Channel Coding Theorems in Non-stochastic Information Theory

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Abstract—**Recently, the δ-mutual information between uncertain variables has been introduced as a generalization of Nair's non-stochastic mutual information functional [1, 2]. Within this framework, we introduce four different notions of capacity and present corresponding coding theorems. Our definitions include an analogue of Shannon's capacity in a non-stochastic setting, and a generalization of the zero-error capacity. The associated coding theorems hold for stationary, memoryless, non-stochastic uncertain channels. These results establish the relationship between the δ-mutual information and our operational definitions, providing a step towards the development of a complete non-stochastic information theory.**

Index Terms—**Shannon capacity, Kolmogorov capacity, zero-error capacity, (ϵ, δ)-capacity, mutual information, coding theo-**

capacity is the supremum of the mutual information between the input and the output of the channel [3]. In the context of control and estimation of dynamical systems, Nair introduced a non-stochastic mutual information functional and established an analogous coding theorem for the zero-error capacity in a non-stochastic setting [9].

A parallel non-stochastic approach is due to Kolmogorov who, motivated by Shannon's results, introduced the notion of ϵ-capacity in the context of functional spaces [10]. He gave an operational definition of the ϵ-capacity as the logarithm base two of the *packing number* of the space, namely the logarithm of the maximum number of balls of radius ϵ that can be

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