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Flexible drinking water pumping to provide multiple grid services☆

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ABSTRACT

Drinking water distribution networks (WDNs) can be operated as flexible, controllable loads. In this paper, we consider using WDNs to provide local and grid level services simultaneously to the power grid. We formulate a robust water pumping problem to determine the amount of voltage support and frequency regulation that can be provided subject to network constraints while managing power demand uncertainty. We tractably reformulate the problem as a sequential optimization problem and solve for the scheduled water pumping operation, the frequency regulation capacity, and the optimal control policy parameters that update the pump operation based on the frequency regulation signal and power distribution network demand forecast error. We demonstrate our approach through detailed case studies. Additionally, we evaluate the performance of the reformulation and discuss the benefits and trade-offs of WDNs providing multiple services.

1. Introduction

Increasing amounts of flexibility are needed in the power grid to ensure reliable grid operation in the presence of higher penetrations of uncertain renewable energy sources. There is a significant body of work demonstrating that the control of demand-side resources can reduce the need for or replace traditional sources of flexibility in the power grid [1]. Typical sources of demand-side flexibility include energy storage units, thermostatically controlled loads, and plug-in electric vehicles. However, additional resources can also be leveraged, such as drinking water distribution networks (WDNs).

WDNs are capable of providing flexibility to the power grid. By leveraging the capacities of storage tanks and the flexibility of water pumps at different locations in the WDN, water pumping loads in the power distribution network (PDN) can be shifted both temporally and spatially. Most WDNs have supervisory control and data acquisition (SCADA) systems that enable water system operators to provide fast operational control [2]. PDNs and WDNs are traditionally operated independently; however, both systems can benefit from coordinated operation. These benefits include improving system reliability, integrating larger quantities of renewable energy resources, and reducing operational costs [3]. The WDN can provide several local and grid level services to the power grid. However, for any one service, the flexibility in the WDN may be underutilized. To improve utilization, the WDN can be operated to simultaneously provide local and grid-level services, which provides more overall benefit to the power system and also can increase the value proposition to the WDN operator.

The goal of this paper is to use WDNs to provide multiple simultaneous grid services. In particular, we focus on providing voltage support to the power distribution network and frequency regulation to the bulk transmission system. In order to provide robust guarantees on the safe operation of the PDN and WDN, we need to account for network demand uncertainty. To do this, we formulate a robust water pumping problem to simultaneously provide voltage support and frequency regulation subject to power and water distribution network constraints in the presence of power demand uncertainty. We first develop the full optimization problem where the scheduled WDN operation, voltage support, and frequency regulation are co-optimized. The formulation is nonconvex and mixed-integer, and so we reformulate the problem into multiple sub-problems and solve the problems sequentially. We then evaluate the performance trade-offs of the co-optimized and sequential problems.

There is a growing body of research that considers the integrated optimization of coupled PDN-WDN systems for power system services. In [4], water pumps are controlled in real-time to consume surplus energy from renewable energy sources. Refs. [5] and [6] solve for the demand response capacities of WDNs. In [7], the authors co-optimize WDN and PDN operation in order to minimize power loss in the PDN. Our previous work on coupled PDN-WDN systems considered providing a single local-level service (i.e., voltage support) while managing water and power demand uncertainties [8,9]. Most research assumes that the network demands are known and, to the best of our knowledge, no papers consider optimally controlling the water distribution network

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to provide multiple services while managing demand uncertainty. However, research on the use of flexible loads like battery energy storage to provide multiple simultaneous services [10] demonstrates the value of more fully utilizing available flexibility and harnessing multiple value streams.

The contributions of this paper are the (1) formulation of a robust optimization problem to simultaneously solve for the pump schedule, voltage support control actions, and frequency regulation capacity subject to WDN and PDN constraints while managing uncertainty in power demand and the frequency regulation signal, (2) tractable reformulation of the problem into a robust, mixed-integer convex sequential problem, (3) evaluation of the challenges associated with providing voltage support and frequency regulation together, and (4) demonstration of the performance of the proposed solution approach through a case study.

2. Problem description

Our goal is to optimize the WDN's operation and capacity allocated to grid services subject to PDN and WDN constraints while managing power demand uncertainty over the scheduling horizon $\mathcal{T}$. Specifically, the operation of the supply pumps in the WDN should not violate WDN or PDN constraints. We co-optimize the WDN's scheduled operation and the capacities reserved for voltage support and asymmetric up and down frequency regulation.

There are several sources of uncertainty in this problem. The power demand at each bus, phase, and time period $t \in \mathcal{T}$ is uncertain but bounded. We define the power demand at time t as the sum of the known, forecasted demand and the uncertain power demand forecast error vector $\boldsymbol{\Delta\rho}^t$, which includes the error at all buses and phases. Additionally, in order to provide frequency regulation, the WDN pumps need to adjust their power consumption based on uncertain up and down frequency regulation signals $\tilde{s}^t_{\text{up}} \in [-1,0]$ and $\tilde{s}^t_{\text{dn}} \in [0,1]$, respectively. The frequency regulation signals are scaled by the up and down frequency regulation capacities, which are decision variables, to obtain the required change in power. We define the uncertainty as $\boldsymbol{\omega}$, i.e., $\boldsymbol{\omega} = [\boldsymbol{\Delta\rho}^t, \tilde{s}^t_{\text{up}}, \tilde{s}^t_{\text{dn}}]_{t\in\mathcal{T}} \in \mathcal{U}$ where $\mathcal{U}$ is the uncertainty set.

We formulate the problem as an adjustable robust optimization problem [11] where $\mathbf{x}$ contains the operational variables and $\mathbf{y}$ contains the adjustable variables that are dependent on the uncertainty $\boldsymbol{\omega}$, specifically,

$$
\begin{aligned}
\min_{\mathbf{x}} \quad & F(\mathbf{x}, \mathbf{y}(\boldsymbol{\omega}, \mathbf{x})) && \textbf{(Co-optimized)}\\
\text{s.t.} \quad & \forall \boldsymbol{\omega} \in \mathcal{U}, \exists\, \mathbf{y},\\
& \Psi_1(\mathbf{x}, \mathbf{y}(\boldsymbol{\omega}, \mathbf{x}), \boldsymbol{\omega}),\\
& \Psi_2(\mathbf{x}, \mathbf{y}(\boldsymbol{\omega}, \mathbf{x}), \boldsymbol{\omega}),\\
& \Psi_3(\mathbf{x}, \mathbf{y}(\boldsymbol{\omega}, \mathbf{x}), \boldsymbol{\omega}),\\
& \Psi_4(\mathbf{x}, \mathbf{y}(\boldsymbol{\omega}, \mathbf{x}), \boldsymbol{\omega}).
\end{aligned}
$$

The cost function $F(\cdot)$ includes the costs of scheduled pump operation and real-time adjustments to provide voltage support and frequency regulation. The constraint sets $\Psi_1(\cdot)$ and $\Psi_2(\cdot)$ include the voltage support and frequency regulation capacities and control actions, respectively. The constraint sets $\Psi_3(\cdot)$ and $\Psi_4(\cdot)$ are the quasi-steady state PDN and WDN constraints for every time step $t \in \mathcal{T}$ of duration ΔT.

In our problem, the operational variable $\mathbf{x}$ includes the WDN schedule (in particular, the scheduled pump power consumption), the pump power capacity needed to provide voltage support, and the up and down frequency regulation capacity. Additionally, we solve for the affine control policy parameters used for voltage support and frequency regulation real-time adjustments. While an affine control policy restricts the feasible space of $\mathbf{y}(\boldsymbol{\omega}, \mathbf{x})$, a computationally tractable problem can be formulated and the real-time implementation of the control policy is simple for water utilities. The adjustable variables $\mathbf{y}$ are dependent on the control policy adjustments, e.g., the real-time pump power consumption and the bus voltages. We can define the real-time single-phase pump power consumption in terms of its schedule and real-time voltage support and frequency regulation adjustments

$$p^t_e = p^t_{\text{nom},e} + \Delta p^t_{\text{vs},e}(\boldsymbol{\omega}^t) + \Delta p^t_{\text{fr},e}(\boldsymbol{\omega}^t) \quad \forall\, \omega \in \mathcal{U}, e \in \mathcal{P}, t \in \mathcal{T}, \tag{1}$$

where $\mathcal{P}$ is the set of pumps in the WDN, $\Delta p^t_{\text{vs},e}$ is the real-time voltage support adjustment based on the power demand forecast error, and $\Delta p^t_{\text{fr},e}$ is the real-time frequency regulation adjustments based on the up and down frequency regulation signals from the bulk transmission system. We assume that the pumps are balanced three-phase loads and therefore we do not specify the phase of the pump power consumption; the power consumed in each phase is equal. These real-time voltage support and frequency regulation control policies are described in Sections 2.1 and 2.2. We next model the voltage support $\Psi_1(\cdot)$, frequency regulation $\Psi_2(\cdot)$, PDN $\Psi_3(\cdot)$, and WDN $\Psi_4(\cdot)$ constraints. The constraints are semi-infinite, since they must hold true for all uncertainty realizations. However, we discuss how to tractably reformulate the constraints in Sections 2.4 and 3.

2.1. Voltage support, $\Psi_1(\cdot)$

We first consider the set of constraints that make up $\Psi_1(\cdot)$. In order to ensure that voltages in the PDN are within safe operating conditions for all power demand uncertainty, we formulate a control policy to adjust the pump power setpoints in response to the real-time power demand forecast error realizations, leveraging the approach in [8]. The voltage support pump power adjustment in (1) can then be written as

$$\Delta p^t_{\text{vs},e} = \boldsymbol{C}^t_{\text{vs},e} \boldsymbol{\Delta\rho}^t \quad \forall\, e \in \mathcal{P}, t \in \mathcal{T}, \tag{2}$$

where decision variable $\boldsymbol{C}^t_{\text{vs},e}$ is the voltage support control policy parameter row vector. The control policy relates the power demand forecast error at each bus and phase to a change in pump e's power consumption. We can define the range of up and down pump power adjustments needed for voltage support by bounding the control policy

$$-R^t_{\text{vs,dn},e} \le \boldsymbol{C}^t_{\text{vs},e} \boldsymbol{\Delta\rho}^t \le R^t_{\text{vs,up},e} \qquad \forall\, e \in \mathcal{P}, t \in \mathcal{T}, \tag{3a}$$

$$R^t_{\text{vs,up},e}, R^t_{\text{vs,dn},e} \ge 0 \qquad \forall\, e \in \mathcal{P}, t \in \mathcal{T}, \tag{3b}$$

where $R^t_{\text{vs,dn},e}$ and $R^t_{\text{vs,up},e}$ are the largest decrease and increase in single-phase pump power consumption due to voltage support services.

2.2. Frequency regulation, $\Psi_2(\cdot)$

We next define the set of frequency regulation constraints that make up $\Psi_2(\cdot)$. We consider both up and down frequency regulation services. Using generator sign convention, up frequency regulation corresponds to a decrease in pump power consumption and down frequency regulation corresponds to an increase in pump power consumption. We solve for the amount of capacity that the WDN can provide at each time period as well as the participation of each pump in response to the up and down frequency regulation signals. The frequency regulation pump power adjustment in (1) can be written as

$$\Delta p^t_{\text{fr},e} = C^t_{\text{fr,up},e} R^t_{\text{fr,up}} \tilde{s}^t_{\text{up}} + C^t_{\text{fr,dn},e} R^t_{\text{fr,dn}} \tilde{s}^t_{\text{dn}} \quad \forall\, e \in \mathcal{P}, t \in \mathcal{T}, \tag{4}$$

where decision variables $R^t_{\text{fr,up}}$ and $R^t_{\text{fr,dn}}$ are the up and down single-phase frequency regulation capacities at time t. Decision variables $C^t_{\text{fr,up},e}$ and $C^t_{\text{fr,dn},e}$ are the up and down control policy parameters of pump e at time t. The frequency regulation capacities are non-negative and the frequency regulation control policy parameters must sum to one to ensure that the requested power adjustment is being fully met by the pumps, i.e.,

$$R^t_{\text{fr,up}}, R^t_{\text{fr,dn}} \ge 0 \qquad \forall\, t \in \mathcal{T}, \tag{5}$$

$$\sum_{e\in\mathcal{P}} C^t_{\text{fr,up},e} = 1 \qquad \forall\, t \in \mathcal{T}, \tag{6}$$

$$\sum_{e\in\mathcal{P}} C^t_{\mathrm{fr,dn},e} = 1 \qquad \forall t \in \mathcal{T}. \tag{7}$$

To remove the bilinear terms in the control policy, we can replace the frequency regulation control policy parameters and capacity terms in (4)–(7) with

$$F^t_{\mathrm{up},e} := C^t_{\mathrm{fr,up},e} R^t_{\mathrm{fr,up}} \qquad \forall e \in \mathcal{P}, t \in \mathcal{T}, \tag{8}$$

$$F^t_{\mathrm{dn},e} := C^t_{\mathrm{fr,dn},e} R^t_{\mathrm{fr,dn}} \qquad \forall e \in \mathcal{P}, t \in \mathcal{T}, \tag{9}$$

$$F^t_{\mathrm{up},e}, F^t_{\mathrm{dn},e} \geq 0 \qquad \forall e \in \mathcal{P}, t \in \mathcal{T}, \tag{10}$$

where $F^t_{\mathrm{up},e}$ and $F^t_{\mathrm{dn},e}$ are decision variables in $\Psi_2(\cdot)$. Then we can recover the up and down frequency regulation capacities and control policy parameters *a posteriori*, e.g., the recovered down frequency regulation variables are

$$R^t_{\mathrm{fr,dn}} := \sum_{e\in\mathcal{P}} F^t_{\mathrm{dn},e} \qquad \forall t \in \mathcal{T}, \tag{11}$$

$$C^t_{\mathrm{fr,dn},e} := \frac{F^t_{\mathrm{dn},e}}{\sum_{e\in\mathcal{P}} F^t_{\mathrm{dn},e}} \qquad \forall e \in \mathcal{P}, t \in \mathcal{T}. \tag{12}$$

2.3. Power distribution network modeling, $\Psi_3(\cdot)$

Next, we define $\Psi_3(\cdot)$, the power distribution network model. We consider a radial power distribution network that contains uncontrollable net loads (i.e., actual loads minus distributed generation) and controllable pumping loads that are connected to a set of buses $\mathcal{K}$ and phases $\boldsymbol{\Phi} = \{a, b, c\}$. We must ensure a feasible power flow where the minimum and maximum voltage limit constraints are satisfied, i.e.,

$$\underline{\boldsymbol{V}}^2 \leq \boldsymbol{Y}^t_k \leq \overline{\boldsymbol{V}}^2 \quad \forall k \in \mathcal{K}, t \in \mathcal{T}, \tag{13}$$

where $\boldsymbol{Y}^t_k$ is the three-phase voltage magnitude squared at bus k and time t. The voltage magnitude squared is calculated from the linearized, three-phase unbalanced power flow equations, which are commonly referred to as Lin3DistFlow [12]

$$\boldsymbol{Y}^t_k = \boldsymbol{Y}^t_n - \boldsymbol{M}_{kn}\boldsymbol{P}^t_n - \boldsymbol{N}_{kn}\boldsymbol{Q}^t_n \qquad \forall k \in \mathcal{K}, t \in \mathcal{T}, \tag{14}$$

$$\boldsymbol{P}^t_k = \boldsymbol{\rho}^t_k + \boldsymbol{p}^t_e + \sum_{n\in\mathcal{I}_k} \boldsymbol{P}^t_n \qquad \forall k \in \mathcal{K}, t \in \mathcal{T}, \tag{15}$$

$$\boldsymbol{Q}^t_k = \boldsymbol{\zeta}^t_k + \eta_e \boldsymbol{p}^t_e + \sum_{n\in\mathcal{I}_k} \boldsymbol{Q}^t_n \qquad \forall k \in \mathcal{K}, t \in \mathcal{T}, \tag{16}$$

where $\boldsymbol{P}^t_k$ and $\boldsymbol{Q}^t_k$ are the real and reactive three-phase power flows entering bus k at time t, $\boldsymbol{M}_{kn}$ and $\boldsymbol{N}_{kn}$ are 3×3 parameter matrices formed from the line impedance matrices, and $\mathcal{I}_k$ is the set of buses directly downstream of bus k. The three-phase real and reactive uncontrollable power demand at bus k and time t is denoted $\boldsymbol{\rho}^t_k$ and $\boldsymbol{\zeta}^t_k$, respectively. The variable $\boldsymbol{p}^t_e$ is the three-phase pump power consumption vector of pump e at time t. In (15) and (16), the pump power consumption is zero if there are no pumps present at bus k. We model the pumps as three-phase balanced loads, with a constant power factor (i.e., η_e is the real-to-reactive power ratio of pump e). We note that other three-phase unbalanced linearized power flow formulations could be used, e.g., [13].

2.4. Water distribution network modeling, $\Psi_4(\cdot)$

Last, we define $\Psi_4(\cdot)$, the water distribution network model. The WDN can be represented as a directed graph composed of a set of nodes $\mathcal{N}$ and a set of edges $\mathcal{E}$. The nodes are made up of disjoint sets of junctions $\mathcal{J}$, elevated storage tanks $\mathcal{S}$, and reservoirs $\mathcal{R}$. The edges, or pipes, are bidirectional and connect nodes in the network, e.g., $ij \in \mathcal{E}$ is a pipe going from node i to node j. A pipe may contain a supply pump, i.e., $\mathcal{P} \subseteq \mathcal{E}$. A WDN's water flow can be described by the hydraulic head H^t_j for each node $j \in \mathcal{N}$ and the volumetric flow rate x^t_{ij} of water through each pipe $ij \in \mathcal{E}$. We do not explicitly consider water demand uncertainty. In [8], we found it reasonable to assume that a portion of the tank volume is reserved to hedge against water demand uncertainty. To ensure safe operation, the hydraulic heads (which are composed of the elevation and pressure head) must be between the minimum and maximum head limits, i.e.,

$$\underline{H}_j \leq H^t_j \leq \overline{H}_j \quad \forall j \in \mathcal{N}, t \in \mathcal{T}. \tag{17}$$

Additionally, the tank water levels ℓ^t_j and supply pump flow rates are bounded

$$\underline{\ell}_j \leq \ell^t_j \leq \overline{\ell}_j \qquad \forall j \in \mathcal{S}, t \in \mathcal{T}, \tag{18}$$

$$0 \leq \underline{x}_{ij} \leq x^t_{ij} \leq \overline{x}_{ij} \qquad \forall ij \in \mathcal{P}, t \in \mathcal{T}, \tag{19}$$

where $\underline{\ell}_j$ and $\overline{\ell}_j$ are the minimum and maximum tank levels of tank j, and $\underline{x}_{ij}$ and $\overline{x}_{ij}$ are the minimum and maximum flow rates of pump $ij \in \mathcal{P}$. The hydraulic heads and flow rates are governed by the water flow equations

$$\sum_{i:ij\in\mathcal{E}} x^t_{ij} = -d^t_j \qquad \forall j \in \mathcal{N}, t \in \mathcal{T}, \tag{20}$$

$$x^t_{ij} = -x^t_{ji} \qquad \forall ij \in \mathcal{E}, t \in \mathcal{T}, \tag{21}$$

$$H^t_j - H^t_i = k_{ij} x^t_{ij} |x^t_{ij}| \qquad \forall ij \in \mathcal{E} \setminus \mathcal{P}, t \in \mathcal{T}, \tag{22}$$

$$H^t_i - H^t_j = m^1_{ij} x^t_{ij} + m^0_{ij} \qquad \forall ij \in \mathcal{P}, t \in \mathcal{T}, \tag{23}$$

$$H^t_j = \bar{h}_j \qquad \forall j \in \mathcal{R}, t \in \mathcal{T}, \tag{24}$$

$$\ell^t_j = \ell^{t-1}_j + \frac{\Delta T}{\gamma_j} \sum_{i:ij\in\mathcal{E}} x^t_{ij} \qquad \forall j \in \mathcal{S}, t \in \mathcal{T}, \tag{25}$$

where d^t_j is the water injection at node j and time t, k_{ij} is the resistance coefficient of pipe ij, m^1_{ij} and m^0_{ij} are head loss parameters of pump ij, γ_j is the cross-sectional area of tank j, and $\bar{h}_j$ is the elevation of node j. Conservation of water is ensured by (20) and (21) specifies skew symmetry of water flow through the pipes. In (22)–(23), the head loss and head gain as a function of flow rate is defined for pipes and pumps that are operating, respectively. The pipe head loss function is modeled using the Darcy–Weisbach formulation [14]. When a pump is on, the pump head gain of a fixed speed pump is generally modeled in the literature with either a linear or quadratic function of the flow rate through the pump. Here, we use a linear form. When the pump is off, the pump behaves like a closed valve and the pump head gain is arbitrary. To formulate this, we can introduce a binary pump status variable and use the big-M method to formulate this equation. Reservoirs are modeled as infinite sources of water with a fixed head, which is specified in (24). In (25), the water level of tank j is calculated based on the tank level in the previous time period and the net flow of water into and out of the tank.

The single-phase pump power consumption is generally modeled as a linear, quadratic, or cubic function of the flow rate through the pump. Here, we model it with a linear function

$$p^t_e = h^1_{ij} x^t_{ij} + h^0_{ij} \quad \forall e = ij \in \mathcal{P}, t \in \mathcal{T}, \tag{26}$$

where h^1_{ij} and h^0_{ij} are parameters. The water distribution network constraints are semi-infinite since the constraints must hold for all realizations of uncertainty (which enters the WDN constraints in the real-time pump power consumption equation (26)).

We tractably reformulate the semi-infinite WDN constraints into three sets of deterministic constraints using the monotonicity properties of dissipative flow networks [15]. In order to apply the monotonicity properties to the WDN, several assumptions must be made [9]. We assume that the tank head is not strictly dependent on the tank level (i.e., there is either a booster pump and/or valve connected to the tank inlet and outlet pipe), the head loss functions are increasing in flow rate, and that an increase in reservoir water injections cause the deviation in tank water injections to be non-positive for all tanks (i.e., the tanks do not also increase their water injection). We also assume that the pump statuses do not change in real-time (i.e., the real-time on/off pump statuses are the same as the schedule) to minimize pump wear-and-tear and ensure monotonicity. The robustness proof and implications of these assumptions are further discussed in [9]. With these assumptions, we can prove that the hydraulic heads at all nodes, the tank levels, and pump flows are monotonic functions of the reservoir water injections (which vary based on the voltage support and frequency regulation control actions). The WDN constraints can then be reformulated as the schedule and extreme cases of the pump power consumption

$$\Gamma_{\text{scheduled}}(\boldsymbol{p}_{\text{nom}}) \leq 0, \tag{27a}$$

$$\Gamma_{\text{extreme}}(\overline{\boldsymbol{p}}) \leq 0, \tag{27b}$$

$$\Gamma_{\text{extreme}}(\underline{\boldsymbol{p}}) \leq 0, \tag{27c}$$

where $\Gamma_{\text{scheduled}}(\boldsymbol{p}_{\text{nom}})$, $\Gamma_{\text{extreme}}(\overline{\boldsymbol{p}})$, and $\Gamma_{\text{extreme}}(\underline{\boldsymbol{p}})$p are the sets of deterministic WDN constraints (17)–(26) given the scheduled, minimum, and maximum power consumption

$$\overline{p}^t_e = p^t_{\text{nom},e} + R^t_{\text{vs,up},e} + R^t_{\text{fr,dn},e} \qquad \forall\, e \in \mathcal{P}, t \in \mathcal{T}, \tag{28}$$

$$\underline{p}^t_e = p^t_{\text{nom},e} - R^t_{\text{vs,dn},e} - R^t_{\text{fr,up},e} \qquad \forall\, e \in \mathcal{P}, t \in \mathcal{T}, \tag{29}$$

where the pumps are balanced three-phase loads.

2.5. Providing both voltage support and frequency regulation

When treating the WDN as a flexible load, we must ensure that the tanks are not simply depleted over the scheduling horizon. This issue is magnified when we include asymmetric frequency regulation services. We address this by specifying a total volume of water that must be in the storage tanks at the end of the scheduling horizon

$$\sum_{j\in\mathcal{S}} \gamma_j \ell_j^{t=|\mathcal{T}|} = \hat{v} + \sum_{j\in\mathcal{S}} \gamma_j \ell_j^{t=0}, \tag{30}$$

where $\hat{v}$ is the water deficit from the previous scheduling horizon that must be recovered. We include (30) in (27a). Alternatively, if we wished to correct the tank levels in each time period, we could incorporate a random variable that compensates for the previous time period's water deviation from the scheduled operation, but we leave this to future work.

A challenge with providing both voltage support and frequency regulation simultaneously is to ensure that the frequency regulation services are not creating voltage issues within the PDN. The goal of voltage support is to provide the smallest pump power adjustments to ensure that the bus voltages are within their limits. Therefore, any additional pump adjustment in the opposite direction of the voltage support adjustments will counteract the voltage support control action and either require more voltage support capacity or cause voltage limit violations. To address this, we consider asymmetric frequency regulation services and require indicator functions to ensure up or down frequency regulation are only provided if it does not cancel out the voltage support control action. This can be done by checking whether there are maximum or minimum voltage limit violations given the scheduled pump operation and power demand uncertainty. For example, if a PDN is experiencing voltages that violate the minimum voltage limit, the voltage support control policy would reduce the pump power consumption which would then increase the voltages. In this case, no down frequency regulation services (increase pump power consumption) can be provided without requiring a larger voltage support capacity to counteract it. However, up frequency regulation (decrease pump power consumption) can still be provided. If there are no voltage limit violations given the scheduled pump operation and power demand uncertainty, both up and down frequency regulation can be provided. In Section 3, we tractably reformulate the problem as a robust, mixed-integer convex sequential problem.

3. Sequential reformulation

The formulation presented in Section 2 is a mixed-integer adjustable robust optimization problem due to the presence of the indicator functions and binary pump status variables. While there are some related approaches and results to tractably reformulate mixed-integer robust problems, they do not appear to directly apply to our problem. Instead, we solve this problem sequentially as three sub-problems. We first solve the robust voltage support and pump scheduling problem. Next, we identify whether the WDN is capable of providing up and/or down frequency regulation at each time period by solving for the worst-case voltages (i.e., minimum and maximum voltages) given the pump schedule and power demand forecast error. Last, we solve the appropriate robust frequency regulation problem.

By separating **(Co-optimized)** into three sub-problems, we are able to eliminate the indicator functions needed to identify the direction(s) of frequency regulation that the WDN can provide and allows us to solve computationally tractable robust reformulations. Each sub-problem is described in the subsections below. It should be noted that the solution of the sequential problem will be a feasible solution of the **(Co-optimized)** problem; however, it may not be the optimal solution. The separate optimization problems no longer experience a trade-off between the cost of the WDN schedule and the profit of providing frequency regulation. In Section 4.2, we explore this trade-off by comparing the solutions of the sequential problem with special cases of **(Co-optimized)** problem, specifically, those in which we know in advance the type of frequency regulation that can be provided. This allows us to neglect the indicator functions so we can tractably reformulate **(Co-optimized)**. Further investigation of how to tractably reformulate the adjustable robust optimization problem while ensuring that the voltage support and frequency regulation do not cancel each other out is a subject for future research.

3.1. Step 1: Voltage support problem

In the first sub-problem, we solve the scheduled pump power consumption and voltage support control policy parameters while satisfying WDN and PDN constraints and managing power demand uncertainty. The decision variables are the scheduled pump power consumption $p^t_{\text{nom},e}$, the voltage support control policy parameters $\boldsymbol{C}^t_{\text{vs},e}$, and the voltage support capacities $R^t_{\text{vs,up},e}$ and $R^t_{\text{vs,dn},e}$ for all pumps $e \in \mathcal{P}$ and all time periods $t \in \mathcal{T}$. The optimization problem can be written as

$$\begin{aligned}
\min_{\mathbf{x}} \quad & \sum_{t\in\mathcal{T}} \sum_{e\in\mathcal{P}} 3\pi^t_e p^t_{\text{nom},e} + 3\pi^t_{\text{vs},e} \left(R^t_{\text{vs,up},e} + R^t_{\text{vs,dn},e} \right) \\
\text{s.t.} \quad & \hat{\Psi}_{\text{vs}}(\mathbf{x}) \leq 0, \qquad\qquad \textbf{(Seq-VS)} \\
& \overline{p}^t_e = p^t_{\text{nom},e} + R^t_{\text{vs,up},e} \quad \forall\, e \in \mathcal{P}, t \in \mathcal{T}, \\
& \underline{p}^t_e = p^t_{\text{nom},e} - R^t_{\text{vs,dn},e} \quad \forall\, e \in \mathcal{P}, t \in \mathcal{T}, \\
& (27),
\end{aligned}$$

where π^t_e and $\pi^t_{\text{vs},e}$ are the costs of electricity and voltage support capacity. We can substitute the power flow constraints (14)–(16), the voltage support control policy constraints (2)–(3b), and the coupling constraint between the pump load and power demand forecast error (i.e., $p^t_e = p^t_{\text{nom},e} + \Delta p^t_{\text{vs},e}(\boldsymbol{\Delta\rho}^t)$) into (13). Since the resulting inequalities are linear in the decision variables and uncertainty, we can use explicit maximization [16] to robustly reformulate the problem. We denote the robust reformulation of the power constraints as $\hat{\Psi}_{\text{vs}}(\mathbf{x})$.

In (27), we approximate the pipe head loss constraints using a quasi-convex hull proposed in [17]. While this approximation is not necessary to reformulate the semi-infinite water constraints as deterministic sets of constraints, it does make the formulation mixed-integer convex.

3.2. Step 2: Frequency regulation preprocessing

Before solving the frequency regulation problem, we need to identify the direction(s) of frequency regulation the WDN can provide. We robustly solve for the worst-case minimum and maximum voltages within the PDN at each time period given the pump schedule and all power demand forecast error uncertainty realizations. For example, to solve for the minimum voltage over all buses and phases at time t, the robust problem is

$$\max_{Y^t_{\min}} \quad Y^t_{\min} \qquad \textbf{(Seq-Vmin)}$$
$$\text{s.t.} \quad Y^t_{\min} \leq \mathbb{Y}^t_{k,\phi}(\boldsymbol{p}^t_{\text{nom}}, \boldsymbol{\Delta\rho}^t) \quad \forall \boldsymbol{\Delta\rho} \in \mathcal{U}.$$

where $\mathbb{Y}^t_{k,\phi}(\boldsymbol{p}^t_{\text{nom}}, \boldsymbol{\Delta\rho}^t)$ is the voltage magnitude squared at bus k and phase ϕ which is an affine function of the power demand forecast errors $\boldsymbol{\Delta\rho}^t$ and the scheduled three-phase pump power consumption vector $\boldsymbol{p}^t_{\text{nom}}$. The robust problem can be reformulated as the deterministic robust counterpart and solved for the worst-case minimum and maximum voltage magnitudes. At each time period, there are four possible cases: (i) if the voltage limits are satisfied, then the WDN can provide both up and down frequency regulation; (ii) if the minimum and maximum voltage limits are violated, then the WDN cannot provide frequency regulation; (iii) if only the maximum voltage limits are violated, then the WDN can only provide down frequency regulation; and (iv) if only the minimum voltage limits are violated, then the WDN can only provide up frequency regulation.

3.3. Step 3: Frequency regulation

In the third and final sub-problem, we solve for the up and down frequency regulation capacities subject to the WDN and PDN constraints while managing power demand forecast error and frequency regulation signals. If the WDN cannot provide either up or down frequency regulation, we force the respective up or down frequency regulation capacity to zero. We solve for $F^t_{\text{fr,up},e}$ and $F^t_{\text{fr,dn},e}$ and recover the frequency regulation capacity and control policy parameters *a posteriori*. The robust frequency regulation optimization problem is then

$$\max_{\mathbf{x}} \quad \sum_{t\in\mathcal{T}} \sum_{e\in\mathcal{P}} \pi^t_{\text{fr,up},e} F^t_{\text{fr,up},e} + \pi^t_{\text{fr,dn},e} F^t_{\text{fr,dn},e} \qquad \textbf{(Seq-FR)}$$
$$\text{s.t.} \quad (27b)\text{–}(29),$$
$$\hat{\Psi}_{\text{fr}}(\mathbf{x}) \leq 0$$

where $\pi^t_{\text{fr,up},e}$ and $\pi^t_{\text{fr,dn},e}$ are the prices associated with providing up and down frequency regulation, $\hat{\Psi}_{\text{fr}}(\mathbf{x})$ is the robust reformulation of the power flow constraints (13)–(16), the frequency regulation control policy constraints (4), (8)–(10), and the coupling constraint between the pump load and real-time uncertainty (1).

4. Case study

In our case study, we consider a coupled PDN-WDN system, shown in Fig. 1. We first describe the setup of the case study and then present the results of the sequential problem. Additionally, we explore the value of co-optimizing the WDN schedule and the frequency regulation capacity.

4.1. Set up

The WDN is an example network (NET3) included in the EPANET software, a WDN simulator [18]. The network parameters are from the EPANET input file with several modifications. The pump parameters are $h^1_{ij} = [0.12, 0.08]$ kW/CMH, $h^0_{ij} = [53.22, 8.42]$ kW, $m^1_{ij} = [-1.09 \times 10^{-2}, -1.3 \times 10^{-2}]$ m/CMH, and $m^0_{ij} = [60.96, 31.70]$m with a minimum and maximum flow rate of $\underline{x}_{ij} = [0, 0]$ CMHx and $\overline{x}_{ij} = [2700, 905]$ CMH for pumps 1 and 2, respectively. The minimum head at each node is the sum of the elevation and a minimum pressure head of 15 m.

For the PDN, we use the IEEE-13 bus topology [19] with the same modifications and assumptions as [8]. Pumps 1 and 2 are connected to buses 10 and 5, respectively. The voltage is constrained to 0.95–1.05 pu. We multiply the power demand loads by 1.4 in order to have a heavily loaded network that is close to the minimum voltage limit. The power demand forecast error at each bus, phase, and time period is unknown but bounded by a percentage of the forecasted load $\bar{\rho}^t_{k,\phi}$, i.e., $[-\sigma\bar{\rho}^t_{k,\phi}, \sigma\bar{\rho}^t_{k,\phi}]$ where σ is a user-specified percentage. We select different σ values to change the size of the uncertainty set.

We consider a 12-hour scheduling horizon. We set $\hat{v} = 10$ m^3, $\pi^t_{\text{vs},e} = 0.025$ \$/kWh, and $\pi^t_{\text{fr},e} = 0.025$ \$/kWh. The electricity prices are from the Midcontinent Independent System Operator (MISO) on July 21st, 2021 [20]. We solve the mixed-integer convex sequential problem using the Gurobi solver [21] and the JuMP package in Julia.

4.2. Results

We first solve the sequential problem where $\sigma = 7.5\%$. In Fig. 2, the scheduled pump power consumption, the range of voltage support capacity and frequency regulation capacity around the schedule are depicted for each pump. In this case, we see that the voltage support capacity is nonzero. This indicates that, given the pump schedule and the power demand uncertainty set, voltage limit violations would occur without real-time voltage support control actions. In this case, the preprocessing sub-problem found that the PDN would violate the minimum voltage limit without the voltage support control actions. The voltage support control policy is responsible for taking the smallest pump power control action to reduce pumping so that the voltages are within the safe operating range. Any increase in pump power consumption would counteract the voltage support control policy. As a result, the WDN can only provide up frequency regulation capacity.

Table 1 evaluates the robust sequential solutions as we vary the size of the uncertainty set (i.e., by varying σ). We present the average range of three-phase voltage support pump power adjustments (i.e., $R^t_{\text{vs}} = \sum_{e\in\mathcal{P}} 3 \cdot (R^t_{\text{vs,up},e} + R^t_{\text{vs,dn},e})$) and the average three-phase up and down frequency regulation capacity over the scheduling horizon. For $\sigma = 3.5$–4.5%, there are no power demand uncertainty realizations that cause voltage limit violations (i.e., the voltage support control policy parameters and R_{vs} are zero). As a result, the WDN can provide both up and down frequency regulation capacity. We observe that the WDN is generally able to provide more up capacity (consume less power) than down capacity (consume more power) since the network is closer to the minimum voltage limit. As σ increases, the worst-case voltages given the scheduled pump power consumption and power demand uncertainty set are closer to or at the minimum voltage limit, reducing the amount of down frequency regulation that the WDN can provide. For $\sigma = 5.5$–8.0%, there are now minimum voltage limit violations and the voltage support control policy has non-zero parameters. Because of this, the WDN can only provide up frequency regulation. As σ increases, the frequency regulation capacity decreases since an increased amount of capacity is needed for voltage support and larger power demand uncertainty realizations cause the PDN to be closer to voltage limit violations. For σ values larger than 8.0%, the WDN is unable to provide voltage support fully and so the problem is infeasible.

One drawback with the sequential problem formulation is that the optimization problem no longer considers the trade-off between the pump scheduling cost and frequency regulation profit since they are now two separate optimization problems. In Fig. 2, the WDN operates pump 2 at its maximum setpoint. Pump 1 is used to provide the remaining water demand since it is more expensive than pump 2. In a sequential problem, the WDN cannot evaluate the cost/profit trade-offs of operating at a slightly more expensive schedule and providing more frequency regulation.

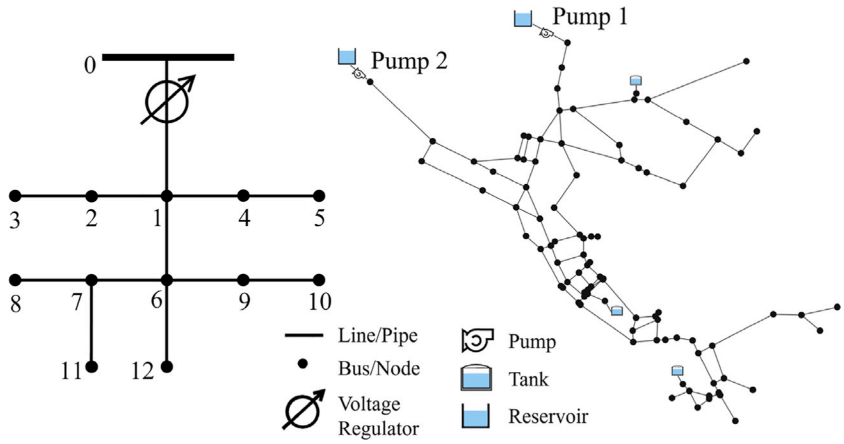

Fig. 1. Coupled power (left) and water (right) distribution networks. Pumps 1 and 2 are connected to buses 10 and 5, respectively.

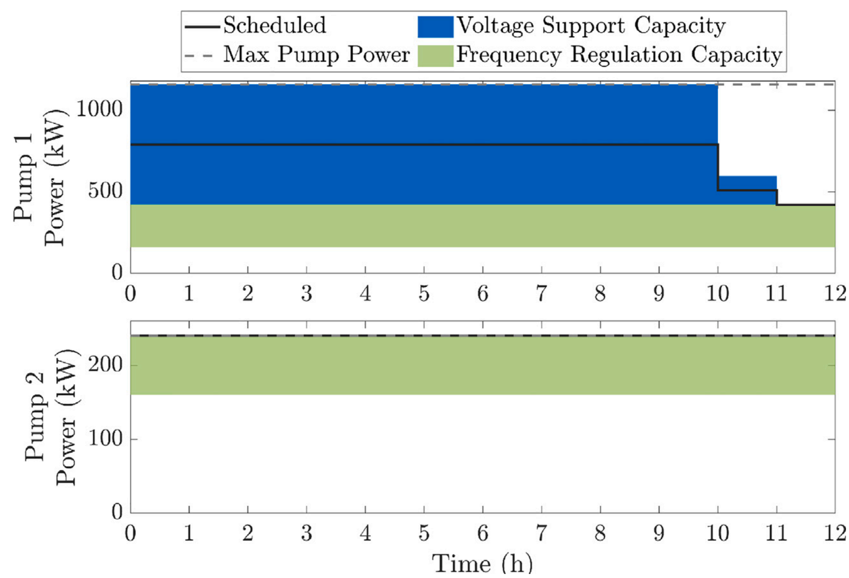

Fig. 2. Three-phase pump power consumption in the sequential problem (σ = 7.5%). The solid black lines indicates the schedule, blue and green bands indicate the range of pump power adjustments allocated for voltage support and frequency regulation.

We next investigate the affect that the WDN schedule has on the frequency regulation capacity. We do this by comparing the solutions of the sequential problem with a special case of the co-optimized problem where the type of frequency regulation that the WDN can provide is known. Under this assumption, the sequential problem can be tractably reformulated into a deterministic, mixed-integer convex program. We focus on the specific case where the power demand uncertainty does not cause voltage violations in the PDN. In this case, the WDN does not provide voltage support (i.e., zero voltage support control policy parameters). We compare the solutions of the sequential problem and three co-optimized problems with differing up and down frequency regulation prices.

Table 2 reports the average three-phase up and down frequency regulation capacity over the scheduling horizon. Since the WDN can provide both up and down frequency regulation when σ = 4.5%, we can observe the trade-off between minimizing the cost of the pumping schedule and the profit from allocating frequency regulation capacity. In the first co-optimized case, the WDN provides more frequency regulation capacity than in the sequential case. As we decrease the frequency regulation prices $\pi_{\mathrm{fr,up}}$ and $\pi_{\mathrm{fr,dn}}$, the co-optimized problem prioritizes minimizing the scheduled pumping operation cost over the profit from frequency regulation. In the third co-optimized case, the schedule and frequency regulation capacity are the same as in the sequential case. While not illustrated in this example, we have also found that if the profit from up and down frequency regulation are different, the co-optimized problem may shift the scheduled pumping operation to a more expensive operating point to provide larger levels of the more profitable type of frequency regulation. When the WDN can only provide up or down frequency regulation, we can expect to see the co-optimized problem shifting the pump operation away from the least expensive operating point to realize higher profits from frequency regulation services.

Table 1
Average three-phase results over 12-hour scheduling horizon.

σ (%)	R_{vs} (kW)	$R_{\mathrm{fr,up}}$ (kW)	$R_{\mathrm{fr,dn}}$ (kW)
3.5	0	355.4	128.5
4.5	0	401.5	65.7
5.5	53.4	465.8	0
6.5	340.6	465.8	0
7.5	627.7	341.7	0
8.0	771.3	269.9	0

Last, we discuss the impact of the WDN approximations on our solution. While it is not required in the analytical reformulation of the semi-infinite robust water flow constraints, we employ approximations

Table 2
Comparison of sequential and co-optimized solutions ($\sigma = 4.5\%$).

Case	$\pi_{\mathrm{fr,up}}$ (\$/kWh)	$\pi_{\mathrm{fr,dn}}$ (\$/kWh)	$R_{\mathrm{fr,up}}$ (kW)	$R_{\mathrm{fr,dn}}$ (kW)
Sequential	0.025	0.025	401.5	65.7
Co-optimized-1	0.025	0.025	465.8	116.9
Co-optimized-2	0.005	0.005	452.6	116.9
Co-optimized-3	0.001	0.001	401.5	65.7

(i.e., quasi-convex hull relaxation of the pipe head loss Eq. (22) and an affine approximation of the pump performance curves (26)) to make the WDN mixed-integer convex. Ref. [17] empirically observed that the quasi-convex hull relaxation was exact when minimizing the pump power consumption. However, our formulation also maximizes a feasible range of pump power consumption and may cause the hydraulic heads to be inexact. We compared the scheduled pump power consumption of our solution using the approximated model with the original nonlinear pump curves. We found that the maximum relative error for pumps 1 and 2 was 8% and 15%, respectively. This motivates future work to evaluate and improve the accuracy of the approximated model.

5. Conclusion

In this paper, we formulated a robust water pumping problem subject to WDN and PDN constraints that provides voltage support and frequency regulation concurrently. We separate the problem into three sub-problems and solve for the solution to the robust reformulation sequentially. In a case study, we demonstrated the ability of the WDN to provide multiple services at the same time. One drawback we found was that the sequential formulation no longer considers the trade-offs between the cost of the pump schedule and the profits from frequency regulation services. However, the sequential solution will always be feasible within the co-optimized formulation. Future research includes exploring approaches to solve the co-optimized robust optimization problem with mixed integer adjustable variables in order to incorporate a binary voltage support control policy that is only implemented when needed and indicator functions to determine the type of frequency regulation to apply.

CRediT authorship contribution statement

Anna Stuhlmacher: Conceptualization, Methodology, Software, Data curation, Visualization, Writing – original draft , Writing – review & editing. **Johanna L. Mathieu:** Conceptualization, Methodology, Funding acquisition, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] J.S. MacDonald, P. Cappers, D.S. Callaway, S. Kiliccote, Demand Response Providing Ancillary Services: A Comparison of Opportunities and Challenges in the US Wholesale Markets, Tech. Rep., (LBNL-5958E) 2012.
[2] D. Denig-Chakroff, Reducing electricity used for water production: questions state commissions should ask regulated utilities, Tech. Rep., Water Research and Policy, 2008.
[3] P. Mancarella, G. Andersson, J.A. Peças-Lopes, K.R.W. Bell, Modelling of integrated multi-energy systems: drivers, requirements, and opportunities, Proc. Power Syst. Comput. Conf. (PSCC) (2016).
[4] D. Fooladivanda, J.A. Taylor, Energy-optimal pump scheduling and water flow, IEEE Trans. Control Netw. Syst. 5 (3) (2018) 1016–1026.
[5] K. Oikonomou, M. Parvania, Optimal coordination of water distribution energy flexibility with power systems operation, IEEE Trans. Smart Grid 10 (1) (2019) 1101–1110.
[6] Y. Liu, C. Barrows, J. Macknick, M. Mauter, Optimization framework to assess the demand response capacity of a water distribution system, J. Water Resour. Plan. Manag. 146 (8) (2020) 1–13.
[7] K.S. Ayyagari, S. Wang, N. Gatsis, A.F. Taha, M. Giacomoni, Co-optimization of interdependent water and power distribution networks, in: IEEE PES Innov. Smart Grid Technol., ISGT, IEEE, 2021, pp. 1–5.
[8] A. Stuhlmacher, J.L. Mathieu, Chance-constrained water pumping to manage water and power demand uncertainty in distribution networks, Proc. IEEE 108 (9) (2020).
[9] A. Stuhlmacher, L.A. Roald, J.L. Mathieu, Tractable robust drinking water pumping to provide power network voltage support, in: Proc. IEEE Conf. Decis. Control (CDC), 2021, pp. 4206–4213.
[10] O. Mégel, J.L. Mathieu, G. Andersson, Scheduling distributed energy storage units to provide multiple services under forecast error, in: Proc. Power Syst. Comput. Conf., Vol. 72, PSCC, 2015, pp. 48–57.
[11] A. Ben-Tal, L. El Ghaoui, A. Nemirovski, Robust Optimization, Vol. 28, Princeton University Press, 2009.
[12] D.B. Arnold, M. Sankur, R. Dobbe, K. Brady, D.S. Callaway, A. Von Meier, Optimal dispatch of reactive power for voltage regulation and balancing in unbalanced distribution systems, in: Proc. IEEE Power Energy Soc. Gen. Meeting, 2016.
[13] A. Bernstein, C. Wang, E. Dall'Anese, J. Le Boudec, C. Zhao, Load flow in multiphase distribution networks: Existence, uniqueness, non-singularity and linear models, IEEE Trans. Power Syst. 33 (6) (2018) 5832–5843.
[14] P.R. Bhave, R. Gupta, Analysis of Water Distribution Networks, Alpha Science International Ltd., Oxford, U.K., 2006.
[15] S. Misra, M. Vuffray, A. Zlotnik, Monotonicity properties of physical network flows and application to robust optimal allocation, Proc. IEEE 108 (9) (2020) 1558–1579.
[16] J. Löfberg, Automatic robust convex programming, Optim. Methods Softw. 27 (1) (2012) 115–129.
[17] Q. Li, S. Yu, A. Al-Sumaiti, K. Turitsyn, Micro water-energy nexus: optimal demand-side management and quasi-convex hull relaxation, IEEE Trans. Control Netw. Syst. (2018).
[18] OWA-EPA, EPANET, 2018, GitHub Repository, GitHub, http://github.com/OpenWaterAnalytics/EPANET.
[19] W.H. Kersting, Radial distribution test feeders, in: Proc. IEEE Power Eng. Soc. Winter Meet., Vol. 2, 2001, pp. 908–912.
[20] MISO actual energy price, 2022, http://www.energyonline.com/Data/Default.aspx (Accessed: 25 Feb 2022).
[21] Gurobi Optimization (Ed.), Gurobi Optimizer 8.1, URL http://www.gurobi.com.