2022 American Control Conference (ACC)
Atlanta, USA, June 8-10, 2022

EPROACH: A Population Vaccination Game for Strategic Information Design to Enable Responsible COVID Reopening

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Abstract— **The COVID-19 lockdowns have created a significant socioeconomic impact on our society. In this paper, we propose a population vaccination game framework, called EPROACH, to design policies for reopenings that guarantee post-opening public health safety. In our framework, a population of players decides whether to vaccinate based on the public and private information they receive. The reopening is captured by the switching of the game state. The insights obtained from our framework include the appropriate vaccination coverage threshold for safe-reopening and information-based methods to incentivize individual vaccination decisions. In particular, our framework bridges the modeling of the strategic behaviors of the populations and the spreading of infectious diseases. This integration enables finding the threshold which guarantees a disease-free epidemic steady state under the population's Nash equilibrium vaccination decisions. The equilibrium vaccination decisions depend on the information received by the agents. It makes the steady-state epidemic severity controllable through information. We find that the externalities created by reopening lead to the coordination of the players in the population and result in a unique Nash equilibrium. We use numerical experiments to corroborate the results and illustrate the design of public information for responsible reopening.**

I. Introduction

The COVID-19 pandemic has created a significant socioeconomic impact on our society. Due to the massive infections, many cities have been locked down. Restrictions, including social distancing, closure of restaurants and schools, and mask mandates, have been in place for nearly two years. With the advent of vaccines, many cities are considering reopening plans. One essential issue concerning the reopening policies is to reach a reasonable vaccination coverage rate so that the return to normal social activities does not generate new outbreaks. Once a threshold is determined, cities can focus on the post-opening policies on mask-wearing and social distancing to reduce further risks threatening public health safety. Apart from them, a concomitant question is to find ways to incentivize or nudge individuals to vaccinate. The decision of an appropriate coverage threshold and the ways to reach it are high-priority for the cities to prepare for reopening.

They, however, are accompanied by several challenges. First, the threshold should take into account strategic decisions of human behaviors and guarantee effective herd immunity in the population. Second, the post-opening policies and the individual vaccination decisions are interdependent. Reopening of the city signals to the public that the pandemic is ending and makes people less compliant to mandates and restrictions. The threshold or the reopening policies should take into account the human behaviors before and after the announced policies.

Challenges also arise when we aim to reach the determined threshold. Due to many reasons, people may choose not to vaccinate. For example, misinformation plays an important role in making people believe the conspiracy theory behind the vaccination. People may seek for a free ride hoping that others are vaccinated. These facts make the individual independent vaccination choices hard to predict.

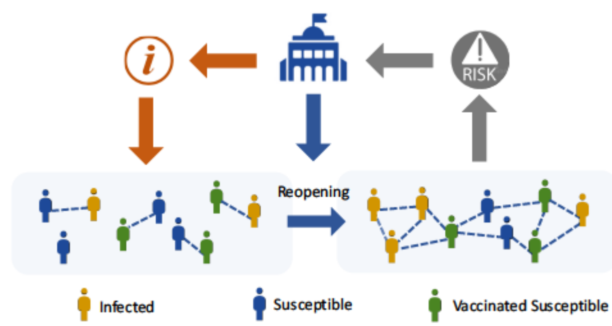

Fig. 1. During the pandemic, an authority determines whether or not to reopen a city. The risk of a post-reopening outbreak is the key concern for this decision. The authority designs and disseminates information to incentivize vaccination to reduce the risk.

To address the above challenges, we propose a framework called the epidemiological reopening game of connected health (EPROACH). EPROACH is a vaccination game framework integrated with the compartmental epidemic models. We use a population game to capture the behavioral patterns resulting from individual decision-making. In particular, we focus on vaccination decisions based on the well-being of individuals in the epidemic over a period of time. We incorporate public and private information into the game. These two types of information shape individual beliefs of the severity of the epidemic and guide the vaccination decisions. They also serve as the means to control the epidemic. To capture the relations between the vaccination decisions and the effects of social policies before and after the reopening, we consider two regimes for the game. These two regimes model the restricted and the reopened cities, respectively. The regimes influence the population's behavior patterns through the spreading of the epidemic. We use two distinct

This work is partially supported by grants SES-1541164, ECCS-1847056, CNS-2027884, and BCS-2122060 from National Science Foundation (NSF), grant 20-19829 from DOE-NE, and grant W911NF-19-1-0041 from Army Research Office (ARO).

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978-1-6654-5196-3/$31.00 ©2022 AACC

epidemic models to quantify the risks of the outbreaks under various social policies adopted in the regimes. These risks drive people's vaccination decisions and determine the accompanying social policies such as mask-wearing. The direction of the evolution of the epidemic states eventually explains whether we arrive at herd immunity. The integration of individual rational decision-making, private and public information, epidemic dynamics, and regime-switching in EPROACH helps quantify the outcomes of reopening and the potential risks attached. We propose incentive mechanisms based on the Nash equilibrium (NE) of EPROACH to ensure public health safety after the reopening of cities.

We break the analysis of EPROACH into three parts. The first part studies the externalities of the game. While the decisions are made by individuals under either restricted or reopened regimes, they arrive at a pattern of coordination when an appropriate regime-switching is triggered. In the second part, we show the conditions for a unique NE. The NE characterizes the individual behaviors at the equilibrium under the perceived infection risks in different regimes. The analysis of the NE provides a way to understand and predict the behavioral patterns of the population under various reopening policies. In addition, we study the interdependence between information and the NE and show the impact of information on the outcomes of both vaccination coverage threshold and epidemic status. The third part leverages this observation to create an informational epidemic control mechanism to achieve social good by manipulating public and private information. Our method serves as a scalable and low-cost tool to incentivize vaccinations.

We provide a brief literature review in the next section. The formulation of EPROACH is discussed in Section III. Section IV presents the game analysis. Section V introduces the informational epidemic control method, which is demonstrated in Section VI using numerical experiments.

II. Related work

One constituent of EPROACH is the compartmental epidemic model. We use the degree-based mean-field model over complex networks [1] to capture the average effect of the contagion within a networked population. Control of epidemics has been discussed in many recent works. The authors in [2] consider epidemic control of two competing viruses. Recently, [3] proposes a framework unifying individual decision-making processes and the epidemic dynamics to study herd behavior. The review [4] summarizes and classifies popular game-theoretic models. Our game-theoretic framework emphasizes the interplay of human behavior and epidemics and investigates the role of information in epidemic control.

EPROACH builds on population games [5]. By adopting public and private signals, we can study herd behaviors in the setting of incomplete information. This approach has been investigated in [6] for understanding equilibrium selection. The monograph [7] motivates a class of global games and their applications in macroeconomics. Its variants include [8], [9], [10]. We consolidate complex networks into this class of games to capture strategic population-level interactions over networks.

III. Formulation of EPROACH

Consider a population of heterogeneous players with mass 1 exposed to the infections of a virus. Each player is associated with a distinct degree $d \in \mathscr{D} := \{1,2,...,D\}$. The mass of players with degree d is $m^d \in (0,1)$ with $\sum_{d\in\mathscr{D}} m^d = 1$ and $\bar{d} := \sum_{d\in\mathscr{D}} dm^d$. The degrees capture the players' intensities of interacting with other players over the complex network defined by the degree distribution $[m^d]_{d\in\mathscr{D}}$. The interactions affect the likelihood of getting infected. We focus on the mean effect of interactions among the players over the complex network.

Let $\beta \in (0,1)$ denote the probability that a vaccinated individual is protected from being infected. Each individual makes a one-time choice of whether or not to vaccinate at the beginning of the game. The decision is based on the anticipations of the player's individual infection risk at time $T > 0$. Let the time interval $(0,T), T > 0$, be the period of interest. The action set of the players is given by $\mathscr{A} := \{0,1\}$. Action $i = 0$ means that a player does not take the vaccination, and action $i = 1$ means that she takes the vaccination.

Vaccination decisions influence the regime of the game s chosen from the set $\{\text{'+'}, \text{'-'}\}$. Regime $s =$'+' is the reopened state where there is no social restriction on the players. Regime $s =$'-' is the restricted state where the intensity of social activities is reduced to $\alpha^- = \alpha \in (0,1)$. By default, we set $\alpha^+ = 1$ to denote the intensity of restriction-free social activities. The switching of the regimes depends on two factors: the average action of players $A \in \mathbb{R}$ and the vaccination coverage threshold θ, which is a random variable with support $\mathbb{R}$. Since A captures the vaccination coverage rate, the state is '+' if $A \geq \theta$ and is '-' if $A < \theta$.

In this work, we view θ as the public signal of the game. The distribution of θ is common knowledge, but players perceive θ heterogeneously. This heterogeneity is captured by the information type space $\mathscr{K} := \{1,2,...,K\}$. A player with information type $k \in \mathscr{K}$ receives a signal $x_k = \theta + \xi_k$, where ξ_k is the random perception bias of type k. We call x_k the private signal of type k since the distribution of ξ_k is only observable to type-k players. The private signals are essential in shaping players' behaviors since they correlates with the threshold θ.

We assume that players with the same degree $d \in \mathscr{D}$ and the same information type $k \in \mathscr{K}$ are statistically equivalent. This assumption is motivated by the following facts. Firstly, the payoffs of the players depend on their risks of getting infected. This risk depends on a player's intensity of interactions captured by the degree. Secondly, players' decisions are made based on their anticipation of the regime changes. It varies for players with different types, since the beliefs of regimes are formed according to the correlations of the public signal θ and the private signals x_k. Let $m_i^{d,k}$ denote the proportion of players with degree $d \in \mathscr{D}$ and type $k \in \mathscr{K}$ who select action $i \in \mathscr{A}$. Let $m_i^d = \sum_{k\in\mathscr{K}} m_i^{d,k}$ and $m_i^k = \sum_{d\in\mathscr{D}} m_i^{d,k}$. The mean action of the population can be expressed as

$A=\sum_{d\in\mathscr{D}}\sum_{k\in\mathscr{K}}m_1^{d,k}=\sum_{d\in\mathscr{D}}m_1^d=\sum_{k\in\mathscr{K}}m_1^k$. Note that when the subscript i is dropped, we do not distinguish the players who play either $i=1$ or $i=0$.

Under the statistical equivalence assumption, we specify the payoffs of players under different regimes. Let $c\in(0,1)$ denote the relative cost of vaccination. Let $r\in\mathbb{R}$ denote the morbidity risk of the virus. When the regime is $s=$'-', *i.e.*, $A<\theta$, a player with degree d and type k observes payoff $u_0^{d,k,-}=-rI_0^{d,k,-}(T)$ under action 0 and payoff $u_1^{d,k,-}=-c-rI_1^{d,k,-}(T)$ under action 1, where $I_i^{d,k,-}(T)$ denotes the infection probability at the end of the period of interest when she plays action i under regime $s=$'-'. The infection probability $I_i^{d,k,-}$ in the population is governed by the epidemic process with recovery rate $\gamma\in(0,1)$ and contagion rate $\lambda\in(0,1)$ as follows:

$$\dot{I}_i^{d,k,-}(t)=-\gamma I_i^{d,k,-}(t)+\lambda_i(1-I_i^{d,k,-}(t))\alpha^- d\Theta^-(t), \quad (1)$$

where $\lambda_i=\lambda$ if $i=0$ and $\lambda_i=\beta\lambda$ if $i=1$. In (1), $\Theta^-(t):=\bar{d}^{-1}\sum_{d\in\mathscr{D}}\sum_{k\in\mathscr{K}}\sum_{i\in\mathscr{A}}dm_i^{d,k}I_i^{d,k,-}(t)$ denotes the probability that a link is connected to an infected player at time t. For the consistency of $\Theta^-(t)$, see [11]. When the regime is $s=$'+', *i.e.*, $A\geq\theta$, a player with degree d and type k observes payoff $u_0^{d,k,+}=-rI_0^{d,k,+}(T)+g^d$ under action 0 and payoff $u_1^{d,k,+}=-c-rI_1^{d,k,+}(T)+g^d$ under action 1, where g^d denotes the utility gain a degree d player generates after reopening. Note that this utility gain can be affected by psychological issues caused by quarantines and isolation. Its dependence on d characterizes the differences in the gains received by players having different degrees of social connections. The term $I_i^{d,k,+}$ is the counterpart of $I_i^{d,k,-}$ under state $s=$'+'. The corresponding epidemic process is

$$\dot{I}_i^{d,k,+}(t)=-\gamma I_i^{d,k,+}(t)+\lambda_i(1-I_i^{d,k,+}(t))\alpha^+ d\Theta^+(t), \quad (2)$$

where $\Theta^+(t):=\bar{d}^{-1}\sum_{d\in\mathscr{D}}\sum_{k\in\mathscr{K}}\sum_{i\in\mathscr{A}}dm_i^{d,k}I_i^{d,k,+}(t)$. The initial infection probabilities are assumed to be increasing in players' degrees and independent of the types. It is based on the practical concern that a player with a higher degree should have a higher initial infection probability.

IV. Equilibrium Analysis

In this section, we first analyze the structural property of (1) and (2). It helps understand the incentives of the players and makes (1) and (2) more tractable. Then, we prove the uniqueness of the equilibrium based on the incentives.

Assume that the vaccination coverage threshold θ follows a normal distribution with mean μ and variance $\frac{1}{\sigma^2}$, *i.e.*, $\theta\sim\mathscr{N}(\mu,\frac{1}{\sigma^2})$. We set the nominal threshold represented by μ to be in $(0,1)$. Among all the possible realizations of θ from the support $(-\infty,+\infty)$, the negative values mean that reopening does not rely on vaccinations; values that are larger than 1 mean that the city is always restricted no matter what the vaccination rate is. The perception bias of type k is assumed to have a precision level σ_k^2, *i.e.*, $\xi_k\sim\mathscr{N}(0,\frac{1}{\sigma_k^2})$. The results in this section also hold with general distributions [7].

A. Incentive Analysis

Externality plays an important role in game-theoretic situations. Network effects generate externalities. Based on different communication structures, the externalities can be either global or local. The systems of coupled differential equations (1) and (2) describing the epidemic evolution over the complex network are the sources of externalities of the players in our population game. While (1) and (2) capture the local interactions of players in an averaged sense, the externalities of players show global patterns of incentives.

The following result shows that when social distancing is sufficiently effective, the decisions in the game with potential regime switching are strategic complements [5]; i.e., a player's incentive to raise her action is non-decreasing in the other player's actions. This fact implies that $u_1^{d,k,+}-u_0^{d,k,+}\geq u_1^{d,k,-}-u_0^{d,k,-}$, for all $d\in\mathscr{D}$ and $k\in\mathscr{K}$.

Proposition 1. *Assume that the initial conditions of (1) and (2) are non-decreasing in players' degrees d and independent of the types k. Then, if $\Theta^+(t)\geq\alpha\Theta^-(t)$ is satisfied for $t\in[0,T]$, the decisions in the population game with the possibility of regime-switching are strategic complements.*

The proof follows similar steps as Theorem 6 of [3]. Note that the condition $\Theta^+(t)\geq\alpha\Theta^-(t)$ means that the effect of social restriction policy α makes the likelihood of linking to an infected player in the restricted state $s=$"-" lower than in the reopened state $s=$"+".

Strategic complementarity can be considered as a coordination of individual decisions. It indicates that the public health connected through the complex network is coordinated. A game is supermodular when the decisions of the players in this game are strategic complements [12]. In a supermodular game, there are often multiple equilibria and simple evolutionary dynamics converge monotonically to these equilibria. Proposition 1 lays the foundations for both analyzing the game from the perspective of players' incentives and solving the game using computational methods. Note that from Proposition 1, the property of strategic complements does not depend on the utility gain g^d of switching from $s=$'-' to $s=$'+'. This utility gain makes the payoffs under the state $s=$'+' more attractive for the players. However, it is the effectiveness of social policies, *i.e.*, α, which shapes players' decisions to become strategic complements.

The following result follows from a similar reasoning as in Proposition 1.

Proposition 2. *Assume that the initial conditions of (1) and (2) are non-decreasing in the players' degrees d and independent of their types k. If the switching of the regimes is absent, or, in other words, when the state is always either $s=$'+' or $s=$'-', the decisions in the population game are strategic substitutes.*

The conflict in incentives shown in Proposition 1 and Proposition 2 arises from the possibility of the regime switching. On the one hand, players' incentives to vaccinate exhibit rationality in the scenario of Proposition 2. This rationality nudges individuals to protect themselves from getting infected without taking others' health conditions into consideration. On the other hand, players' incentives to vaccinate show a pattern of coordination in the scenario of

Proposition 1. The coordination effect is the congruence with the anticipations of the game. The players tend to mimic others' vaccination decisions since under condition $\Theta^{+}(t) \geq \alpha\Theta^{-}(t)$, the only incentive to vaccinate is the possibility of switching to the reopening regime.

The coordination phenomenon found in Proposition 1 implies a convenient structure of the incentives of the players when the outcome is captured by the NE. Our equilibrium analysis in the next section leverages this structural property.

B. Equilibrium Analysis

Strategic complementarities generate multiple equilibria [12]. The next result shows that a unique NE is selected when the public and private signals in our game satisfy certain conditions. This equilibrium describes the vaccination decisions of the players when they observe their private signals. The result extends Proposition 1 of [10] to the case containing multiple private signals. We omit the proof due to the page limit.

Proposition 3. *The global vaccination game admits a unique equilibrium in switching strategies if*

$$\sum_{k\in\mathscr{K}} \frac{m^k}{\sigma_k} \leq \frac{\sqrt{2\pi}}{\sigma^2}. \tag{3}$$

Condition (3) requires that the private signals are sufficiently precise compared to the public signal. Since the population's mean action involves the terms $\Phi(\sigma_k(x_k^* - \theta))$ for $k \in \mathscr{K}$, the public information is scaled by σ_k. Hence, (3) involves variance $\frac{1}{\sigma^2}$ of the public signal but the standard deviation $\frac{1}{\sigma_k}$ of private signals. The reason why (3) is independent of players' degrees is as follows. Firstly, the strategic complements property allows us to focus on the behavioral patterns of the population rather than the strategy revisions of individuals. It means that the property has already taken into account the degree-dependent effects of infection risks captured by the coupled epidemic processes. Secondly, the independence of degrees and types allows us to describe the behaviors of players with different degrees using one single posterior probability $P(x_k \leq x_k^*|\theta)$ for the players who have the same type k.

We have consolidated many elements into our framework including switching regimes, public and private signals, and epidemic processes to capture the multifaceted behaviors of the players. Under such complex settings, Proposition 3 shows that the players' vaccination decisions turn out to be predictable; *i.e.*, the actions are captured by $([x_k^*]_{\forall k\in\mathscr{K}}, \theta^*)$.

V. Informational Epidemic Control

The long-term behaviors of the epidemic processes (1) and (2) are tightly connected with practical epidemic control policies. When a non-trivial steady state of the epidemics is considered, we often seek social policies which either decrease the total proportion of the infected or reduce the period required for reaching a desired level of the infected. The reason lies in the fact that the disease-free steady state often requires strong assumptions on the contagion rate and the recovery rate of the epidemics. On the contrary, the integration of the epidemic spreading and the population game of incomplete information in our framework makes the disease-free steady state reachable through proper designs of the information. In particular, we will show in the following that the public and private signals serve as tools to nudge people to vaccine.

We assume $\frac{\gamma}{d\lambda} \leq 1, \forall d \in \mathscr{D}$. It is the condition that guarantees that the virus does not die out by itself [3].

Let $(\bar{\Theta}^+, \bar{I}^+)$ denote the steady-state pair of $(\Theta^+(t), I^+(t))$ when the regime is '+'. The pair $(\bar{\Theta}^+, \bar{I}^+)$ satisfies

$$\bar{\Theta}^+ = \bar{d}^{-1} \sum_{d\in\mathscr{D}} \sum_{k\in\mathscr{K}} \sum_{i\in\mathscr{A}} d m_i^{d,k} \bar{I}_i^{d,k,+}, \tag{4}$$

and

$$\gamma \bar{I}_i^{d,k,+} = \lambda_i (1 - \bar{I}_i^{d,k,+}) d\bar{\Theta}^+. \tag{5}$$

Equations (4) and (5) yield

$$\bar{\Theta}^+ = \bar{d}^{-1} \sum_{d\in\mathscr{D}} \sum_{k\in\mathscr{K}} \sum_{i\in\mathscr{A}} d m_i^{d,k} \frac{\bar{\Theta}^+}{\frac{\gamma}{\lambda_i d} - \bar{\Theta}^+}. \tag{6}$$

We observe from (6) that $(\bar{\Theta}^+, \bar{I}^+) = (0,0)$ is the disease-free steady-state pair. The next result shows a sufficient condition under which a disease-free steady state in a "reopened" regime is approachable through a proper design of players' accuracy of signals $\sigma_d, \forall d \in \mathscr{D}$.

Proposition 4. *There exists $\Sigma^{d,k} \subset \mathbb{R}, \forall d \in \mathscr{D}, \forall k \in \mathscr{K}$, such that if $\sigma_k \in \cap_{d\in\mathscr{D}} \Sigma^{d,k}$, the disease-free steady-state pair $(\bar{\Theta}^+, \bar{I}^+) = (0,0)$ is globally asymptotically stable (GAS).*

Proof. Using the technique introduced in Theorem 1 of [3], we can show that the disease-free steady-state pair is GAS if

$$m_1^{d,k} \geq \frac{m^{d,k}(\frac{\gamma}{d\lambda} - 1)}{\beta - 1}, \forall d \in \mathscr{D}, \forall k \in \mathscr{K}. \tag{7}$$

Observing that x_k follows a normal distribution with mean μ and variance $\frac{1}{\sigma^2} + \frac{1}{\sigma_k^2}$, we obtain that the cumulative distribution function of x_k is given by

$$\Phi_k(x_k) := \Phi\left((x_k - \mu) \Big/ \sqrt{\frac{\sigma^2 + \sigma_k^2}{\sigma^2 \sigma_k^2}} \right).$$

We know that $m_1^{d,k} = m^{d,k} \Phi_k(x_k^*)$ with

$$x_k^* = \frac{\sqrt{\sigma^2 + \sigma_k^2} \Phi^{-1}(c) + \sigma^2 \mu - (\sigma^2 + \sigma_k^2)\theta^*}{-\sigma_k^2},$$

where θ^* satisfies the following fixed-point equation:

$$\theta^* = \sum_{k\in\mathscr{K}} m^k \Phi\left(\frac{\sqrt{\sigma^2 + \sigma_k^2} \Phi^{-1}(c) + \sigma^2\mu - \sigma^2\theta^*}{-\sigma_k} \right). \tag{8}$$

Therefore, combining (7) with $\Phi_k(x_k^*)$ yields, for all $k \in \mathscr{K}$:

$$\theta^* \geq \hat{\theta}^* := \mu + \frac{\Phi^{-1}(c)}{\sqrt{\sigma^2 + \sigma_k^2}} + \frac{\sigma_k}{\sigma\sqrt{\sigma^2 + \sigma_k^2}} \Phi^{-1}(e_d), \tag{9}$$

where $e_d := \frac{\frac{\gamma}{d\lambda} - 1}{\beta - 1}$. Next, we need to guarantee that θ^* solves the fixed-point equation (8) and satisfies the inequalities (9) simultaneously. Recall from the proof of Proposition 3 that θ^* solves the fixed-point equation (8) if and only if $W(\theta^*) = 0$, and that $W(\cdot)$ is a continuous function and $\lim_{\theta\to 1} W(\theta) < 0$. Hence, it suffices to show that for $\theta^* = \hat{\theta}^*$, $W(\theta^*) \geq 0$. By combining the definition of $W(\theta)$ and equation (9), we obtain the following limit points for all $k \in \mathscr{K}$:

$$\lim_{\sigma_k\to 0} W(\theta^*) = \sum_{k\in\mathscr{K}} m^k e_d - \mu - \frac{1}{\sigma}\Phi^{-1}(c), \tag{10}$$

and

$$\lim_{\sigma_k\to \infty} W(\theta^*) = \sum_{k\in\mathscr{K}} m^k (1-c) - \mu - \frac{1}{\sigma}\Phi^{-1}(e_d), \tag{11}$$

It can be shown that $\lim_{\sigma_k\to 0} W(\theta^*) < 0$ and $\lim_{\sigma_k\to\infty} W(\theta^*) > 0$ if $\Phi(\sigma(1-c-\mu)) < \mu + \frac{1}{\sigma}\Phi^{-1}(c)$; and that $\lim_{\sigma_d\to 0} W(\theta^*) > 0$ and $\lim_{\sigma_d\to\infty} W(\theta^*) < 0$ if $\Phi(\sigma(1-c-\mu)) > \mu + \frac{1}{\sigma}\Phi^{-1}(c)$. Therefore, we conclude that the disease-free steady state is GAS. The existence of $\Sigma^{d,k}, \forall d \in \mathscr{D}, \forall k \in \mathscr{K}$ follows from the fact that $W(\theta^*)$ is continuous in σ_k and the limit points of $W(\theta^*)$ have opposite signs. This completes the proof. □

Proposition 4 has corroborated the existence of private signals that guarantee the disease-free steady state of the epidemic process to be GAS under the reopened regime. The private signals, when chosen from the sets $\cap_{d\in\mathscr{D}}\Sigma^{d,k}$, nudge players to take the vaccine and drive the epidemic to extinction. The vaccination coverage threshold which guarantees safe reopening is obtained in (9).

Next, we provide another sufficient condition for the stability of the disease-free steady state.

From the proof of Proposition 4, we know that a sufficient condition for $(\bar{\Theta}^+, \bar{I}^+) = (0,0)$ to be GAS is $W(\hat{\theta}^*) \geq 0$. From the definition of $W(\cdot)$, we obtain

$$W(\hat{\theta}^*) = \sum_{k\in\mathscr{K}} m^k \Phi\left(-\frac{\sigma_k}{\sqrt{\sigma^2+\sigma_k^2}}\Phi^{-1}(c) + \frac{\sigma}{\sqrt{\sigma^2+\sigma_k^2}}\Phi^{-1}(e_d)\right) - \left(\mu + \frac{\Phi^{-1}(c)}{\sqrt{\sigma^2+\sigma_k^2}} + \frac{\sigma_k}{\sigma\sqrt{\sigma^2+\sigma_k^2}}\Phi^{-1}(e_d)\right). \tag{12}$$

The first term on the right-hand side of (12) is always positive. Define $Y(\sigma_k) := \mu + \frac{\Phi^{-1}(c)}{\sqrt{\sigma^2+\sigma_k^2}} + \frac{\sigma_k}{\sigma\sqrt{\sigma^2+\sigma_k^2}}\Phi^{-1}(e_d)$. If the condition $Y(\sigma_k) \leq 0$ is satisfied, the disease-free steady state is GAS. Observing that $Y(\cdot)$ is continuously differentiable on $(0,+\infty)$, we obtain $Y(\sigma_k) \leq 0$ if and only if $\lim_{\sigma_k\to 0} Y(\sigma_k) \leq 0$, $\lim_{\sigma_k\to+\infty} Y(\sigma_k) \leq 0$, and $Y(\hat{\sigma}_k) \leq 0$, where $\hat{\sigma}_k = \sigma\frac{\Phi^{-1}(e_d)}{\Phi^{-1}(c)}$ solves $\frac{\partial Y(\sigma_k)}{\partial \sigma_k} = 0$ being the stationary point of $Y(\cdot)$. By expressing explicitly these three conditions, we arrive at the following result.

Proposition 5. *The disease-free steady state $(\bar{\Theta}^+, \bar{I}^+) = (0,0)$ is GAS if the following condition holds for all $d \in \mathscr{D}$:*

$$\sigma\mu \leq \min\{-\Phi^{-1}(c), -\Phi^{-1}(e_d), -\frac{1+\Phi^{-1}(e_d)}{\sqrt{(\Phi^{-1}(c))^2 + (\Phi^{-1}(e_d))^2}}\}. \tag{13}$$

The product $\sigma\mu$ in (13) measures the concentration of the distribution of θ. Its reciprocal $\frac{1}{\sigma\mu}$, called the coefficient of variation, measures the dispersion of a probability distribution. Condition (13) requires the value of $\sigma\mu$ to be small. This indicates that it is the informational uncertainty about the vaccination coverage threshold which drives the epidemic process to the disease-free steady-state globally.

Since $\mu \in (0,1)$, the interesting case happens when the precision of the public signal goes to infinity, *i.e.*, $\sigma \to +\infty$. We leave the case with $\sigma \to +\infty$ and fixed μ and σ_k in the numerical experiments. Now, suppose that the precision of the private signals also goes to infinity, but the ratio of the precisions of the public signal and the private signals satisfies (3), *i.e.*, $\sigma_k \to +\infty$, $\frac{\sigma^2}{\sigma_k} = l \leq \frac{\sqrt{2\pi}}{m_k}$. Then, (8) becomes

$$\theta^* = \sum_{k\in\mathscr{K}} m^k \Phi\left(l\theta^* - l\mu + \Phi^{-1}(c)\right). \tag{14}$$

Leveraging the implicit function theorem, we express the change of θ^* with respect to μ as:

$$\frac{\partial\theta^*}{\partial\mu} = -\left(\sum_{k\in\mathscr{K}} m^k \phi(l\theta^* - l\mu + \Phi^{-1}(c))l - 1\right)^{-1} \cdot \left(\sum_{k\in\mathscr{K}} m^k \phi(l\theta^* - l\mu + \Phi^{-1}(c))l\right) \geq 0.$$

Hence, the solution to (14) increases in μ. In this extreme scenario where $\sigma \to +\infty$, if (13) is satisfied, we need $\mu \to 0$ to hold. Then, the vaccination coverage threshold θ^* solves

$$\theta^* = \sum_{k\in\mathscr{K}} m^k \Phi\left(l\theta^* + \Phi^{-1}(c)\right), \tag{15}$$

which yields the minimum threshold when the public signal is infinitely precise and concentrates approximately at 0.

Another perspective toward (13) is by focusing on the vaccination cost c, which is also a parameter that we can control. From (13), we obtain the following inequality of c:

$$\Phi\left(-\sqrt{\frac{1+(\Phi^{-1}(e_d))^2}{(\sigma\mu)^2} - (\Phi^{-1}(e_d))^2}\right) \leq c \leq \Phi(-\sigma\mu). \tag{16}$$

The upper bound of c in (16) suggests the pricing of the vaccines to the authorities. A high vaccine price demotivates people from vaccination. It results in a low vaccination coverage rate which fails to subdue the virus in the long run. The lower bound of c in (16) appears since (13) is only a sufficient condition. The sufficiency arises from using the Lyapunov's method and the relaxation of the condition (12).

VI. Numerical experiments

In this section, we continue the discussion of Proposition 5 using numerical experiments. We provide a suggested region of the precision of the public information which guarantees the disease-free epidemic steady-state while maintaining a high probability of reopening under the NE. Our goal in the experiments is to study the effect of public information. Hence, we set the degrees of all players to be equal for illustration purposes. We consider two information types. The parameters $c, \mu, \sigma_k, \beta, e_d$ are chosen so that there is a unique NE and $-\Phi^{-1}(c)$ is the minimum element in the right-hand side of (13). Note that when the parameters does not satisfy (3), simple learning algorithms still converge to the extreme points of the set of NE [12].

In Fig. 2, we plot two indices for a given reopening plan. The first index, the reopening probability, increases

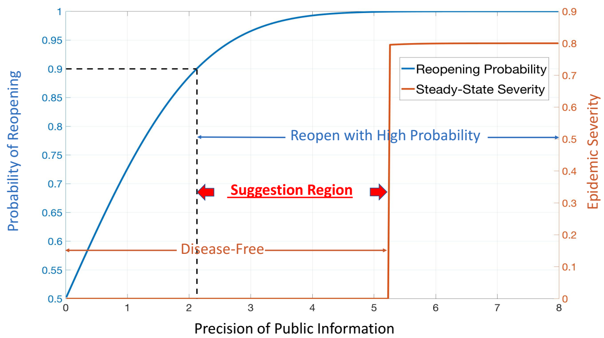

Fig. 2. The changes of the reopening probability and the steady-state epidemic severity are plotted for varied precision of the public information. Let the desired probability of reopening be 0.9, and we find the region of reopening with high probability. Aiming to eliminate the virus, we find the region of disease-free steady state. The suggested region of the precision of the public information is the intersection of these two regions.

monotonically as the precision of public information becomes higher. The second index, the epidemic severity at the steady state, is a piece-wise curve increasing in the precision of public information. Suppose that the authority wants a high probability (chosen as 0.9 in Fig. 2) for reopening so that the city has a higher likelihood of resuming its activities. Then, the desired public information lies above the value of the precision of public information which yields the reopening probability of 0.9 (shown using the dashed lines). Meanwhile, the way to control the outbreaks and the spreading of the virus affects the reopening plans. Hence, the authority keeps the public information in the disease-free region. As a consequence, our framework suggests a region of the precision of public information obtained at the intersection of the region of reopen with high probability and the disease-free region. Within this suggested region, condition (13) is satisfied and the disease-free epidemic steady state is GAS. The public information in this region is also precise enough, since individuals, upon receiving their private signals, have a posteriori belief that the probability of regime-switching is high. These beliefs will lead to affirmative vaccination decisions of a large proportion of the population, leading to the reopening of the city.

VII. Conclusion

In this paper, we have proposed EPROACH to capture the interplay between the population-level vaccination decisions under private and public information and the regime-switching of the epidemics. This framework has been motivated by developing reopening policies for cities when an increasing number of people become vaccinated. The analysis of the externalities has shown that the self-centered individual vaccination decisions become coordinated because of their anticipation of reopening. Leveraging the coordination effect, we have found a unique Nash equilibrium of the game. We have characterized players' vaccination decisions and the vaccination coverage threshold for safe reopening at the equilibrium. The uniqueness of the Nash equilibrium has informed the design of reopening plans with the proposed informational epidemic control method. We have observed that by choosing the resolution of the information, we can nudge the population to achieve the targeted vaccination threshold. In the numerical experiments, the informational epidemic control method has provided a suggested region of the resolution of the public signal. In this region, the reopening probability is high and the steady-state epidemic is eliminated after the reopening.

References

[1] R. Pastor-Satorras, C. Castellano, P. Van Mieghem, and A. Vespignani, "Epidemic processes in complex networks," *Reviews of modern physics*, vol. 87, no. 3, p. 925, 2015.

[2] J. Chen, Y. Huang, R. Zhang, and Q. Zhu, "Optimal curing strategy for competing epidemics spreading over complex networks," *IEEE Transactions on Signal and Information Processing over Networks*, vol. 7, pp. 294–308, 2021.

[3] S. Liu, Y. Zhao, and Q. Zhu, "Herd behaviors in epidemics: A dynamics-coupled evolutionary games approach," *Dynamic Games and Applications*, pp. 1–31, 2022.

[4] Y. Huang and Q. Zhu, "Game-theoretic frameworks for epidemic spreading and human decision-making: A review," *Dynamic Games and Applications*, pp. 1–42, 2022.

[5] W. H. Sandholm, *Population games and evolutionary dynamics*. MIT press, 2010.

[6] H. Carlsson and E. Van Damme, "Global games and equilibrium selection," *Econometrica: Journal of the Econometric Society*, pp. 989–1018, 1993.

[7] S. Morris and H. S. Shin, "Global games: Theory and applications," 2001.

[8] D. M. Frankel, S. Morris, and A. Pauzner, "Equilibrium selection in global games with strategic complementarities," *Journal of Economic Theory*, vol. 108, no. 1, pp. 1–44, 2003.

[9] S. Morris and H. S. Shin, "Coordination risk and the price of debt," *European Economic Review*, vol. 48, no. 1, pp. 133–153, 2004.

[10] G.-M. Angeletos, C. Hellwig, and A. Pavan, "Dynamic global games of regime change: Learning, multiplicity, and the timing of attacks," *Econometrica*, vol. 75, no. 3, pp. 711–756, 2007.

[11] R. Pastor-Satorras and A. Vespignani, "Epidemic spreading in scale-free networks," *Physical review letters*, vol. 86, no. 14, p. 3200, 2001.

[12] D. M. Topkis, "Equilibrium points in nonzero-sum n-person submodular games," *Siam Journal on control and optimization*, vol. 17, no. 6, pp. 773–787, 1979.

[13] D. Acemoglu, A. Fallah, A. Giometto, D. Huttenlocher, A. Ozdaglar, F. Parise, and S. Pattathil, "Optimal adaptive testing for epidemic control: combining molecular and serology tests," *arXiv preprint arXiv:2101.00773*, 2021.