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# Broadening views of mathematical creativity: Inclusion of the undergraduate student perspective

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ABSTRACT

Numerous conceptions of creativity exist in the literature; yet these are commonly based on the perspectives of professional mathematicians. Including students' perspectives in creativity is crucial not only for a more robust picture of what it means to be creative but also to combat damaging dominant narratives about who can be creative. We examined calculus students' views of mathematical creativity, a group not often considered in the creativity literature, to broaden future considerations of creativity. Interviews with N=55 calculus students across various institutions were conducted. Results show six emergent wide-ranging themes of these students' creativity views: actions and attitudes, application, different ways, originality, outside authority, and understanding. Of these six themes, understanding was striking due to a clear distinction between students who felt understanding was required first to be creative and students who felt creativity could lead to better understanding. Our themes provide insight into what may resonate for some students, which may serve as coding parameters in qualitative and quantitative studies for researchers conducting future work about mathematical creativity.

Creativity has emerged as both a skill sought-after by employers (Schöning & Witcomb, 2017) and a necessary component of a robust mathematical curriculum (Askew, 2013). At the same time, research on mathematical creativity and how to foster it are becoming more voluminous (Singer et al., 2017). A necessary component for this research is understanding the construct of mathematical creativity by those being studied. Yet, while such conceptions of creativity abound in the literature (Mann, 2006), the vast majority are based on mathematicians' and mathematics instructors' views (Borwein et al., 2014; Leikin et al., 2013; Sriraman, 2009). Research on students' perspectives of mathematical creativity remains sparse (Cilli-Turner et al., 2019).

Inclusion of the student perspective provides a fuller picture of mathematical creativity–what it is, and who can embody it–but also challenges some pervasive dominant perspectives, such as the *genius view* (as described in Silver, 1997). A focus on Calculus I students is consequential as Calculus I is a required course for most STEM majors, thus acting as a gatekeeper in STEM (Ellis, Fosdick, & Rasmussen, 2016). This makes Calculus I an ideal course in which to foster undergraduate students' mathematical creativity. Indeed, recent studies are emerging about cultivating creativity in calculus (Arsyad et al., 2017; El Turkey et al., 2020; Tang et al., 2020). We need insight, however, into how calculus students think about and interpret mathematical creativity in order to conduct creativity research studies with this population.

In this paper, we share results of exploring students' views on creativity and show that students' perspectives are wide-ranging and complex. Our findings counter claims that students do not see science as creative (Valenti et al., 2016) or that one must be a genius to be creative in mathematics (Silver, 1997; Moore-Russo & Demler, 2018). We illustrate six themes present in Calculus I students' responses to what it means to be creative in mathematics and discuss how these themes supplement those present in the literature. We also provide two cases of a student who did not and one who did feel creative, to illustrate the six themes in context and how multiple themes appear together.

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## Background literature

### *Dominant narratives of mathematical creativity*

Embedded in views and conceptions of mathematical creativity are two dominant narratives: the first about who determines what mathematical creativity is (which we will call the "expert view") and the second about who can be mathematically creative (which we will call the "genius view"). We use the definition by Berry III et. al (2011) that dominant narratives "embody and dictate expectations about how things work and how stories are framed" (p.11). In what follows, we outline the development of these two dominant narratives. First, we provide a short chronology of conceptions of mathematical creativity to show that nearly all have been developed exclusively by studying experts (i.e., mathematicians) and their mathematical problem-solving processes. Second, we examine the genius view of creativity, which propagates ideas about who can and cannot be creative.

Research on perspectives of general creativity started with examining what experts (e.g. scientists and mathematicians) do when problem solving, a tradition that has continued with mathematical creativity researchers. One of the first documented models of mathematical creativity was that of the four-stage creative process (preparation, incubation, illumination, and verification) developed by Wallas (1926). Development of this model was based on the mathematical research process described by Henri Poincare (Riling, 2020). Hadamard (1945) built on Wallas' model, exploring mathematical creativity by surveying prominent mathematicians at the time. The first standard description of creativity in psychology was provided by Guilford (1950), who saw it as the "abilities that are most characteristic of creative people" (p. 444). When referring to these creative people, Guilford again described experts, using a thought experiment about habits of scientists and technologists. Using characteristics of these hypothetical creative people, Guilford (1950) developed fluency (number of ideas), flexibility (changing ideas or approaches), and novelty (unique or original ways) as components of creativity. Torrance (1974) expanded on the vision of Guilford's by creating a fourth component of creativity, elaboration (describing or elaborating on those ideas), and developing tests to measure a person's creativity. It is important to note that many subsequent studies use a conception of creativity based on one of the *expert* creativity views listed above (e.g., Leikin, 2013).

More recent works continue to use the voices and experiences of experts to define creativity. For example, Sriraman (2005) interviewed five research mathematicians about their creativity and found Hadamard's four stages were still applicable to modern day mathematicians, while Borwein et al. (2014) used quotes and excerpts from mathematicians to paint a picture of how to think about mathematical creativity in their book "Mathematicians on Creativity." A recent meta-analysis of views of creativity in the mathematics education literature (Joklitsche et al., 2022) examined these views using published creativity definitions, further perpetuating the expert view. While numerous studies assess creativity of students at all educational levels (Haylock, 1987; Liljedahl, 2013; Singer et al., 2017), very few exist that explicitly ask students about their views of the nature of mathematical creativity to determine how the students are defining such a concept.

Another dominant narrative in the literature regarding who can be mathematically creative (Moore-Russo & Demler, 2018) is the association between mathematical creativity and giftedness (Sriraman, 2005) or genius (as described in Silver, 1997). In the United States, equating innate mathematical ability with creativity ignores the symbolic and material racism (Battey and Leyva, 2016) involved in declaring students as "mathematically gifted". This approach perpetuates the *genius view* of mathematical creativity, as termed by Silver (1997). While the genius view of creativity is losing favor among education researchers (Silver, 1997; Moore-Russo & Demler, 2018), this discourse is still strong amongst mathematicians and mathematics educators. Haavold (2016) wrote that "a widespread belief among both teachers and students is that mathematics is created only by very prodigious and creative people; others just have to try to learn what is handed down" (p. 233). Along the same lines, Leikin et al. (2013) found strong agreement amongst K-12 teachers from six different countries with sentiments of creativity as a fixed ability. Interestingly, while the genius view of creativity persists in mathematics, creativity in other disciplines is seen as developed by cultures, personality traits, and collaboration (Riling, 2020).

The historical development given above of describing mathematical creativity shows that the literature has thus far been concerned with only the voices of experts and those deemed "geniuses". However, if helping students to develop their mathematical creativity is our overall goal, then incorporating the voices of mathematics students is crucial for studying creativity from an educational standpoint. There are several reasons for this. First, we risk omitting aspects of mathematical creativity that may most resonate with students' creative development. Second, if we limit descriptions of creativity to the experiences of experts, then we risk perpetuating the dominant views that dictate expectations from an expert lens. This is more inclusive of students who internally feel creative but identify it differently. Given the history of academia, it is a well-known fact that early university mathematics education was limited to mostly White male scholars with wealth (Leyva et al., 2021). Thus, the bulk of the scholarship and theories that constitute what we consider as knowledge reflect the experiences and ideas of this singular group of individuals (Borum & Walker, 2012). This includes the early scholarship on what constitutes mathematical creativity, which many of the current research views are based on. Riling (2020) raised a concern about the literature's tendency to use Wallas' Four Stages to describe mathematicians' creative process: "Is it possible that the four-stage process appears to be widespread because it is an approach that professional mathematicians learn from their teachers and mentors, rather than because it is the only form of mathematical creativity?" (p. 9). Finally, student views on mathematical creativity are worth discerning, to inform instructors of these student perspectives so that they can be incorporated into curriculum and classroom instruction. This would assist with dismantling the tradition of preserving "curricular models [that are] based upon the thinking of White elites" (Martin, 2013, p. 323, as cited by Battey & Leyva, 2016, p. 60) and specifically, "at the tertiary levels… the structure imposed… and limitations of 'narrow, profoundly Western centric attitudes' (Creme, 2003, p. 273)" (Sriraman, 2005, p. 21), in favor of educational instruction rooted in promoting creativity for all.

### *Students' conceptions of mathematical creativity*

Despite the need to broaden views of creativity in the literature, there are few studies that ask non-experts specifically about their definitions of mathematical creativity and almost none that ask students. Moore-Russo and Demler (2018) investigated perceptions of creativity by interviewing U.S. faculty and staff participants from K-12 "gifted" mathematics programs. This study found that only 7 of the 13 participants thought of flexibility, fluency, and novelty as essential to creativity. Additionally, all participants expressed beliefs that all students were capable of being mathematically creative, eschewing the genius view, despite being part of a "gifted" program in mathematics. These results suggest that views about mathematical creativity are already shifting away from the genius view to a more student-friendly perspective.

Investigating students' views of mathematical creativity, Tang et al. (2015) contrasted professional mathematicians' and transition-to-proof students' definitions of mathematical creativity. Six research mathematicians and eight undergraduates were interviewed and stark differences between the students' and mathematicians' views were discovered. For instance, only 9% of students' codes associated making connections with mathematical creativity compared to 38% of the mathematicians' responses. A large proportion (64%) of students' responses referred to creation of ideas as a critical component of creativity, while this code occurred in less than half of mathematicians' utterances (45%). This again shows that there are fundamental differences in the views of students as compared to experts.

Cilli-Turner et al. (2019) also focused on exploring students' definitions of mathematical creativity and how they shifted during a transition-to-proof course. Seven students were interviewed at the conclusion of this course. While many of the students expressed views akin to fluency, flexibility, and originality, four of the seven students relayed components of mathematical creativity not well reflected in current literature. One student defined creativity as stemming from identifying and revising mistakes in one's work. Three other students thought of mathematical creativity as akin to efficiency and implied that shorter proofs were more creative. Three of the seven students recognized that their views on mathematical creativity had evolved during the course and attributed this shift to elements of the classroom. One student pinpointed the proof-based nature of the course as opening more opportunities for her to be mathematically creative, while two other students said the inquiry-based teaching style of the course and the instructor's pedagogical choices, such as having students present proofs to the class, affected the evolution of their conceptions of creativity.

In this study, we aim to bring forth calculus students' views of mathematical creativity to provide a more inclusive and robust understanding of the topic and add the student perspective and voice to the growing body of literature on mathematical creativity. The research questions guiding this work are:

1) What are calculus students' views about mathematical creativity?
2) How do students view themselves as mathematically creative in an introductory calculus course?

## Research perspective: creativity as self-reported and anti-deficit

We take the following perspectives on creativity, that students' perceptions of their own creativity and experience is worthy of study and can be taken as valid. This is appropriate as these perceptions communicate students' lived realities (Abakpa et al., 2017) and we are interested in students' experiences as lived and retained (e.g., Gholson & Martin, 2019). Students carry their perceptions of their experiences with them into future mathematical situations, so these self-perceptions are relevant for study. Therefore, this lens takes as an underlying assumption that students' self-reported views of creativity and whether they themselves feel creative to be valid and appropriate data. In the general creativity literature, an analogous concept, Implicit Theories, is presented as "constructions by people (whether psychologists or laypersons) that reside in the minds of the individuals" (Sternberg, 1985, p. 608). Runco (1999) stated that "As it happens, implicit theories of creativity do differ from explicit theories in several ways, and they should be examined and respected. They tell us how people in the natural environment really think about creativity" (p. 646). This lens is necessary to ensure the student voice is retained and highlighted in reporting on undergraduates' views of mathematical creativity; it is important we capture students' own definitions of mathematical creativity and not project dominant views about creativity onto them

We also adopt an anti-deficit approach to students' creativity in mathematics. Anti-deficit approaches focus on what students do know rather than what they do not, as areas on which further knowledge can be built on (Langer-Osuna et al., 2016). Specific to creativity, the anti-deficit approach to creativity taken in this paper starts with the assumption that all students can be mathematically creative and that they bring productive resources for being creative in mathematics (adapted from Adiredja, 2019). In our work, we consider all students to be doers of mathematics who make up an important part of the mathematical community. Therefore, what students say about what it means to be creative (to them) is worth listening to, taking seriously, and useful for guiding future students' creativity. Dominant views of mathematical creativity provide deficit discourse about students' mathematical creativity, as such narratives assume that only some students (those labeled as geniuses) are capable of creativity. These dominant views serve to "justify attitudes and behaviors that reproduce systems of domination, to legitimize oppression as the natural and moral consequence of dominant-group merits and subordinate-group deficiencies" (Adiredja & Louie, 2020, p. 46).

In summary, deficit discourses create a set of master-narratives based on dominant views about calculus students and their mathematical creativity. We share student quotes to provide a more inclusive approach to what constitutes mathematical creativity – that these students do indeed hold robust views about mathematical creativity and can identify course experiences that allowed their views of creativity to flourish.

## Method

### *Participants & context*

This paper reports a subset of data from a larger study conducted at several universities across the United States. The participants were $N$=55 undergraduate students enrolled in Calculus I across these institutions. Calculus I in the United States typically covers the major topics of limits, derivatives, and basic integrals. We report here on data from two cohorts of students; cohort 1 was enrolled in Calculus I during Spring 2019 and cohort 2 was enrolled during Spring 2020.

The goal of larger study was to determine impacts on calculus students (e.g., greater self-efficacy, persistence, affect) of explicitly valuing and fostering mathematical creativity. This study included three cohorts (Spring 2019, Spring 2020, and Fall 2021) of participant instructors and their Calculus I students. Participant instructors implemented at least six *creativity-fostering* tasks and used a reflection tool called the Creativity-in-Progress Reflection in their Calculus I course. Additionally, instructors attended a weekly professional development (PD) session (online) with the research team.

### *Data collection*

The larger research project collected several different data sources (e.g., professional development videos, interviews with students and instructors, surveys, etc.). However, in this paper, we report on results from post-semester semi-structured interviews conducted with 55 students. These 60–90-minute interviews were conducted over Skype or Zoom and then transcribed. We conducted 17 interviews with cohort 1 students and 38 interviews with cohort 2 students. In this paper, the bulk of the results came from responses originating from these interview questions: *What is your definition of mathematical creativity?, Did you feel creative in your Calculus I course?, Do you think it's important to be creative in mathematics?*, and *Give an example of a task where you felt creative*. Given the semi-structured format, follow-up questions specific to students' responses were asked to delve deeper into students' personal definitions and experiences around creativity.

Using students' responses on the interview or survey, we categorized their self-identified gender and racial identities. These categorizations resulted in 35 female, 19 male, and one non-binary participant. Participants included 34 People of Color, categorized as a student who self-identified as a race or ethnicity that was not only White or Caucasian, and 21 White students.

### *Data analysis*

The first three authors conducted an open *in vivo* coding (Saldaña, 2015) on the interview transcripts from cohort 1. This method aligns with our phenomenological approach as the main characteristic of *in vivo* coding is to label each theme using the exact words of the interviewee. Once open coding on all data from cohort 1 was complete, there were many codes from combining all three coders. We organized the codes by collating and relabeling; through axial coding (Saldaña, 2015),

**Table 1**
Coding manual excerpts

| Theme | In Vivo Codes (Excerpts) |
|---|---|
| Actions and Attitudes | Answer own questions; Answer without help; Ask questions; Closer look; Comfortable; Confidence; Curiosity; Figuring out by yourself; First step is not right; First try; Genius; Learn way you want to; My own; Making up my own; Not fear of messing up; Not understanding but try; Overcoming; Persistent; Play around; Practice out of box; See what fits; Solving concept for yourself; Think out of the box; Willing to try; Wondering; Working hard; |
| Application | Apply rules to a problem; Help people; New theorems for application; Other applications; Outside of class; Useful; Using previous math |
| Different Ways | Another way; Different perspectives/processes/ways; Diverse route; Exhausting ways; Find another way; Many ways; Multiple theorems; Normal way not working; Not known methods; Not one way |
| Originality | Advance math; Conjecturing; Create new; Don't know how; New; No one ever thought of; Problem posing; People haven't thought [of]; Uncertainty; Want to find new |
| Against Authority | Breaking; Not book problem; Not do what taught; Not exact formula; No formula; Not rules; Not step by step |
| Understanding | Different levels of understanding; Further than laws; Help understand; Incorporates more sectors; Intuition; Know what to do; Recognizing; Relationships; Understand; Understanding needed; Utilize knowledge; Wrong in past |

we sorted codes into eight identified themes. Data from cohort 2 students were then coded using these eight themes as a coding framework; no additional themes were identified in the cohort 2 data.

For norming purposes, all three coders coded the same four students in cohort 2. Then, another six students were coded by two of the three coders, and the final 28 students were coded by one of the coders after significant discussion and norming was completed. After completion of coding of data from both cohorts, we examined the eight themes and determined that two were sub-themes of one of the others and were not distinct. This allowed us to collapse the eight themes to six. In a few instances, a coded quote appeared in more than one theme. We chose not to use inter-rater reliability and instead "rel[ied] on intensive group discussion and simple group 'consensus' as an agreement goal" (Harry, Sturges, & Klingner 2005, p. 6). We used nVivo$^{TM}$ software to organize data, code, and run counts. An excerpt of our coding manual, with a selection of the in vivo codes and corresponding theme, is provided in Table 1.

## Results

We discuss the results in three subsections. First, we define the six themes that emerged from the data, with examples. We delve into one theme, *Understanding,* to show the contrast of viewpoints embedded. Finally, two cases are introduced to illustrate the multiple themes in context and as examples of a student who did not versus did feel creative in their Calculus I course, respectively.

### *Six themes in students' views of creativity*

Data coding resulted in six themes exhibited in students' views on mathematical creativity: *Actions and Attitudes, Application, Different Ways, Originality, Against Authority,* and *Understanding.* As a person's experiences with mathematics and creativity are inextricably linked with their identities, when a student participant is quoted, we give the self-identified gender and racial description of each participant, as suggested by Adiredja et al. (2015), along with their self-chosen pseudonym.

#### *Actions and attitudes*

The *Action and Attitudes* theme consisted of characteristics and habits that are enacted as either an action one could take (e.g. asking questions) or as an attitude one could hold (e.g. willingness to try). This is an extension of Rhodes (1961)'s *Persons* category of creativity, with our inclusion of actions as personal as well. Students talked about these actions and attitudes in reference to themselves, implying some level of self-efficacy; they were in control of being creative depending on what they did and how they felt. Creativity as "genius" fell under this theme, but only one student referenced genius in their views of creativity. An example of creativity as an attitude is "the willingness to keep trying something" (Frost, White/Spanish/Indigenous, Male), which implies that creativity comes from underlying persistence. An example of creativity as an action was "getting to play around with math" (Bryan, White, Female).

#### *Application*

The theme *Application* was mathematical creativity as applying mathematics to other mathematics or "real life" situations. For example, Shy-Ann (White, Female) stated, "I feel like [math] creativity is putting those words into play. Putting those equations into a real-life aspect." Some of those real-life aspects were also career objectives to the student: "I'm planning on becoming a veterinarian, I'm gonna have to find math when I reset a bone or have to do anything like that… just realizing that math isn't just on a piece of paper. It's all around you" (Estella, Latinx, Female).

#### *Different ways*

The *Different Ways* theme was mathematical creativity as multiple (or varied) techniques or solution approaches. For instance, Winston (Native American/White, Male) commented that creativity to him was "being able to ... tackle something from different angles." Danger (White, Female) emphasized multiple approaches, saying that "Calculus is, in my mind, is just like, you know, 'how many different ways can we analyze and explore this pattern that we're looking at?'" However, there were some instances where students spoke about multiple solutions, not just approaches, to a problem: Caydicle (White, Non-binary) expressed, "But there is math now … well, there's not always an exact answer anymore. And that means there's not always exactly one way to find it."

#### *Originality*

*Originality* was mathematical creativity as creating new mathematics, either new to the student as an individual or in general. Situations that fell under *Originality* included problem posing, advancing mathematics, and doing something no one else had thought of before. Abbie (White, Female) described a situation that showcased multiple opportunities for original work: "We had to create and solve most of our own problems based on problems in the textbook and then we had a quiz where we had to create new mathematics, where we had to create new theorems". *Originality* could refer to mathematics new to oneself: "you yourself come up with problems where you can't answer" (Ensigo, Mexican, Female). *Originality* could also refer to mathematics that was new to everyone: "It would be like coming up with things that you know people haven't thought about before" (Bryan, White, Female). *Originality* was its own separate category, as students with coded statements frequently referenced something new or novel; this is different from *Different Ways* which need not be new or unusual but only different to the student.

#### *Against Authority*

*Against Authority* was any view of mathematical creativity as juxtaposed with an authority outside of one's self. This authority could be the discipline, class, textbook, or teacher, and *Against Authority* would be going against this authority in some manner. Examples of this category often included some variation of Caydicle's view (White, Non-Binary): "I think the only way to advance in math at a certain point is to get creative about it. You can't just follow the formulas at a certain point." This view of creativity was against following formulas, rules, examples, textbook, and the teacher. For example, JD (Black, Female) expressed, "I wasn't doing it myself, I was just learning the process of my professor. So, I didn't feel very creative"; she felt she was not creative because she was following the teacher's ways. *Against Authority* is differentiated from *Different Ways* and *Originality* when the student clearly referenced an artifact of the classroom (e.g., a particular method shown by the instructor or textbook) and stated that to be mathematically creative, they must go explicitly against that artifact.

*Understanding*

*Understanding* was when a student's view of mathematical creativity included references to learning mathematical concepts, such as when Vivala (Latinx, Female) said "I feel like once you have an understanding of the material, you have more flexibility to be creative." This code was also used if a student spoke about some component of understanding: organizing, preparing, building on, or leveraging prior knowledge. 007 (Asian, Female) illustrated this when she defined being creative as "making connections between things ... you want to take, like, rudimentary knowledge and ... through connection building … find a completely different approach."

*Creativity and understanding - a deeper analysis*

In exploring our themes, the *Understanding* theme required a deeper dive as it incorporated two contrasting viewpoints regarding mathematics vis a vis creativity: a disparity that was not observed strongly in any of the other themes. For example, the following excerpt by Angus O'Sullivan was coded as *Understanding*: "I have a **hard enough time... understanding the concept** as is, much less trying...**I don't spend any time trying to think of a different or better way to do it**". The following quote by Aon (Female, African American) was also coded under the *Understanding* theme:

> When I'm [having a hard time] I'm going to go on YouTube and find different ways of doing it. I'm pretty sure there should be different ways to do it and **that creativity will just help you. Just help you learn it in a different way and maybe understand it better.**

However, even though each of the above excerpts fell under this same theme, we can see that they are remarkably different in spirit. Angus O'Sullivan espoused the view that understanding is necessary before one can be creative. Aon's quote, however, suggests a contrasting view in which she emphasized that creativity is needed for better understanding. This realization urged us to complete a second-level coding of this theme, grouping students who discussed understanding as a prerequisite for creativity (*understanding* → *creativity*) and those who felt creativity led them to more understanding (*creativity* → *understanding*). There were also utterances coded as *Understanding* that fell into neither of these categories.

Thirteen students fell into the *understanding* → *creativity* group, expressing a view that proficiency in mathematical concepts was helpful before attempting to be mathematically creative. Six students fell into the *creativity* → *understanding* group. These students expressed a view consistent with the literature that says mathematical creativity can lead to learning mathematics (Leikin 2007; Mann 2006). It is interesting to note that five of the six students in the *creativity* → *understanding* group were women of color, while the *understanding* → *creativity* group was more evenly split by gender and primarily White. Further research should investigate whether we can leverage creativity for mathematical understanding for women of color.

*Aggregate themes*

As an aggregate (see Table 2), the most frequent student view of mathematical creativity was *Different Ways*, mentioned at least once from 46 of the 55 students (84%). *Against Authority* and *Application* were the least coded, with 22 students (40%) and 21 students (38%) respectively. Additionally, across all 55 students in our study, 33 students said they felt creative in their calculus course, while 22 students said they did not. We discuss more about the popularity of themes and feeling creative versus not feeling creative in Satyam et al. (2021).

*Cases: illustration of themes & feeling creative*

We now illustrate our multiple themes by delving further into two cases: Angus O'Sullivan and JCRU. We chose these two students because they each talked about multiple views of creativity, but one did not feel creative whereas the other did. Inclusion of these cases is not to claim causality about what led to them feeling creative (or not) but rather to showcase the student voice in our results.

*Angus O'Sullivan: case of not feeling creative in calculus*

We first discuss the case of Angus O'Sullivan, to showcase the following views of creativity, as *Different Ways, Against Authority, Application, Originality, and Understanding.* Angus was a student in cohort 1 who identified as a White male. His major was biomedical sciences. When asked what it meant to him to be creative in mathematics, he said the following:

> ANGUS O' SULLIVAN: Finding a different means or method [*Different Ways*] … or means, method, or mechanism to evaluate an expression.
>
> INTERVIEWER (INT): OK. What do you mean by "different" from?
>
> ANGUS: What's basically the written way [*Against Authority*]. So, a lot of things we do is, he'll show us. I'm trying to think of the most recent example. Like for an integral… we're first taught the Riemann Sum. And then after we work on Riemann sum for a bit and the indefinite integrals, he then shows us definite integrals, he's like 'well, this is a better way to do it.' Well, that's...why didn't you do that the first time? But it's part of, it's part of learning the history of it and knowing that it's justified, and it works because you can apply multiple theorems to get the same value, and the creativity in math is finding other applications for theorems [*Application*] or developing new theorems [*Originality*] that can also be applied.

Angus indicated his view of creativity was of *Different Ways,* albeit in carrying out a procedure, based on his word choice of "evaluate an expression." When pressed further about what he meant by the word "different," he explained that he meant the written method of Riemann Sums provided by the instructor. This was also an example of *Against Authority* because this method was the artifact to which all other methods were compared to. Thus, to use different methods (his view of creativity) was to work against this normative first method. Note that in his view of *Against Authority,* he held the method as the authoritative artifact, not the instructor, as he talked about the instructor showing other ways. While he expressed some frustration in wanting to know "better" methods earlier ("why didn't you do that the first time?"), he was aware of why pedagogically. Angus also viewed creativity in mathematics as *Application*; learning different methods from the instructor over time was acceptable to him because it would lead to different applications within mathematics or outside of it.

When asked whether he felt creative in his calculus course, he said "No, not at all" and shared his view about the relationship between creativity and understanding.

> INT: Why?
>
> ANGUS: I have a hard enough time doing...understanding the concept as is, much less trying...I don't spend any time trying to think

**Table 2**
Percentage of students who viewed creativity by theme

| | Actions and Attitudes | Application | Different Ways | Originality | Against Authority | Understanding |
|---|---|---|---|---|---|---|
| Total | 31 (56%) | 21 (38%) | 46 (84%) | 23 (42%) | 22 (40%) | 36 (65%) |

of a different or better way to do it. I'm trying to understand the fleshed-out known methods. [*Understanding*]

Angus viewed *Understanding* as related to creativity, because his rationale for why he did not feel creative was that he struggled with understanding. For Angus, understanding was needed before one could be creative. He said that understanding the content was difficult enough for him, so being creative (trying different methods, for him) was not something he had time for. He felt that creativity was something extra for him, after one understood the content.

However, he did not hold this view of how understanding and creativity related to each other in general for all people; it was specific to him and his career goals.

INT: Okay. So, you try...so do you think you always have to know the known methods before doing a different method?

ANGUS: Not always. And if it was something that I had…if it were something that I had as a profession or doing in-work experience where it was about efficiency, I would take a lot more liberties with trying to be creative, but because it's a class that I am paying for and have very little time outside of the material that's covered, my priority is more of trying to understand what I'm being...the criteria that I'm being graded over.

INT: OK so do you think it's important to try to be creative in mathematics?

ANGUS: If you were a math major or even like an engineer then yes.

INT: OK. Why, why only if you're a math major or engineer?

ANGUS: Because we have such…we as individuals have such finite time and energy available that we should probably just focus on being creative in the fields we're interested in.

For other students for whom calculus and mathematics in general was in their interests, he could see them undertaking creative endeavors earlier. But because mathematics was not Angus' major nor career goal, his priority was to do what was needed to understand the content that was being graded. He repeatedly referred to limited time: "have very little time outside of the material" and "as individuals [we] have such finite time and energy." For Angus, creativity would take extra time that he felt he did not have, nor would it help him in his career goals. Ultimately, despite the multiple themes comprising his view of creativity, calculus was not a place to be creative for Angus, and he felt he just needed to get through it.

*JCRU: case of feeling creative in calculus*

JCRU was a student in cohort 2 who identified as an African American female. Her major was biomedical science and Spanish. When asked "What does it mean to you to be creative in mathematics?", she said:

JCRU: I think it means to just be open and … recognizing that [an approach] doesn't have to … fit in the box [*Actions and Attitudes*] …I mean, sometimes I think having a set of rules does help, but the problems can be approached differently [*Different Ways*]. And, yeah, that you can just go different ways about solving it or even thinking about math and how it can apply [*Application*].

The overall view of creativity expressed here was about the *Different Ways* one can approach a problem. The view of *Actions and Attitudes* was also apparent when she said, "I think it means to just be open and... recognizing that it doesn't have to be…this fits in the box". Her perspective was that openness is an attribute exhibited by a creative person, which allows one to think outside the box.

JCRU responded affirmatively when asked if she felt creative, but she did not see creativity as essential to mathematics. She responded to "Do you think it is important to be creative in mathematics?" with "I think at least to some extent. Well, I guess it depends. I feel like if it's something that you don't want to look at as a dreadful task, then it helps to be creative." Here she referred to creativity as necessary for the enjoyment of otherwise boring or tedious tasks. Even though her response was similar to Angus O'Sullivan's in that creativity is needed in mathematics in special circumstances only, JCRU was able to develop enjoyment when being creative in mathematics, whereas Angus deemed enjoyment as unimportant. For instance, we see this understanding using creativity emerge when JCRU was asked about an example of a task on which she was creative.

JCRU: Yeah. So, we were talking about the different rules, like derivative rules we can use. And one of our quizzes, I think it kind of encouraged you to be creative because it was basically like, "Create a function that can be solved using both the product rule or the power rule and show how you can do it either way." And so, I feel like that gave me a chance to be creative. One because I was just making up the problem on my own. And, also, I was able to approach it. And I... I guess just really understand it to be able to apply two different rules to it.

It was clear that JCRU felt a sense of ownership in this task as she was asked to create a problem that could be solved in multiple ways. Once she demonstrated the two derivative rules on *her* function, she felt like she truly understood the concepts; she took away *Different Ways* as being creative from the task. This likely contributed to JCRU disclosing that she felt the most confident she ever had in a mathematics course.

Like Angus, JCRU initially said that she didn't associate mathematics with creativity, but unlike him, her view changed after her experience with the course:

JCRU: I guess just generally that I can be...way more creative than I thought because I didn't really, like I wouldn't, in the past have associated the word creativity in math, in the same sense necessarily or the same concept. So, I think just… being more open to that idea that math can be creative.

When asked what she had learned about her mathematical creativity from this course, she reiterated her view that she had been creative but also that her view on the relationship between creativity and mathematics had shifted.

## Discussion

Our results indicate that our sample of calculus students have rich and varied views of mathematical creativity, based on the six found themes of *Actions and Attitudes, Application, Different Ways, Originality, Against Authority,* and *Understanding* and their frequencies. In delving into our *Understanding* theme, we found two contrasting viewpoints: the view that understanding was needed first before one could be creative and a belief that creativity could occur first without a requisite understanding. We also illustrated how multiple themes were woven through students' views of creativity, through the cases of Angus O' Sullivan and JCRU, as one student who did not feel creative and another who did, respectively.

Our results regarding the themes in students' views of creativity align with our previous work about transition-to-proof students' views of creativity (Cilli-Turner et al., 2019), in that our sample of calculus students had fully formed and wide-ranging perspectives of mathematical creativity. Given the presumed lower mathematical sophistication of Calculus students compared to that of students in more advanced mathematical classes, our work shows Calculus students do hold views about mathematical creativity and can articulate them. Our work is significant then in its focus on an empirical investigation into Calculus students' views of mathematical creativity.

First, we found that our sample of students held some views that did generally align with the dominant conceptions of creativity. This is not surprising because our conceptions of creativity that stem from expert views may have influenced our analysis. We did however notice some new ideas about creativity from students, which we discuss later.

Second, given our explicit goal to help students develop a sense of creativity, we stressed the inclusion of students' views of mathematical creativity, as fellow doers of mathematics and thus members of mathematical communities. This is as opposed to mainly considering two specific dominant views: the expert and genius view. Our research counters these dominant narratives: for one, our sample of beginning calculus students articulated six different themes of views on creativity, and second, only one student ever stated the notion that one must be a genius to be creative. If our goal is to nourish mathematical creativity in all students, then it is imperative that we listen and take their views into account. This work contributes to the ongoing shift in expanding who is deemed creative and what counts as creativity, that anyone may take actions and engage in internal processes to be creative. It also can help future educators leverage the students' six notions of creativity to further foster their own students' creativity.

## *Contributions: extending literature with student-driven themes*

We now discuss some of our found themes in alphabetical order, with respect to similar ideas in the literature, for how our work extends what is known. *Actions and Attitudes* extends the Persons category of the 4 Ps of creativity (Rhodes, 1961) to include not only attitudes but also actions. The actions part is key, as it suggests that anyone can take actions, regardless of any inherent abilities, to be creative. Moore-Russo and Demler (2018) discussed the "subjective experience" or AHA! Moment (Hadamard, 1945), which has connections to the "wondering" cited by students in the *Actions & Attitudes* section. We note that the genius view of creativity appeared in this theme for us and for Moore-Russo and Demler. However, most codes were of an affective nature, including "willing to try," "confidence," and "comfortable," and the literature is scarce with affective aspects of mathematical creativity (Goldin, 2017). Recently Kozlowski and colleagues (2019) have attempted to theoretically characterize how affect and mathematical creativity intertwine and stated that there is much more work to be done by the field.

Our analysis shows that *Application* is another component of our sample of students' conceptions of creativity. The students in our study were concerned with creativity as ways of applying their math to other aspects, including real-life scenarios. Runco and Jaeger's (2012) standard conception of creativity had effectiveness which included utility. Sriraman (2009), on the other hand, has argued against including utility in mathematical creativity as "mathematicians would raise several arguments with this conception, simply because the results of creative work may not always have implications that are 'useful' in terms of applicability in the real world" (p. 14-15), but our findings show this to be an important category when listening to the student voice. This example highlights why student views are listened to in this work, if the goal is to foster widespread creativity in all students, for their individual betterment.

Our findings within the theme *Against Authority* answers Chamberlin and Mann (2014)'s call for more empirical evidence of iconoclasm: "challeng[ing] conventions in MPS [mathematical problem solving]" (p. 36). They stated that iconoclasts "challenge the system, e.g., teacher or textbook, to identify a greater number of solution paths" (p. 38). Many students, like Caydicle, talked about diverging from the rules, procedures, textbook, examples, or the professor themselves to be creative. Although it was the least-coded theme, we believe the lack of codes was a product of the focus on creativity in many calculus courses since at least nine professors were actively promoting, and thus permitting, creativity in their classrooms.

*Understanding* has connections to both Torrance's elaboration and Wallas' preparation stage. For example, 007 stated that one needs to make connections, which can be an elaboration on previous ideas and Vivala stated that one needs to understand (know material) to be creative, akin to the preparation stage of determining a problem's constraints. However, through an in-depth analysis of the understanding theme, we've found that creativity can lead to understanding a concept.

Another contribution to creativity literature is that students showed that they did not need to proceed linearly through Wallas' four stages to be creative. For instance, Aon showed that her deliberate action of looking for different ways, which she associated with creativity, helped her better understand the content. Moreover, using creativity to understand mathematics can lead to confidence, as seen in JCRU. The view that understanding is needed first to be creative could prevent a student from taking deliberate steps to be creative. Angus did not feel creative because, in his mind, there is a requirement to "understand the fleshed-out known methods." These students' views provide empirical evidence for Riling's (2020) hypothesis that "it is not clear that the mathematical creativity of ... students will always occur in a process similar to Wallas' four sequential stages" (p. 9).

## *Limitations*

The validity of the results in this study may be affected by several factors. First, we asked students about their views on mathematical creativity at the end of their calculus course, so we do not have data about how these views evolved over the semester. Second, the general limitations of interviews and self-response apply, where respondents may feel pressure to answer in ways that they perceive are favorable. For example, students may believe they must provide some conception. This is still a student view, even if it may not be wholly authentic to the student and provides information to the field about what resonates with students. Lastly, we acknowledge that instructors participating in the PD of the larger project could have influenced students' view of mathematical creativity differently than instructors that were not participants in the study. This is possible as the PD was explicitly focused on mathematical creativity and instructors volunteered to be participants, which could both impact how these instructors talked to their students about creativity. However, as the mathematical creativity community lacks studies that ask students about their views on creativity, these findings are still relevant in adding to the body of creativity literature. These limitations can also provide future research questions for the field of creativity.

Our work shows that not only do calculus students have sophisticated and nuanced views of mathematical creativity, but also that they can recognize their own creativity in a classroom. Unpacking the views of mathematical creativity of students shows that some students have recognized what creativity they wield and how that may contribute to their mathematical identities. Focusing on this connection in future research is especially important, as it has been established that a positive mathematics identity can lead to success in mathematics (Froschl & Sprung, 2016; Oppland-Cordell & Martin, 2016). Creativity-fostering teaching actions taken by the instructor need to be investigated to determine how they connect to these views. Furthermore, the tasks and teaching actions that promote students' creativity as well as the linkage between such tasks and students' view of creativity need to be uncovered. We are currently investigating the teaching actions that students stated led to fostering their feelings of creativity in the classroom and their positive affective outcomes, including enjoyment, self-efficacy, and comfort.

## Declaration of Competing Interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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