



Budget-Constrained Rail Electrification Modeling Using Symmetric Traffic Assignment: A North American Case Study

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Abstract: We consider a budget-constrained rail network electrification problem with associated changes in costs of energy usage (via path gradient and curvature), operations, and long-term maintenance. In particular, we consider that freight flows on such a network form a user equilibrium. Interactions between electric and diesel trains on the same corridor are represented with nonseparable link performance functions that nevertheless have a symmetric Jacobian. This bilevel formulation is solved for the North American railroad network using a genetic algorithm (GA), incorporating domain-specific insights to reduce the number of solutions that must be considered. We analyze solution characteristics and decision-making implications. Results show that broad connectivity would be beneficial for the most impact. Increasing demand shifts electrified corridors toward the more populous east and gulf coasts, while increased operational costs results in the electrification of routes through mountainous terrains. DOI: 10.1061/(ASCE)IS.1943-555X.0000682. © 2022 American Society of Civil Engineers.

Introduction

Rail networks play a vital role in local and national economic structures. In many countries such as Japan and Switzerland, they constitute a significant share of passenger transport mode share. Rail freight transit also accounts for a large portion of total freight transit, exceeding 50% modal share in large economies like Canada and Russia. Therefore, a significant opportunity exists for government policies incentivizing rail network improvements to provide for larger economic, environmental, and social returns.

Rail network electrification and the accompanying transition from diesel-electric to fully electric locomotives are important steps toward sustainable systems and renewable fuel sources for the United States of America. There are many studies on the impact and cost-benefit analysis of rail electrification (USDOT—Federal Railroad Administration 2015, 2019). Advantages of electric locomotion include lower long-term energy and locomotive maintenance costs, lower noise and air pollution levels, faster acceleration, and more flexibility in the primary power source, leading to less volatility from fuel price fluctuations. These benefits must be balanced with significant upfront investment for infrastructure upgrades, higher infrastructure maintenance costs, vulnerability of overhead architecture, higher property tax obligations for private rail companies, and the general uncertainty in the investment (Walthall 2019).

We consider a rail network design problem (RNDP) where parts of a freight rail network can be electrified, subject primarily to budget constraints. The RNDP is formulated as a bilevel problem: the upper-level problem deciding the optimal subset of links for electrification, and the lower-level problem calculating link flows as well as associated network metrics. Our upper-level formulation uses the objective of minimizing private costs, laying a foundation that can be adapted to improve net social benefit and reflect where subsidies ought to be directed. Given that there are many rail operators, and shippers can choose which operator to use (not necessarily aligning with net social benefit), we model the lower-level problem as a user-equilibrium traffic assignment problem (TAP). Genetic algorithms (GA) solve the higher-level problem of finding the optimal links to electrify.

In this setting, rather than apportioning an electrification budget to each rail operator independently, a utilitarian schema allocates the electrification budget for specific link electrification in order to bring about the greatest possible cost reductions across the network. The rail operators and shippers then respond to these changes by altering their scheduling and flow patterns to minimize their individual costs. Given multiple operators, the lower-level problem is a setting where flow is directed “selfishly” to minimize shipment costs. For the lower-level problem, we assume that these activities lead to an equilibrium where the costs of shipment flows cannot be lowered unilaterally. With shipment flow expressed in tons, as a continuous quantity over the long term, we model flows as the solution to a user-equilibrium traffic assignment model, reflecting decentralized decision making with each unit of flow seeking a minimum-cost route. This assumption is consistent with prior literature on the topic (Uddin and Huynh 2015; Wang et al. 2018). An alternative approach would be to model a smaller number of self-optimizing fleet owners, resulting in a Nash–Cournot equilibrium. However, Van Vuren and Watling (1991) show that the difference in the resulting assignments is less than 5% for aggregate measures in large networks, with this difference decreasing to zero as the number of owners increases (indeed, the user equilibrium is the limiting case of infinitely many owners). The Nash–Cournot equilibria are harder to compute and require more data to calibrate; given the

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relatively small observed differences in the aggregate metrics of interest in this paper, we opt to study only user equilibria.

NDPs are widely studied in road networks. Rail NDPs vary in two significant ways. First, most of the freight rail network is privately owned by the user or contracted out for usage, which leads to nonsocially-optimal usage restrictions. Rail electrification has highly uncertain, and possibly negative, rates of return when external benefits are not accounted for. This paper provides a framework for future analysis of policy interventions in the United States to internalize the benefits to the private companies that own the rails and would be responsible for implementing electrification. Second, the characteristics of individual network links (track curvature and gradient) affect maintenance and operating costs more so than in road networks.

With these distinctions in mind, the main contributions of this article are as follows:

- We formulate the rail electrification NDP as a bilevel optimization problem incorporating electrification costs; fuel, locomotive, and operational costs; and train resistance (bearing, flange, air, grade, curve, braking, and inertia) costs. The result is a novel instance of the TAP including (symmetric) link interactions.
- We derive the appropriate flow-shift formula for Algorithm B for solving our formulation (traffic assignment with symmetric link interactions) and show that it meets the optimality conditions required for convergence.
- We use a general-purpose metaheuristic (a GA) based on problem-specific insights to generate high-quality solutions and solve this problem on a large-scale North American rail network.
- We conduct sensitivity analysis and analyze the resulting solutions to draw insights and policy conclusions.

The rest of this article is organized as follows. We first provide background information on rail electrification and discuss advances in traffic assignment, NDPs, and solution methods for both problems. We then describe the formulation for the rail electrification NDP and associated model components. We then describe the North American rail network dataset and demand data we use and outline our experiment design. We follow this with a summary of the results from our experiments and draw practical insights. We conclude by summarizing our findings and suggest avenues for future work.

Literature Review

Rail Electrification in the United States

The oil crisis of the 1970s spurred research into the economics of rail electrification in the United States. In 1977, Schwarm (1977) examined the detailed costs associated with rail electrification infrastructure, finding three primary categories: power delivery, public works compatibility, and signaling systems compatibility. Power delivery was used to calculate electrified link capacity. Kneschke (1986) then detailed the design requirements of the traction substations that affect the electrified lines' capacities. Meanwhile, other researchers looked at the benefits of electrification. Whitford (1952) examined the energy savings from electrifying the high-density portions of the rail network, and Ditmeyer et al. (1985) incorporated ancillary benefits and costs, such as those accruing to maintenance, reliability, and fuel handling. At the time, emissions were not a major concern. Additionally, computational costs and the relative youth of network optimization techniques meant that

these analyses primarily used traffic density as a heuristic for link selection.

As computational resources and optimization algorithms progressed, the oil crisis abated and deregulation led to a decrease in rail network mileage and interest in rail electrification. In 2012, Cambridge Systematics (2012) conducted an extensive study on electrification for the Southern California Association of Governments. RailTEC (2016) conducted a similar study for the California Air Resource Board in 2016. Both studies were primarily motivated by reducing rail emissions in dense urban areas. The latter study highlighted the problem of network connectivity: because rail freight routes are particularly long and the costs of switching from electric to diesel-electric locomotives en route are high (in terms of delay and logistical costs of ensuring locomotives are available), confining electrification to relatively small regions is not an efficient way to limit emissions. Outside of regulatory constraints or internalized emissions costs, rail companies would likely send large numbers of diesel-electric trains through electrified portions of the rail network in order to limit overall route costs. We thus face an optimization problem impacted by policy: how should the initial links be selected for electrification, given budget constraints and a desire to reduce emissions from the rail network?

There are very few studies to draw on for link improvement under a budget constraint, such as Mishra et al. (2016), which applied numerical methods toward "[making] optimal investment decisions . . . in moderate and large transportation networks." Mishra utilized travel costs to road users, primarily consisting of temporal costs. Applying a similar framework to a rail network considers similar costs, although fuel costs for rail constitute a relatively larger portion, and time costs a relatively smaller one.

Rail Electrification of Other National Networks

While the European rail network is extensively electrified, most of it is used for passenger transport and comparisons to the US network are difficult. This can be attributed to deliberate prioritization of passenger rail efficiency over freight, as evidenced by limits on maximum permissible train lengths, maximum axle loads, and vertical car height (Walthall 2019). Similarly, the Japanese rail network is used primarily for passengers, accounting for less than 1% of the total freight load.

India and China both offer examples of extensive electrification of freight rail systems relevant to the US network. China has extensively electrified both its passenger and freight networks in the past decades, with over 70% total electrification and about 25% of its network electrified between 1990 and 2007 (Lawrence et al. 2019). Much of this electrification has coincided with major construction to upgrade capacity and signaling infrastructure, especially in mountainous areas.

Similarly, India has extensively electrified its network (about 70%), prioritizing selected main lines and high-density routes [Ministry of Railways (India) 2020]. India has constructed high overhead contact systems in order to accommodate double-stacked freight. Similar construction would be required in the United States, where double-stacked container freight is a large component of American railroads' revenues.

Traffic Assignment Problem Background

This lower-level assignment problem is a generalized version of the static TAP formulated as a convex program by Beckmann et al. (1956). Specifically, the rail traffic assignment can be modeled as a variant of static TAP with symmetric link interactions if the

flows refer to the tonnage of goods (Alliance Transportation Group 2013; Uddin and Huynh 2015; Wang et al. 2018). We therefore assume the units of flow to be tons, allowing us to use existing solution methods to solve this problem. Given this formulation, the assignment problem follows Wardrop's first principle (Wardrop and Whitehead 1952): "All used paths between each origin and destination must have equal and minimal cost." Static TAP is a well-studied problem; for more on its properties and solution methods, see the books by Patriksson (1994) and Boyles et al. (2020b). Static TAP solution methods include specialized algorithms to obtain the optimal solution. Some widely used methods are gradient projection (Jayakrishnan et al. 1994), Algorithm B (Dial 2006), and TAPAS (Bar-Gera 2010), among others. A recent computational comparison by Perederieieva et al. (2015) concludes that Algorithm B and TAPAS outperform other tested methods.

However, solution existence and optimality for these methods have been shown only for TAP with separable link costs, where the cost of each link depends only on its own flow. Generalizations have been explored for cases with link interactions, i.e., the cost of a link depends on the flow of many links. The convex programming formulation is valid for symmetric interactions, i.e., when the Jacobian of the cost mapping is symmetric everywhere. This extension to TAP was first mentioned by Prager (1954), discussing the need for traffic interactions on a two-way street. Dafermos (1971, 1972) first modeled symmetric interactions using the entire flow vector and showed multiclass TAP and symmetric TAP as equivalent. Dafermos also presented an iterative flow update algorithm to obtain user equilibrium and system optimal flows. Research since then has focused on the even more general asymmetric traffic assignment problem, and there have been no recent notable advances focusing specifically on symmetric TAP. A core contribution of this study is adapting Algorithm B for the symmetric TAP by proving the correctness of the flow shifts and ensuring that required optimality conditions are met.

The solution to static symmetric TAP is unique if the link performance functions are strictly monotone and the cost Jacobian is positive semidefinite. These conditions are satisfied by the cost functions we describe in the Cost Formulas Section. This study uses our implementation of Algorithm B to solve this assignment problem. Algorithm B selects the shortest and longest paths to each destination node within a bush (acyclic graphs rooted at each zone), and shifts flow using Newton's method, iterating over all bushes till convergence. The source code is available at the SPARTA lab github repository (Boyles et al. 2020a) under the *pd-word* branch.

Network Design Problem Background

Network design problems (like many other bilevel problems) are intractable to solve exactly. Such methods (such as branch-and-bound or Benders decomposition) have been proposed, as in Leblanc (1975), Chen and Alfa (1991), Drezner and Wesolowsky (1997), and Long et al. (2010). However, the largest network tested in any of these studies has 40 nodes and 99 links.

Heuristic methods are standard for this class of problems. GAs, simulated annealing, and tabu search are examples of such methods that have been applied to traffic NDPs. For details on such methods, see the review papers by Farahani et al. (2013) and Iliopoulou et al. (2019). They study the urban transportation NDPs and provide an overview of the types of problem variations as well as solution methods and some applications.

Alternatively, the discrete NDP can be transformed into a single-level problem. Gao et al. (2005) transformed the problem into a nonlinear single-level program utilizing support functions, which was solved by existing techniques. A third approach involves

formulating the problem with equilibrium constraints and then using branch-and-bound or reduced gradient-based methods (Lo and Szeto 2004, 2009; Szeto and Lo 2006, 2008; Szeto et al. 2010). Other methods, such as Lagrangian relaxation and column generation, have also been used as exact solution methods for small NDP instances (Meng et al. 2001; Borndörfer et al. 2008). As this approach is not viable for large instances, we focus on the nonexact methods, which trade-off guaranteed optimality for tractability.

In our experiments, we use a GA as a solution heuristic. Katoch et al. (2021) provide a recent review of GAs with applications in transportation network design problems (road, transit, and multimodal network design), facility layout, inventory and scheduling, and other domains. Key advantages of GAs for our application are easy parallelization, only requiring objective function values (and not gradients or Hessians), and the ability to quickly obtain quality solutions in practice. These advantages help us exploit the subproblem independence for parallelization. Additionally, this avoids first- and second-order information calculation for the line integral objective function. Other heuristics exploiting these properties can work well for this problem structure and can be used instead of GAs.

Model Formulation

Let $\mathbf{A}$ denote the set of rail links traversable by diesel-electric trains and $\mathbf{N}$ denote the set of nodes (yards and stations). Each link $i \in \mathbf{A}$ currently has a flow-dependent usage cost $c_i(x_i)$ per unit flow. The link can be electrified for a cost of c_i^e , changing the usage cost function to $c_i^e(x_i)$ per unit flow. The usage costs for diesel and electrified links differ due to technological differences and fuel costs and are lower for electrified links. The set of candidate links eligible for electrification is denoted by $\mathbf{A}^E$, a subset of $\mathbf{A}$. The flow on link i is given by x_i tons per day. The capacity of the link is denoted by u_i . Let Π denote the set of paths in the network and h_π denote the flow on a particular path $\pi \in \Pi$.

The set of nodes (e.g., denoting stations, yards, and interchanges) is denoted by $\mathbf{N}$. The demand between each origin r and destination s node is given by d_{rs} . The demand information between all origin-destination (OD) pairs is stored in the OD matrix $\mathbf{D}$. The total budget for upgrades is B . The decision variable y_i for $i \in \mathbf{A}$ equals 1 if link i is chosen for upgrades (electrification) and 0 otherwise. The cost for switching the mode of operations from diesel to electric (and vice versa) at yard u is w_u (and infinitely high at other nodes without switching facilities).

The practical interpretation of link electrification involves additional electric infrastructure on a link allowing electric trains to use the same physical link with different fuel costs while maintaining the same total link capacity. Our model captures this using the network transformation shown in Fig. 1 with a node, an incoming link, and an outgoing link. Each physical link is split into two parallel sublinks, one for diesel flow (x_{Di}) and one for electric flow (x_{Ei}). The congestion cost c_i' on each electrified link is not separable, depending on the sum of diesel and electric flows, thus capturing the physical link capacity constraint. The fuel (track resistance) costs c_i'' on these two sublinks are separable, depending only on the sublinks own flow (diesel or electric). The capacity of the electric sublink is set infinitely high for links not chosen for electrification. Therefore, the total link cost of any link i is $c_i(x_{Di} + x_{Ei}) = 2c_i'(x_{Di} + x_{Ei}) + c_i''(x_{Di}) + c_i''(x_{Ei})$. Each node u in the network is expanded to incorporate switching costs w_u . This switching cost is set infinitely high at nonyard nodes to allow switching only at yards.

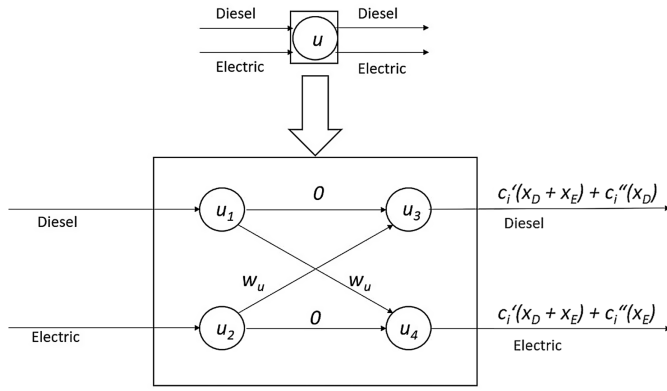


Fig. 1. Network transformation for every node.

The RNDP is then formulated as follows:

$$\min \sum_{i \in \mathbf{A}} c_i(x_i)x_i \quad (1)$$

subject to:

$$\sum_{i \in \mathbf{A}} c_i^e y_i \leq B \quad (2)$$

$$x_i = x_{Di} + x_{Ei} \quad \forall i \in \mathbf{A} \quad (3)$$

$$y_i \in \{0, 1\} \quad \forall i \in \mathbf{A} \quad (4)$$

where y_i are found from the upper-level problem and x_i values are found by solving the following lower-level assignment problem, in which M is a sufficiently large constant

$$\min_{\mathbf{x}, \mathbf{h}} \int_0^x \mathbf{c}(\mathbf{s}) \cdot d\mathbf{s} \quad (5)$$

subject to:

$$x_i = \sum_{\pi \in \Pi: (r,s) \in \pi} h_\pi \quad \forall (r,s) \in \mathbf{A} \quad (6)$$

$$x_i \leq M y_i \quad \forall i \in \mathbf{A}^E \quad (7)$$

$$\sum_{\pi \in \Pi^{rs}} h_\pi = d_{rs} \quad \forall (r,s) \in \mathbf{N}^2 \quad (8)$$

$$h_\pi \geq 0 \quad \forall \pi \in \Pi \quad (9)$$

Eq. (1) minimizes the total usage cost over electrified and non-electrified links. Eq. (2) enforces the electrification budget constraint. Eq. (4) indicates that y_i is a binary indicator variable. Eq. (5) is the modified Beckmann function for symmetric link interactions, described more in the next subsection. Eq. (6) states the relation between link and path flows, and Eq. (7) ensures that the electrified links with flow are the ones actually chosen for construction. Eq. (8) ensures that the demand is met across all used paths for each O-D pair, and Eq. (9) states flow nonnegativity.

This formulation is presented with fixed demand. If elastic-demand functions are known, the fixed-demand assumption can be relaxed by applying the Gartner network transformation (Gartner 1980; Boyles et al. 2020b). This transformation allows for the elastic-demand equilibrium problem to be solved as a fixed-demand problem, introducing new artificial links with specially-designed cost functions based on the demand functions.

Since the elastic-demand case can be handled by the same solution methods as the fixed-demand case, for simplicity of exposition, we do not discuss it further in our presentation of the formulation and solution methods.

It is also straightforward to calculate costs for multiple years, accounting for forecasted future changes by solving a lower-level TAP for each future year and computing a total discounted cost. This approach would significantly increase computational requirements, however, and we do not pursue it in the experiments we report.

Cost Formulas

This section provides the cost calculation formulas for the link electrification costs, track resistance, and generalized link costs. Owing to a significant number of constants and rates within these formulas, they are omitted here for brevity but can be found on the author's github repository (Patil 2020). In order to update cost values for inflation, this study uses industry-appropriate producer price indices from the US Bureau of Labor Statistics (2020).

There are two main costs to be considered, roughly corresponding to the capital expenses and operational expenses. Electrification costs (or capital expenses) refer to all the costs associated with converting a link from standard operations to electric operations that we considered. The generalized link costs (operational expenses) refer to the costs of traversing a link and incorporate both time and fuel (whether diesel or electricity) costs based on the link's speed and resistance, calculated on a per-ton basis.

Link Electrification Cost

The link electrification cost does not depend on the link's traffic/tonnage (costs associated with rerouting during construction are not considered), but rather the link's location and existing characteristics. This cost can be divided into five components:

- Overhead contact system (OCS)— c_{OCS}
- Electrical substations— c_{sub}
- Transmission lines— c_{trn}
- Public works— c_{pub}
- Signaling— c_{signal}

The first four cost components are positively correlated with the roughness of the link's terrain (e.g., the electrification infrastructure is more expensive to construct in mountainous segments than in plains segments). For each of those four categories, the cost calculations utilize two parameters: a low value for ideal conditions, and a high value for the worst possible condition. In order to create an upgrade cost estimate for all links in the network, the difficulty of the terrain is scaled linearly based on the ratio, α , between each link's actual length and its straight-line length. α has a minimum value of one, which represents a straight track. The cost of signaling depends on the complexity of replacing the link's existing signal systems and is independent of terrain.

The cost of upgrading link i to be electrified is then written as

$$c_i^e = l_i \left(\left(\frac{\alpha_i - \alpha_{\min}}{\alpha_{\max} - \alpha_{\min}} \right) (c_{OCS,\max} + c_{sub,\max} + c_{transmission,\max} + c_{pub,\max}) + \left(1 - \frac{\alpha_i - \alpha_{\min}}{\alpha_{\max} - \alpha_{\min}} \right) (c_{OCS,\min} + c_{sub,\min} + c_{trn,\min} + c_{pub,\min}) + c_{signal,i} \right) \quad (10)$$

where l_i = length of link i ; and $\alpha_{\min}$ and $\alpha_{\max}$ = smallest and largest terrain difficulty values across the network, respectively.

The parameters are derived from Schwarm (1977), Kneschke (1986), and Gattuso and Restuccia (2014).

Rail Link Delay Function

To assign freight flows to the network, a link performance function is required, accounting for the rising cost of using a link as link congestion increases. This subsection details the formulation of the function as well as the process used to obtain the coefficients and constants. The final form of the equation is provided in Eq. (13) at the end of this subsection.

Several studies have applied a rail link performance function similar in form to the Bureau of Public Roads (1964) function used in road assignment (Clarke 1995). Uddin and Huynh (2015) proposed a link delay function of the form

$$t_i = t_i^0 \left(1 + \left(\frac{x_i}{u_i} \right)^\beta \right) \quad (11)$$

where x_i = flow on link i ; u_i = link's capacity; and t_i^0 = free-flow travel time. Uddin suggested a value of 4 for the parameter β . u_i is a function of the link's class, as well as the number of tracks in the link and the frequency of sidings and switches.

The link's total generalized cost is a function of the travel time, crew and cargo costs, as well as the fuel cost. The fuel cost is the product of the link travel time, cost per unit energy for the fuel source used, and power level required for the link divided by the efficiency of the locomotive. For this analysis, an average train unit is the basis of analysis, and the generalized cost is based on the cost for that train unit to traverse a link divided by that train unit's cargo mass. In this way, the network flows can be assigned as tons of cargo. The power level used on each link is calculated using the link's total resistance, as outlined in the next subsection. The generalized cost function can then be written as follows:

$$c_i(\text{or } c_i') = t_i \left(\text{crewRate} + \text{cargoRate} + \frac{P_i}{\eta} \text{fuelCost} \right) / (n_c m_{\text{cargo}}) \quad (12)$$

where $n_c m_{\text{cargo}}$ = mass of the cargo hauled by a train unit used in the analysis.

In order to separate the congestion cost and the resistance cost, we make the assumption that the fuel consumption on each link is fixed, meaning the generalized cost function changes as follows:

$$c_i(\text{or } c_i') = \left(t_i(\text{crewRate} + \text{cargoRate}) + t_0 \left(\frac{P_i}{\eta} \text{fuelCost} \right) \right) / (n_c m_{\text{cargo}}) \quad (13)$$

Track Resistance

In this paper, we assume that trains have n_L locomotives and n_c railcars. Each railcar has a tare weight of m_c and a gross weight of $m_g = m_c + m_{\text{cargo}}$. Each locomotive has a weight of m_L . A loaded train then has a mass of $m_T = n_L m_L + n_c m_g$. Time and energy exertions associated with switching between diesel-electric and electric locomotives are handled separately and discussed in the switching costs section. The resistance on the train is separated into bearing resistance, flange resistance, air resistance, grade resistance, curve resistance, brake resistance, and inertial resistance. Each of these quantities is discussed and specified in the following sections.

Bearing resistance: Assuming a relatively new train (less than 50 years old), the bearing resistance on each railcar will be $(a + (bN_{ax}/m_g))m_g$, where N_{ax} is the number of axles on the railcar. According to Gillespie and Hayes (2003), when m_g is measured in tons, $a = 2.9(\text{N/ton})$ and $b = 97.3\text{N}$. Therefore, the total bearing resistance is calculated as $\sum_{k=1}^{n_L+n_c} (am_{g,k} + bN_{ax,k})$.

Flange resistance: The bearing resistance on each component of the train is $m(Bv)$, where v is the speed of the train relative to the track, and m is the mass of the railcar or locomotive (Gillespie and Hayes 2003). $B = 0.329 \text{ N} \cdot \text{s/m} \cdot \text{ton}$ for locomotives(B_L), and $0.494 \text{ N} \cdot \text{s/m} \cdot \text{ton}$ for railcars(B_C). The total flange resistance is $v(B_L n_L + B_C n_c)$. The flange resistance varies based on the track quality, so a factor $k_{f,i}$ can be applied to adjust the flange resistance for link i .

Air resistance: The air resistance (Hay 1982) on each railcar is proportional to the square of the train's speed relative to the wind. For the purposes of this study, the wind speed is assumed to be zero, so that the train's speed relative to the track is equal to its speed relative to the air. The air resistance on each component of the train becomes Kv^2 . When v is measured in m/s, K takes on a value of $1.56 \text{ N} \cdot \text{s}^2/\text{m}^2$ for conventional equipment, or $2.06 \text{ N} \cdot \text{s}^2/\text{m}^2$. The total air resistance is $\sum_{k=1}^{n_L+n_c} K_k v^2$. A factor $k_{a,i}$ can be used to adjust the air resistance based on the average air density for link i .

Grade resistance: The grade resistance is the resistance due to gravity. Unlike the other resistances discussed, grade resistance can be positive (upgrades) or negative (downgrades). The grade resistance for the train is given by $m_T g \sin(\theta)$, where g is the acceleration due to gravity and θ is the angle of incline. Rail inclines are small, allowing the small-angle approximation that $\sin(\theta)$ is approximately equal to the track grade gr .

Curve resistance: Curve resistance arises from the force of the track on the wheels within a curve. According to AREMA (Gillespie and Hayes 2003), curve resistance is approximately equivalent to a grade of 0.04% per degree of curvature. That heuristic would put the curve resistance at $0.45836 m_T g \arcsin(30.48/2r)$, where r is the link's radius of curvature in meters (an average radius of curvature is assigned to each link based on the link's α value).

Brake resistance: The brake resistance is the force of the brakes applied to the train. This force is applied to maintain control of the train along down grades and to stop the train. Trains have three braking systems, the most powerful of which, the air brake, takes large amounts of time to engage or disengage. Electric locomotives have regenerative braking systems that allow them to recover some braking energy. The regenerative braking system is more reliable than a diesel-electric locomotive's rheostatic braking system, allowing electric locomotives to rely less so on the air brake.

The cost parameters for fuel efficiency and maintenance are derived from Whitford (1952), Fritz (2000), and Nektalova (2008). Because we model trains as flows, it is beyond our scope to determine the actual brake usage on each link. We assume that on level terrain, links with an average positive grade, or links with a negative grade below a certain threshold, the incidental brake usage will be equivalent to the resistance of a 0.1% grade. A threshold is determined using the grade that would cause the train to exceed its desired speed along the link when utilizing the minimum throttle level. Beyond that threshold, the brake force is set to the level that will allow the train to reach its desired speed along the link while utilizing the minimum throttle position. The throttle is not set to zero because the grade utilized is only the average along the link, and even when a train is going downhill for a significant distance, the motors can be operating to prevent the railcars from bunching together.

Inertial resistance: Inertial resistance is the positive or negative impedance from the train changing its velocity. Trains must use energy to accelerate, and much of that energy is not regained when slowing down due to friction or the need to slow down faster than would otherwise be necessary. The inertial resistance is $m_T a$. This study assumes that the positive and negative inertial resistances will cancel each other out. In reality, more energy is used in accelerating a train than the energy saved as a train decelerates, so this is one category where the total energy consumption is understated and future analysis could improve upon.

Total resistance: Combining the preceding formulas yields the total average resistance along a link i as

$$R_i = \sum_{k=1}^{n_L+n_C} (am_{g,k} + bN_{ax,k} + K_k k_{a,i} v_i^2) + v_i (B_L n_L + B_C n_C) k_{f,i} + m_T g(gr_i) + 0.4536 m_T g \arcsin\left(\frac{15.24}{r_i}\right) + R_{brake,i} + m_T a \quad (14)$$

where k = each rail vehicle in the train unit.

Power level and speed: The power level used along the link, P_i is a function of the link's resistance and the train's speed along the link

$$P_i = R_i v_i \quad (15)$$

The train unit has a discrete number of possible power levels, which it uses to approach the desired speed along each link. Eqs. (14) and (15) are solved iteratively to determine each link's associated power level, P_i ; and base travel time, $t_i^0 = l_i/v_i$. Those values are used in Eq. (13).

Switching Costs

For many O-D pairs, the lowest-cost route for the shipper may involve switching between electric and diesel-electric operations en route. This might occur when a small portion of the overall network is electrified and an electrified path does not exist between on O-D pair, or the electrified route is far enough away from the direct path that the savings from electric operations are not worth the cost in added time. If the network does not allow any switching, electric trains would only be assigned between O-D pairs that have fully electrified, mostly direct, paths between them, which is not realistic. The network allows flows to switch from electric to diesel-electric operations (and vice versa) at certain nodes in order to simulate more realistic routing.

461 nodes in the rail network have facilities adequate to allow for switching. At those nodes, links between the electric and diesel-electric links represent the cargo time, crew time, and energy costs associated with switching locomotives. The cost assumes an hour-and-a-half per switch and that six employees will be involved (including the train operators and the yard workers). Whenever the path switches between operations, each train accrues a cost of roughly \$3,800. This cost represents in part the cost of yard electrification, i.e., additional switching using locomotive caller switchers.

Subproblem Solution

Most algorithms for TAP are designed for the separable cost case. In this subsection, we describe the changes that need to be made for our cost structure and derive a new flow-shift formula for this case. Our solution method is based on Algorithm B, briefly described in

the literature review. The main points of this algorithm are disaggregating link flows by origin, where the sets of used links form acyclic subnetworks; shifting flows from longer-cost segments to shortest-cost segments within each subnetwork; and updating these subnetworks by removing unnecessary links and adding new ones which provide shorter paths.

Of these three components, the original derivation of only the second component (flow shift) relies on separable link costs and must be reexamined in light of interactions. We first show that link interactions in our formulation satisfy the symmetry condition needed for the Eqs. (5)–(9) to hold and then rederive the flow-shift formula for Algorithm B with interactions.

As discussed in the Cost Formulas Section, all flow continuing on the same fuel type has zero switching cost, whereas all flow changing fuel type has predefined switching costs. Note that only the parallel diesel-electric link pair has a cost function dependent on the sum of the flows (Fig. 1), and $c'(x)$ is identical for both links. The Jacobian takes the form

$$J_{\mathbf{x}} = \begin{pmatrix} \frac{\partial(c_1(\mathbf{x}))}{\partial x_1} & \frac{\partial(c_2(\mathbf{x}))}{\partial x_1} & \dots & \frac{\partial(c_n(\mathbf{x}))}{\partial x_1} \\ \frac{\partial(c_1(\mathbf{x}))}{\partial x_2} & \frac{\partial(c_2(\mathbf{x}))}{\partial x_2} & \dots & \frac{\partial(c_n(\mathbf{x}))}{\partial x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial(c_1(\mathbf{x}))}{\partial x_n} & \frac{\partial(c_2(\mathbf{x}))}{\partial x_n} & \dots & \frac{\partial(c_n(\mathbf{x}))}{\partial x_n} \end{pmatrix}$$

The off-diagonal elements are zero whenever the two links do not interact. In our case, the only nonzero off-diagonal elements are for parallel diesel-electric pairs. These elements are calculated using the chain rule

$$\begin{aligned} \frac{\partial c'(x_D + x_E) + c''(x_D)}{\partial x_E} &= \frac{\partial c'(x_D + x_E)}{\partial (x_D + x_E)} \cdot \frac{\partial (x_D + x_E)}{\partial x_E} \\ &= \frac{\partial c'(x_D + x_E)}{\partial (x_D + x_E)} \end{aligned} \quad (16)$$

$$\begin{aligned} \frac{\partial c'(x_D + x_E) + c''(x_E)}{\partial x_D} &= \frac{\partial c'(x_D + x_E)}{\partial (x_D + x_E)} \cdot \frac{\partial (x_D + x_E)}{\partial x_D} \\ &= \frac{\partial c'(x_D + x_E)}{\partial (x_D + x_E)} \end{aligned} \quad (17)$$

As can be seen, these two derivatives are identical, proving that the Jacobian is a symmetric matrix and establishing the validity of the Eqs. (5)–(9).

We next show that the Algorithm B flow-shift procedure (equalizing cost between routes) is valid even in the presence of link interactions. This result is more general and can be applied to any instance of symmetric costs depending on at most two links, not just the ones we adopt in our study. We separate out the terms involving interactions as follows: $c_i = f_1(x_i, x_j) + f_2(x_i)$ and $c_j = g_1(x_i, x_j) + g_2(x_j)$, with $\partial f_1(x_i, x_j)/\partial x_j = \partial g_1(x_i, x_j)/\partial x_i$. For our rail electrification formulation, $f_1(x_i, x_j) = g_1(x_i, x_j) = c_{i'}(x_D, x_E)$, resulting in a simpler problem.

In this flow-shift operation, we have identified two paths (a lower-cost path π_L and a higher-cost π_U) and wish to shift flow between them to minimize the objective. Recall that the line integral formulation can now be written as

$$F(\mathbf{x}) = \oint_0^x \mathbf{c}(\mathbf{s}) \cdot d\mathbf{s} \quad (18)$$

Partition the arc set $\mathbf{A}$ into three subsets A_1 , A_2 , and A_3 . Links in set A_3 have separable cost, depending only on their own flow.

The travel time on the links in sets A_1 and A_2 depend on the flows on two links: the link itself, and exactly one link in the other set. Mathematically, there is a bijection between sets A_1 and A_2 , with the notation $i(j)$ and $j(i)$ used to denote the counterparts of links in the other set. The notation for these cost functions lists the link's own flow first, and its counterpart's flow second, that is, $c_i(x_i, x_{j(i)})$ and $c_j(x_j, x_{i(j)})$. Let $p = |A_1| = |A_2|$ and $m = |A_1| + |A_2| + |A_3|$. The index a will be used to denote a generic link if it does not matter which set it is from.

As the line integral is path-independent, we choose the following integration path

$$(0, 0, 0, \dots, 0) \rightarrow (x_1, 0, 0, \dots, 0) \rightarrow (x_1, x_2, 0, \dots, 0) \\ \rightarrow \dots \rightarrow (x_1, x_2, \dots, x_m)$$

The line integral then decomposes into a sum of ordinary integrals

$$F(\mathbf{x}) = \sum_{i=1}^p \int_{(x_1, \dots, x_{i-1}, 0, \dots, 0)}^{(x_1, \dots, x_{i-1}, x_i, 0, \dots, 0)} c_i(\mathbf{s}) \cdot d\mathbf{s} \\ + \sum_{j=p+1}^{2p} \int_{(x_1, \dots, x_{j-1}, 0, 0, \dots, 0)}^{(x_1, \dots, x_{j-1}, x_j, 0, \dots, 0)} c_j(\mathbf{s}) \cdot d\mathbf{s} \\ + \sum_{k=2p+1}^m \int_{(x_1, \dots, x_{k-1}, 0, 0, \dots, 0)}^{(x_1, \dots, x_{k-1}, x_k, 0, \dots, 0)} c_k(\mathbf{s}) \cdot d\mathbf{s} \\ = \sum_{i=1}^p \int_0^{x_i} c_i(s, 0) ds + \sum_{j=p+1}^{2p} \int_0^{x_j} c_j(s, x_{i(j)}) ds \\ + \sum_{k=2p+1}^m \int_0^{x_k} c_k(s) ds$$

Let I_a be +1 if link a is on π_L , -1 if a is on π_U , and 0 otherwise. Then for a flow shift Δx from the longest to the shortest path, the change on each link's flow is given by $\Delta x_a = I_a \Delta x$ and $I_a = \partial(\Delta x_a) / \partial(\Delta x)$. (This latter equation will be used when differentiating via the following chain rule.) The objective in terms of a flow shift of size Δx is now

$$F(\Delta x) = \sum_{i=1}^p \int_0^{x_i + \Delta x_i} c_i(s, 0) ds \\ + \sum_{j=p+1}^{2p} \int_0^{x_j + \Delta x_j} c_j(s, x_{i(j)} + \Delta x_{i(j)}) ds \\ + \sum_{k=2p+1}^m \int_0^{x_k} c_k(s) ds$$

and when we differentiate with respect to Δx , we obtain the following (the second term splits into two terms from the Leibniz rule)

$$\frac{dF}{d(\Delta x)} = \sum_{i=1}^p c_i(x_i + \Delta x_i, 0) I_i + \sum_{j=p+1}^{2p} c_j(x_j + \Delta x_j, x_{i(j)} + \Delta x_{i(j)}) I_j \\ + \sum_{j=p+1}^{2p} \int_0^{x_j + \Delta x_j} \frac{\partial c_j}{\partial x_{i(j)}}(s, x_{i(j)} + \Delta x_{i(j)}) I_{i(j)} ds \\ + \sum_{k=2p+1}^m c_k(x_k + \Delta x_k) I_k$$

We will show that $dF/d(\Delta x) = \sum_{a \in (A \cup A^E)} c_a(\mathbf{x} + \Delta \mathbf{x}) I_a$. The interpretation is that the derivative vanishes if Δx equalizes the cost

difference on the longest and shortest segments, thus minimizing the Beckmann function. The second and fourth terms in the sum are exactly what we need for the links in A_2 and A_3 . We need to show

$$\sum_{i=1}^p c_i(x_i + \Delta x_i, 0) I_i + \sum_{j=p+1}^{2p} \int_0^{x_j + \Delta x_j} \frac{\partial c_j}{\partial x_{i(j)}}(s, x_{i(j)} + \Delta x_{i(j)}) I_{i(j)} ds \\ = \sum_{i=1}^p c_i(x_i + \Delta x_i, x_{j(i)} + \Delta x_{j(i)}) I_i$$

Using the symmetry condition $\partial c_j / \partial x_{i(j)} = \partial c_i / \partial x_{j(i)}$, we have

$$\sum_{i=1}^p c_i(x_i + \Delta x_i, 0) I_i \\ + \sum_{j=p+1}^{2p} \int_0^{x_j + \Delta x_j} \frac{\partial c_i}{\partial x_{j(i)}}(s, x_{i(j)} + \Delta x_{i(j)}) I_{i(j)} ds$$

Using the fundamental theorem of calculus and one-to-one correspondence of terms in the two summations, it is clear that it equals $\sum_{i=1}^p c_i(x_i + \Delta x_i, x_{j(i)} + \Delta x_{j(i)}) I_i$, which is the required form. Therefore, $dF/d(\Delta x) = \sum_a c_a(\mathbf{x} + \Delta \mathbf{x}) I_a$ and Algorithm B flow shifts are valid in our setting. Most implementations of Algorithm B (including ours) use a Newton method to equalize the costs on these links. The denominator in Newton's method (the flow-shift scaling factor) can also be adjusted to reflect interactions (Patil and Boyles 2022), but even without this change, existing implementations still converge to the equilibrium.

Experiment Design

After obtaining solutions and validating the solution method for our formulation, the second set of experiments involves policy testing and sensitivity analysis. These experiments vary parameters such as total budget, demand data, electrification costs, electricity costs, crew/cargo costs, policy changes in the form of monetary incentives. The base electrification budget is \$30 billion, roughly equivalent to electrifying 65,000 km of track. This is varied by up to $\pm 20\%$, or \$24–36 billion. The analysis involves studying the return on investment (ROI) in the form of reduced costs, incentivizing policymakers and private stakeholders to upgrade infrastructure.

Freight Analysis Framework, version 4 (FAF4) provides demand data from 2010 and 2020, as well as forecasts for 2030 and 2040 (Bureau of Transportation Statistics 2020). In addition, lowering the usage cost of rail freight allows for a modal shift from trucking to rail. This potential demand variation is considered by increasing the demand data by up to 25% from base values. Lastly, electrification costs and crew/cargo costs are also tested at increased values (+25%) to gauge effects on the electrification solution. Note that under the current experiment formulation, adjusting the electrification costs is equivalent to adjusting the budget for electrification. We solve our problems with fixed demand since FAF4 does not contain the information needed to estimate elastic-demand functions.

The North American railroad network has been extracted from the statewide analysis module (Alliance Transportation Group 2013) provided by the Texas Department of Transportation, originally based on the CTA rail network developed at Oak Ridge National Laboratory (Center for Transportation Analysis 2014). The dataset includes 35,424 links and has geographical information (length, latitude, longitude, grade category), ownership information, and other auxiliary information. There are 28,289 nodes connected by these links, denoting stations, yards, and interchanges.

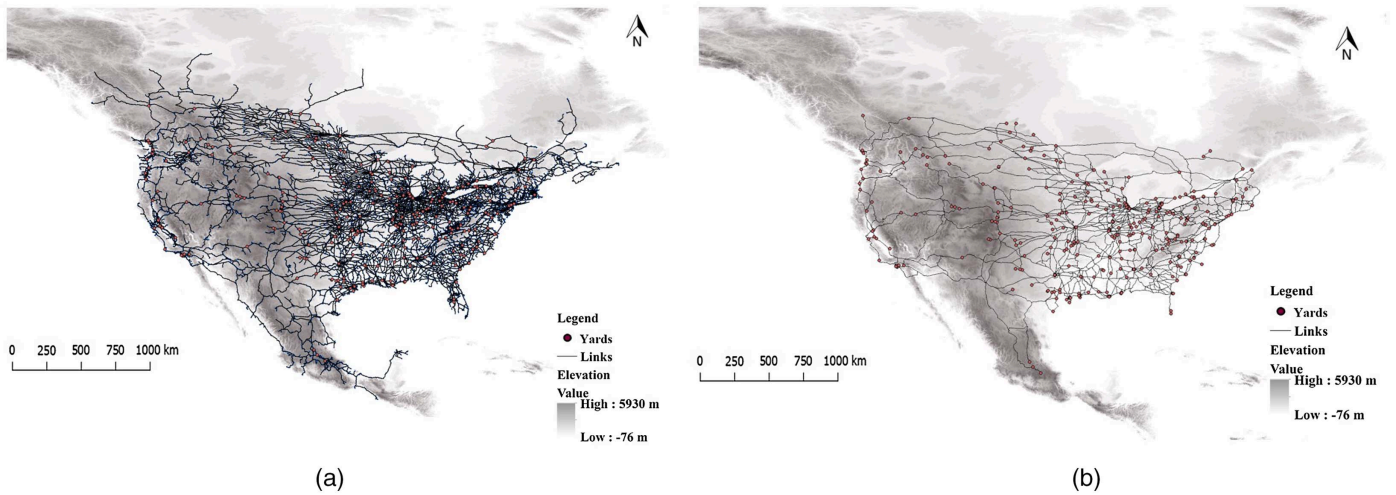


Fig. 2. US rail network visualization: (a) base network with elevation data; and (b) shortest path network with elevation data.

Elevation data were obtained by overlaying the network on the North American elevation grid (USGS 2007) using ArcGIS [Fig. 2 (a)]. The demand data (in tons) has been obtained from FAF4 (Bureau of Transportation Statistics 2020). We would like to note that directional running (like implemented by Union Pacific in Texas) can result in increased flow efficiency and can be modeled using increased link density. Barring detailed information on directional running, we have not included it in our model.

As specified, operational mode changes can only occur at yards. Therefore, corridor electrification is more sensible than individual link electrification. Candidate corridors were obtained by connecting each yard to the nearest neighboring yards. Specifically, the following method was employed to find the set of candidate corridors, and the results can be seen in Fig. 2(b). This new shortest corridor network has tracks totaling 170,000 km of track available for electrification, down from over 305,000 km in the full network. The switching costs were obtained using the following assumptions for the locomotives: 1.5 h to switch, throttle position 1 for diesel-electrics, 10% power for electrics, and 6 crew-equivalents manpower.

1. Calculate all pair shortest paths (APSP) from all yards.
2. Calculate APSP from all yards, but prune search branch after reaching a yard.
3. Compare the two sets and keep the paths common to both sets.

Results and Sensitivity Analysis

All result visualizations are shown in Figs. 3–6. The low budget, base case (medium budget), and high budget cases lead to electrification of about 49,000 (30,000), 61,000 (36,000), and 70,500 (42,000) km (mi), respectively. Only the base case results are shown with the full network backdrop; all other results are shown on the shortest path corridor network for better visibility. Figs. 4–6 show the results as the overlap of different scenarios. Fig. 4 is a cumulative plot, where the medium budget (base case) and high budget results are shown as increments over the low budget case. Figs. 5 and 6 pivot off the base case; therefore, we highlight the overlapping links as well as the differences.

The first observation is that the GA retained all corridors selected for the low budget case in the base case and high budget case. However, this changes for the increased demand and increased costs scenarios, given that the network OD matrix and costs change in the two scenarios. This allows us to identify the most impactful corridors and stations, such as all three of the major transcontinental routes (LA–Chicago, Oakland–Chicago, and Seattle–Chicago), which the algorithm chooses for electrification in each scenario. There is a wide variety of corridors selected across the entire network. The algorithm selects quite a few mountainous

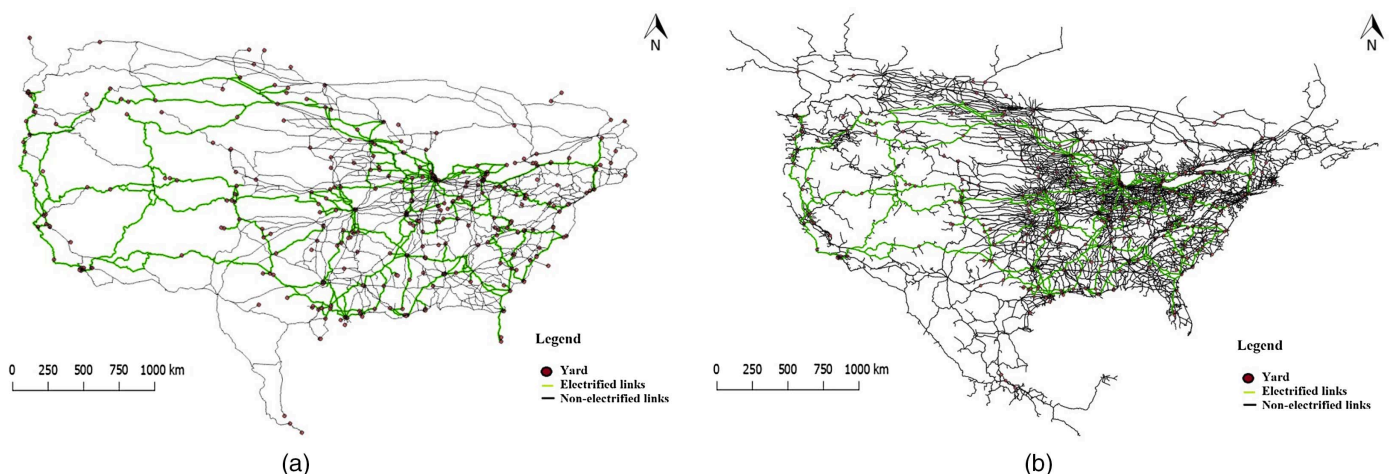


Fig. 3. Base case visualizations: (a) base case results with shortest path network; and (b) base case results with the full network.

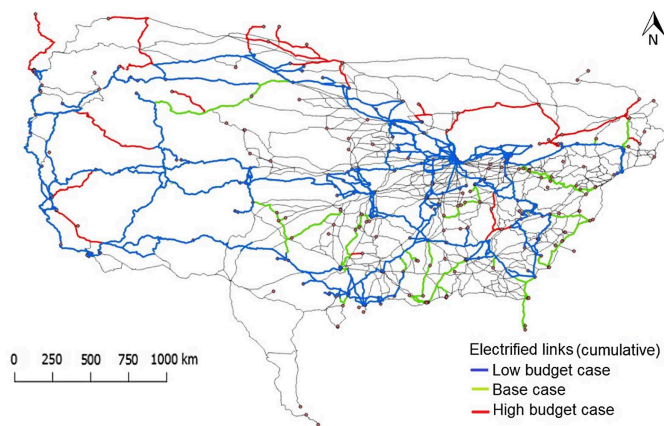


Fig. 4. Results for varying electrification budgets.

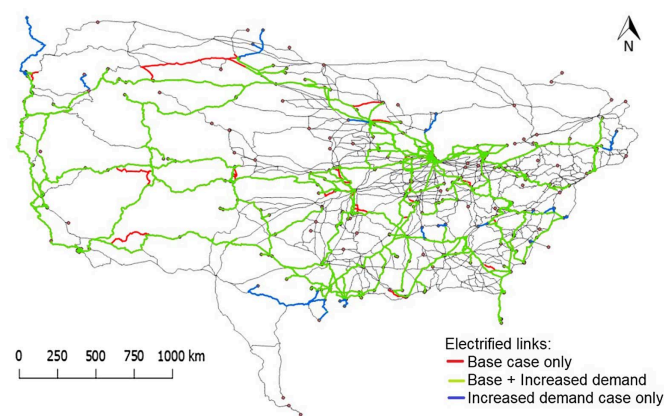


Fig. 5. Results for increased demand case.

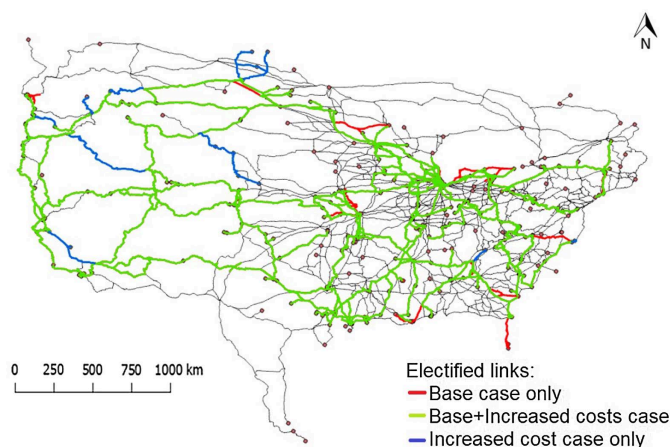


Fig. 6. Results for increased operations cost case.

routes where electrification provides the most benefits per train, but the algorithm also appears to select links that provide connectivity throughout the network. This trend is best illustrated in Fig. 6 and seems to show that the increased demand case shifts the selection from the more mountainous west to the more populous east and

gulf coasts. With higher demand, links that provide smaller benefits per train are more likely to provide greater benefits overall.

Examining the base case in further detail, the algorithm selects 13.2% of the network (by line-miles) for electrification and predicts that 15.5% of final flow (by tonnage) will use electrical traction. This reflects that busier corridors benefit more from electrification. This also implies that most of the traffic along electrified links would be using electric rather than diesel-electric locomotives.

The increased demand scenario increases electrified corridor connectivity to several new yards/stations. The increased cost scenario reduces selection from the east and central United States and chooses corridors from the western United States. We hypothesize this selection occurs due to the higher elevations and grades for these tracks seeing benefit from electrification. The current formulation does not incorporate any regional variations in the wholesale cost of electricity or diesel.

Each subproblem took 5 min to converge, and each GA instance took 36 h to finish on a Linux PC with i7 Processor (2.6 GHz) and 16 GB RAM. Given the long-term planning horizon, runtimes are not crucial but still useful to the scalability of the heuristic for a large network.

An important question is the validation of the lower-level flows. Uddin and Huynh (2015) have shown the comparison of static traffic assignment (w/o electrification) patterns for major links, given default link performance functions. Our model for the lower-level problem adapts the cost function parameters to incorporate additional cost factors. Aggregate metrics are easily available and therefore can be compared to or modeled. We present a comparison between our aggregate assignment metrics for the lower-level problem and those provided by Bureau of Transportation Statistics data as well as Commodity Flow Survey data. The total rail freight ton-miles (in millions) from our model is 1,627,854 as compared to 1,712,567 for BTS data and 1,387,777 from the CFS data. Thus, it is within 5% of the BTS estimates and within 15% of the CFS estimates. Note that BTS and our model uses FAF4 demand with different assignment models, while CFS data rely on survey responses which can differ significantly.

Conclusions and Future Directions

This study presents a novel way of looking at the rail network electrification problem using connections to the road TAP. We formulate the problem as a bilevel network design problem, where the lower-level problem is a symmetric TAP, assigning goods flow instead of traffic. We then show the correctness of Algorithm B flow-shift formula for the symmetric TAP, using it to solve this subproblem. The costs and network parameters incorporate electrification costs; fuel, locomotive, and operational costs; and train resistance costs.

The North American railroad network is used as a test network to demonstrate our method, show scalability for large networks and draw insights. We observe and note some corridors chosen in all different testing scenarios (varying budgets, opex, and demand), as well as provide policy insights for planning. The key observations are as follows:

- While there seems to be some evidence of corridors in more mountainous terrain producing better cost savings, overall, there is a wide variety of corridors selected across the entire network. This implies that providing broad connectivity might be more important than electrifying the corridors with the best savings on a per-train basis.

- The results for the increased demand case, shown in Fig. 5, indicate a shift in chosen corridors from the more mountainous west toward the more populous east and gulf coasts. This is probably a result of the increased demand causing those corridors to generate higher savings, even though they have fewer savings per train. This is also evidence that the highest priority corridors might be routes through rough terrain operating at or near capacity.
- Sensitivity to demand means long-term trends in the trucking industry could have a substantial influence on the highest priority corridors for electrification.

The main limitations of this work are: (1) static power levels (once calculated), (2) static brake usage assumption on links, (3) static electricity and diesel costs, and (4) lack of incorporation of track ownership restrictions on path selection. The first three limitations are relatively straightforward to tackle, although they increase problem complexity and computation time significantly by introducing another layer of iterative calculations.

An additional complication is partly based on the data available: because this analysis uses large zones with few centroids, the simulations might overestimate the benefit of electrifying well-traveled corridors that connect multiple zones. Studying how the selection changes based on varying switching costs might show whether the results are skewed by simulating a higher amount of connectivity than would actually exist. This problem could partially be ameliorated by applying a cost to electric trains reaching centroid connectors to reflect that some electric trains might require additional drayage over diesel-electric trains. Properly calibrating such a cost would remain a difficult problem.

The methods formulated in this study can be adapted to consider social benefits from changes in emissions, formulating the problem as one of social benefit maximization rather than private cost minimization. The composition of the power grid affects the emissions an electric locomotive causes, meaning the marginal social benefits of electrification and the marginal private cost savings can vary substantially from link to link. In some parts of the power grid where most of the electricity is provided by coal, an electric train might even produce more emissions than an equivalent diesel-electric train (Walthall 2019). Understanding social benefits is important because the high variability of private ROI from electrification might necessitate public subsidies before capital construction becomes possible. Prior studies have formulated social benefit maximization problems as single-level problems, thus reducing computational complexity.

Our formulation could also be generalized to reflect investments made at different points in time, along with variations in future demand. Such a dynamic model could present valuable insights about the proper timing of electrification but poses challenges in estimating future demand, costs, and budgets.

Data Availability Statement

All data, models, and code generated or used during the study appear in the published article.

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