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Experimental and Analytical Prescribed-Time Trajectory Tracking Control of a 7-DOF Robot Manipulator

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Abstract— **We present an analytical design and experimental verification of trajectory tracking control of a 7-DOF robot manipulator, which achieves convergence of all tracking errors to the origin within a finite terminal time. A key feature of this control strategy is that this terminal convergence time is explicitly prescribed by the control designer, and is thus independent of the initial conditions of the tracking errors. In order to achieve this beneficial property of the proposed controller, a scaling of the state by a function of time that grows unbounded towards the terminal time is employed. Through Lyapunov analysis, we first demonstrate that the proposed controller achieves regulation of all tracking errors within the prescribed time as well as the uniform boundedness of the joint torques, even in the presence of a matched, non-vanishing disturbance. Then, through both simulation and experiment, we demonstrate that the proposed controller is capable of converging to the desired trajectory within the prescribed time, despite large initial conditions of the tracking errors and a sinusoidal disturbance being applied in each joint.**

I. INTRODUCTION

In many applications where robot manipulators are utilized, the convergence time of the underlying controller plays a crucial role. In many tasks, there are strict requirements on the maximum duration of convergence, and thus a failure to achieve convergence by the required time could lead to the inability of the robot manipulator to perform its task. Convergence time also plays a role in the planning and reliability of robot manipulators. When multiple robot manipulators are used cooperatively, such as in an assembly line in industrial applications, having reliable estimates of the completion time of each individual task is crucial in order to effectively plan the operation of each manipulator. A considerable amount of research has been devoted towards the development of control methods for robot manipulators which are capable of guaranteeing an upper bound on the convergence time (potentially dependent on initial conditions), achieving convergence to zero within a finite period of time.

The literature on finite-time convergence methods concerning robot manipulators can be broadly organized into 3 distinct categories, finite-time methods [1]–[3], fixed-time methods [4]–[7], and prescribed-time methods [8]–[16]. Finite-time methods are characterized by a finite convergence time that is bounded by the norm of the initial condition, as well as a function of the controller parameters. Thus, in order to complete a larger set of tasks, with different initial conditions and maximum allowable operation times, separate controller parameters must be determined for each task. Fixed-time methods are characterized by a finite convergence time that is bounded by a function of the controller parameters which is independent of the initial conditions. Thus, the process for tuning the controller parameters for a specific task is considerably simplified, as one no longer needs to consider the maximum expected initial conditions of the task. However, it is important to note that the upper bound of the convergence time is typically conservative and may not be able to be arbitrarily set.

Prescribed-time methods are characterized by a finite convergence time that is explicitly prescribed as a controller parameter. This desirable property of prescribed-time methods enables the same set of controller parameters to be utilized for a wide variety of tasks with different required completion times. Due to this desirable property, the development of prescribed-time methods has become an active research topic in recent years. One of the first examples of a prescribed-time method was introduced by Song *et al.* [8], in which a scaling of the state of a normal-form nonlinear system by a function of time that grows unbounded towards the terminal time was employed. By stabilizing the system in the scaled representation, regulation in prescribed finite time is achieved for the original state, along with a smooth, uniformly bounded control input and the rejection of a matched non-vanishing disturbance. Another important class of prescribed-time controllers for robot manipulators, which was first introduced by Becerra *et al.* [10] and improved upon by Obregón-Flores *et al.* [11], utilizes time base generators, which are state trajectories designed such that the system state smoothly converges to zero at the prescribed terminal time. A key feature of this control method is the explicit use of the initial conditions in the controller structure as a feedforward term, along with a sliding-mode control scheme to correct for a uniformly bounded matched uncertainty. Notably, this scheme exhibits prescribed-time convergence in the ideal case of no matched uncertainty, and finite-time convergence when uncertainties are present. In a separate approach, Cao *et al.* [14] utilizes a scaling system transformation technique to transform the Euler-Lagrange system considered into a new set of variables, in which the boundedness of the variables ensures that both partial and full state constraints will not be violated. In addition, this transformation also ensures

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that for any time greater than the prescribed convergence time, the remaining tracking errors will be less than a prescribed value. This approach is notable in that the scaling transformation utilized does not approach infinity as the terminal time is approached, and thus numerical difficulties caused by an unbounded gain are avoided in this method. However, a potential drawback to using this method is that the controller does not allow for separate control gains for each joint, meaning that aggressive torques are likely applied to certain joints of the robot manipulator when there is a large difference in inertia between joints, which is typically the case for high-DOF robot manipulators.

In this effort, we reformulate the prescribed-time controller initially developed by Song *et al.* [8] in order to handle the case of trajectory tracking with a robot manipulator. This formulation yields convergence of the tracking errors to the origin within the prescribed terminal time, even in the presence of model uncertainties and a non-vanishing matched disturbance. Furthermore, through the experimental verification of this prescribed-time control strategy on Baxter, a 7-DOF redundant robot manipulator, we demonstrate convergence of the tracking errors to a small neighborhood of zero by the prescribed terminal time, despite a significant initial angular position tracking error of 20 degrees on each joint, as well as a sinusoidal torque disturbance of $0.1\sin 5t$ applied to each joint. Thus the prescribed-time control strategy studied here is both theoretically sound and effective in practice.

Notations: In the following, we use the common definitions of class $\mathcal{K}$ and $\mathcal{KL}$ given in [17]. $|\cdot|$ refers to the Euclidean norm, the matrix norm is defined accordingly, for $M \in \mathcal{M}_\ell(\mathbb{R})(\ell \in \mathbb{N}^*)$, as $|M| = \sup_{|x|\leq 1} |Mx|$ and the spatial norm is defined as follows:

$$\|f\|_{[a,b]} = \sup_{t\in[a,b]} \left|f(t)\right|$$

II. Mathematical Modeling

The redundant manipulator, which is being studied here, has 7-DOF as shown in Figure 1. The Euler-Lagrange formulation leads to a set of 7 coupled nonlinear second-order ordinary differential equations:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + D(t) = \tau \quad (1)$$

where, $q, \dot{q}, \ddot{q} \in \mathbb{R}^7$ are angles, angular velocities and angular accelerations of joints, respectively, and $\tau \in \mathbb{R}^7$ indicates the vector of joints' driving torques. Also, $M(q) \in \mathbb{R}^{7\times 7}$ is a symmetric mass-inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{7\times 7}$ is a matrix of Coriolis coefficients, $G(q) \in \mathbb{R}^7$ is a vector of gravitational loading, $F(\dot{q}) \in \mathbb{R}^7$ represents a vector of frictional torques, and $D(t) \in \mathbb{R}^7$ is a vector of disturbance torques with an unknown bound applied to the system.

Our verified coupled nonlinear dynamic model of the robot [18]–[31] is used as the basis of the prescribed-time approach. In order to formulate a controller that is robust to modeling uncertainty, the values of the mass matrix, gravity vector, and frictional torques derived from this dynamic model are treated as estimates, and are denoted as $\hat{M}(q), \hat{G}(q), \hat{F}(\dot{q})$, respectively. We make the following assumptions concerning the difference between our dynamic model and the true dynamics of Baxter:

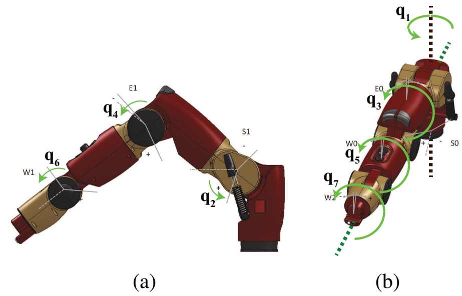

Fig. 1. The joints' configuration: (a) sagittal view; (b) top view

Assumption 1. *The true and estimated values of the mass matrix, Coriolis matrix, gravity vector, frictional torques, and the disturbance torques satisfy the following inequalities:*

$$\left|M^{-1}(q)\hat{M}(q) - I\right| \leq c_1 \quad (2)$$

$$\left|M^{-1}(q)C(q,\dot{q})\dot{q}\right| \leq c_2|\dot{q}|^2 \quad (3)$$

$$\left|M^{-1}(q)(\hat{G}(q) - G(q))\right| \leq c_3 \quad (4)$$

$$\left|M^{-1}(q)(\hat{F}(\dot{q}) - F(\dot{q}))\right| \leq c_4|\dot{q}| \quad (5)$$

$$\left|M^{-1}(q)D(t)\right| \leq c_5\left|D(t)\right| \quad (6)$$

Assumption 2. *The true mass matrix $M(q)$, and the estimated mass matrix $\hat{M}(q)$ are symmetric and positive definite.*

Furthermore, we make the following assumption regarding the reference joint trajectories:

Assumption 3. *The desired joint trajectories are designed such that $q_r(t)$, $\dot{q}_r(t)$, and $\ddot{q}_r(t) \in \mathbb{R}^7$ exist and are uniformly bounded for all $t \in [0,T]$, where $T > 0$ is the prescribed terminal time.*

III. Prescribed-Time Tracking for Robot Manipulators

We consider the following trajectory tracking system:

$$\dot{E} = \begin{bmatrix} \dot{\epsilon} \\ M^{-1}(\tau - C\dot{q} - G - F - D) - \ddot{q}_r \end{bmatrix} \quad (7)$$

$$\epsilon = q - q_r \quad (8)$$

$$E = [\epsilon, \dot{\epsilon}]^T \quad (9)$$

where $E \in \mathbb{R}^{14}$ is the state error vector, and $\epsilon \in \mathbb{R}^7$ is the vector of joint angular position tracking errors.

In order to regulate this system in prescribed-time, we first introduce the following monotonically increasing scaling function, as well as it's inverse:

$$\mu_1(t) = \frac{T}{T-t}, \quad t \in [0,T) \quad (10)$$

$$\nu_1(t) = \mu(t)_1^{-1} = \frac{T-t}{T}, \quad t \in [0,T) \quad (11)$$

where $T > 0$ is the prescribed terminal time, with the properties $\mu(0) = 1$, $\mu(T) = +\infty$, $\nu(0) = 1$ and $\nu(T) = 0$. To achieve prescribed-time regulation of the tracking errors, we introduce the following change of coordinates:

$$w(t) = \mu(t)\epsilon(t) \tag{12}$$

$$z(t) = \dot{w}(t) + \alpha w(t) \tag{13}$$

where

$$\mu(t) = \mu_1(t)^2 \tag{14}$$

and $\alpha > 0$. This change of coordinates results in the following forward and inverse scaling transforms:

$$Z = \mu \begin{bmatrix} I & 0 \\ (\alpha + \mu_1 \frac{2}{T})I & I \end{bmatrix} E = P(\mu_1)E \tag{15}$$

$$E = \nu_1 \begin{bmatrix} \nu_1 I & 0 \\ (-\nu_1\alpha - \frac{2}{T})I & \nu_1 I \end{bmatrix} Z = Q(\nu_1)Z \tag{16}$$

$$Z = [w, z]^T \tag{17}$$

where $I \in \mathbb{R}_{7\times 7}$ is the identity matrix, $P(\mu_1) \in \mathbb{R}^{7\times 7}$ is the forward scaling transform, $Q(\nu_1) \in \mathbb{R}^{7\times 7}$ is the inverse scaling transform, and $Z \in \mathbb{R}^{14}$ is the scaled state error vector. By taking the time derivative of (15) and substituting the inverse transformation (16), the dynamics of the scaled state error vector are obtained:

$$\dot{w} = z - \alpha w \tag{18}$$

$$\dot{z} = \mu \left[\ddot{\epsilon} - \left(\alpha^2 \nu_1^2 + \alpha \nu_1 \frac{4}{T} + \frac{2}{T^2} \right) w + \left(\nu_1 \frac{4}{T} + \alpha \nu_1^2 \right) z \right] \tag{19}$$

where

$$\ddot{\epsilon} = M^{-1}(\tau - C\dot{q} - G - F - D) - \ddot{q}_r \tag{20}$$

Before presenting the design of the prescribed-time control law, it is first necessary to present several definitions concerning notions of stability within a finite prescribed interval of time.

Definition 1 (FT-ISS [8])**.** The system $\dot{x} = f(x, t, d)$ (of arbitrary dimensions of x and d) is said to be *fixed-time input-to-state stable in time T* (FT-ISS) if there exists a class $\mathcal{KL}$ function β and a class $\mathcal{K}$ function γ, such that, for all $t \in [0, T)$:

$$\left|x(t)\right| \leq \beta\left(|x_0|, \mu_1(t) - 1\right) + \gamma\left(\|d\|_{[0,t]}\right) \tag{21}$$

Definition 2 (FT-ISS+C [8])**.** The system $\dot{x} = f(x, t, d)$ (of arbitrary dimensions of x and d) is said to be *fixed-time input-to-state stable in time T and convergent to zero* (FT-ISS+C) if there exists class $\mathcal{KL}$ functions β and β_f, and a class $\mathcal{K}$ function γ, such that, for all $t \in [0, T)$:

$$\left|x(t)\right| \leq \beta_f\left(\beta\left(|x_0|, \mu_1(t) - 1\right) + \gamma\left(\|d\|_{[0,t]}\right), \mu_1(t) - 1\right) \tag{22}$$

As the function $\mu_1(t) - 1$ starts at zero and grows monotonically to infinity as $t \to T$, a system that is FT-ISS is also ISS, with the additional property that in the absence of a disturbance d, it is fixed-time globally asymptotically stable in time T. Additionally, a system that is FT-ISS+C is also FT-ISS, with the additional property that the state converges to zero even in the presence of a disturbance.

Now, we present the design of the prescribed-time control law.

Theorem 1. *Under Assumptions 1-3, consider the system (7) with the controller:*

$$\tau(t) = -\hat{M}(q)\left[(k + \theta + \eta\psi(\dot{q})^2)z(t) + \ddot{q}_r(t)\right] + \hat{G}(q) + \hat{F}(\dot{q}) \tag{23}$$

where

$$\psi(\dot{q}) = |\dot{q}|^2 + |\dot{q}| + 1 \tag{24}$$

If the controller gains are chosen such that $\rho, k, \eta > 0$*,*

$$\rho k \alpha^2 > \frac{1}{4\lambda_M^2}, \tag{25}$$

and

$$\theta \geq \frac{1}{\lambda_M}\left(\alpha + \frac{4}{T}\right) + \rho\left(\alpha^2 + \alpha\frac{4}{T} + \frac{2}{T^2}\right)^2 \tag{26}$$

where

$$\lambda_M = \min_{q \in [0, 2\pi)} \lambda_{\min}\left(\frac{M^{-1}(q)\hat{M}(q) + \hat{M}(q)M^{-1}(q)}{2}\right) \tag{27}$$

then the closed loop system (7) with (23) is FT-ISS+C and the joint torques τ *remain bounded over* $[0, T)$*.*

IV. Lyapunov Analysis

For the purpose of the Lyapunov analysis, we propose the following Lyapunov function:

$$V = \frac{1}{2}|z|^2 \tag{28}$$

Taking the derivative of this function yields:

$$\begin{aligned} \dot{V} = \mu z^T \Big[& - M^{-1}\hat{M}(k + \theta + \eta\psi^2)z + (M^{-1}\hat{M} - I)\ddot{q}_r \\ & + M^{-1}\left(\hat{G} - G + \hat{F} - F - C\dot{q} - D\right) \\ & - \left(\alpha^2\nu_1^2 + \alpha\nu_1\frac{4}{T} + \frac{2}{T^2}\right)w + \left(\frac{4}{T}\nu_1 + \alpha\nu_1^2\right)z \Big] \end{aligned} \tag{29}$$

First, we seek to obtain an upper bound for the 1st term of $\dot{V}$. Utilizing the positive definite symmetric property of the mass matrices as stated in Assumption 2:

$$z^T M^{-1}\hat{M}z = z^T\left(\frac{M^{-1}\hat{M} + \hat{M}M^{-1}}{2}\right)z \geq \lambda_M |z|^2 \tag{30}$$

where λ_M is first defined in (27).

Next, we examine the 2nd and 3rd terms of $\dot{V}$. Through the application of Assumptions 1 and 3, the following inequality can be obtained:

$$(M^{-1}\hat{M}-I)\ddot{q}_r+M^{-1}\left(\hat{G}-G+\hat{F}-F-C\dot{q}-D\right)\leq\psi d \tag{31}$$

where

$$d(t)=\max\left\{c_1\|\ddot{q}_r\|_{[0,t]}+c_3+c_5\|D\|_{[0,t]},c_2,c_4\right\} \tag{32}$$

Applying (30) and (31) to (29), along with Young's inequality yields the following inequality:

$$\begin{aligned}\dot{V}\leq&-\mu\lambda_M|z|^2(k+\theta+\eta\psi^2)+\mu\eta\lambda_M\psi^2|z|^2\\&+\frac{\mu}{4\eta\lambda_M}d^2+\mu\rho\lambda_M|z|^2\left(\alpha^2\nu_1^2+\alpha\nu_1\frac{4}{T}+\frac{2}{T^2}\right)\\&+\frac{\mu}{4\rho\lambda_M}|w|^2+\mu|z|^2\left(\frac{4}{T}\nu_1+\alpha\nu_1^2\right)\end{aligned} \tag{33}$$

Through the application of (26), this inequality can be further reduced:

$$\dot{V}\leq-2\mu\lambda_M kV+\frac{\mu}{4\eta\lambda_M}d^2+\frac{\mu}{4\rho\lambda_M}|w|^2 \tag{34}$$

In order to proceed with the Lyapunov analysis, it is necessary to introduce a technical lemma from the work of Song *et al.* [8].

Lemma 1. *If a continuously differentiable function $V:[0,T)\to[0,+\infty)$ satisfies:*

$$\dot{V}(t)\leq-2k\mu(t)V(t)+\frac{\mu(t)}{4\lambda}d(t)^2 \tag{35}$$

for positive constants k,λ, where $\mu(t)$ is defined in (10), then:

$$V(t)\leq\xi(t)^{2k}V(0)+\frac{\|d\|_{[0,t]}}{8k\lambda},\quad\forall t\in[0,T) \tag{36}$$

where ξ is the monotonically decreasing function:

$$\xi(t)=e^{T(1-\mu_1(t))} \tag{37}$$

with the properties that $\xi(0)=1$ and $\xi(T)=0$.

Through the application of this lemma to (34), it can be seen that:

$$\left|z(t)\right|\leq\xi(t)^{\lambda_M k}|z_0|+\frac{1}{2\lambda_M\sqrt{k}}\left(\frac{\|w\|_{[0,t]}}{\sqrt{\rho}}+\frac{\|d\|_{[0,t]}}{\sqrt{\eta}}\right) \tag{38}$$

and thus the z-system is FT-ISS w.r.t. the w-input with a gain of $\frac{1}{2\lambda_M\sqrt{k\rho}}$ and is also FT-ISS w.r.t the d-input. In order to obtain the behavior of the w-system, one can rearrange (13) to obtain $\dot{w}(t)=-\alpha w(t)+z(t)$. From this point, it is straightforward to obtain a bound on w:

$$\left|w(t)\right|\leq|w_0|e^{-\alpha t}+\frac{1}{\alpha}\|z\|_{[0,t]} \tag{39}$$

and thus the w-system is ISS w.r.t the z-input with a gain of $\frac{1}{\alpha}$. Thus by the small-gain theorem, if condition (25) is satisfied, then the combined system Z is ISS w.r.t. d and thus there exist constants $\Gamma,\delta,\gamma>0$ such that:

$$\left|Z(t)\right|\leq\Gamma|Z_0|e^{-\delta t}+\gamma\|d\|_{[0,t]} \tag{40}$$

Through the substitution of the scaling transformation (15) into the right side of (40), followed by the substitution of the resulting inequality into the right side of the inverse scaling transformation (16), the following inequality is obtained:

$$\left|E(t)\right|\leq\nu_1(t)\left[\breve{\Gamma}|E_0|e^{-\delta t}+\breve{\gamma}\|d\|_{[0,t]}\right] \tag{41}$$

where

$$\breve{\Gamma}=\Gamma\left|P(1)\right|\max_{\nu_1\in[0,1]}\left|Q(\nu_1)\right| \tag{42}$$

$$\breve{\gamma}=\gamma\max_{\nu_1\in[0,1]}\left|Q(\nu_1)\right| \tag{43}$$

Due to the fact that $\nu_1(T)=0$, this inequality establishes that the closed loop system (7) with (23) is FT-ISS+C. Due to the boundedness of $Z(t)$ established in (40), the boundedness of $E(t)$ established in (41), and the boundedness of q_r, $\dot{q}_r$, and $\ddot{q}_r$ established in Assumption 3, the uniform boundedness of the input τ is established from (23).

V. Remarks on Prescribed-Time Control Law

Through the substitution of the scaling transform (15) to the control law (23), it is possible to obtain an expression for the control law in terms of the joint angular position and velocity errors $\epsilon,\dot{\epsilon}$ rather than the scaled state z:

$$\begin{aligned}\tau=&-\mu_1^2\hat{M}\left[(k+\theta+\eta\psi^2)\left((\alpha+\mu_1\frac{2}{T})\epsilon+\dot{\epsilon}\right)+\ddot{q}_r\right]\\&+\hat{G}+\hat{F}\end{aligned} \tag{44}$$

From this representation, the role of the controller parameters k, θ, η, and α can be observed. $k+\theta$ is a scaled PD gain, and thus is the primary driver of the error signal to zero, η is the gain of the nonlinear damping term ψ, which aims to attenuate the effects of uncertainties on the control law, and α is a weighting factor which determines the ratio between the proportional and derivative gains of the control law. Thus, implementing the proposed prescribed-time control law control law requires the tuning of just 3 parameters (treating $k+\theta$ as 1 parameter), whose effect on the control law is readily observed. Furthermore, due to the direct dependence of the control law on the prescribed final time T, these 3 controller parameters need only be determined once for a given robot manipulator, regardless of the specific tasks the manipulator needs to perform. Thus, the proposed control law can be readily applied to a wide variety of tasks with different convergence time constraints.

A potential barrier to the practical application of this proposed method is the consequences of employing an unbounded gain $\mu_1(t)$. While the proposed control law guarantees boundedness of the control torques $\tau(t)$ even in the presence of non-vanishing uncertainties, problems may still arise due to measurement noise, numerical issues when multiplying large gains with small errors, and a finite controller frequency. In order to combat these practical issues, one effective strategy that can be employed is gain clipping. Using this strategy, we define a constant $\zeta\in(0,1)$

and redefine $\mu_1(t)$ in (44) as:

$$\mu_1(t) = \frac{T}{T - \min\{t, \zeta T\}} \tag{45}$$

This redefinition of (10) upper bounds the scaling gain μ_1 by the value $\frac{1}{1-\zeta}$, ensuring that the controller gains do not grow past the point where the previously mentioned issues begin to noticeably affect the closed-loop system. A consequence of this modification is that the regulation of the tracking errors is to a small neighborhood of zero, rather than exactly zero as when utilizing an unbounded gain. Employing a ζ that is sufficiently large can ensure that this neighborhood is negligible, achieving performance that is qualitatively similar to that of utilizing an unbounded gain.

VI. Simulation and Experimental Results

In order to assess the performance of the proposed prescribed-time approach, we perform both a simulation using ODE methods on Baxter's dynamic equation (1), as well as an experiment. In both the simulation and experiment, Baxter must track a trajectory designed for a pick and place task in [24], while under the influence of a torque disturbance of $D(t) = 0.1\sin(5t)$ applied to each joint. In addition, this task is purposely started from a large initial angular position error of 20 degrees for each joint. Thus, this simulation and experiment demonstrates the ability of the proposed method to converge from a large initial condition to the desired trajectory within the prescribed finite time, while rejecting a large torque disturbance. The controller parameters used in both the simulation and experiment are $T = 6$, $k + \theta = 5$, $\eta = 0.005$, $\alpha = 2$, and $\zeta = 0.4$.

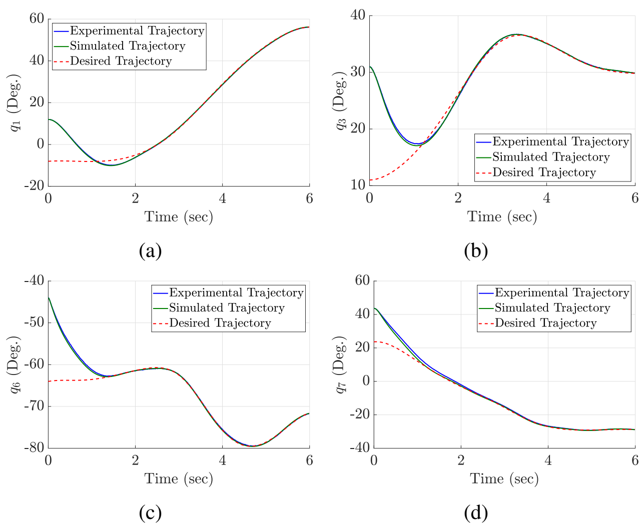

Fig. 2. The experimental (blue line), simulated (green line), and desired (red dashed line) joint trajectories of Baxter: (a) Joint 1, (b) Joint 3, (c) Joint 6, (d) Joint 7

The experimental, simulated, and desired joint trajectories for several select joints can be seen in Figure 2. Despite the large initial joint tracking errors, as well as the large sinusoidal disturbance applied to the system, negligible tracking errors are achieved after around 2.5 seconds of operation.

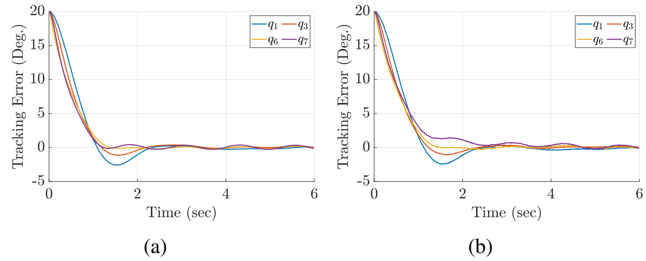

Fig. 3. The simulated (a) and experimental (b) joint tracking errors of select joints of Baxter (Joints 1, 3, 6 and 7).

Throughout the procedure, oscillations in the joint angular positions can not be observed from this figure, indicating that the nonlinear-damping method employed was effective at absorbing the effect of the sinusoidal disturbance. Furthermore, minimal overshoot is observed during the 1st 2.5 seconds of operation, indicating that the proposed control law is acting neither too aggressively or too leniently in the beginning of the task. Observing Figure 3, it is possible to see the convergence behavior of the proposed method in more detail. After 2.5 seconds of operation, roughly coinciding to the time of $\zeta T = 2.4$ seconds where the gain multiplier μ_1 stops increasing, the majority of the tracking errors have already been significantly attenuated. From 2.5 seconds onward, the residual tracking errors, mostly resulting from the sinusoidal torque disturbance, are attenuated to an acceptably small value of less than 0.2 degrees.

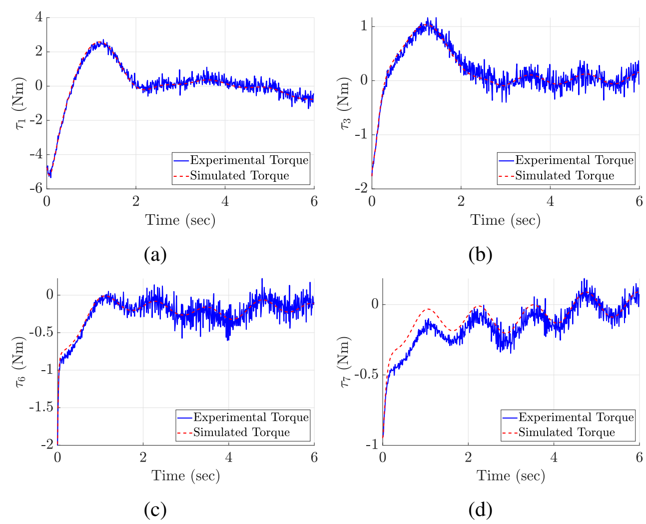

Fig. 4. The experimental (blue line) and simulated (red dashed line) joint torque input signals of Baxter: (a) Joint 1, (b) Joint 3, (c) Joint 6, (d) Joint 7

The experimental and simulated joint toque input signals for several select joints can be seen in Figure 4. It is important to note that these torques are significantly lower than the maximum torque output of Baxter's joints, which are 50 Nm for joints 1-4, and 15 Nm for joints 5-7. Thus, the prescribed-time approach is able to correct for a large initial error without producing excessive joint torques. Additionally, the simulated torques remain smooth throughout the procedure and do not display chattering, which can negatively affect the lifespan of the actuators used to control the robot manipulator.

VII. Conclusion

In this research effort, we formulated and experimentally verified the prescribed-time trajectory tracking control of a 7-DOF robot manipulator. In order to ensure regulation of the tracking errors by the prescribed final time, we employed a scaling of the state by a function of time that grows unbounded towards the terminal time. Through Lyapunov analysis, we demonstrated that the proposed controller achieves regulation of all tracking errors within the prescribed time with a torque that is uniformly bounded, even in the presence of a matched non-vanishing disturbance. Through inspection of the control law, we demonstrated that the choice of parameters for the proposed control law is intuitive and straightforward, and that the controller could be implemented in a practical system with minimal modifications. Then, through both simulation and experiment, we demonstrated that the proposed controller is capable of converging to the desired trajectory within the prescribed time, despite large initial conditions of the tracking errors and a sinusoidal disturbance being applied in each joint.

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