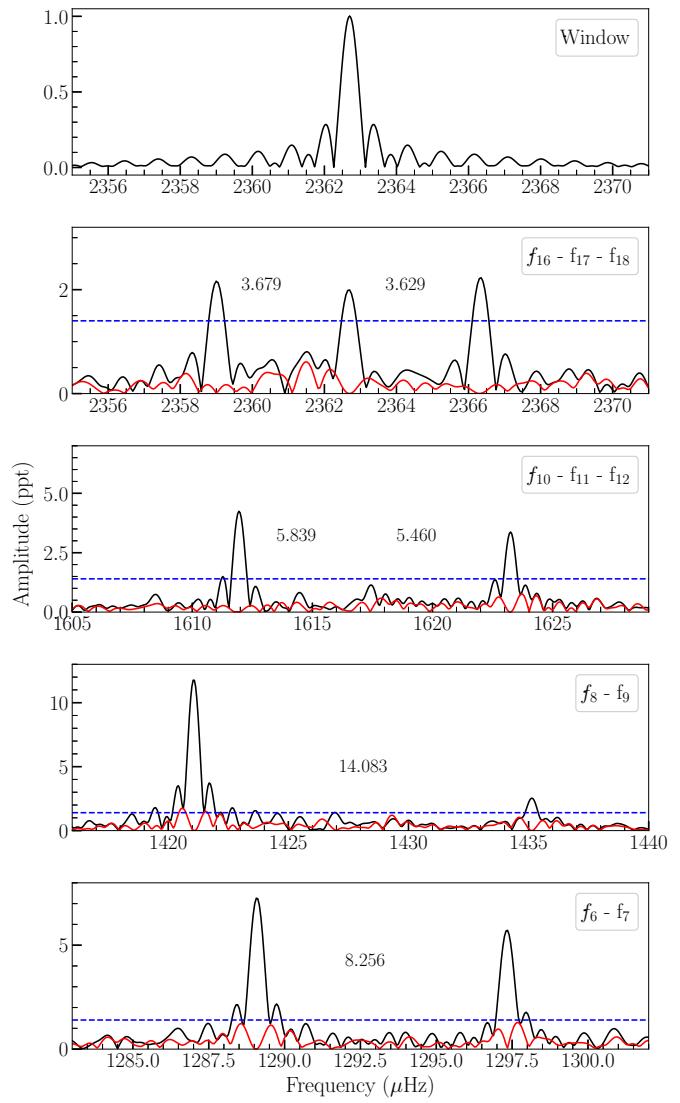
Ledoux coefficients (Ledoux & Walraven 1958), that can be assessed as  $C_{\ell,k} \sim [\ell(\ell + 1)]^{-1}$  in the asymptotic limit of high radial-order  $g$  modes ( $k \gg \ell$ ). In the specific case of dipole ( $\ell = 1$ ) and quadrupole ( $\ell = 2$ ) modes, we have  $C_{1,k} \sim 0.5$  and  $C_{2,k} \sim 0.17$ , respectively. Besides allowing an estimate of the rotation speed of the star, multiplets in the frequency spectrum of a pulsating WD are very useful to identify the harmonic degree of the pulsation modes. This approach to derive the rotation period has been successfully applied to several pulsating WD stars (see, for instance, Hermes et al. 2017a, for the case

of ZZ Ceti stars observed during the *Kepler* and *K2* missions). [Winget et al. \(1994\)](#) detected 10 complete (that is, with all the three components) rotational triplets of frequencies in GD 358 as a result of an intensive scrutiny of this star with the *WET* collaboration. These authors found that the frequency splittings ( $\delta\nu$ ) are not constant, but vary with the radial order, which led them to conclude that the star could be experiencing differential rotation, with the outer envelope rotating twice as fast as the core (but see also [Kawaler et al. 1999](#)). [Kepler et al. \(2003\)](#) reported the absence of triplets in the 2000 data of GD 358, except for one clear triplet centered at  $\sim 2154 \mu\text{Hz}$ . Later, [Provencal et al. \(2009\)](#) found only two clear rotational triplets, centered at  $\sim 2154 \mu\text{Hz}$  and  $\sim 2363 \mu\text{Hz}$ , and their analysis from 1990 to 2008 revealed a long-term change in the multiplet splittings coinciding with the 1996 *sforzando* event, where the star dramatically altered its pulsation characteristics on a timescale of hours. These phenomena could be attributed to the interaction between convection and/or magnetic fields and pulsations.

The *TESS* data of GD 358 presented in this work reveal the presence of 4 of the 10 triplets found in [Winget et al. \(1994\)](#). At variance with the results of that paper, in the *TESS* data we find two complete triplets and two incomplete triplets. One of the complete triplets has frequencies  $2359.010 \mu\text{Hz}$ ,  $2362.689 \mu\text{Hz}$ , and  $2366.318 \mu\text{Hz}$ . Going to smaller frequencies, we find the other complete triplet with frequencies  $1623.248 \mu\text{Hz}$ ,  $1617.409 \mu\text{Hz}$ , and  $1611.949 \mu\text{Hz}$ , an incomplete triplet with frequencies  $1435.142 \mu\text{Hz}$  and  $1421.059 \mu\text{Hz}$ <sup>1</sup>, and other incomplete triplet with frequencies  $1297.338 \mu\text{Hz}$  and  $1289.082 \mu\text{Hz}$ . These rotational triplets are emphasized with boldface in Table 2. We show the rotational triplets in Fig. 3. Similar to the findings of [Winget et al. \(1994\)](#), we find that the frequency splittings  $\delta\nu$  in these 4 triplets are not constant, as we will show in Sect. 5.1.

#### 4. Evolutionary models, numerical codes, and spectroscopic masses

We employ a set of fully evolutionary DB WD stellar models that consider the whole evolution of the progenitor stars. Specifically, the stellar models were taken from the evolutionary calculations presented by [Althaus et al. \(2009\)](#) produced with the LPCODE evolutionary code. For details about the input physics and evolutionary code, and the numerical simulations performed to obtain the DB WD evolutionary sequences employed here, we refer the interested reader to that paper. These evolutionary tracks have been employed in the asteroseismic analyses of the DBV stars KIC 8626021 ([Córsico et al. 2012](#)), KUV 05134+2605 ([Bognár et al. 2014](#)), PG 1351+489 ([Córsico et al. 2014](#)), and WD 0158–160 ([Bell et al. 2019](#)). The sequences of DB WD models have been computed considering a detailed treatment of the prior evolution, starting from the zero-age main sequence (ZAMS), taking into account the thermally pulsing asymptotic giant branch (TP-AGB) and born-again (VLTP; very late thermal pulse) phases, the stage of



**Fig. 3.** Rotational triplets in the frequency spectrum of GD 358. The FT are shown with black lines, and residuals with red lines. The blue-dashed horizontal lines correspond to 0.1% FAP confidence levels. The upper panel is the window function.

the PG 1159 stars, and finally the DB WD phase. By virtue of this, the models have evolving chemical profiles consistent with the prior evolution. The models assume the ML2 prescription of convection with the mixing length parameter,  $\alpha$ , fixed to 1 ([Bohm & Cassinelli 1971](#); [Tassoul et al. 1990](#)). Specifically, we considered nine DB WD sequences with stellar masses of:  $0.515, 0.530, 0.542, 0.565, 0.584, 0.609, 0.664, 0.741$ , and  $0.870 M_{\odot}$ . These DB WD sequences are characterized by the maximum He-rich envelope that can be left by prior evolution if we assume that they are the result of a born-again episode. The value of envelope mass ranges from  $M_{\text{He}}/M_{\ast} \sim 2 \times 10^{-2}$  ( $M_{\ast} = 0.515 M_{\odot}$ ) to  $M_{\text{He}}/M_{\ast} \sim 1 \times 10^{-3}$  ( $M_{\ast} = 0.870 M_{\odot}$ ). In Figure 1 we show the complete set of DB WD evolutionary tracks (with different colors according to the value of the stellar mass) along with the location of all the DBVs known to date ([Córsico et al. 2019](#)), including GD 358.

The  $\ell = 1, 2$  g-mode pulsation periods employed in this work were computed with the adiabatic and nonadiabatic versions of the pulsation code LP-PUL ([Córsico & Althaus 2006](#);

<sup>1</sup> Between these two frequencies there is a frequency of  $1426.986 \mu\text{Hz}$  but with a very low amplitude (1.294 ppt), which could be the  $m = 0$  component of the triplet, although the splittings of the triplet would be quite different ( $\delta\nu \sim 8 \mu\text{Hz}$  and  $\delta\nu \sim 6 \mu\text{Hz}$ ). A peak of similar frequency ( $1427.27 \mu\text{Hz}$ ) has been reported by [Winget et al. \(1994\)](#) with appreciable amplitude (19 ppt). However, we will not include the frequency of  $1426.986 \mu\text{Hz}$  in this work since all periods detected in GD 358 from ground-based observations such as those of [Winget et al. \(1994\)](#) are taken into account through the "mean periods" calculated by [Bischoff-Kim et al. \(2019\)](#); see Section 5.1.

**Table 2.** Identified frequencies (combination frequencies), periods, and amplitudes (and their uncertainties) and the signal-to-noise ratio in the data of GD 358.

Peak	$\nu$ ( $\mu$ Hz)	$\Pi$ (s)	$A$ (ppt)	S/N
$f_1$	$919.507 \pm 0.018$	$1087.538 \pm 0.021$	$2.521 \pm 0.18$	10.5
$f_2$	$1038.177 \pm 0.026$	$963.226 \pm 0.025$	$1.739 \pm 0.18$	7.2
$f_3$	$1082.770 \pm 0.008$	$923.556 \pm 0.007$	$5.723 \pm 0.18$	23.8
$f_4$	$1170.146 \pm 0.015$	$854.593 \pm 0.011$	$3.115 \pm 0.18$	12.9
$f_5$	$1232.928 \pm 0.004$	$811.076 \pm 0.003$	$10.325 \pm 0.18$	43.0
<b><math>f_6</math></b>	$1289.082 \pm 0.006$	$775.745 \pm 0.003$	$7.212 \pm 0.18$	30.0
<b><math>f_7</math></b>	$1297.338 \pm 0.008$	$770.808 \pm 0.005$	$5.574 \pm 0.18$	23.2
<b><math>f_8</math></b>	$1421.059 \pm 0.004$	$703.700 \pm 0.002$	$11.782 \pm 0.18$	49.0
<b><math>f_9</math></b>	$1435.142 \pm 0.018$	$696.794 \pm 0.009$	$2.530 \pm 0.18$	10.5
<b><math>f_{10}</math></b>	$1611.949 \pm 0.010$	$620.367 \pm 0.004$	$4.327 \pm 0.18$	18.0
<b><math>f_{11}</math></b>	$1617.409 \pm 0.034$	$618.272 \pm 0.013$	$1.352 \pm 0.18$	5.6
<b><math>f_{12}</math></b>	$1623.248 \pm 0.013$	$616.048 \pm 0.005$	$3.403 \pm 0.18$	14.1
$f_{13}$	$2024.182 \pm 0.020$	$494.026 \pm 0.005$	$2.280 \pm 0.18$	9.5
$f_{14}$	$2154.064 \pm 0.010$	$464.238 \pm 0.002$	$4.360 \pm 0.18$	18.1
$f_{15}$	$2157.584 \pm 0.042$	$463.481 \pm 0.009$	$1.109 \pm 0.18$	4.6
<b><math>f_{16}</math></b>	$2359.010 \pm 0.021$	$423.906 \pm 0.003$	$2.214 \pm 0.18$	9.2
<b><math>f_{17}</math></b>	$2362.689 \pm 0.021$	$423.246 \pm 0.003$	$2.169 \pm 0.18$	9.0
<b><math>f_{18}</math></b>	$2366.318 \pm 0.019$	$422.597 \pm 0.003$	$2.380 \pm 0.18$	9.9
$2f_1$	$1839.301 \pm 0.031$	$543.684 \pm 0.009$	$1.498 \pm 0.18$	6.2
$f_3+f_5$	$2315.669 \pm 0.036$	$431.840 \pm 0.006$	$1.282 \pm 0.18$	5.3
$2f_5$	$2465.756 \pm 0.026$	$405.555 \pm 0.004$	$1.772 \pm 0.18$	7.3
$f_5+f_7$	$2530.302 \pm 0.034$	$395.209 \pm 0.005$	$1.337 \pm 0.18$	5.5
$f_5+f_8$	$2653.958 \pm 0.028$	$376.795 \pm 0.004$	$1.614 \pm 0.18$	6.7
$f_6+f_{12}$	$2912.290 \pm 0.040$	$343.372 \pm 0.004$	$1.147 \pm 0.18$	4.7
$f_8+f_{12}$	$3044.295 \pm 0.029$	$328.483 \pm 0.003$	$1.586 \pm 0.18$	6.6
$f_5+f_{14}$	$3386.984 \pm 0.038$	$295.247 \pm 0.003$	$1.213 \pm 0.18$	5.0

(Córsico et al. 2006, 2009) and the same methods employed in the previous works of de La Plata Stellar Evolution and Pulsation Research Group<sup>2</sup>.

We assess a value of the spectroscopic mass of GD 358 by interpolation on the evolutionary tracks presented in Fig. 1 and the published values of the spectroscopic surface gravity and temperature. This is a relevant aspect because this same set of DB WD models is employed to derive the stellar mass from the period spacing (next Section). We get an spectroscopic stellar mass of  $M_\star = 0.508 \pm 0.050 M_\odot$  if  $T_{\text{eff}} = 24\,000 \pm 500$  K and  $\log g = 7.80 \pm 0.05$  (Nitta et al. 2012), and  $M_\star = 0.560 \pm 0.028 M_\odot$  if  $T_{\text{eff}} = 24\,937 \pm 1\,018$  K and  $\log g = 7.92 \pm 0.05$  (Bédard et al. 2017). The uncertainties in the stellar mass are derived from the errors in  $T_{\text{eff}}$  and  $\log g$  adopting the extreme values of these parameters when interpolating between the evolutionary tracks of Fig. 1.

## 5. Asteroseismology

In the asymptotic limit of high-radial orders ( $k \gg \ell$ ), the periods of  $g$  modes with consecutive radial order are approximately evenly separated (Tassoul et al. 1990), being the constant period spacing dependent on the harmonic degree:

$$\Delta\Pi_\ell^a = \frac{\Pi_0}{\sqrt{\ell(\ell+1)}}, \quad (1)$$

$\Pi_0$  being a constant value defined as:

$$\Pi_0 = \frac{2\pi^2}{\left[ \int_{r_1}^{r_2} \frac{N}{r} dr \right]}, \quad (2)$$

where  $N$  is the Brunt-Väisälä frequency. The asymptotic period spacing given by Eq. (1) is very close to the computed period spacing of  $g$  modes in chemically homogeneous stellar models without convective regions (Tassoul 1980). In the case of pulsating DB WDs, they may have a surface convective zone, although usually very thin compared to the stellar radius. On the other hand, they have several chemical composition gradients. Mainly for this last reason, the calculated period spacing does not coincide with the asymptotic period spacing given by Eq. (1), but nevertheless the *average* of the calculated spacing is very close to  $\Delta\Pi_\ell^a$  for radial orders high enough. The departures of the period spacing from the averaged period spacing are provoked by the mechanical resonance called "mode trapping". Mode trapping has been intensively studied in the context of DAV, DBV, and GW Vir stars (see, e.g., Brassard et al. 1992; Bradley et al. 1993; Kawaler & Bradley 1994; Córsico et al. 2002; Córsico & Althaus 2006).

The methods we use in this paper to extract information of the stellar mass and the internal structure of GD 358 are the same employed in Córsico et al. (2021) for GW Vir stars observed with *TESS* (see, also, Córsico et al. 2012; Bognár et al. 2014; Córsico et al. 2014; Bell et al. 2019). Specifically, we compare the observed period spacing of GD 358 ( $\Delta\Pi$ ) with the asymptotic period spacing ( $\Delta\Pi_\ell^a$ ) computed with Eq. (1) at the effective

<sup>2</sup> <http://fcaglp.fcaglp.unlp.edu.ar/evolgroup/>

temperature of the star to derive an estimate of the stellar mass. DBV stars generally do not have all of their pulsation modes in the asymptotic regime, so there is usually no perfect agreement between  $\Delta\Pi$  and  $\Delta\Pi_\ell^a$ . Therefore, the derivation of the stellar mass using the asymptotic period spacing may not be entirely reliable in DBV stars that pulsate with modes characterized by low and intermediate radial orders, but it gives a good estimate of the stellar mass for stars pulsating with  $g$  modes of high radial order (see Althaus et al. 2008, for the case of GW Vir stars). A variation of this approach to infer the stellar mass of DBV stars is to compare  $\Delta\Pi$  with the average of the computed period spacings ( $\overline{\Delta\Pi}_k$ ). It is calculated as  $\overline{\Delta\Pi}_k = (n-1)^{-1} \sum_k \Delta\Pi_k$ , where the "forward" period spacing ( $\Delta\Pi_k$ ) is defined as  $\Delta\Pi_k = \Pi_{k+1} - \Pi_k$  ( $k$  being the radial order) and  $n$  is the number of computed periods laying in the range of the observed periods. This method is more reliable for the estimation of the stellar mass of DBV stars than that described above using  $\Delta\Pi_\ell^a$  because, provided that the average of the computed period spacings is evaluated at the appropriate range of periods, the approach is valid for the regimes of short, intermediate and long periods as well. When the average of the computed period spacings is taken over a range of periods characterized by high  $k$  values, then the predictions of the present method become closer to those of the asymptotic period-spacing approach (Althaus et al. 2008). On the other hand, the present method requires detailed period computations, at variance with the method described above, which does not involve pulsational calculations. Note that both methods for assessing the stellar mass rely on the spectroscopic effective temperature, and the results are unavoidably affected by its associated uncertainty. The methods outlined above take full advantage of the fact that the period spacing of DBV stars primarily depends on the stellar mass and the effective temperature, and very weakly on the thickness of the He envelope (see, e.g., Tassoul et al. 1990)<sup>3</sup>.

A powerful approach to study the internal structure of pulsating stars is to search for models that best fit the observed pulsation periods. To quantify the goodness of the match between the theoretical periods ( $\Pi_{\ell,k}$ ) and the observed periods ( $\Pi_i^o$ ), we follow the same procedure as in our previous studies:

$$\chi^2(M_\star, T_{\text{eff}}) = \frac{1}{N} \sum_{i=1}^N \min[(\Pi_{\ell,k} - \Pi_i^o)^2] \quad (3)$$

$N$  being the number of observed periods. In order to find the stellar model that best fits the observed periods exhibited by GD 358 –the “asteroseismological” model– we evaluate the function  $\chi^2$  for stellar masses  $M_\star = 0.515, 0.530, 0.542, 0.565, 0.584, 0.609, 0.664, 0.741, 0.870 M_\odot$ . For the effective temperature, we employ a very fine model grid ( $\Delta T_{\text{eff}} = 10 - 30$  K). The DB WD model that shows the smallest value of  $\chi^2$  is adopted as the best-fit asteroseismological model. Below, we employ the tools described above to extract information of GD 358.

### 5.1. Period spacing and the seismic mass

Ground-based photometric observations of GD 358 span a period of 34 years. No other DBV star has been studied for such

a long period of time, which is why we know the most about this object. A detailed compilation of the observations, that includes archival data from 1982 to 2006, and 1195.2 hr of new observations from 2007 to 2016, has been presented by Bischoff-Kim et al. (2019). Figure 4 of that paper, constructed on the basis of the periods and amplitudes of their Tables 2 and 3, is extremely illustrative of how the periods of pulsation in this star are concentrated in bands with finite widths, rather than discrete periods. Frequencies detected in a given observing season are not found in all observing runs, and most of the detected frequencies are not statistically identical from year to year. Only a few frequencies appear with exactly the same values at various observing runs. Regarding the bands of periods exhibited by GD 358, Bischoff-Kim et al. (2019) find a general increase in their width with decreasing frequency (increasing period), at least until the band at 1238  $\mu$ Hz (807 s). This behaviour could be related to the oscillation of the outer convection zone of the WD during pulsations (Montgomery et al. 2020). Specifically, the oscillation in the base of the convection zone would affect the radial eigenfunction of  $g$  modes that have the outer turning point of oscillation located precisely at the base of the outer convection zone. We note, however, that if the reason for the existence of bands of periods in GD 358 is the oscillation of the base of the outer convective zone, then the *overall structure* of the star remains largely unchanged.

While the origin of this phenomenon is not entirely clear and deserves further exploration, we note that the existence of finite bands of periods poses a problem for applying the asteroseismological tools, because they require a set of *discrete* observed periods (even with possible uncertainties) that must be compared with well-defined periods calculated in stellar models of WDs. To tackle this difficulty, Bischoff-Kim et al. (2019) determined the mean periods for each band to be used in the asteroseismic fits (see their Table 5). The use of the mean values of periods in asteroseismology is well-justified given that the periods do not change secularly, but remain within a limited range of periods forming each band. In the lower and middle panels of Fig. 4, we show schematically all the periods detected from ground-based observations and the mean periods determined by Bischoff-Kim et al. (2019) for each band, respectively. The amplitude has been arbitrarily set to one to facilitate visualization. The period at 1014.35 s in the middle panel, represented with a dashed red line, is not associated to any specific band, but instead corresponds to a single detection in the 2016 ground-based observations. In the upper panel of Fig. 4 we include the 18 periods detected by TESS (Table 2). By comparing the mean periods (middle panel) and the periods detected by TESS (upper panel), we note that there are at least 9 periods that coincide between both sets, at  $\sim 420$  s,  $\sim 465$  s,  $\sim 495$  s,  $\sim 620$  s,  $\sim 700$  s,  $\sim 770$  s,  $\sim 810$  s,  $\sim 855$  s, and  $\sim 970$  s. Other periodicities are present in the ground-based mean periods but not in the TESS periods ( $\sim 540$  s,  $\sim 575$  s,  $\sim 660$  s,  $\sim 730$  s,  $\sim 902$  s,  $\sim 1015$  s, and  $\sim 1063$  s), and vice versa ( $\sim 925$  s and  $\sim 1090$  s). The space-based detection of 9 periods already found from ground-based observations is a great finding itself, because it confirms the results derived from exhaustive previous studies. Likewise, the detection of 2 additional signals which were not detected in previous works, allows us to broaden the spectrum of periods available for the asteroseismological study of this star. In fact, in order to extract as much information as possible with the tools of asteroseismology, it is crucial to employ as many periods (which represent eigenvalues of the star) as possible. In order to identify the pulsation modes and determine the possible period spacing of GD 358, which is essential to estimate the stellar mass, we

<sup>3</sup> These methods cannot, in principle, be directly applied to DAV stars to infer the stellar mass, for which the period spacing depends simultaneously on  $M_\star$ ,  $T_{\text{eff}}$  and  $M_H$  with comparable sensitivity, and this implies the existence of multiple combinations of these three quantities that produce the same spacing of periods.

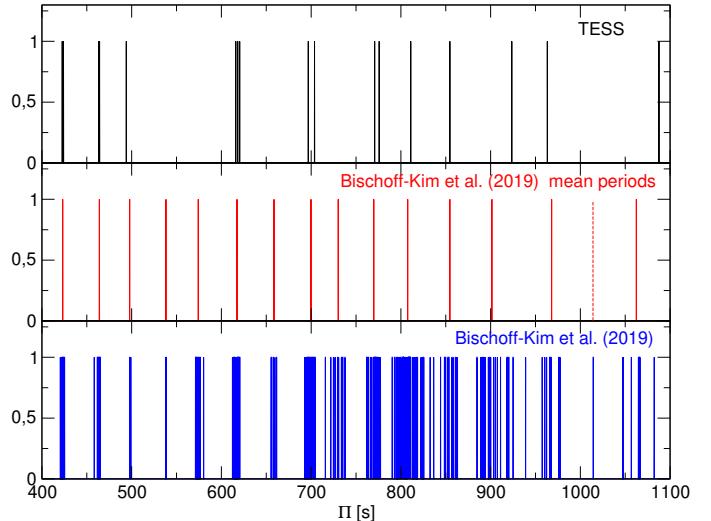
consider an expanded list of periods by combining the 15 dipole  $m = 0$  mean periods found by Bischoff-Kim et al. (2019) to the set of periods measured by *TESS* (Table 2). In the case of the 9 periods close to each other detected in both data sets, we decided to adopt the periods measured by *TESS* because they are in general more accurate. In the case of the periods near 700 s, we adopt the period 699.82 s from Bischoff-Kim et al. (2019), which seems to be the central component of the incomplete rotational triplet  $(+1, 0, -1) = (696.794 \text{ s}, \dots, 703.700 \text{ s})$  detected by *TESS* (see below). The resulting extended list of periods to be used in our analysis contains 26 periods and is shown in the first and second columns of Table 3. Note that we have also considered the period 1014.35 s, which has been detected only in the observations of 2016. This period was not considered in the asteroseismological analysis of Bischoff-Kim et al. (2019). However, since its value seems to fit very well in the apparent pattern of dipole periods with constant separation present in this star, as can be guessed from Fig. 4, we decided to include it in our subsequent analysis. We consider a proper procedure to combine periods detected at different times and both with observations from the ground and from space. We have the excellent example of the DAV star G29–38 (Kleinman et al. 1998) where a global pulsation spectrum for this star was constructed using different ground-based observations from a decade. Whatever the physical mechanism behind the alternate appearance and disappearance of modes, we can perform robust asteroseismological analyses by collecting the data from all epochs and constructing a combined spectrum of pulsations. This has been demonstrated for the DBV star KIC 08626021 (see Bischoff-Kim & Østensen 2011; Bischoff-Kim et al. 2014; Giammichele et al. 2018).

In the case of the rotational triplets, we have assigned the  $m$  value to the different components following Winget et al. (1994). We note that the frequency splittings are not constant among the different triplets. In fact, we have  $\delta\nu = 3.679 \mu\text{Hz}$  and  $\delta\nu = 3.629 \mu\text{Hz}$  for the complete triplet centered at  $2362.689 \mu\text{Hz}$  (423.246 s),  $\delta\nu = 5.839 \mu\text{Hz}$  and  $\delta\nu = 5.460 \mu\text{Hz}$  for the complete triplet centered at  $1617.409 \mu\text{Hz}$  (618.272 s),  $2 \delta\nu = 14.083 \mu\text{Hz}$  (that is,  $\delta\nu = 7.042 \mu\text{Hz}$ ) for the incomplete triplet with side components  $1435.142 \mu\text{Hz}$  (696.794 s) and  $1421.059 \mu\text{Hz}$  (703.700 s), and  $\delta\nu = 8.256 \mu\text{Hz}$  for the incomplete triplet centered at  $1289.082 \mu\text{Hz}$  (775.745 s). There is an apparent trend of larger  $\delta\nu$  for decreasing frequencies (increasing periods), in agreement with Winget et al. (1994) (see their Fig. 6 and Table 2). This dependence of the frequency splittings with the radial order of the modes could indicate differential rotation of GD 358, since each mode samples areas of different depth in the star, and would indicate different speeds of rotation<sup>4</sup>. While these results would imply that GD 358 does not rotate as a rigid body, in order to put this results on a firm basis it would be necessary to make a detailed analysis such as that carried out by Kawaler et al. (1999) (see, also, Córscico et al. 2011, for the specific case of the GW Vir star PG 122+200), which is beyond the scope of this paper.

We looked for a constant period spacing in the data of GD 358 using the Kolmogorov-Smirnov (K-S; Kawaler 1988), and the inverse variance (I-V; O’Donoghue 1994) statistical tests. Fig. 5 displays the results of applying the K-S and I-V significance tests to the period spectrum of GD 358. We show the case in which we adopted the full set of 352 periods of Tables 2 and 3 of Bischoff-Kim et al. (2019) (blue curves), and the

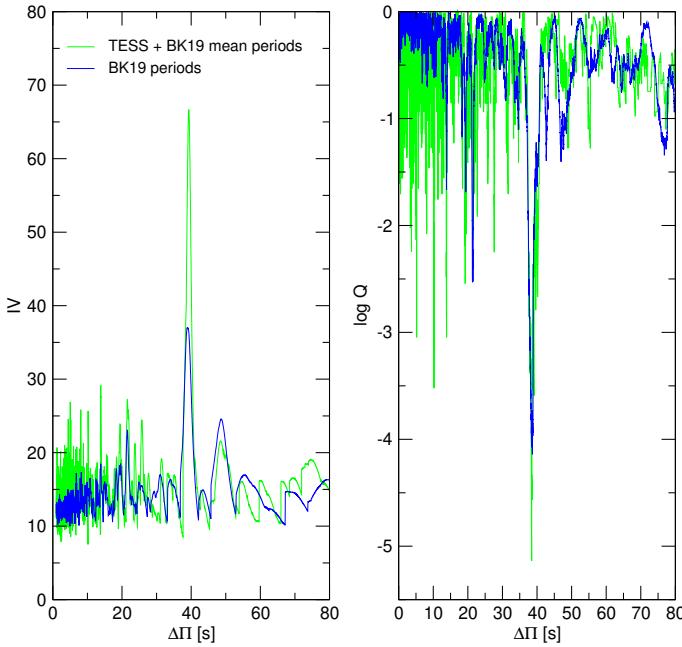
**Table 3.** Enlarged list of periods of GD 358. Column 1 corresponds to seven  $\ell = 1$   $m = 0$  average periods derived by Bischoff-Kim et al. (2019) (BK19). We include also the period at 1014.35 s extracted from the ground based observations of 2016 (see Table 3 of Bischoff-Kim et al. 2019). Column 2 corresponds to 18 periods detected by *TESS* (Table 2). The periods with an asterisk are the 13 periods used in the linear least square fit depicted in Fig. 6.

$\Pi_i^O$ (s) BK19	$\Pi_i^O$ (s) <i>TESS</i>	$\Pi_{\text{fit}}$ (s)	$\delta\Pi$ (s)	$\ell^O$	$m^O$
422.597				1	+1
423.246*	421.697	1.549	1	0	
423.906				1	-1
463.481*	460.942	2.539	1	0	
464.238			?	?	
494.026			?	?	
538.30*	539.433	-1.133	1	0	
574.22*	578.678	-4.458	1	0	
616.048			1	+1	
618.272*	617.923	0.349	1	0	
620.367			1	-1	
658.69*	657.168	1.522	1	0	
696.794			1	+1	
699.82*	696.414	3.406	1	0	
703.700			1	-1	
730.28*	735.659	-5.379	1	0	
770.808			1	+1	
775.745*	774.904	0.841	1	0	
811.076*	814.149	-3.073	1	0	
854.593*	853.394	1.199	1	0	
901.49			?	?	
923.556			?	?	
963.226			?	?	
1014.35*	1010.575	3.775	1	0	
1062.32			?	?	
1087.538*	1088.866	-1.328	1	0	

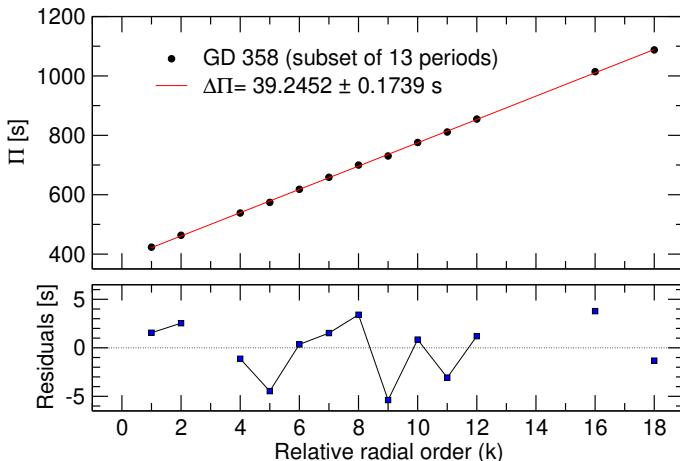


**Fig. 4.** Illustrative distribution of the periods of GD 358 according to *TESS* (19 periods, black lines, upper panel), and according to Bischoff-Kim et al. (2019) (15 mean periods, red lines, middle panel, and 352 periods, blue lines, lower panel). The period at 1014.35 s in the middle panel, represented with a dashed red line, corresponds to a single detection in the 2016 ground-based observations. The amplitudes have been arbitrarily set to one for clarity.

<sup>4</sup> We note that it makes no sense to calculate different rotational periods for GD 358 using the different frequency splittings using an uniform-rotation formula like the one described in Sect. 3.



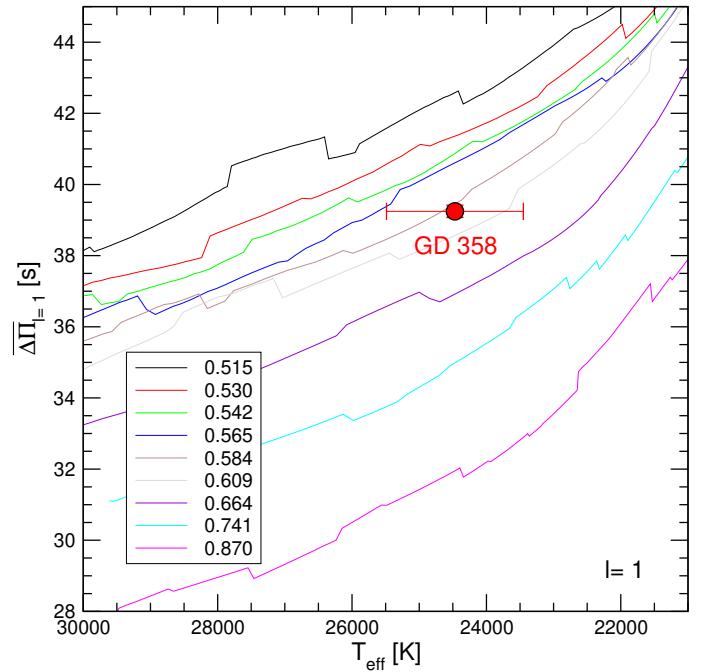
**Fig. 5.** I-V (left panel) and K-S (right panel) significance tests to search for a constant period spacing in GD 358. The tests are applied to the complete set of 352 pulsation periods of Tables 2 and 3 of Bischoff-Kim et al. (2019) (blue curves), and to the combination of the *TESS* periods plus the mean periods of Bischoff-Kim et al. (2019) (green curves), included in Table 3. A clear signal of a constant period spacing at  $\sim 39$  s is evident. See text for details.



**Fig. 6.** Upper panel: linear least-squares fit to the 13 periods of GD 358 marked with asterisks in Table 3. The derived period spacing from this fit is  $\Delta\Pi = 39.25 \pm 0.17$  s. Lower panel: the residuals of the period distribution relative to the mean period spacing, revealing signals of mode trapping in the period spectrum of GD 358. Modes with consecutive radial order are connected with thin black lines.

situation in which the *TESS* periods plus the mean periods of Bischoff-Kim et al. (2019) (green curves) are considered. In this last case we also considered the period at 1014.35 s disregarded in that paper. The two tests point to the existence of a pattern of  $\ell = 1$  constant period spacing of  $\Delta\Pi \sim 39$  s.

To derive a refined value of the period spacing, we have carried out a linear least-squares fit to the 13 periods marked with an asterisk in Table 3 (see Fig. 6). These periods are all supposed to be the  $\ell = 1$   $m = 0$  members of a sequence of periods equally spaced. We obtain a period spacing of  $\Delta\Pi = 39.25 \pm 0.17$  s.



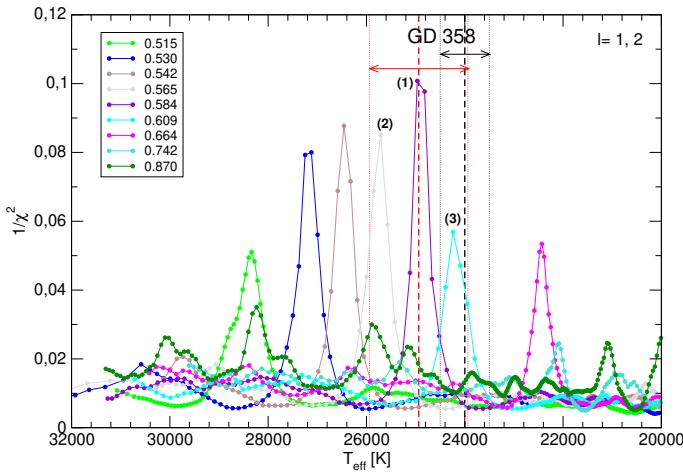
**Fig. 7.** Dipole ( $\ell = 1$ ) average of the computed period spacings,  $\overline{\Delta\Pi}_k$ , evaluated in a range of periods that embraces the periods observed in GD 358, shown as curves of different colors for different stellar masses. We consider a mean effective temperature for the star,  $T_{\text{eff}} = 24\,469 \pm 1018$  K, resulting from averaging  $T_{\text{eff}} = 24\,000 \pm 500$  K (Koester et al. 2014) and  $T_{\text{eff}} = 24\,937 \pm 1\,018$  K (Bédard et al. 2017). We adopt the mean period spacing  $\Delta\Pi = 39.25 \pm 0.17$  s derived in Sect. 5.1. We include the error bars associated to the uncertainties in  $\overline{\Delta\Pi}_k$  and  $T_{\text{eff}}$ . The stellar mass derived by interpolation is  $M_{\star} = 0.588 \pm 0.024 M_{\odot}$ .

This period spacing corresponds to our expectations for a dipole ( $\ell = 1$ ) sequence. On the other hand, if we assume that the  $\sim 39$  s period spacing were due to quadrupole modes, then the dipole period spacing would be  $\sim 68$  s, which is not present in the tests<sup>5</sup>. Thus, the identification of the  $\sim 39$  s period spacing as due to a sequence of dipole modes is robust. This sequence includes the 13 periods marked with an asterisk in Table 3, and the 7 periods that are the  $m = -1$  or  $m = +1$  components of the four triplets. The remaining 6 periods in that table can be associated with  $\ell = 1$  modes which, due to mode trapping effects, deviate substantially from the derived sequence of almost equally-spaced periods, or with modes with  $\ell = 2$  (or possibly higher) modes. In short, we have a total of 20 periods identified with  $\ell = 1$  modes.

In the lower panel of Fig. 6 we show the residuals ( $\delta\Pi$ ) between the dipole observed periods ( $\Pi_{\text{obs}}^0$ ) and the periods derived from the mean period spacing ( $\Pi_{\text{fit}}$ ). The average of the absolute values of the residuals is  $\overline{|\delta\Pi|} = 2.35$  s. The presence of several minima in the distribution of residuals strongly suggests the mode-trapping effects inflicted by the presence of internal chemical transition regions.

We calculated the average of the computed period spacings for  $\ell = 1$ ,  $\overline{\Delta\Pi}_k$ , in terms of the effective temperature for all the masses considered and a period interval of 400 – 1100 s, corresponding to the range of periods exhibited by GD 358. The results are shown in Fig. 7, where we depict  $\overline{\Delta\Pi}_k$  with curves of different colors according to the various stellar masses. For

<sup>5</sup> If such a dipole period spacing of  $\sim 68$  s existed, it would involve an extremely low stellar mass for GD 358, which can safely be ruled out.



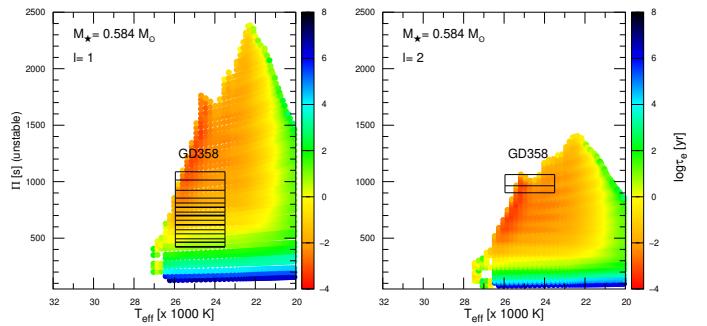
**Fig. 8.** The inverse of the quality function of the period fit in terms of the effective temperature, shown with different colors for the different stellar masses. The vertical black dashed line corresponds to the spectroscopic  $T_{\text{eff}}$  of GD 358 and the vertical dotted lines its uncertainties ( $T_{\text{eff}} = 24\,000 \pm 500$  K; Koester et al. 2014). Analogously, the blue vertical lines correspond to the spectroscopic  $T_{\text{eff}}$  and its uncertainties as given by Bédard et al. (2017) ( $T_{\text{eff}} = 24\,937 \pm 1\,018$  K). Three maxima have been labeled as (1), (2) and (3), corresponding to the three asteroseismological solutions compatible with spectroscopy (see the text).

the location of GD 358, indicated by a small red circle with error bars, we considered the average of the effective temperature,  $T_{\text{eff}} = 24\,469 \pm 1\,018$  K, based on the effective temperatures derived by Bédard et al. (2017) and Koester et al. (2014). We perform a linear interpolation and obtain  $M_{\star} = 0.588 \pm 0.024 M_{\odot}$ .

### 5.2. Period fits and the asteroseismological model

In our analysis of period-to-period fits, we only took into account the components  $m = 0$  present in the frequency spectrum of GD 358, and ignored the  $m \neq 0$  components. In total, we have 19 observed periods as the input for our asteroseismological period fits (see column 1 of Table 4). According to the results derived in Sect. 5.1, we can assume that a subset of 13  $m = 0$  periods are associated with  $\ell = 1$  modes — those marked with an asterisk in Table 3 — and leave free the identification as  $\ell = 1$  or  $\ell = 2$  for the remaining 6 periods. We display our results in Fig. 8. We find three possible solutions, that is, maxima of  $(\chi^2)^{-1}$ , labelled as (1), (2), and (3), which are compatible with both effective temperature determinations of GD 358 and its uncertainties. These solutions are (1)  $T_{\text{eff}} = 24\,967$  K and  $M_{\star} = 0.584 M_{\odot}$  ( $\chi^2 = 9.921$ ), (2)  $T_{\text{eff}} = 25\,712$  K and  $M_{\star} = 0.565 M_{\odot}$  ( $\chi^2 = 11.758$ ), and (3)  $T_{\text{eff}} = 24\,240$  K and  $M_{\star} = 0.609 M_{\odot}$  ( $\chi^2 = 17.595$ ). Clearly, the optimal solution is (1), since the DB WD model associated to it provides the best agreement between the theoretical and observed periods. Note that the effective temperature of this model is very close to that of GD 358 according to the spectroscopic determination of Bédard et al. (2017).

We adopt the model characterized by  $M_{\star} = 0.584 M_{\odot}$ ,  $T_{\text{eff}} = 24\,967$  K, and  $\log(L_{\star}/L_{\odot}) = -1.215$  as the asteroseismological model for GD 358. The location of this model in the  $\log g - T_{\text{eff}}$  diagram is displayed in Fig. 1 with a blue circle. In Table 4 we show a detailed comparison of the observed periods of GD 358 and the theoretical  $m = 0$  periods of the asteroseismological model. According to this model, the periods exhibited by the star correspond to 16 dipole  $m = 0$  modes with



**Fig. 9.** Left panel: periods of excited  $\ell = 1$   $g$  modes as a function of the effective temperature, with the palette of colors (right scale) indicating the logarithm of the  $e$ -folding time ( $\tau_e$  in years), for the DB WD sequence with  $M_{\star} = 0.584 M_{\odot}$ . Right panel: same as left panel, but for  $\ell = 2$  modes. In both panels, the pulsation periods of the DBV star GD 358 with the identification of  $\ell$  according to our asteroseismological model (see Table 4), are shown as horizontal segments, where their widths represent the possible  $T_{\text{eff}}$  interval, according to spectroscopy.

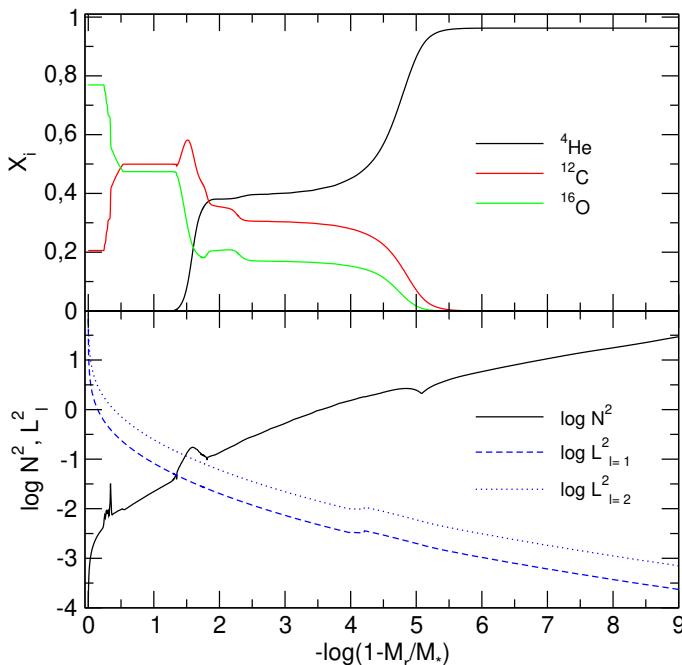
radial order  $k$  in the range  $k \in [8, 25]$ , and 3 quadrupole modes with  $36 \leq k \leq 43$ . The average of the computed  $\ell = 1$  period spacings for this model is  $\overline{\Delta\Pi}_{\ell=1} = 38.926$  s, very similar to the dipole mean period spacing obtained in Sect. 5.1 for this star,  $\Delta\Pi = 39.25 \pm 0.17$  s. In order to quantitatively assess the goodness of our period-to-period fit, we compute the average of the absolute period differences,  $\overline{\delta\Pi}_i = (\sum_{i=1}^n |\delta\Pi_i|)/n$ , where  $\delta\Pi_i = (\Pi_{\ell,k} - \Pi_i^0)$  and  $n = 19$ , and the root-mean-square residual,  $\sigma = \sqrt{(\sum_{i=1}^n |\delta\Pi_i|^2)/n} = \sqrt{\chi^2}$ . We obtain  $\overline{\delta\Pi}_i = 2.56$  s and  $\sigma = 3.15$  s. To have a global indicator of the quality of the period fit that takes into account the number of free parameters, the number of fitted periods, and the proximity between the theoretical and observed periods, we computed the Bayes Information Criterion (BIC; Koen & Laney 2000)<sup>6</sup>:

$$\text{BIC} = n_p \left( \frac{\log N}{N} \right) + \log \sigma^2, \quad (4)$$

$n_p$  being the number of free parameters of the models, and  $N$  the number of observed periods. The smaller the value of BIC, the better the quality of the fit. Note that this criterion penalizes for an excess of parameters. In our case,  $n_p = 2$  (stellar mass and effective temperature),  $N = 19$ , and  $\sigma = 3.15$  s. We obtain  $\text{BIC} = 1.13$  for the asteroseismological model, this value being the smallest among the possible solutions (1), (2) and (3) shown in Fig. 8. Also, the obtained BIC value is similar to that derived by Bischoff-Kim et al. (2019) (BIC = 1.2) for their best period fit to GD 358. The low value of BIC obtained in this work indicates that our period fit is very good.

Table 4 also includes the secular rates of period change ( $\dot{\Pi} \equiv d\Pi/dt$ ) expected for each  $g$  mode of GD 358. Note that all of them are positive ( $\dot{\Pi} > 0$ ), meaning that the periods are lengthening over time. The rate of change of periods in WDs and pre-WDs is related to  $\dot{T}$  ( $T$  being the temperature at the region of the period formation) and  $\dot{R}_{\star}$  ( $R_{\star}$  being the stellar radius) through the approximate expression  $(\dot{\Pi}/\Pi) \approx -a(T/T) + b(\dot{R}_{\star}/R_{\star})$  (Winget et al. 1983a). According to our asteroseismological model, the star is cooling with approximately constant stellar radius. As a consequence,  $\dot{T} < 0$  and  $\dot{R}_{\star} \sim 0$ , and then,  $\dot{\Pi} > 0$ . Can we compare these theoretical estimates of the

<sup>6</sup> See Liddle (2004, 2007) for an equivalent formulation of the BIC index.



**Fig. 10.** Chemical profiles (upper panel) and the squared Brunt-Väisälä and Lamb frequencies for  $\ell = 1$  and  $\ell = 2$  (lower panel) corresponding to our asteroseismological DB WD model with a stellar mass  $M_\star = 0.584M_\odot$  and an effective temperature  $T_{\text{eff}} = 24967$  K.

secular drift of the periods with the true rates of period change of GD 358? The frequencies (periods)  $2362.689\ \mu\text{Hz}$  ( $423.246\ \text{s}$ ) and  $2154.064\ \mu\text{Hz}$  ( $464.238\ \text{s}$ ) of GD 358, corresponding to the  $g$  modes with  $k = 8$  and  $k = 9$  according to our asteroseismological model, are the most stable frequencies of this star (Kepler et al. 2003; Provencal et al. 2009). However, the frequency shifts are large enough to mask any possible signs of evolutionary period change. Thus, we are forced to conclude that the observed periods of GD 358 are not stable enough to be able to measure the rate of change of periods due to the evolution of the star. This means that the values derived for the rate of change of periods of our seismological model of GD 358 remain (for now) only of academic interest.

We have also studied the pulsational stability/instability nature of the modes associated with the periods fitted to the observed ones. We assume the frozen-in convection approximation (Unno et al. 1989). In particular, we examine the sign and magnitude of the linear nonadiabatic growth rates,  $\eta_k = -\Im(\sigma_k)/\Re(\sigma_k)$ , where  $\Re(\sigma_k)$  and  $\Im(\sigma_k)$  are the real and the imaginary parts, respectively, of the complex eigenfrequency  $\sigma_k$ . A positive value of  $\eta_k$  means that the mode is linearly unstable (eighth column of Table 4). In the left (right) hand panel of Fig. 9, we show the instability domain of  $\ell = 1$  ( $\ell = 2$ ) periods as a function of the effective temperature for the DB WD model sequence with  $M_\star = 0.584M_\odot$ . The palette of colors (right-hand scale) indicates the logarithm of the  $e$ -folding time,  $\tau_e$  (in years) of each excited mode, being  $\tau_e = 1/|\Im(\sigma_k)|$ . Many pulsation modes are excited, and the interval of periods corresponding to unstable modes of our asteroseismological model is nearly coincident with the range of the periods exhibited by GD 358 for most of the range of possible effective temperatures. In particular, the pulsation periods of GD 358 fall into the highest excitation regimen (that is, shortest  $e$ -folding times), which reflects an excellent agreement between our nonadiabatic calculations and the observational data.

In Table 5, we list the main characteristics of GD 358 according to the previous studies and the present work. Most of the data of this table are extracted from Table 7 of Bischoff-Kim et al. (2019). In the case of the results of the present work, the errors in  $T_{\text{eff}}$  and  $\log(L_\star/L_\odot)$  are estimated from the width of the maximum in the function  $(\chi^2)^{-1}$  vs  $T_{\text{eff}}$  and  $\log(L_\star/L_\odot)$ , respectively, and the error in the stellar mass comes from the grid resolution in  $M_\star$ . Errors in the remainder quantities,  $\log g$  and  $\log(R_\star/R_\odot)$ , are derived from these values. The seismological stellar mass ( $0.584^{+0.025}_{-0.19} M_\odot$ ) is somewhat larger than the values derived from spectroscopy, although still compatible with them ( $0.508 \pm 0.050 M_\odot$  and  $0.560 \pm 0.028 M_\odot$ ) within their uncertainties. In addition, the stellar mass of the asteroseismological model is in excellent agreement with the stellar mass value derived from the mean period spacing ( $0.588 \pm 0.024 M_\odot$ ). We can conclude that the three approaches to determine stellar mass of GD 358 give very similar results, which implies that it is a well-constrained quantity.

In comparison with previous seismological studies, the effective temperature of our asteroseismological model is the largest one, although very similar to that derived by Fontaine & Brassard (2002), and in excellent agreement with the spectroscopic  $T_{\text{eff}}$  inferred by Bédard et al. (2017). Regarding the stellar mass of our asteroseismological model, its value is in excellent agreement with the values derived in all previous studies. Direct comparison of other quantities such as the central abundance of O ( $X_O$ ), the thickness of the envelope rich in O, C, and He,  $\log(1 - M_{\text{env}}/M_\star)$ , or the thickness of the pure helium envelope,  $\log(1 - M_{\text{He}}/M_\star)$ , becomes less clear because the chemical structure of our DB WD models is (by construction) substantially different from those used in previous studies. Even so, we can note that the envelope of our asteroseismological model is somewhat thicker than in the previous studies, and that the pure-He envelope has a thickness quite similar to the thickness derived in other works.

We succinctly describe the main properties of our asteroseismological DB model for GD 358. We display in Fig. 10 the internal chemical profiles of this model (upper panel), where the abundance by mass of the main constituents ( $^4\text{He}$ ,  $^{12}\text{C}$ , and  $^{16}\text{O}$ ) is depicted as a function of the outer mass fraction  $[-\log(1 - M_r/M_\star)]$ . The chemical structure of the model is characterized by a C/O core –resulting from the core He-burning phase of the prior evolution– shaped by extra-mixing processes such as overshooting. The core is surrounded by a layer rich in He, C, and O, which results from the nucleosynthesis during the TP-AGB stage. Above this shell, there is a pure He mantle resulting from gravitational settling that causes He to float to the surface and heavier species to sink. The lower panel of Fig. 10 shows the squares of the two critical frequencies of nonradial stellar pulsations, that is, the Brunt-Väisälä frequency and the Lamb frequency  $L_\ell$  for  $\ell = 1$  and  $\ell = 2$ . The shape of the Brunt-Väisälä frequency largely defines the properties of the  $g$ -mode period spectrum of the model. In particular, each chemical transition region in the model contributes locally to the value of  $N$ . The most notable characteristic is the highly peaked structure at the C/O chemical transition  $[-\log(1 - M_r/M_\star) \sim 0.34]$ . On the other hand, there is the He/C/O interface at  $-\log(1 - M_r/M_\odot) \sim 1.3 - 1.8$  that causes the presence of a notable bump in the Brunt-Väisälä frequency and affects the mode-trapping properties of the model.

**Table 4.** Observed and theoretical  $m = 0$  periods of the asteroseismological model for GD 358 [ $M_\star = 0.584M_\odot$ ,  $T_{\text{eff}} = 24\,967$  K,  $\log(L_\star/L_\odot) = -1.215$ ] corresponding to solution (1) in Fig. 8.  $\delta\Pi_k = \Pi_i^0 - \Pi_k$  represents the period differences,  $\ell$  the harmonic degree, and  $k$  the radial order. The last column gives information about the pulsational stability/instability nature of the modes.

$\Pi_i^0$ (s)	$\ell^0$	$\Pi_k$ (s)	$\ell$	$k$	$\delta\Pi_k$ (s)	$\dot{\Pi}_k$ ( $10^{-13}$ s/s)	Unstable
423.246	1	420.975	1	8	2.272	0.725	yes
463.481	1	462.511	1	9	0.970	1.354	yes
464.238	?	462.511	1	9	1.727	1.354	yes
494.026	?	499.054	1	10	-5.028	1.146	yes
538.300	1	538.126	1	11	0.174	1.394	yes
574.220	1	572.947	1	12	1.273	1.041	yes
618.272	1	617.647	1	13	0.625	1.319	yes
658.690	1	657.517	1	14	1.173	1.735	yes
699.820	1	696.097	1	15	3.723	1.650	yes
730.280	1	732.797	1	16	-2.517	1.506	yes
775.745	1	770.251	1	17	5.494	1.621	yes
811.076	1	812.662	1	18	-1.586	2.014	yes
854.593	1	853.775	1	19	0.818	2.026	yes
901.490	?	894.617	2	36	6.873	2.176	yes
923.556	?	926.073	1	21	-2.517	1.900	yes
963.226	?	965.149	2	39	-1.923	2.704	yes
1014.350	1	1010.499	1	23	3.851	2.470	yes
1062.320	?	1063.670	2	43	-1.350	3.004	yes
1087.538	1	1082.719	1	25	4.819	2.420	yes

**Table 5.** The main characteristics of the DBV star GD 358. The second column corresponds to spectroscopic and astrometric results, whereas the third, fourth, fifth, sixth, seventh, and eighth columns present the results from the seismological studies of BW94 (Bradley & Winget 1994), DK95 (Dehner & Kawaler 1995), MEA00 (Metcalfe et al. 2000), MEA01 (Metcalfe et al. 2001), FB02 (Fontaine & Brassard 2002), M03 (Metcalfe 2003), and BKEA19 (Bischoff-Kim et al. 2019), respectively, and the last column presents the seismological results from the present work.

Quantity	Spectroscopy Astrometry	BW94	DK95	MEA00	MEA01	FB02	M03	BKEA19	Seismology (this work)
$T_{\text{eff}}$ [K]	$24\,000 \pm 500^{(a)}$ $24\,937 \pm 1018^{(b)}$	24 000	24 121	22 600	22 600	24 800	22 900	23 650	$24\,967 \pm 200$
$\log g$ [cm/s <sup>2</sup> ]	$7.78 \pm 0.05^{(a)}$ $7.92 \pm 0.05^{(b)}$	8.0	...	...	...	8.02	...	...	$7.964_{-0.043}^{+0.048}$
$M_\star$ [ $M_\odot$ ]	$0.508 \pm 0.050^{(c)}$ $0.560 \pm 0.028^{(d)}$	0.61	0.58	0.605	0.65	0.625	0.66	0.571	$0.584_{-0.019}^{+0.025}$
$\log(L_\star/L_\odot)$	...	-1.30	...	...	...	-1.25	...	-1.287	$-1.215 \pm 0.015$
$\log(R_\star/R_\odot)$	...	-1.90	...	...	...	-1.89	...	...	$-1.880 \pm 0.014$
$X_\odot$ (center)	...	0.50	0.50	0.80	0.84	0.00	0.67	0.50	0.77
$\log(1 - M_{\text{env}}/M_\star)$	...	...	-2.6	-2.74	-2.74	-2.97	...	-2	-1.6
$\log(1 - M_{\text{He}}/M_\star)$	...	-5.70	-6.0	-5.97	...	-5.80	-2.0	-5.5	-5.98
$d$ [pc]	$36.6 \pm 4.5^{(b)}$ $43.02 \pm 0.04^{(e)}$	42	...	...	...	43	...	44.5	$42.85 \pm 0.73$

References: (a) Nitta et al. (2012); (b) Bédard et al. (2017); (c) From the evolutionary tracks and the  $T_{\text{eff}}$  and  $\log g$  values of Nitta et al. (2012); (d) From the evolutionary tracks and the  $T_{\text{eff}}$  and  $\log g$  values of Bédard et al. (2017); (e) Gaia.

### 5.3. Asteroseismological distance

We can estimate an asteroseismological distance to GD 358 on the basis of the luminosity of the asteroseismological model [ $\log(L_\star/L_\odot) = -1.215 \pm 0.015$  and  $T_{\text{eff}} = 24\,967 \pm 200$  K] and a bolometric correction  $BC = -2.71$  (Bradley & Winget 1994). The absolute magnitude can be assessed as  $M_V = M_B - BC$ , where  $M_B = M_{B,\odot} - 2.5 \log(L_\star/L_\odot)$ . We employ the solar bolometric magnitude  $M_{B,\odot} = 4.74$  (Cox 2000). The seismological distance  $d$  is derived from the relation:  $\log d = [m_V - M_V + 5]/5$ . We use the apparent visual magnitude  $m_V = 13.65 \pm 0.01$  (Winget et al. 1982a), and obtain the seismological distance and parallax  $d = 42.85 \pm 0.73$  pc and  $\pi = 23.33 \pm 0.41$  mas. The uncer-

tainty in the seismological distance comes from the uncertainty in the luminosity of the asteroseismological model. These values are consistent with the results of Bradley & Winget (1994) and Bischoff-Kim et al. (2019), although a bit larger than the distance derived by Bédard et al. (2017). A very important check for the validation of the asteroseismological model is the comparison of the seismological distance with the distance derived from astrometry. We have available the estimates from Gaia EDR3,  $d_G = 43.02 \pm 0.04$  pc and  $\pi_G = 23.244 \pm 0.024$  mas. They are in excellent agreement with the asteroseismological derivations. This adds robustness to the asteroseismological model we

found for GD 358. Also, the match of seismological and trigonometric parallax confirm that we are seeing dipole modes.

## 6. Summary and conclusions

In this paper, we have presented new space observations of the already known DBV star GD 358. This is the first time that this star is intensively examined by a space mission such *TESS*, which provides high-quality time-series photometry for asteroseismic purposes. The results of our analysis broadly confirm the previous observations from extensive ground-based observational campaigns. We also carried out a detailed asteroseismological analysis employing fully evolutionary models of DB WDs. We find that the evolutionary and pulsational properties of GD 358 according to our analysis based on space data combined with ground-based data are in line with the results of previous asteroseismological analyzes of this star based on data from the ground alone. The present study is the third part of a series of papers devoted to the study of pulsating H-deficient WDs observed with *TESS*. We extracted 26 periodicities including 8 combination frequencies from the *TESS* light curve of GD 358 using a standard pre-whitening procedure to derive the potential pulsation frequencies. The oscillation frequencies, associated to *g*-mode pulsations, have periods from  $\sim 422$  s to  $\sim 1087$  s. We combined these space data with the abundant ground-based observations available and found a constant period spacing of  $39.25 \pm 0.17$  s, which allowed us to infer its stellar mass ( $M_\star = 0.588 \pm 0.024 M_\odot$ ) and constrain the harmonic degree  $\ell$  of some of the modes. We performed a period-to-period fit analysis on GD 358, which provides us with an asteroseismological model with a stellar mass ( $M_\star = 0.584^{+0.025}_{-0.019} M_\odot$ ) in agreement with the stellar-mass value inferred from the period spacing, and also compatible with the spectroscopic mass ( $M_\star = 0.560 \pm 0.028 M_\odot$ ). In agreement with previous works, we found that the frequency splittings vary according to the radial order of the modes, suggesting differential rotation and preventing us to derive a reliable representative rotation period of the star. The seismological model derived from our analysis allowed us to estimate the seismological distance ( $d_{\text{seis}} = 42.85 \pm 0.73$  pc) of GD 358, which is in excellent agreement with the precise astrometric distance measured by *GAIA* EDR3 ( $d_{\text{GAIA}} = 43.02 \pm 0.04$  pc).

In accordance with the findings of our recent works focused on pulsating H-deficient WDs (Bell et al. 2019; Uzundag et al. 2021) we conclude that the high-quality data collected by the *TESS* space mission, combined with ground-based photometric data, are able to provide a reliable input to the asteroseismology of WD stars. The *TESS* mission, along with future space missions and upcoming surveys, will allow an unprecedented boost to the stellar seismology of these ancient stars.

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