Improving the Computational Efficiency of Optimal Transmission Switching Problems

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***Abstract*—Transmission switching is widely used in the electric power industry for both preventive and corrective purposes. Optimal transmission switching (OTS) problems are usually formulated based on optimal power flow (OPF) problems. OTS problems are originally nonlinear optimization problems with binary integer variables indicating whether a transmission line is in or out of service, however, they can be linearized into mixed-integer linear programs (MILP) through the big-M method. In such big-M-based MILP problems, the value of M can significantly affect their computational efficiency. This paper proposes a method to find the optimal big-M values for OTS problems and studies the impact of big-M values on the computational efficiency of OTS problems. The model was implemented on a modified RTS-96 test system, and the results show that the proposed model can effectively reduce the computational time by finding an optimal big-M value which ensures optimal switching solutions while maintaining numerical stability.**

***Index Terms*—Big-M method, economic dispatch, mixed-integer programming, optimal power flow, optimal transmission switching, stochastic optimization, unit commitment, wind energy.**

I. Nomenclature

Indices

b	Bus.
g	Generator.
k	Transmission Line.
s	Scenario.
seg	Segments for piece-wise linear cost function.
t	Time.

Sets

σ_b^+	Transmission lines with their "to" bus connected to bus b.
σ_b^-	Transmission lines with their "from" bus connected to bus b.
g_b	Generators connected to bus b.

Variables

$\theta_{b,t,s}$	Voltage angle of bus b at time t in scenario s.
$\theta_{fr,k,t,s}$	Voltage angle at the "from" bus of line k at time t in scenario s.
$\theta_{to,k,t,s}$	Voltage angle at the "to" bus of line k at time t in scenario s.
M_t	Big-M value at time t.
$P_{g,t}$	Real power generation of generator g at time t.
$P_{g,t}^{seg}$	Real power generation of generator g at time t in segment seg.
$P_{b,t,s}^{W}$	Wind generation of wind farm at bus b at time t in scenario s.
$P_{b,t,s}^{WC}$	Wind curtailment of wind farm at bus b at time t in scenario s.
$P_{b,t,s}^{LC}$	Flexible load curtailment of bus b at time t in scenario s.
$F_{k,t,s}$	Real power flow through transmission line k at time t in scenario s.
$SR_{g,t}^{D}$	Spinning down reserve available through generator g at time t.
$SR_{g,t}^{U}$	Spinning up reserve available through generator g at time t.
$sr_{g,t,s}^{D}$	Spinning down reserve deployed by generator g at time t in scenario s.
$sr_{g,t,s}^{U}$	Spinning up reserve deployed by generator g at time t in scenario s.
$Z_{k,t}$	Transmission switching binary status (1: line k is on at time t; 0: line k is off at time t.

Parameters

$\gamma_{t,s}$	Probability of scenario s at time t.
θ_k^{min}	Minimum voltage angle difference at line k.
θ_k^{max}	Maximum voltage angle difference at line k.
$\theta_{1,t,s}$	Voltage angle at the slack bus at time t in scenario s.
B	Total number of buses.
b_k	Susceptance of transmission line k.
G	Total number of generators.
M_{max}	The maximum value of the big M.
$P_{b,t}^{L}$	Load at bus b at time t.
F_k^{max}	Upper real power flow limit of transmission line l.

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N Number of piece-wise linear segments for the generators.
P_g^{max} Upper generation limit of generator g.
P_g^{min} Lower generation limit of generator g.
P_{rate} Rated output of the wind farm.
$P_g^{seg,max}$ Upper generation limit of generator g in segment seg.
R_g Per minute ramp rate for generator g.
S Total number of scenarios.
T Length of investigated time period.
$u_{g,t}$ Generator binary status (1: generator g is on at time t; 0: generator g is off at time t.
$u_{g,t}^{SD}$ Shutdown binary indicator (1: generator g shuts down at time t; 0: generator g does not shut down at time t).
$u_{g,t}^{SU}$ Startup binary indicator (1: generator g starts up at time t; 0: generator g does not start up at time t).
Z_k^F Transmission line failure at line k.
Z_k^{max} Maximum lines permitted for transmission switching.
μ_g^{seg} Linear cost of generator g in segment seg.
μ^M M penalty cost.
μ_g^{NL} No load cost of generator g.
μ_g^{sr} Spinning reserve deployment cost of generator g.
μ_g^{SR} Spinning reserve capacity cost of generator g.
μ_g^{SD} Shutdown cost of generator g.
μ_g^{SU} Startup cost of generator g.
μ^{WC} Wind curtailment compensation rate.
μ_b^{FLC} Flexible load curtailment compensation rate for bus b.
v Wind speed.
$f(v)$ The frequency rate of wind speed.
v_{ci} The cut-in speed of the wind turbine.
v_{co} The cut-out speed of the wind turbine.
v_{rated} The rated speed of the wind turbine.

II. Introduction

The rising environmental awareness has boosted the usage of renewable energy sources (RESs), which plays a crucial role in reducing the usage of fossil fuel-based electric power as well as providing economic benefits to the users. However, RESs introduces significant uncertainty to power systems, resulting in power system stability and reliability concerns, and various techniques are currently being used to improve system flexibility for increased RES penetration to improve the reliability and cost efficiency of power systems [1]-[3]. Among different techniques, appropriately adjusting the network topology of electric power transmission systems through optimal transmission switching (OTS) can result in great benefits such as reduced transmission congestion, improved voltage profiles, better system security, and reduced operating costs [4]-[8].

Currently, OTS problems are usually formulated based on optimal power flow (OPF) problems by modifying the power flow constraints with the introduction of binary integer variables that indicate whether the transmission line is in or out of service. The OTS problem is originally a nonlinear optimization problem, and the big-M method is usually used to linearize the problem, making the problem a mixed-integer linear program (MILP). Although the linearization helps improve the computational efficiency of the OTS problem, the MILP is still relatively computation, especially for large-scale, real-world power systems [9]. Thus, improving the computational efficiency of the OTS problem is paramount.

In the big-M method, binary variables are used to enable or disable constrains, leading to big-M constraints in the form of (1), where x represents a binary variable and M corresponds to a "big constant" [10]:

$$a'y + M(1-x) \geq b \tag{1}$$

The selection of M has a significant impact on the computational efficiency of MILP problems. A poorly selected M value can lead to non-optimal solutions or an unnecessarily long solution time [11], [13]. It is common in literature and research to advocate selecting a very large number as the value of M, however, an unnecessarily large value of M can expand the feasible region of the MILP problem, resulting in an increased number of iterations required to obtain the optimal solution [11], [12] and thus negatively affect the computational efficiency of the problem. Furthermore, an extremely large value of M can even cause a loss of solver's precision and numerical instability [13], [14]. In additional to the previously mentioned scenarios, a small M value limits the feasible region space. Consequently, a small M does not guarantee the convergence to a global optimum and can potentially lead to infeasible models [13].

Real-world OTS problems are part of the power system operations model, and the OTS problems have to be solved within the operational time frame. Thus, computational efficiency are extremely important for OTS problems. For large-scale OTS problems, a moderate change in the value of the big M can result in a considerable difference in the computational efficiency. Although the big-M-based OTS model has been widely used [15]-[18], there still lacks a method to search for the optimal value for the big M-based OTS problem.

To fill this gap, this paper aims to propose a model to find the optimal big M values for the big M-based OTS problems. The contributions of this paper are as follows. First, an evaluation on the effects of different big M values on the computational efficiency of OTS problems is presented. Second, a model to find the optimal big M values is proposed. The model was implemented on a modified RTS-96 test system with different contingency scenarios, and results show that the proposed model can effectively find the optimal big M values for OTS problems, reducing the solution time of the big M-based OTS problems while achieving optimality and maintaining numerical stability.

The remainder of this paper is organized as follows. Section III describes the proposed big M value optimization model for

OTS problems. Model setup, cases descriptions, and results discussion is presented in Section IV. Finally, Section V summarizes the main conclusions.

III. The Big M Value Optimization Model

This section presents the proposed big M value optimization model for OTS problems, and the model is presented in equations (2)-(26). The model is based on a multi-hour OTS model, with M being a variable, while allowing different transmission contingency scenarios to be considered. To better observe the detrimental consequences of an erroneous big M selection, load shedding and wind power curtailment are allowed with a penalty in the model.

$$min\left\{\sum_{t=1}^{T}\left[\mu^{M}M_{t} + \sum_{g=1}^{G}\left(\mu_{g}^{NL}u_{g,t} + \mu_{g}^{SD}u_{g,t}^{SD} + \mu_{g}^{SU}u_{g,t}^{SU} + \mu_{g}^{SR}\left(SR_{g,t}^{U} + SR_{g,t}^{D}\right) + \sum_{seg=1}^{N}\mu_{g}^{seg}P_{g,t}^{seg} + \sum_{s=1}^{S}\gamma_{t,s}\mu_{g}^{sr}\left(sr_{g,t,s}^{D} + sr_{g,t,s}^{U}\right)\right) + \sum_{b=1}^{B}\sum_{s=1}^{S}\gamma_{t,s}\left(\mu_{b}^{FLC}P_{b,t,s}^{FLC} + \mu^{WC}P_{b,t,s}^{WC}\right)\right]\right\} \tag{2}$$

$$u_{g,t}P_{g}^{min} \leq P_{g,t} \leq u_{g,t}P_{g}^{max} \tag{3}$$

$$0 \leq P_{g,t}^{seg} \leq P_{g}^{seg,max} \tag{4}$$

$$P_{g,t} = \sum_{seg=1}^{N}P_{g,t}^{seg} \tag{5}$$

$$-F_{k}^{max}Z_{k}^{F}Z_{k,t} \leq F_{k,t,s} \leq F_{k}^{max}Z_{k}^{F}Z_{k,t} \tag{6}$$

$$-b_{k}\left(\theta_{fr,k,t,s} - \theta_{to,k,t,s}\right) - \left(1 - Z_{k,t}\right)M \leq F_{k,t,s} \tag{7}$$

$$-b_{k}\left(\theta_{fr,k,t,s} - \theta_{to,k,t,s}\right) + \left(1 - Z_{k,t}\right)M \geq F_{k,t,s} \tag{8}$$

$$\sum_{k=1}^{K}\left(1 - Z_{k,t}\right) \leq Z_{k}^{max} \tag{9}$$

$$\sum_{g\in g_{b}}\left(P_{g,t} + sr_{g,t,s}^{U} - sr_{g,t,s}^{D}\right) + \sum_{k\in\sigma_{b}^{+}}F_{k,t,s} - \sum_{k\in\sigma_{b}^{-}}F_{k,t,s} + P_{b,t,s}^{W} - P_{b,t,s}^{WC} + P_{b,t,s}^{LC} = P_{b,t}^{L} \tag{10}$$

$$0 \leq M_{t} \leq M_{max} \tag{11}$$

$$u_{g,t}^{SU} - u_{g,t}^{SD} = u_{g,t} - u_{g,t-1} \tag{12}$$

$$u_{g,t}^{SU} + u_{g,t}^{SD} \leq 1 \tag{13}$$

$$\sum_{t=m}^{m+T_{g}^{up-1}}u_{g,t} \geq T_{g}^{up}\left(u_{g,m} - u_{g,m-1}\right), \quad 2 \leq m \leq T - T_{g}^{up} + 1 \tag{14}$$

$$\sum_{t=m}^{m+T_{g}^{down-1}}\left(1 - u_{g,t}\right) \geq T_{g}^{down}\left(u_{g,m-1} - u_{g,m}\right), \quad 2 \leq m \leq T - T_{g}^{down} + 1 \tag{15}$$

$$-60R_{g}u_{g,t} \leq P_{g,t} - P_{g,t-1} \leq 60R_{g}u_{g,t} \tag{16}$$

$$P_{g,t} - SR_{g,t}^{D} \geq P_{g}^{min}u_{g,t} \tag{17}$$

$$P_{g,t} + SR_{g,t}^{U} \leq P_{g}^{max}u_{g,t} \tag{18}$$

$$0 \leq SR_{g,t}^{U} \leq 10R_{g} \tag{19}$$

$$0 \leq SR_{g,t}^{D} \leq 10R_{g} \tag{20}$$

$$0 \leq sr_{g,t,s}^{U} \leq SR_{g,t}^{D} \tag{21}$$

$$0 \leq sr_{g,t,s}^{D} \leq SR_{g,t}^{U} \tag{22}$$

$$0 \leq P_{b,t,s}^{WC} \leq P_{b,t,s}^{W} \tag{23}$$

$$0 \leq P_{b,t,s}^{FLC} \leq P_{bt}^{L} \tag{24}$$

$$\theta_{k}^{min} \leq \theta_{fr,k,t,s} - \theta_{to,k,t,s} \leq \theta_{k}^{max} \tag{25}$$

$$\theta_{1,t,s} = 0 \tag{26}$$

The objective function (2) has the goal of minimizing the total cost. Equation (2) considers big M as an hourly variable instead of a constant predetermined value. Furthermore, a minor penalty cost is added to the big-M variable to avoid unnecessarily large M values while keeping the generation dispatch cost the dominant objective. Additionally, (2) considers the piece-wise linear generation cost, spinning reserve cost, start-up, shutdown, and no-load cost, as well as the load curtailment and wind power curtailment penalties. Generation limits are modeled in (3) and the generation piece-wise linear segments constraints are described in (4) and (5). Equation (6) describes the transmission capacity limits considering transmission contingencies and transmission switching. Additionally, Equations (7) and (8) represent the DC power flow constraints considering a big-M formulation and the maximum permitted transmission line switching is modeled in (9). Equation (10) describes the nodal power balance constraint and (11) sets an upper limit for M. The start-up and shutdown constraints are modeled in Equations (12) and (13), while Equations (14) and (15) model the minimum up and down time for the generators. The hourly generation ramp constraint is modeled in (16), Equations (17) and (18) describe the spinning up and down reserves correspondingly. The reserve energy deployment time is described in Equations (19) and (20), while the energy deployment limits are shown in Equations (21) and (22). The wind power curtailment and load curtailment constraints are modeled in Equations (23) and (24), respectively, and Equations (25) and (26) are the constraints for the bus voltage angle.

IV. Case Study Results and Discussion

A. Case Studies and Model Setup

The proposed model was implemented on the RTS-96 test system with a minor modification; two wind energy farms located at bus 3 and bus 24, each with a 200-MW rated power similar to the model in [19]. Two different conditions were used to evaluate the effects of different big M values in the OTS model: (1) One wind power output scenario was considered, and (2) 25 wind power output scenarios were considered. Under each condition, two sub-conditions are considered: (a) No transmission contingency exists in the system; (b) Three highly utilized transmission lines fail in the system. Furthermore, under each sub-condition, three cases were carried out, allowing a maximum of 1, 2, and 3 transmission lines to be switched out, respectively. In total, 12 test cases were carried out. A small penalty cost of \$1E-7/unit was added to the M variable in the objective function. Additionally, the load curtailment and wind power curtailment penalties were set at \$10,000/MW and \$30/MW, respectively. The model proposed in Section III was implemented using a two-step approach. In the first step, the unit commitment variable was solved without considering transmission switching. In the second step, the model was solved with predetermined values for the unit commitment variables, start-up and shut-down variables.

B. The Impact of the Big M Value on the Objective Value

Choosing an extremely large or a small value of M can have undesirable consequences to the optimization problem [9]. To study the impact of the big M value on the objective value, the OTS model in Section III was solved using predetermined values of M without the penalty for the big M. The M values used in this study ranged from 1E2 to 1E12 for the 1-scenario cases and from 1E2 to 1E9 for the 25-scenario cases. The big M values in the latter cases were smaller because the problem became very computationally burdensome with such large M values, and we only chose cases that could be solved within 7200 seconds. The objective values of different cases obtained with different big M values are shown in Fig. 1 and Fig. 2.

From the two figures, it can be seen that, with a small value of M (less than 10E3), the search space for the solution was limited, thus the OTS problem could not converge to the global optimum. With an excessively large M value (greater than 10E7), unreasonable objective values were produced because of the numerical instability. Thus, the values of the big M have to be chosen properly to avoid instability in the optimization model [20].

C. Optimal M Model

Using the model proposed in Section III, the optimal values of the big M were solved for 24 hours in the 12 cases described in section IV.A, and the optimal big M values are shown in Fig. 3 and Fig. 4. It can be observed that the optimal big M values vary with time, the maximum number of lines to be switched, and transmission contingencies.

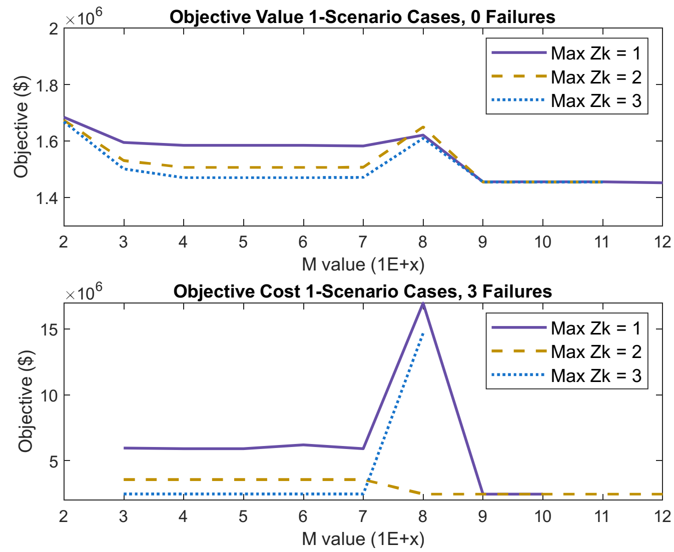

Fig. 1. Objective Value for the 1-Scenario Cases under different M values

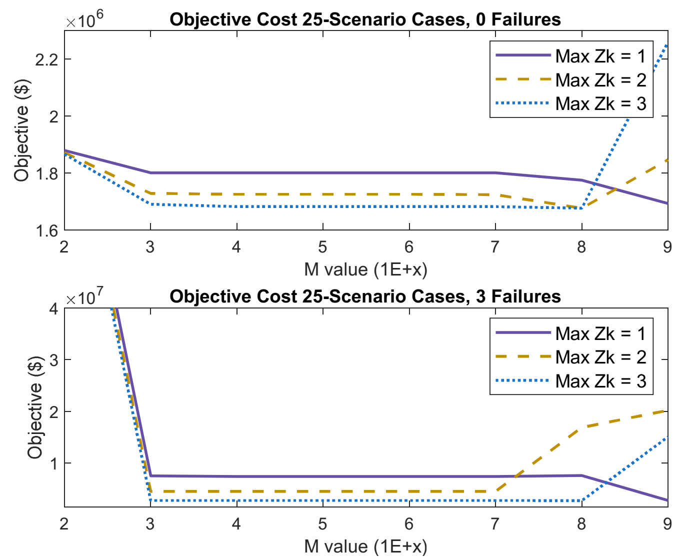

Fig. 2. Objective Value for the 25-Scenario Cases under different M values

D. Computational Efficiency

To study the impact of big M values on the computational efficiency of the OTS model, the solution time of the OTS cases were obtained with the M values being 1000, 5000, 10000, and the obtained optimal M values for each of the 12 cases. The solution times for the 1-scenario and 25-scenario cases are listed in Table I and Table II, respectively. The cases were implemented using Python and Gurobipy on a computer with an Intel Core i7-1065G7 CPU and 16GB of RAM.

From Table I and Table II it can be observed that in most of the cases the computational time increased with the increase of the M values. With the optimal M values, the solution time was always the shortest among the cases where the global optimum could be reached under each condition. This verifies the effectiveness of the proposed model in obtaining optimal big M values for OTS problems.

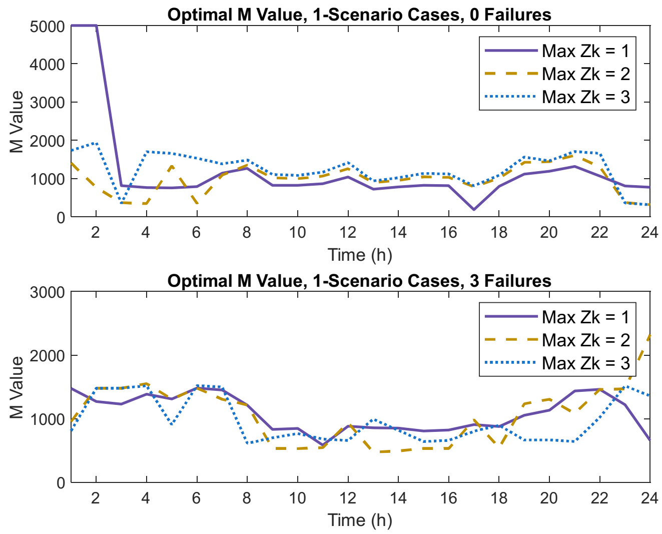

Fig. 3. Optimal M Values for the 1-Scenario Cases

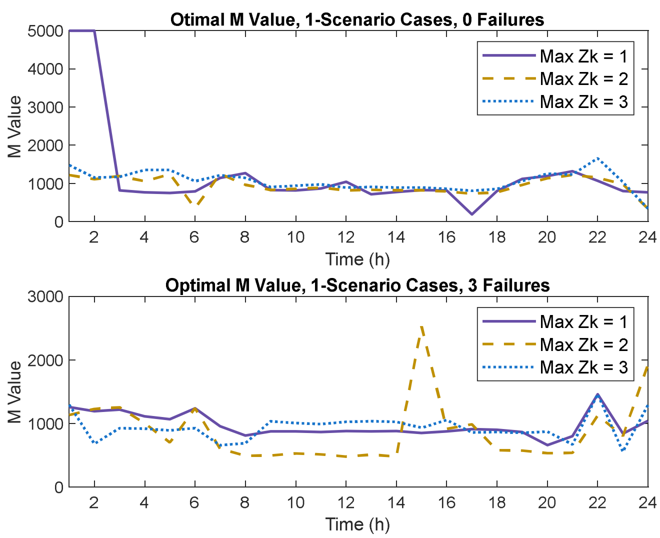

Fig. 4. Optimal M Values for the 25-Scenario Cases

V. Conclusions

This paper proposes a big M value optimization model for big M-based OTS problems and evaluates the impact of the big M values on the optimality and computational efficiency of the OTS problems. The model was implemented on a modified RTS-96 test system under 12 conditions. The results show that properly chosen big M values are critical to the computational efficiency, optimality, and stability of the OTS problems, and the proposed model can effectively find a set of optimal values for the big M, allowing the OTS problem to be solved fast while converging to the global optimum.

TABLE I
COMPUTATIONAL TIMES FOR 1-SCENARIO OTS

M	*1000*	*5000*	*10000*	*Optimal M*
	1 Transmission Switch Allowed – No failures			
Time (s)	3.157	3.355	3.376	3.063
Objective ($)	1,595,271	1,584,756	1,584,756	1,584,756
	1 Transmission Switch Allowed – 3 failures			
Time(s)	2.266	2.567	2.703	2.008
Objective ($)	5,964,818	5,911,480	5,911,480	5,911,480
	2 Transmission Switch Allowed – No failures			
Time(s)	4.235	4.579	5.547	4.207
Objective ($)	1,530,968	1,506,473	1,506,473	1,506,473
	2 Transmission Switch Allowed – 3 failures			
Time(s)	2.938	3.360	3.579	2.859
Objective ($)	3,569,559	3,564,907	3,564,907	3,564,907
	3 Transmission Switch Allowed – No failures			
Time(s)	7.097	6.646	7.754	5.709
Objective ($)	1,501,527	1,470,217	1,470,217	1,470,217
	3 Transmission Switch Allowed – 3 failures			
Time(s)	3.391	4.016	4.126	3.193
Objective ($)	2,465,004	2,464,802	2,464,802	2,464,802

TABLE II
COMPUTATIONAL TIMES FOR 1-SCENARIO OTS

M	*1000*	*5000*	*10000*	*Optimal M*
	1 Transmission Switch Allowed – No failures			
Time(s)	162.617	260.895	249.264	158.593
Objective ($)	1,801,245	2,464,802	2,464,802	2,464,802
	1 Transmission Switch Allowed – 3 failures			
Time(s)	63.322	132.663	135.299	52.653
Objective ($)	7,489,953	7,399,367	7,399,367	7,399,367
	2 Transmission Switch Allowed – No failures			
Time(s)	149.566	195.236	203.369	192.162
Objective ($)	1,728,716	1,725,476	1,725,476	1,725,476
	2 Transmission Switch Allowed – 3 failures			
Time(s)	106.641	113.697	130.817	85.121817
Objective ($)	4,489,102	4,488,312	4,488,312	4,488,312
	3 Transmission Switch Allowed – No failures			
Time(s)	186.261	252.698	266.511	216.309
Objective ($)	1,690,485	1,682,310	1,682,310	1,682,310
	3 Transmission Switch Allowed – 3 failures			
Time(s)	112.588	125.534	139.057	121.698
Objective ($)	2,739,361	2,739,222	2,739,222	2,739,222

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