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# Measurements of propane–O<sub>2</sub>–Ar laminar flame speeds at temperatures exceeding 1000 K in a shock tube

Adam J. Susa\*, Lingzhi Zheng, Ronald K. Hanson

*Stanford University, Department of Mechanical Engineering, Stanford, CA, USA*

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## Abstract

Laminar flame speed ( $S_L$ ) measurements of stoichiometric propane in an oxygen-argon oxidizer were performed in a shock tube at unburned-gas temperatures of 296–1234 K and near-atmospheric pressures. Non-intrusive laser-induced breakdown is used to ignite expanding flames following the reflected-shock passage. Flame propagation is recorded using schlieren imaging in a recently implemented side-wall imaging flame test section (SWIFT). In a refined approach to account for flame distortion and the slight residual motion of the post-reflected-shock gas, an area-averaged formulation of the linear-curvature model (the AA-LC model) is derived for use extrapolating flame data to zero stretch. Measured  $S_L$  values extracted using the AA-LC model closely agree with previous experimental measurements performed in a conventional kinetics shock tube (CKST) using much smaller flame kernels. Below the chemistry-affected limit of 1050 K, experimental  $S_L$  values agree well with simulations performed using the detailed AramcoMech 3.0 kinetic mechanism;  $S_L$  values simulated with propane mechanisms from NUIG and San Diego are found to differ from the measurements by 10% or more. Over the wide temperature range of the present data, the ubiquitous power-law form of empirical fit is shown to be inadequate for capturing the  $S_L$  temperature dependence; a non-Arrhenius form is shown to perform favorably. The uncertainties of flame speed measurements performed in the SWIFT average 3.0% and 4.4% for experiments performed under static and post-reflected-shock conditions, respectively, a reduction from the 5.8% average uncertainty of CKST experiments. This work represents a significant step forward in the development of experimental capabilities for high-temperature flame speed measurements. The present results illustrate the potential value of the shock-tube flame speed method to provide measurements useful for informing kinetic model tuning and validation at conditions for which experimental data were not previously obtainable.

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**Keywords:** Flame speed; Shock tube; Schlieren; Propane

\* Corresponding author.

E-mail address: [asusa@alumni.stanford.edu](mailto:asusa@alumni.stanford.edu) (A.J. Susa).

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## 1. Introduction

At the extreme conditions reached within modern combustion engines, coupled ignition chemistry and flame propagation can jointly govern engine performance [1]. Zeldovich first considered the effect of unburned-gas reactivity on the flame propagation speed, determining that a continuum of solutions to the chemistry-affected deflagration speed are possible, bounded from below by the unaffected flame speed [2]. Numerous efforts to predict the behavior of laminar flames at reactive unburned-gas conditions have employed simulation-based methods [e.g., [1,3–6], 7]. However, while experimental advancements have extended the range of unburned-gas pressure ( $P_u$ ) accessible to laminar flame speed ( $S_L$ ) measurements [e.g., 8], practical limitations have made progress extending the accessible unburned-gas temperature ( $T_u$ ) range more protracted.

To the authors' knowledge, the 873-K data reported by Kurata et al. using a Bunsen-flame configuration [9] represent the  $S_L$  measurements at the highest  $T_u$  reported in the literature prior to the introduction of the shock-tube flame speed method. The confined spherically expanding flame (CSEF) method has been demonstrated up to 720 K [10]. Heated, diverging channels have been employed in  $S_L$  measurements up to about 650 K [e.g., 11]. Rapid compression machines (RCMs) have been employed to measure burning velocities at  $T_u$  up to about 1000 K, but flame wrinkling and pressure rise typically precludes the determination of  $S_L$  values [12].

The introduction of the shock-tube flame speed method by Ferris et al. promised to enable experimental study at never-before-accessible, high- $T_u$  conditions, beginning with their initial demonstration of propane/air flame speed measurements at  $T_u$  up to 832 K [13]. In early shock-tube flame speed studies, the onset of instabilities limited the highest  $T_u$  at which  $S_L$  could be measured. This limitation was finally overcome through the use of argon (Ar) dilution and ignition nearer to the end wall in [14], measurements which serve as a point of comparison to the present work. Early applications of side-wall imaging to flame experiments additionally provided insights into the dynamics of flames generated in a shock-tube environment [15–17].

In the present study, a combination of experimental and analytical enhancements to the shock-tube flame speed method are applied in propane  $S_L$  measurements at temperatures in excess of 1000 K. Large-field-of-view (-FOV) side-wall schlieren imaging is applied in a novel, side-wall imaging flame test section (SWIFT) featuring large side-wall windows [18, Ch. 6]. In this work, a refined analysis method is derived and applied to extract the area-averaged burning velocities ( $\bar{S}_b$ ) and curvatures ( $\bar{\kappa}$ ) from rotationally symmetric flames

typical in the shock tube, which are used to determine unstretched, unburned flame speeds ( $S_u^0$ ) at  $T_u$  conditions reaching the limit of spontaneous unburned-gas reactivity.

## 2. Theoretical development

Within a spherically expanding flame (SEF) ignited in quiescent unburned gas, the burned gas remains stationary. For this reason, the displacement speed observed as the time derivative of the flame radius ( $\dot{r}_f$ ) can be taken as the burned flame speed ( $S_b$ ), the flame speed relative to the burned gas [19]. Unlike the unstretched, planar flames for which  $S_L$  is defined, conditions which cannot be experimentally realized, SEFs are subject to positive flame stretch ( $K$ ) that causes  $S_b(K)$  to differ from the unstretched, burned flame speed ( $S_b^0$ ). In order to estimate  $S_b^0$ ,  $S_b$  measurements recorded over a range of  $K$  conditions are commonly projected to zero stretch using physics-based extrapolations.

The choice of extrapolation function is the topic of ongoing debate. A discussion of the selection of the linear-curvature (LC) model in this work follows; a more complete review of the literature can be found in [18, pp. 32–40]. Markstein [20] first proposed the LC model:

$$S_b = S_b^0(1 - L_b\kappa), \quad (1)$$

where  $\kappa$  is the total curvature of the surface ( $\kappa = 2/r_f$  for a spherical flame) and  $L_b$  is the Markstein length. The linear-stretch (LS) model:

$$S_b = S_b^0 - L_bK, \quad (2)$$

was proposed somewhat later [e.g., 21] but gained dominance in the interpretation of experimental measurements. For a spherical flame,  $K = S_b(2/r_f)$ .

More recently, the need to account for nonlinear stretch effects became apparent [22], and numerous extrapolation models (including LC) have been considered for their ability to capture nonlinear stretch effects. The nonlinearity of the LC model is seen by recasting the spherical form of Eq. (1) in terms of  $K$ , giving:

$$S_b = S_b^0 - (S_b^0/S_b)L_bK. \quad (3)$$

Here, it is apparent that the LC model is nonlinear in terms of  $K$  as a result of the additional  $(S_b^0/S_b)$  term compared to the LS model. Analyses by Chen [23] and Cai et al. [24] both conclude that the LC model provides more accurate results than the more commonly used expression proposed by Kelley and Law [22] when the Lewis number is greater than unity ( $L_b > 0$ ). For this reason, the LC model (Eq. (1)) is utilized in the present study.

In practice, a perfectly spherical flame is rarely realized. Within static vessels ignited using electric sparks, the interaction of the flame with the electrodes locally perturbs the flame surface. For low-speed flames, buoyant effects can lead to mild flame

distortion [e.g., 25]. When laser-induced plasma ignition (LIPI) is used to ignite a flame, the toroidal flow field produces an aspherical flame kernel [e.g., 13]. Recently, expanding flames in a shock tube were found through side-wall imaging to exhibit sometimes-significant axial distortion [15,17].

Distorted flames often exhibit locally smooth surfaces, such that correct handling of their structure may allow them to be used as a basis for  $S_L$  measurements. Considering an expanding flame of total flame-front area  $A_f$ , the total mass burning rate ( $\dot{M}_f$ ) is defined as the surface integral of the local burning flux,  $\dot{m}_f = \rho_b S_b$ , where  $\rho_b$  is the burned-gas density:

$$\dot{M}_f = \int_{A_f} \dot{m}_f dA = \rho_b \int_{A_f} S_b dA. \quad (4)$$

Substitution of the LC model (Eq. (1)) provides a functional form of  $S_b$  in the expression:

$$\dot{M}_f = \rho_b \int_{A_f} S_b^0 (1 - L_b \kappa) dA \quad (5)$$

where  $L_b$  is constant for a given unburned-gas state and  $\kappa$  is the local, total curvature of the flame front.

From the simple expression for the mass of burned gas,  $M_b = V_b \rho_b$ , where  $V_b$  is the burned-gas volume, the rates  $\dot{M}_b$  and  $\dot{V}_b$  can likewise be related:

$$\dot{M}_b = \dot{V}_b \rho_b. \quad (6)$$

Recognizing  $\dot{M}_b = \dot{M}_f$  by continuity, Eqs. (5) and (6) can be equated and simplified:

$$\frac{\dot{V}_b}{A_f} = S_b^0 \left( 1 - \frac{L_b}{A_f} \int_{A_f} \kappa dA \right). \quad (7)$$

Defining the area-averaged burning speed,  $\bar{S}_b$ , and total curvature,  $\bar{\kappa}$ :

$$\bar{S}_b \equiv \dot{V}_b / A_f \quad (8)$$

$$\bar{\kappa} \equiv \frac{1}{A_f} \int_{A_f} \kappa dA, \quad (9)$$

Equation (7) can be converted to a form equivalent to the original LC model (Eq. (1)):

$$\bar{S}_b = S_b^0 (1 - L_b \bar{\kappa}) \quad (10)$$

The basis of Eq. (10) on area-averaged properties is significant in that it allows for exact accounting of local propagation characteristics, unlike the approximate form of a previous aspherical correction [13]. The relevant parameters  $V_b$ ,  $A_f$ , and  $\bar{\kappa}$  are inherently insensitive to bulk motion of the flame, an advantageous property when applied to shock-tube experiments where small residual velocities do exist [16].

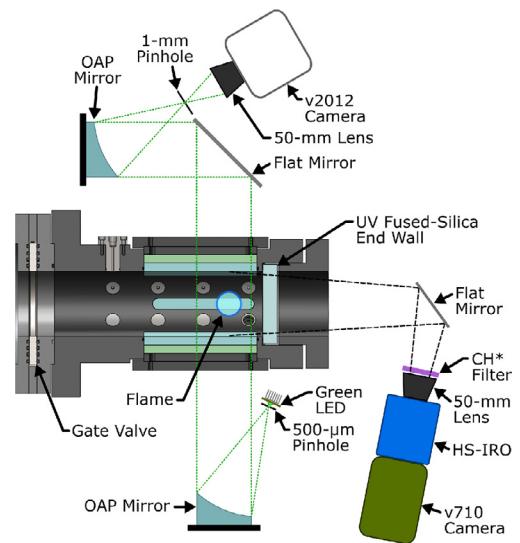


Fig. 1. SWIFT experimental configuration shown as a top-down cross-sectional view. Curved side-wall windows provide a 18- x 6.4-cm lateral field of view. The ignition laser (not shown) is orthogonal to the view. Cross sections to scale; imaging instrumentation and ray tracing approximate.

### 3. Methodology

#### 3.1. Experimental setup

The shock tube used throughout this work features a 11.53-cm inner diameter, 9.76-m-long driven section, and variable length driver section. Previous shock-tube flame studies [e.g., 13,14] were performed in the conventional kinetics shock tube (CKST) configuration, as described by Campbell et al. [26], which afforded very limited side-wall optical access. The new SWIFT, shown schematically in Fig. 1, replaces the old test section (between the gate valve and driven end wall) with a new, anodized-aluminum, optically accessible test section.

Stoichiometric ( $\phi = 1$ ) unburned-gas mixtures of propane ( $C_3H_8$ ) in an oxidizer comprised of 21% oxygen ( $O_2$ ) and 79% argon (Ar) are prepared manometrically in a stainless-steel mixing tank. Following at least one hour of mixing by a mechanical vane, the premixed test-gas is introduced into the evacuated shock-tube driven section to an initial (region-1) pressure,  $P_1$ , and ambient temperature,  $T_1$ . The driver section, initially separated from the driven section by a polycarbonate diaphragm, is then filled with helium and nitrogen until the diaphragm ruptures.

The sudden diaphragm rupture produces a shock wave in the test gas, accelerating it and elevating  $T$  and  $P$  to post-incident-shock (region-2) conditions. Upon reaching the driven-section end wall, the shock wave reflects, further elevating  $T$

and  $P$  as the gas is stagnated at the post-reflected-shock (region-5) conditions. The incident shock speed is measured by a series of pressure transducers positioned along the driven section. The region-5 state, which defines the unburned-gas conditions for flame experiments, are calculated using the chemically frozen shock solver FROSH with the end-wall extrapolated shock speed and assuming vibrational equilibrium in regions 2 and 5 [27]. The uncertainty of the region-5 conditions is calculated at the 95% confidence level based contributions from the initial conditions and measured shock speed. The fuel mole fraction is measured *in situ* using a 3.41  $\mu\text{m}$  laser absorbance diagnostic [28].

Expanding flames are ignited behind the reflected shock by LIPI [13]. A Q-switched, frequency-doubled Nd:YAG laser (Solo PIV 120, 532 nm) serves as the laser source. A  $f = 15$  cm lens focuses the beam to a waist of  $\mathcal{O}(10 \mu\text{m})$  diameter where laser-induced breakdown occurs, igniting a flame. Spark delays following the reflected shock were less than 0.5 ms for experiments with  $T_u > 1000$  K and less than 1 ms for those performed at lower  $T_u$ . Laser-energy control is achieved with a variable attenuator. Spark energies in SWIFT experiments were estimated using differential energy measurements to be 1–2 mJ;

Schlieren imaging is performed through the side walls of the shock tube. In the previous work utilizing the CKST [14], the small ports restricted the schlieren FOV to a diameter of about 1.5 cm. In the SWIFT, large side-wall windows designed as afocal cemented-doublet cylindrical lenses provide optical access for large-FOV schlieren imaging [18, Ch. 6] and allow flames to be observed to much larger  $r_f$ . A high-power light-emitting diode (LED) paired with a pinhole serves as the schlieren light source; while a white LED was used with the CKST, a green LED is used in the SWIFT to minimize chromatic aberrations. Both the CKST and SWIFT experiments utilize symmetric schlieren stops (slit and pinhole, respectively) to produce isotropic responses to density gradients [29], which is preferable for the consistent detection of a circular flame front. A detailed comparison of the schlieren systems used with the CKST [14] and SWIFT (this work) can be found in Supplement 1, Table S1. Further discussion of the use of off-axis parabolic (OAP) mirrors in the SWIFT schlieren arrangement can be found in [30].

### 3.2. Image processing

Flame front positions are extracted from each schlieren video frame using active-contouring [31], implemented in Scikit-Image [32]. Prior to contouring, images are first background normalized to homogenize non-uniformity in the illumination intensity. Next, sequential images are differenced, and positive values in the resulting images are trun-

cated, to eliminate slow-varying background artifacts while enhancing the fast-moving flame front. Finally, a Gaussian blurred copy of the difference image is superimposed onto the difference image to provide a spatially distributed gradient for the contour to follow.

Active contouring is applied sequentially to each frame of the video sequence in reverse order, with the optimal contour of each frame serving as the initial guess for the previous frame of the sequence. After each contour is optimized, the X-Y coordinates are parameterized and interpolated to a fixed spacing to prevent the clustering of the points through successive optimization steps. Sample optimized contours overlaid on processed schlieren images are provided in Supplement 1, Fig. S1. Example images at all processing steps can be found in [18, pp. 121–128].

### 3.3. Flame speed extraction

Prior works have shown flames ignited in a shock tube tend to exhibit substantial radial symmetry about the axis of the tube even in the presence of significant axial distortion [15,17]. This behavior was confirmed for flames studied in the present investigation through a qualitative assessment of end-wall emission images, in which flames appear generally circular regardless of the presence of axial distortion as seen in the side-wall view. As such, an approximation of cylindrical symmetry is adopted for representing the morphology in order to perform the property integrals. Defining the horizontal direction,  $z$ , as the axis of symmetry, the flame radius,  $r_f(z)$ , is approximated as:

$$r_f(z) = \delta(z)/2, \quad (11)$$

where  $\delta(z)$  is the total vertical extent of the optimal contour (Supplement 1, Fig. S2a).

Values of  $V_b$ ,  $A_f$ , and  $\bar{\kappa}$  are evaluated as follow:

$$V_b = \pi \int_0^{z_f} r_f^2 dz \quad (12)$$

$$A_f = 2\pi \int_0^{z_f} r_f \sqrt{1 + r_f'^2} dz \quad (13)$$

$$\bar{\kappa} = \frac{1}{A_f} \int_0^{z_f} \kappa(z) r_f \sqrt{1 + r_f'^2} dz, \quad (14)$$

with numerical integration performed using Simpson's method as implemented in Scipy [33]. The local, total curvature ( $\kappa$ ) in Eq. (14) is taken as:

$$\kappa(z) = \frac{-r_f''}{(1 + r_f'^2)^{3/2}} + \frac{1}{r_f(1 + r_f'^2)^{1/2}} \quad (15)$$

where  $r_f'$  and  $r_f''$  are the first and second derivatives of  $r_f$  with respect to  $z$ , respectively. Equation (15) is

adapted from Spivak [34] and confirmed to recover the expected result of  $\kappa = 2/r_f$  for a sphere.

Time-series data for properties  $\chi \in \{\bar{\kappa}, A_f, V_b\}$  are converted to effective radii,  $\mathcal{R}_\chi$ :

$$\mathcal{R}_\kappa = 2/\kappa \quad (16)$$

$$\mathcal{R}_A = (A_f/4\pi)^{1/2} \quad (17)$$

$$\mathcal{R}_V = (3V_b/4\pi)^{1/3} \quad (18)$$

before being smoothed with a second-order Savitzky-Golay filter [33,35]. The conversions of Eqs. (16)–(18) are then reversed, converting the smoothed  $\mathcal{R}_\chi$  to smoothed time-series data,  $\chi$ .  $\dot{V}_b$  values (used in Eq. (8)) are calculated from smoothed  $V_b$  data using a third-order, first-derivative Savitzky-Golay filter.

A linear fit of the form of Eq. (10) applied to the smoothed  $\bar{S}_b$  and  $\bar{\kappa}$  is used to extract  $S_b^0$  and  $L_b$ . The subset of data over the  $\bar{\kappa}$  range 1.15–4 cm<sup>-1</sup> are typically used in performing the fit, corresponding to an equivalent  $r_f$  range 0.5–1.75 cm. The 1.75-cm upper radius limit is selected as 30% that of the shock-tube based on the cylindrical confinement criteria of Burke et al. [36]. The 0.5-cm lower limit is selected to avoid fitting data showing evidence of ignition effects or for which the active contour solution becomes unstable. The linear fit is found to be inherently insensitive to residual waviness in the  $\bar{S}_b$  data, which tends to be present at periods longer than the filter window.

Extrapolated values  $S_b^0$  are converted to the unburned-gas reference frame using the density ratio:

$$S_u^0 = (\rho_b/\rho_u)S_b^0. \quad (19)$$

Region-5 conditions provide the unburned-gas density ( $\rho_u$ );  $\rho_b$  is estimated from constant- $P$  thermo-chemical equilibrium, evaluated with Cantera [37].

## 4. Results and discussion

In this section, the results of propane–O<sub>2</sub>–Ar ( $\phi = 1$ ) flame-speed experiments are presented and discussed. The present discussion focuses primarily on the experimental determination of  $S_L$ ; values of  $L_b$  are presented for completeness but not discussed in depth. Static flame experiments exhibiting nearly spherical flames are first evaluated to assess the AA-LC model under ideal conditions. Flames ignited behind reflected shocks ( $T_u \sim 650$  K) and exhibiting differing degrees of distortion are next evaluated to assess the performance of the AA-LC model when distortion is present. Finally, measurements obtained from new experiments performed at higher  $T_u$  are reported along with a detailed treatment of uncertainty.

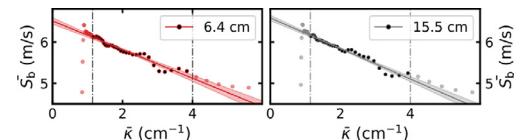


Fig. 2. Sample  $\bar{S}_b$ – $\bar{\kappa}$  extrapolations for two static flame experiments ( $T_u = 296$  K,  $P_u = 1$  atm). The gray, vertical lines represent the limits of data used in the AA-LC fit.

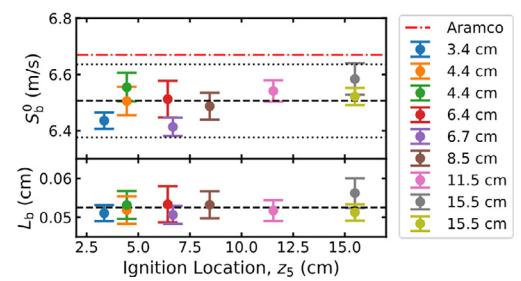


Fig. 3.  $S_b^0$  (top) and  $L_b$  (bottom) measurements extracted from static-flame experiments using the AA-LC model. The black dashed line represents the mean of the measured values; the dotted lines mark  $\pm 2\%$ . The red dot-dashed line shows the value  $S_b^0$  calculated with AramcoMech 3.0 [38]. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

### 4.1. Static, spherical-flame validation

In order to evaluate the performance and repeatability of the AA-LC model as presently implemented, it is first applied in the analysis of static experiments ( $T_u = 296 \pm 1$  K) exhibiting nearly spherical flames borrowed from a prior study [17]. Figure 2 shows example AA-LC extrapolations of  $\bar{S}_b$ – $\bar{\kappa}$  data obtained from two of the nine static experiments analyzed (all fits can be seen in Supplement 1, Fig. S2). The legends indicate the ignition locations, measured from the end wall. Lines mark best fits and shaded region represent the 95% confidence intervals.

Values of  $S_b^0$  and  $L_b$  obtained from the static experiments are shown in Fig. 3. Error bars in Fig. 3 represent only the uncertainty of the linear fits evaluated at the 95% level. Black dashed lines mark the mean values. The mean uncertainty of the  $S_b^0$  fits is 0.7% across static experiments. The standard deviation of the  $S_b^0$  values is 0.8%, from which the 95% confidence interval of the random error is calculated to be 1.8% using Student-t statistics [39]. The combined uncertainty of the fit and random error is found to be 2%, marked by black dotted lines in Fig. 3 which nearly encompass the value  $S_b^0$  simulated by AramcoMech 3.0 [38] (red dashed line).

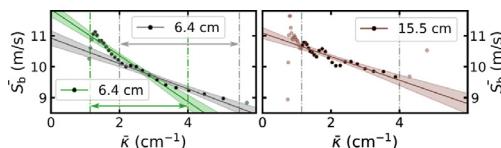


Fig. 4.  $\bar{S}_b$ – $\bar{\kappa}$  extrapolations for post-shock validation experiments. Plot legends indicate the ignition location. Two fits are shown for the 6.4-cm flame (left); the green fit covers the same  $\bar{\kappa}$  range as other experiments, while the gray line is fit over a range  $2\text{--}5.5\text{ cm}^{-1}$ . (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

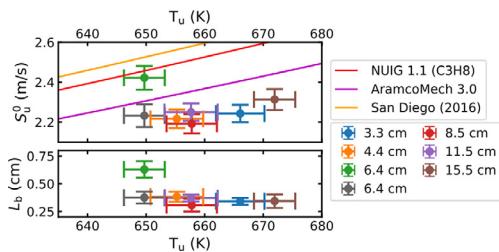


Fig. 5.  $S_u^0$  (top) and  $L_b$  (bottom) measurements from post-shock validation experiments. Simulated  $S_L$  values are shown for comparison. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

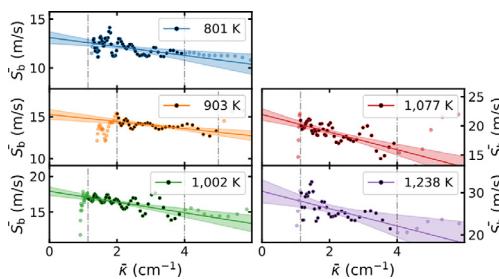


Fig. 6.  $\bar{S}_b$ – $\bar{\kappa}$  extrapolations for high- $T_u$  experiments.  $T_u$  (i.e.,  $T_5$ ) is labeled in the legends.

#### 4.2. Post-shock, distorted-flame validation

A primary motivation of the AA-LC model is to enable the use of distorted flames in reliable  $S_u^0$  measurements. This ability is evaluated using six flames ignited in a post-reflected-shock environment at  $T_u \sim 650$  K and exhibiting differing degrees of distortion [17].  $\bar{S}_b$ – $\bar{\kappa}$  data and AA-LC extrapolations for two experiments are shown in Fig. 4; plots for all experiments can be found in Supplement 1, Fig. S3. Values of  $S_u^0$  and  $L_b$  are shown in Fig. 5, along with curves of simulated values using AramcoMech 3.0 [38] and propane-specific skeletal mechanisms from NUIG 1.1 [40] and San Diego [41]. In Fig. 5, vertical error

bars contain contributions from the fit uncertainty and random error (taken as 1.8% from static experiments); horizontal error bars show the  $T_5$  uncertainty.

The 6.4-cm ignition location (Fig. 4, left) represents the single worst-case of flame distortion. For the 6.4-cm flame, two fits are performed: one over the typical range  $\bar{\kappa}$  and a second over the range  $2\text{--}5.5\text{ cm}^{-1}$ . Using the standard  $\bar{\kappa}$  range (green), the value  $S_u^0$  exceeds that obtained from the other experiments by about 10%; the corresponding value  $L_b$  is over twice that of the other experiments. When higher- $\bar{\kappa}$  data is instead fit (*i.e.*, smaller  $r_f$ ) when the flame is less distorted; shown as gray), the values obtained for both  $S_u^0$  and  $L_b$  are found to closely agree with those of the other experiments. As such, the ability of the AA-LC model to correctly account for all but the most severe distortion is confirmed. Consistency of extracted  $L_b$  values is additionally found to be a useful check that appropriate ranges of data are used in performing AA-LC extrapolations.

#### 4.3. High-temperature results

Having validated the AA-LC model,  $S_u^0$  measurements are reported and discussed for propane–O<sub>2</sub>–Ar experiments ( $\phi = 1$ ) at highest-ever- $T_u$  conditions. Five experiments were performed in the SWIFT over the range  $801\text{ K} \leq T_u \leq 1238\text{ K}$ . In the 1238-K experiment, fuel loss was observable in the 3.41- $\mu\text{m}$  measurement beginning immediately after passage of the reflected shock, confirming the spontaneously reactive nature of the unburned at the highest  $T_u$ .

Combined with the validation experiments of the previous sections,  $S_u^0$  and  $L_b$  measurements are reported over the unprecedented range 296–1238 K (Fig. 7). Across the post-reflected-shock SWIFT experiments, the mean  $P_u$  was 1.03 atm, with a standard deviation of 0.03 atm. The mean, measured  $\phi$  was 1.02 with a standard deviation of 0.01. Vertical error bars for  $L_b$  values are shown based on the 95% confidence interval of the fit parameters. Error bars for  $S_u^0$  values are not shown in Fig. 7a, as they would be too small to see clearly, but are presented following a discussion of  $S_u^0$  uncertainty in Fig. 8. Details of each SWIFT experiment are tabulated in Supplement 1, Table S2. Shown for comparison are results from earlier shock-tube flame speed measurements performed in the CKST [14].<sup>1</sup> While the values  $L_b$  extracted from CKST experiments using radius-based  $S_b$  and  $\kappa$  data obtained

<sup>1</sup>  $T_5$  values reported for CKST experiments were recalculated in this work using the correct equilibrium-equilibrium FROSH constraint [27] and are thus higher by 30–50 K than  $T_5$  values reported in [14], which were calculated using the incorrect frozen-equilibrium assumption.

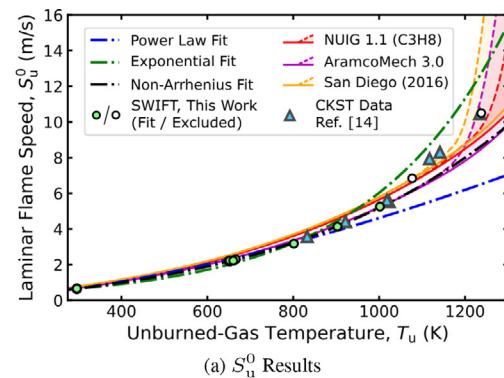
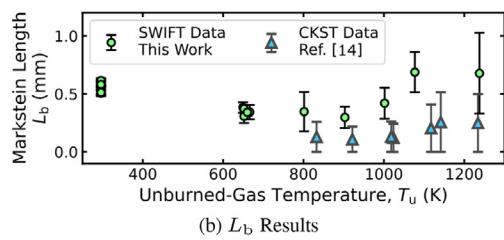
(a)  $S_u^0$  Results(b)  $L_b$  Results

Fig. 7. Flame speed and Markstein length results obtained from shock-tube flame speed experiments. Experimental data are shown as markers. Lines show flame speeds simulated using two different domain lengths: 1-cm (solid) and 6-cm (dashed). Dot-dashed lines represent empirical fits to the  $T_u < 1050$  K SWIFT data (green-filled circles). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

from very small flame kernels (2–7 mm) are found to be systematically smaller than those obtained from SWIFT experiments, the values  $S_u^0$  obtained from the two sets of experiments are found to agree quite closely.

$S_u$  values simulated using the AramcoMech 3.0 detailed mechanism [38] and skeletal propane mechanisms from NUIG 1.1 [40] and San Diego [41] are shown in comparison to measured  $S_u^0$  data. In order to provide a first-order accounting of chemistry effects on the flame speed at reactive values  $T_u$ , simulations were performed using two different calculation domain lengths ( $\ell_{\text{sim}}$ ) in the Chemkin-Pro PREMIX solver, a method previously reported in the literature [e.g. 3, 4]. With the flame centered in the simulation domain, the induction time ( $\tau_{\text{in}}$ ) between when the unburned gas enters the simulation domain and when it reaches the flame is  $\tau_{\text{in}} \approx (\ell_{\text{sim}}/2)/S_u$ . Considering high- $T_u$  conditions with  $S_u \sim \mathcal{O}(10 \text{ m/s})$ , the chosen domain lengths of 1 cm (solid lines) and 6 cm (dashed lines) represent  $\tau_{\text{in}} \sim 0.5$  and 3 ms, time scales representative of time of the spark after the reflected shock on the low end and an upper bound on the total measurement time on the high end.

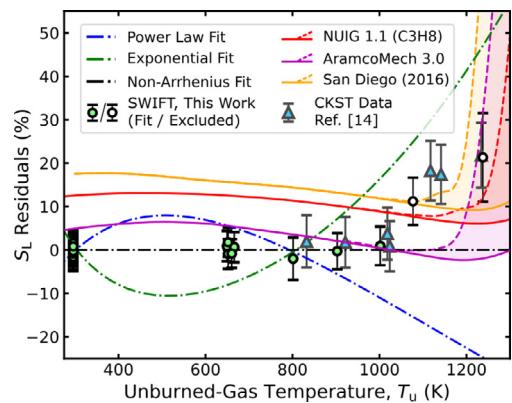


Fig. 8.  $S_L$  residuals of measurements, simulations, and fits all computed relative to the non-Arrhenius empirical fit. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The difference between the values  $S_L$  simulated with different  $\ell_{\text{sim}}$  (shaded regions) bound a continuum of flame speeds that may be encountered as a result of chemistry effects on the relevant time scale [2]. At temperatures below  $T_u \sim 1050$  K, both values  $\ell_{\text{sim}}$  return effectively the same simulated value  $S_L$  for all mechanisms, indicating the unburned gas is chemically frozen over the experimental timescale of  $\tau_{\text{in}}$  at these temperatures. However, values  $S_L$  simulated with different  $\ell_{\text{sim}}$  diverge at higher  $T_u$ , demonstrating that chemistry effects become relevant beyond that  $T_u$ . For this reason, only that data for which  $T_u < 1050$  K (green-filled circles) are considered in the empirical fits discussed next. Chemistry-affected data at higher  $T_u$  (white-filled circles) are included in the plots to illustrate the manifestation of chemistry effects in measurements.

The dot-dashed lines in Fig. 7a show empirical fits of the SWIFT  $S_u^0$  measurements ( $T_u < 1050$  K) following three functional forms. The empirically determined power-law model (blue line):

$$S_u = S_{u,0}(T_u/T_0)^\alpha, \quad (20)$$

is ubiquitous in the literature, with considerable effort having been made to quantify and report values of the temperature exponents ( $\alpha$ ) [42]. Two other functional forms are also evaluated, an exponential form:

$$S_u = S_{u,0} \exp\left(\frac{T_u - T_0}{T_{\text{exp}}}\right), \quad (21)$$

and a non-Arrhenius form that combines the exponential and power-law forms:

$$S_u = S_{u,0} \left(\frac{T_u}{T_0}\right)^\alpha \exp\left(\frac{T_u - T_0}{T_{\text{exp}}}\right). \quad (22)$$

In the latter two forms,  $T_{\text{exp}}$  is a characteristic temperature. Parameters of all three empirical fits are

Table 1  
Empirical Fitting Parameters.

Empirical Form	Eqs.	$S_{L,0}$ (cm/s)	$\alpha$ (–)	$T_{\text{exp}}$ (K)
Power Law	(20)	66.5	1.61	–
Exponential	(21)	66.9	–	318
Non-Arrhenius	(22)	66.5	0.95	767

tabulated in Table 1; an evaluation of empirical forms' performances is provided in the discussion of Fig. 8.

#### 4.4. Measurement uncertainty

Sources and magnitudes of uncertainty are next considered for the  $S_u^0$  measurements. In the discussion that follows, all sources of uncertainty are quantified at the 95% confidence level using small-population (Student-t) statistics [39] where appropriate. Already discussed in the static validation section, each  $S_u^0$  measurement has an uncertainty associated with the linear fit to the  $\bar{S}_b - \bar{\kappa}$  data; in this work, the mean fit uncertainty is found to be 0.7% across static experiments and 2.6% in post-reflected-shock experiments. The random measurement error was estimated from the static validation experiments to be 1.8%; this value is assumed to apply across all  $S_u^0$  measurements reported here. The choice of the functional form used to extrapolate data (LC in this work) has itself been argued to be a source of uncertainty. Using the correlation reported by Huo et al. [43], the uncertainty of SWIFT experiments attributed to the extrapolation method (for which the  $R_{f,\text{new}}$  evaluates to 0.67 cm) is found to be 2.3%. By comparison, CKST experiments have a 5.4% extrapolation-method uncertainty due to the use of smaller flames ( $R_{f,\text{new}} = 0.24$  cm).

Uncertainty in the experimental conditions ( $T_5$  and  $P_5$ ) also contribute to the overall measurement uncertainty and are evaluated for the SWIFT experiments. For shocked experiments, uncertainties of  $T_5$  and  $P_5$  include contributions from the incident-shock velocity ( $V_{is}$ , evaluated as the 95% confidence interval of extrapolated end-wall value) as well as the initial conditions —  $T_1$ ,  $P_1$ , and the test-gas composition ( $x_i$ ) — for which uncertainties are estimated as 0.5 K, 0.5%, and 0.5%, respectively. The contributions of  $V_{is}$ ,  $T_1$ ,  $P_1$ , and  $x_i$  on  $T_5$  and  $P_5$  are propagated using numerical derivatives and combined as a root-sum-square of the components, providing mean  $T_5$  and  $P_5$  uncertainties of 0.6% and 1.5%, respectively, in post-reflected-shock experiments.

Contributions to  $S_u^0$  uncertainty from the experimental conditions are propagated using derivative of the empirical non-Arrhenius fit for  $T_5$  and a power-law fit for  $P_5$  (taking  $\beta \approx 0.35$  from Konnov et al. [42]), leading to the finding that uncertainty of  $T_5$  contributes an average 1.3% uncertainty to  $S_u^0$ ,

while  $P_5$  contributes only 0.5% in shocked experiments. In static experiments,  $T_u$  contributes 0.5% and  $P_u$  contributes 0.2%. In general, uncertainty in  $\phi$  would also contribute to the  $S_u^0$  uncertainty; however, at  $\phi \approx 1$ ,  $dS_u^0/d\phi \approx 0$  such that the contributions of the average 0.7%  $\phi$  uncertainty and the 2% deviation of the mean measured  $\phi$  from the nominal value of unity are negligible.

An alternative view of the  $S_u^0$  results is provided in Fig. 8, where residuals calculated against the non-Arrhenius empirical fit are presented for all values shown in Fig. 7a. Vertical error bars represent the total uncertainty related to both the extrapolation and experimental conditions for SWIFT results; those for CKST results include only uncertainties associated with the extrapolation (fit, model, random error). Horizontal error bars are not shown, as vertical error bars include uncertainty from  $T_u$ . The mean uncertainty is found to be 3.0% for static experiments in the SWIFT and 4.4% for SWIFT experiments performed behind reflected shocks. The mean uncertainty of CKST experiments is larger, at 5.8%. Uncertainty related to image processing and the calculation of area-averaged properties is not treated in the present analysis but should be considered as a topic for future study.

The non-Arrhenius fit is seen to capture the temperature trends of both the data and the simulations well; the common power-law fit and alternative exponential fit both systematically fail to capture the trend of  $S_u^0$  over the wide  $T_u$  range studied in this work.  $S_L$  values simulated with AramcoMech 3.0 are found to agree very well with the measurements up to 1000 K; at higher  $T_u$ , the measurements diverge from the 1-cm- $\ell_{\text{sim}}$  simulation results, the expected result of chemistry effects. Discrepancies on the order of 10–20% between the measurements and values  $S_L$  simulated using the other two mechanisms illustrate the opportunity for mechanism refinement using experimental  $S_u^0$  measurements.

## 5. Conclusions

Flame speed measurements at extreme temperatures in excess of 1000 K are reported using the shock-tube flame speed method. Schlieren images were used to track the boundary of the flame through time. Area-averaged flame properties were then extracted using a novel analysis methodology (the AA-LC model) to accurately account for the details of aspherical flames. A detailed evaluation found the average  $S_u^0$  uncertainty to be 4.4% in post-reflected-shock measurements performed in the SWIFT; uncertainty was higher (5.8%) for  $S_u^0$  measurements based on smaller flames in the CKST configuration. Measured  $S_u^0$  closely agreed with  $S_L$  values simulated using AramcoMech 3.0; propane-specific mechanisms from both NUIG 1.1 and San Diego were found to systematically over predict

the measured values. Discrepancies between some mechanisms and measurements illustrate the potential value of the shock-tube flame speed method to enable kinetic investigations at previously inaccessible and completely unvalidated conditions. Measurements were also evaluated against three forms of empirical fits, revealing that the common power-law fit fails to capture the  $T_u$  dependence over wide ranges but that an alternative non-Arrhenius form performs much better in capturing the  $T_u$  trend of the new measurements.

The present work provides a step towards enabling several significant opportunities for future study, of which two are highlighted. First, at reactive conditions ( $T_u > 1050$  K), it should be possible to use laser diagnostics or other methods to characterize the evolution of the unburned-gas state, potentially providing a basis for performing reliable chemistry-affected  $S_u^0$  measurements (something first attempted in [44]). The AA-LC model also provides an opportunity for improved interpretation of wall-bounded flames, potentially supporting their use as a basis of reliable  $S_u^0$  measurements in both rapid-compression machines [45] or in the shock tube [17].

### Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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### Supplementary material

Supplementary material associated with this article can be found, in the online version, at [10.1016/j.proci.2022.07.191](https://doi.org/10.1016/j.proci.2022.07.191).

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