

Estimation of Three-Dimensional Center of Gravity Relocation for Ground Vehicles with Tire Blowout

Ao Li, Yan Chen^{*}, Wen-Chiao Lin, and Xinyu Du

Abstract—Influenced by tire effective radius change, suspension rearrangement, and pitch/roll disturbance due to tire blowout, the vehicle center of gravity (CG) can significantly relocate toward the blown-out tire position. This paper proposes an estimation method of the CG relocation for ground vehicles with tire blowout by utilizing vertical force variations and geometric relationships in tire blowout events. Based on a new recursive least square (RLS) formulation in this paper, the three-dimensional (3D) CG relocation (i.e., the height, the longitudinal and lateral positions) can be estimated simultaneously. Matlab/Simulink and CarSim[®] co-simulation results for different tire blowout locations validate that the proposed estimation method can effectively and accurately capture the vehicle 3D CG relocation after tire blowout.

I. INTRODUCTION

Tire blowout of ground vehicles can tremendously endanger vehicle directional stability and road safety [1]. In the explosive air-leaking process of tire blowout, tire friction forces of the blown-out tire(s) would significantly change, which further generates additional lateral force and yaw moment on the vehicle [2]. Without proper controls, the vehicle would rapidly deviate from the driving lane and collide with other vehicles or curbs. In 2015, the National Highway Traffic Safety Administration (NHTSA) estimated that tire blowout could cause more than 400 deaths and over 78,000 crashes every year [3]. Given the severe effects of tire blowout, much research work has been conducted in recent years. To describe tire blowout impacts on vehicle dynamics, various tire blowout models were developed by considering different parameters and their variations [3]-[8]. To ensure vehicle directional stability after tire blowout, shared steering control of tire blowout for partially automated vehicles (SAE Levels 2/3) [9] and fully automatic control algorithms for highly automated vehicles (SAE Levels 4/5) [10]-[13] were proposed.

Modeling and control design of tire blowout are major research problems to be investigated. For modeling, an important issue is the estimation of vehicle center of gravity (CG) relocation after tire blowout. Influenced by the decreased effective radius of the blown-out tire, suspension rearrangement, and pitch/roll disturbance caused by tire blowout, the vehicle sprung mass would eventually settle down on the blown-out tire when the wheel rim bottoms on the ground. Correspondingly, the vehicle CG would relocate toward the blown-out tire side [5][7]. The CG relocation due to tire blowout is typically more significant than that during normal driving. If the CG relocation caused by tire blowout

can be effectively estimated, the obtained geometric parameters can provide many practical benefits. On the one hand, considering tire blowout duration is very short (e.g., typically less than 1 second [14]), a traditional tire pressure monitoring system (TPMS) with a typical sampling time of 15-60 seconds [15] is not suitable to timely detect the occurrence and location of a tire blowout. However, with related information of CG relocation (e.g., convergence trend of the estimation), tire blowout will be identified faster and the corresponding stabilization control can be timely triggered. On the other hand, the knowledge of the actual three-dimensional (3D) CG relocation may largely improve stabilization control algorithms and help to precisely execute control actuation [14][17].

Over the past decades, the estimation of CG position has been intensively investigated for normal driving without tire faults. In [17], the relationship between vertical load transfer and tire instant effective radius was adopted, and a Kalman filter (KF) was applied to estimate the CG height. With the accurate observation of roll angle, the CG height estimation problem was formulated based on vehicle roll dynamics, and a recursive least square (RLS) algorithm with a variable forgetting factor was adopted [18]. In [19], the CG longitudinal position and tire cornering stiffness were estimated from velocity-dependent static gains of a linear bicycle vehicle model by using the least square (LS) algorithm. Based on a vehicle longitudinal dynamic model and a simple linear tire model, both the CG longitudinal position and height were estimated in real-time through an adaptive KF-extended KF (AKF-EKF) approach [20]. In [21], a multiple-model-switching strategy was proposed based on linear lateral and roll vehicle dynamic models to estimate the CG height. The CG longitudinal position was estimated by using the relationship between the ratio of rear-to-front tire longitudinal force and the corresponding wheel slips [22]. In [23], a dynamic detection method using a braking model was developed to estimate the CG height during braking.

Although the estimation of CG position was largely discussed in the literature, the estimation problems were all formulated based on vehicle models and/or physical relationships that are only suitable for normal driving vehicles without tire faults. However, these common models and derived relationships for healthy vehicles will be typically ineffective or inaccurate after tire blowout. Based on the authors' best knowledge, the estimation of CG relocation for ground vehicles with tire blowout has not been explored in the literature yet. Moreover, the vehicle CG relocation due to tire blowout will happen in a 3D space, which should be described by three parameters (i.e., longitudinal and lateral positions, and height). Existing CG estimation works typically focused on

This work was supported in part by the National Science Foundation Grant CMMI-2043286.

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one or two of the three parameters, and the proposed methods could barely estimate full parameters simultaneously.

In this paper, an estimation method of 3D CG relocation is specifically proposed for ground vehicles with tire blowout. Based on the authors' previous work [7], the estimation problem is formulated using specific vertical force variations and geometric relationships in tire blowout events. A new RLS formulation is applied to estimate the 3D CG relocation simultaneously. Matlab/Simulink and CarSim[®] co-simulation is conducted by considering different tire blowout locations with different CG relocations. Simulation results are used to validate that the proposed estimation method can effectively and accurately capture the vehicle 3D CG relocation caused by tire blowout.

The rest of the paper is organized as follows. The coordinate system to describe the CG relocation is discussed in Section II. In Section III, the new RLS-based estimation method of 3D CG relocation is described. Simulation results and analyses about the proposed estimation method are presented in Section IV. In Section V, conclusions are described.

II. COORDINATE SYSTEM FOR THREE-DIMENSIONAL CG RELOCATION

In this section, the coordinate system to describe the 3D CG relocation is introduced.

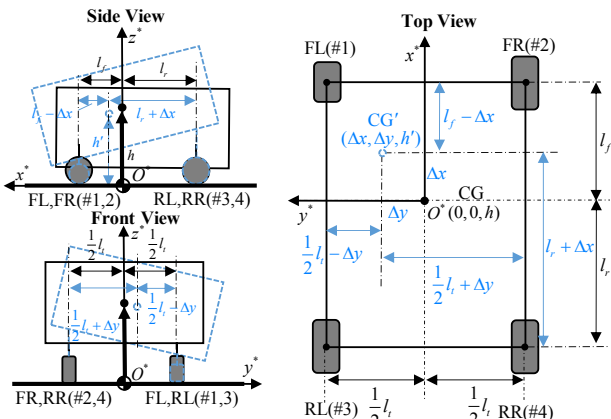


Figure 1. The coordinate system for the 3D CG relocation.

The coordinate system $O^*-x^*y^*z^*$ for the 3D CG relocation is illustrated in Figure 1. The positive x^* axis is in the forward direction, the positive y^* axis is toward the left, and the positive z^* axis is upward. The z^* axis passes through the original CG (the one before tire blowout and labeled in the black dot in Figure 1) and intersects with x^* and y^* axes at the origin O^* locating on the ground. The coordinate system $O^*-x^*y^*z^*$ is attached to the vehicle body and moves with the vehicle.

Before tire blowout, the original CG height is denoted as h . l_f and l_r are the distances from the front and rear axles to the original CG, respectively. l_i is the wheel track. The transversal distances from the left and right two tires to the original CG are both $l_i/2$. Correspondingly, the original CG position with respect to the coordinate $O^*-x^*y^*z^*$ is $(0, 0, h)$.

As an example, consider a front left tire blowout, after which the sprung mass leans toward the blown-out tire side when the wheel rim bottoms on the ground. The original CG relocates to a new position denoted as CG' (labeled as the

blue dashed circle in Figure 1) in the defined coordinate system. The longitudinal and lateral relocation distances are Δx and Δy , respectively. The new CG' height is denoted as h' . Correspondingly, the new CG' position with respect to $O^*-x^*y^*z^*$ is $(\Delta x, \Delta y, h')$. The front and rear axles have distances of $l_f - \Delta x$ and $l_r + \Delta x$ to the new CG' , respectively. The transversal distances from left and right two tires to the new CG' are $l_i/2 - \Delta y$ and $l_i/2 + \Delta y$, respectively. Note that Figure 1 is an example of the front left tire blowout, in which Δx and Δy are both positive. With the same idea, the new CG' position and related distances for other possible tire blowout locations can also be depicted with positive/negative Δx and Δy in different magnitudes.

Remark 1: With the origin always fixed on the CG, a traditional vehicle body coordinate system can only describe the change of the CG height, which is not suitable to describe the 3D CG relocation due to tire blowout. Therefore, the aforementioned coordinate system is specifically applied in this work.

III. ESTIMATION METHOD OF THREE-DIMENSIONAL CG RELOCATION

In this section, the estimation method of the 3D CG relocation is described. The formulation of the estimation problem, the RLS-based estimation method, and a summary will be successively introduced in subsections A, B, and C, respectively.

A. Formulation of the Estimation Problem

For simplicity, let indexes $i=1,2,3,4$ denote the subscripts for the front left (FL), front right (FR), rear left (RL), and rear right (RR) tires, respectively.

In the authors' previous work [7], the 3D CG relocation due to tire blowout was considered in the proposed two-stage variations of tire vertical forces F_{zi} , as described in eq. (1),

$$\left\{ \begin{aligned} F_{z1} &= \frac{mg(l_r + \Delta x)}{2L} + \frac{K_{sf}mg\Delta y}{l_iK_s} - \frac{ma_xh'}{2L} \\ &\quad - \frac{K_{sf}ma_yh'}{l_iK_s} - \frac{\Delta RK_{sf}K_{sr}}{2K_s} \\ F_{z2} &= \frac{mg(l_r + \Delta x)}{2L} - \frac{K_{sf}mg\Delta y}{l_iK_s} - \frac{ma_xh'}{2L} \\ &\quad + \frac{K_{sf}ma_yh'}{l_iK_s} + \frac{\Delta RK_{sf}K_{sr}}{2K_s} \\ F_{z3} &= -\frac{mg(l_r + \Delta x)}{2L} + \frac{K_{sr}mg\Delta y}{l_iK_s} + \frac{ma_xh'}{2L} \\ &\quad - \frac{K_{sf}ma_yh'}{l_iK_s} + \frac{\Delta RK_{sf}K_{sr}}{2K_s} + \frac{1}{2}mg \\ F_{z4} &= -\frac{mg(l_r + \Delta x)}{2L} - \frac{K_{sr}mg\Delta y}{l_iK_s} + \frac{ma_xh'}{2L} \\ &\quad + \frac{K_{sf}ma_yh'}{l_iK_s} - \frac{\Delta RK_{sf}K_{sr}}{2K_s} + \frac{1}{2}mg \end{aligned} \right. \quad (1)$$

where m is the total vehicle mass. L is the wheelbase ($L = l_f + l_r$). K_{sj} ($j=f,r$) denotes the front and rear

suspension stiffness respectively and $K_s = K_{sf} + K_{sr}$. a_x and a_y are the longitudinal and lateral accelerations, respectively. ΔR represents the change of tire effective radius due to tire blowout.

During the 1st stage, as the vertical stiffness of the blown-out tire reduces due to high-pressure tire air leakage, the tire effective radius of the blown-out tire decreases. The vertical forces of the blown-out tire and the corresponding diagonal tire decrease, while those of the other two tires increase. The impact of the changed vertical stiffness of the blown-out tire is indirectly described using the tire effective radius change ΔR , thus, which is the key factor to model the 1st stage.

During the 2nd stage, when the wheel rim bottoms on the ground, the vertical stiffness of the blown-out tire starts to increase sharply (over 100 times [24]) and the effective radius reduces close to the wheel rim radius. Influenced by the pitch/roll disturbance, the sprung mass starts to settle down on the blown-out tire and the vehicle CG leans toward the blown-out tire side, which is described by Δx , Δy , and h' as shown in Figure 1. Consequently, the vertical force of the blown-out tire begins to increase, while that of the diagonal tire continues decreasing. The impact of pitch/roll disturbance is indirectly depicted using CG relocation (Δx , Δy , and h'), which is the key factor to model the 2nd stage. More discussions and results can be referred to the authors' previous work [7].

Remark 2: In [7], the aforementioned two-stage vertical force variations and self-alignment torque (SAT) variations due to tire blowout were explicitly considered to develop a high-fidelity tire blowout model, which is further integrated with CarSim[®]. If the tire blowout location is predefined, by assigning appropriate values of Δx , Δy , and (positive/negative) ΔR together with the vertical force variations to a CarSim[®] vehicle model, the tire blowout impacts on vehicle dynamics for a certain tire blowout location were successfully investigated. Conversely, if the values or the convergence trends of some parameters were estimated, the tire blowout location will be identified, as shown in Table 1. Such characteristics (especially convergence trends of Δy and ΔR) will be utilized in the estimation of the 3D CG relocation, which will be discussed in subsections B and C.

Table 1. Convergence trends vs. tire blowout location

	FL (#1)	FR (#2)	RL (#3)	RR (#4)
($\Delta x, \Delta y, \Delta R$)	(+, +, +)	(+, -, -)	(-, +, -)	(-, -, +)

The geometric relationship between suspension heights h_{si} and the tire effective radii R_{ei} during tire blowout is described in eq. (2) [3][25],

$$R_{e1} + R_{e4} + h_{s1} + h_{s4} = R_{e2} + R_{e3} + h_{s2} + h_{s3}. \quad (2)$$

The initial suspension heights are denoted as h_{s0i} . Considering that the left and right suspensions on the same axle have the same suspension structure, spring, and damper, assume that the initial suspension heights of the front/rear two suspensions are equal (i.e., $h_{s01} = h_{s02}$, $h_{s03} = h_{s04}$). The

relationships between the vertical forces and the suspension heights are given as,

$$h_{si} = h_{s0i} - \frac{F_{zi}}{K_{sj}}. \quad (3)$$

Using eqs. (2) and (3), the relationship between vertical forces and tire effective radii can be obtained,

$$R_{e1} + R_{e4} - R_{e2} - R_{e3} = \frac{F_{z1} - F_{z2}}{K_{sf}} + \frac{F_{z4} - F_{z3}}{K_{sr}}. \quad (4)$$

During driving, the effective radius is given as,

$$R_{ei} = \frac{v_{wi}}{\omega_i}, \quad (5)$$

where v_{wi} is the wheel center velocity of each wheel in the wheel plane. ω_i is the rotational speed of each wheel.

Based on Figure 1, the wheel center velocities corresponding to the CG relocation after tire blowout are described in eq. (6),

$$\begin{cases} v_{w1,3} = v_x - \left(\frac{1}{2}l_t - \Delta y\right)r \\ v_{w2,4} = v_x + \left(\frac{1}{2}l_t + \Delta y\right)r \end{cases}, \quad (6)$$

where v_x is the vehicle longitudinal velocity and r is the vehicle yaw rate.

Therefore, by combining eqs. (1)(4)(5)(6), the parameter estimation problem of the 3D CG relocation due to tire blowout can be specifically formulated in eq. (7),

$$\begin{aligned} & \frac{2v_x - l_t r}{\omega_1} + \frac{2v_x + l_t r}{\omega_4} - \frac{2v_x + l_t r}{\omega_2} - \frac{2v_x - l_t r}{\omega_3} \\ &= \left(\frac{r}{\omega_2} + \frac{r}{\omega_3} - \frac{r}{\omega_1} - \frac{r}{\omega_4}\right)\Delta y + \frac{2(K_{sf} - K_{sr})ma_y}{l_t K_s K_{sr}}h' - \Delta R. \end{aligned} \quad (7)$$

Assume that vehicle and wheel states (ω_i , r , a_y) can be successfully obtained from sensors and vehicle parameters (l_t , m , K_{sf} , K_{sr}) can also be measured offline, two parameters (Δy and h') about the CG relocation and tire effective radius change (ΔR) can be estimated using a certain estimation algorithm together with eq. (7). In subsection B, a full-parameter estimation method will be described.

B. RLS-based Estimation Method

In this work, the RLS method in [26] is adopted, which can recursively estimate the parameters by minimizing a weighted linear least square cost function. A reorganization of eq. (7) results in the regular form for RLS estimation,

$$y = H\theta^T, \quad (8)$$

where the desired output y , regression matrix H , and unknown parameter matrix θ are defined as follows,

$$y = \frac{2v_x - l_t r}{\omega_1} + \frac{2v_x + l_t r}{\omega_4} - \frac{2v_x + l_t r}{\omega_2} - \frac{2v_x - l_t r}{\omega_3}, \quad (9)$$

$$H = \left[\frac{r}{\omega_2} + \frac{r}{\omega_3} - \frac{r}{\omega_1} - \frac{r}{\omega_4} \quad \frac{2(K_{sf} - K_{sr})ma_y}{l_t K_s K_{sr}} \quad -1 \right], \quad (10)$$

$$\theta = [\Delta y \quad h' \quad \Delta R]. \quad (11)$$

From eq. (11), the developed RLS formulation can only estimate two parameters (Δy and h') about the CG relocation. Furthermore, Δx can be calculated based on the estimated parameters. By using the convergence trends of Δy and ΔR as discussed in *Remark 2*, the tire blowout location can be determined. For a healthy tire on the opposite side of the blown-out tire, the relationship between tire effective radius and vertical force is given as [17][27],

$$R_{ei} = R_u - \frac{F_{zi}}{3K_{zi}}, \quad (12)$$

where R_u is the tire unloaded radius of each tire. K_{zi} is the vertical stiffness of a healthy tire.

Note that two effective radius calculations are adopted in this paper, as shown in eqs. (5) and (12), respectively. The former one is to calculate instant tire effective radius, which is valid for both blown-out tires and healthy tires. The latter one is only valid for healthy tires. In the aforementioned discussion about the 2nd stage vertical force variations, the vertical stiffness of the blown-out tire increases tremendously (over 100 times [24]), and the effective radius eventually reduces close to the wheel rim radius in real tire blowout events. However, eq. (12) cannot describe such a change for the blown-out tire. As shown in eq. (12), the tire effective radius will vary to the unloaded radius R_u (not wheel rim radius) with a tremendous increase of the vertical stiffness. Considering that the vertical stiffness of the healthy tires is not changed and can be directly measured for certain tire pressure [17], eq. (12) can be utilized to determine Δx as follows.

Eq. (1) can be rewritten as,

$$\begin{cases} F_{zi} = \frac{mg(l_r + \Delta x)}{2L} + f_i(\Delta y, h', \Delta R), & (i = 1, 2) \\ F_{zi} = -\frac{mg(l_r + \Delta x)}{2L} + f_i(\Delta y, h', \Delta R), & (i = 3, 4) \end{cases} \quad (13)$$

For the healthy tire on the opposite side of the blown-out tire, using eqs. (12)(13), Δx can be obtained as,

$$\begin{cases} \Delta x = \frac{2L[3K_{zi}(R_u - R_{ei}) - f_i(\Delta y, h', \Delta R)]}{mg} - l_r, & (i = 1, 2) \\ \Delta x = \frac{2L[f_i(\Delta y, h', \Delta R) - 3K_{zi}(R_u - R_{ei})]}{mg} - l_r, & (i = 3, 4) \end{cases} \quad (14)$$

C. Summary

The proposed estimation method of the 3D CG relocation can be summarized in Figure 2. First, using the vehicle and wheel states (ω_i , r , a_y), the proposed RLS formulation in eq. (8) can estimate Δy , h' , and ΔR . Next, the tire blowout location can be determined and an opposite healthy tire can be selected based on the convergence trends of Δy and ΔR . Last, Δx can be calculated using the estimated parameters, as shown in eq. (14).

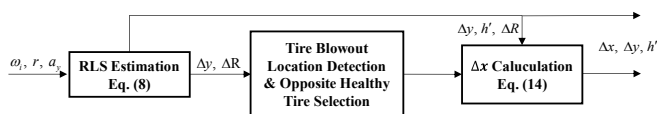


Figure 2. The framework of the proposed estimation method.

Remark 3: Since tire blowout is a hazardous and emergency event, the estimation of the 3D CG relocation should have a fast enough convergence rate for modeling and control purposes. As an initial investigation, this work mainly focuses on the formulation of the estimation problem to preliminarily validate the effectiveness. Therefore, the basic RLS algorithm is utilized. Our future work will investigate advanced estimation algorithms with a finite-time convergence rate. Moreover, the robustness to sensor noises and sensitivity to different/varying driving speeds will also be explored through comprehensive experimental results.

IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, co-simulation between Matlab/Simulink and CarSim[®] is conducted to validate the effectiveness of the proposed estimation method.

The simulation framework is shown in Figure 3. Having been experimentally validated, the aforementioned high-fidelity tire blowout model considering two-stage vertical variation and SAT in the authors' previous work [7] is integrated with CarSim[®] to serve as the ground truth. As discussed in *Remark 2*, by assigning appropriate values of Δx , Δy , and (positive/negative) ΔR together with the vertical force variations to a CarSim[®] vehicle model, the vehicle responses with tire blowout impacts can be obtained. Some wheel and vehicle states, such as ω_i , r , and a_y , will be utilized in the proposed estimation method. Finally, the estimated parameters will be compared with their actual values for validation purposes.

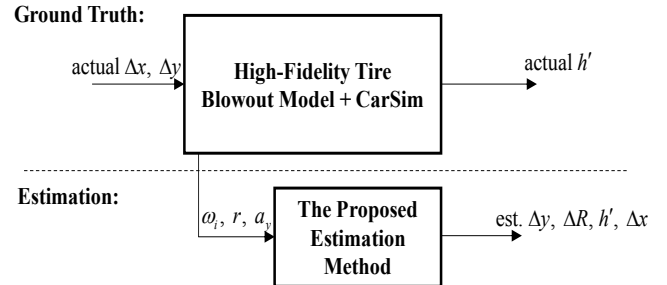


Figure 3. Simulation framework.

A C-Class hatchback vehicle in CarSim[®] is adopted and its parameters are shown in Table 2. The final values of tire parameters are utilized to characterize the tire properties after tire blowout. The tire blowout duration is set as 0.3 seconds. In the following subsections, two tire blowout locations with different CG relocations in a straight-line driving maneuver are considered. The vehicle is driven at a constant speed of 100 km/h during the simulation. At the 5th second of the total 15-second simulation time, the tire blowout is triggered. To achieve a relatively faster convergence rate without loss of accuracy, the forgetting factor in RLS, which can discount the old data in the estimation, is selected as 0.995 in simulation.

Note that tire blowout itself will introduce additional lateral force and yaw moment on the vehicle. Correspondingly, the changed vehicle states can be utilized to excite the estimation. Therefore, no external excitation is required in the simulation.

Table 2. Parameters of the C-Class Hatchback

Parameter	Symbol	Value	Final value
Vehicle Parameters			
Vehicle total mass	m	1412 kg	-
Moment inertia around CG	I_z	1536.7 kg-m ²	-
Distance from front axle to CG	l_f	1.105 m	-
Distance from rear axle to CG	l_r	1.895 m	-
wheel track	l_t	1.675 m	-
Height of CG (original)	h	0.54 m	-
Front suspension stiffness	K_{sf}	27000 N/m	-
Rear suspension stiffness	K_{sr}	30000 N/m	-
Steering system stiffness	K_{ss}	100 Nm/rad	-
Tire Parameters			
Tire unloaded radius	R_u	0.34 m	-
Tire effective radius	R_e	0.325 m	To 2/3
Longitudinal stiffness	C_x	47000 N/slip	To 1/10
Cornering stiffness	C_y	-55000 N/rad	To 1/10
Wheel moment inertia	I_w	0.9 kg-m ²	-
Rolling resistance coefficient	K_r	0.018	To 30 times

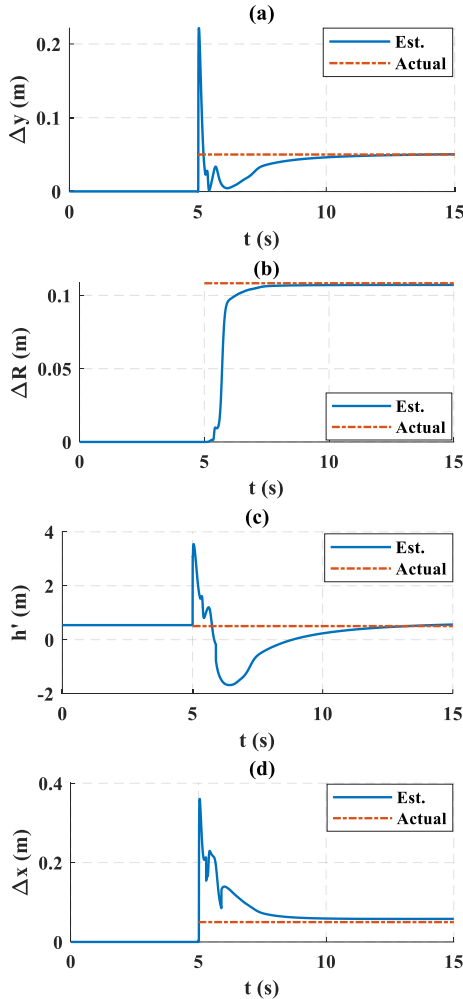


Figure 4. Simulation results of FL tire blowout in a straight-line driving maneuver.

A. FL Tire Blowout in A Straight-line Driving Maneuver

In this subsection, the FL tire blowout is considered. The actual values for $(\Delta x, \Delta y, h')$ are (0.05 m, 0.05 m, 0.504 m). The actual absolute value of ΔR is 0.1083m. As shown in

Figure 4 (a) and (b), the positive convergence trends of Δy and ΔR can indicate the tire blowout location (the FL tire), when the tire blowout occurs at the 5th second. Such observations are consistent with the discussions in *Remark 2* and Table 1. Correspondingly, the healthy FR tire is selected for Δx calculation. Based on Figure 4 (a)-(d), the estimated values eventually converge to the actual values with acceptable accuracy.

B. RL Tire Blowout in A Straight-line Driving Maneuver

In this subsection, the RL tire blowout is considered. Considering the rear suspension stiffness of the adopted C-Class hatchback vehicle is stiffer than the front one, the actual values for $(\Delta x, \Delta y, h')$ are different from those in the FL tire blowout case, which are (-0.03 m, 0.03 m, 0.496 m). The actual absolute value of ΔR is still 0.1083m. As shown in Figure 5 (a) and (b), the positive convergence trend of Δy and the negative convergence trend of ΔR after tire blowout can indicate the tire blowout location is at the RL tire, which is consistent with the discussions in *Remark 2* and Table 1. Correspondingly, the healthy RR tire is selected for Δx calculation. Based on Figure 5 (a)-(d), the estimated values also converge to the actual values with good accuracy.

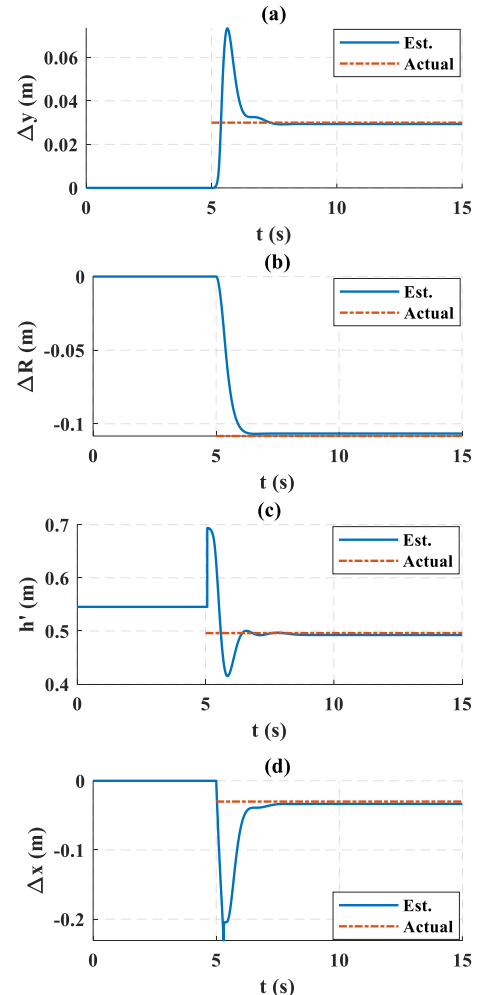


Figure 5. Simulation results of RL tire blowout in a straight-line driving maneuver.

Since tire blowout is the only excitation and different tire blowout locations can have different impacts on vehicle dynamics, different convergence rates are observed in Figure 4 and Figure 5, even if the same forgetting factor (0.995) is applied in both simulation cases.

To sum up, the good agreement between actual and estimated values for two tire blowout locations with different CG relocations can validate that the proposed estimation method can effectively and accurately capture the vehicle 3D CG relocation after tire blowout.

V. CONCLUSION

In this paper, an estimation method of the 3D CG relocation was specifically proposed for ground vehicles with tire blowout by utilizing vertical force variations and geometric relationships. The new recursive least square (RLS) formulation can estimate the three-dimensional (3D) CG relocation (i.e., longitudinal and lateral positions, and height) simultaneously. The simulation results indicated that the proposed estimation method can effectively and accurately capture the vehicle 3D CG relocation after tire blowout. Advanced estimation algorithms with a fast convergence rate, robustness and sensitivity studies, and experimental validations of the proposed estimation method will be explored in the future work.

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