

Performance Modeling of Partitioning and Slicing Spectrum Assignment Schemes in EONs

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Abstract—Routing and Spectrum Assignment (RSA) is a well-known problem in Elastic Optical Networks (EONs), and the literature abounds with numerous heuristic algorithms that attempt to efficiently utilize spectrum and mitigate spectrum fragmentation. However, blocking performance modeling of EONs is a relatively under-explored topic, mainly because of the complexity of handling variable request sizes. In this paper, we analyze three spectrum assignment policies and compare their blocking performance. In the full slicing policy, every request is sliced into multiple one-slot calls. In the partitioning policy, each request size is allocated a dedicated spectrum partition, and each request can be only assigned spectrum from its corresponding partition. The partition with slicing policy is a hybrid of the two – there are dedicated partitions for small request sizes, but large requests are sliced into multiple sub-requests of the smaller sizes and assigned spectrum from those partitions. We obtain expressions for the bandwidth blocking probability for these three policies, and validate the analysis with extensive simulation results. Our results show that partitioning with slicing has the best performance.

I. INTRODUCTION

In recent years, Elastic Optical Networks (EONs) have been widely studied to improve spectrum utilization by apportioning bandwidth in terms of smaller spectrum slices in comparison to fixed-grid wavelength-division multiplexed (WDM) networks. In EONs, the bandwidth is divided into a large number of frequency slots/slices (FSs) and connection requests occupy only as many FSs as needed, based on their data rate requirement and the assigned modulation format. A successful connection must satisfy both the spectrum continuity constraint (same spectrum on all links of the path) and the spectrum contiguity constraint (the assigned FSs must be contiguous).

The blocking caused by unsatisfied spectrum contiguity constraint when there are sufficient FSs for the request is also known as *fragmentation blocking*, as it is due to the fragmentation of free spectrum on the link(s). There are many proposals in the literature that attempt to mitigate fragmentation in the Routing and Spectrum Assignment (RSA) problem. In [1], the authors show that the RSA problem in EONs is an NP-hard problem. A Mixed Integer Linear Program (MILP) model for small networks and a heuristic algorithm for realistic networks which jointly optimizes delay-bandwidth product, fragmentation, and link congestion are proposed in [2]. An indirect approach to avoid fragmentation using offline load balancing and rewarding smaller-indexed spectrum choices is proposed in our previous work [3]. Moreover, many assignment policies that specifically target the fragmentation issue

have been studied. A flexible partition scheme in which the entire fiber bandwidth is partitioned according to request size, but a request can be assigned to its neighbor partition when its corresponding partition is full to reduce the fragmentation effect and improve the utilization of spectrum is introduced in [4]. A joint first-last-fit spectrum assignment policy, based on partition scheme, is presented in [5] to enhance the probability of successful establishment of a request and spectrum efficiency. In [6], the authors propose a metric to quantify fragmentation in a link, and propose a heuristic method based on this metric to decrease the fragmentation. In [7], a partition scheme that considers connection holding time is studied, and a scheme is proposed to jointly optimize the blocking probability and the quality-of-transmission. In [8], [9], the authors introduce *spectrum slicing* technology which allows calls to be sliced into different sub-requests, each of which may then be assigned FSs independently (i.e., without the FSs for the separate sub-requests being contiguous).

On the other hand, the literature is relatively sparse on the topic of blocking probability analysis in EONs, despite numerous modeling efforts for fixed-grid WDM networks. This is mainly because of the complexity of the model induced by two features of EONs - the large number of FSs per fiber (compared to wavelengths in fixed-grid networks) and the variability of the number of FSs requested by connections (as opposed to one wavelength per connection in fixed-grid WDM). One of the earliest efforts was [10], in which the effect of bandwidth fragmentation on the blocking probability is studied for a static model wherein the number of calls on a link are given. Another significant paper is [11], which used defragmentation to decrease the effect of fragmentation blocking in EONs and proposed a reduced-state Markov chain analysis to calculate the blocking probability for Poisson traffic for random fit and first fit SA policies. While this is the best available performance model to date, the approximate model's accuracy has room for improvement, suggesting the inherent difficulty of the problem.

Motivated by SA algorithms that adopt spectrum partitioning and slicing, we analyze three SA policies in this paper for dynamic traffic. In the full slicing policy, every request is sliced into multiple one-slot calls. In the partitioning policy, each request size is allocated a dedicated spectrum partition, and each request can be only assigned spectrum from its corresponding partition. The partition with slicing policy is

a hybrid of the two – there are dedicated partitions for small request sizes, but large requests are sliced into multiple sub-requests of the smaller sizes and assigned spectrum from those partitions. We obtain expressions for the bandwidth blocking probability for these three policies, and validate the analysis with extensive simulation results.

The paper is organized as follows. The system model is presented in Section II. We present the analysis for the 3 SA schemes for a single link in Section III, for a two-link segment in Section IV, and finally extend it to a network in Section V. Results are presented in Section VI, and the paper is concluded in Section VII.

II. SYSTEM MODEL

We consider an EON with W FSs per fiber link to which connection requests arrive according to a Poisson process. Request sizes of calls may be different, and we assume that type i calls require k_i FSs to be assigned. In this paper, we assume that there are 3 types of calls for ease of explanation, but note that this may be generalized to a larger number of types with some increase in computational complexity. Static shortest path routing is assumed, and the arrival of each i th type call follows a Poisson process with rate $\lambda_i^{(l)}$ on path l . The holding time for all types of calls are assumed to be exponentially distributed with identical mean $\frac{1}{\mu}$. To avoid fragmentation blocking, the bandwidth on a link is divided into multiple cells such that a cell is a group of contiguous FSs required by a call based on different assignment policies. The main notations used in this paper are listed in Tab. I. A connection request is assigned FSs according to one of the three SA policies used for the network. The SA policies will be described in more detail in the next section.

TABLE I
NOTATIONS

Notation	Description
W	Bandwidth of a link (number of FSs).
$n_{i,j}$	Number of i th type calls on link j .
k_i	Number of required FSs for type i calls.
B_j	Number of busy cells on link j .
$\bar{B}_j$	Number of free cells on link j .
$\lambda_i^{(l)}$	Arrival rate of i th type calls on path l .
$\frac{1}{\mu}$	Expected holding time for a call.

III. SINGLE-LINK ANALYSIS

We start by describing the 3 SA policies and deriving the blocking probability for a single link for each policy.

A. Partition Assignment

The partition assignment scheme divides the link spectrum into different partitions for different types of calls to avoid fragmentation blocking.

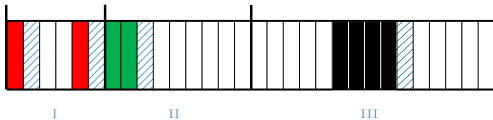


Fig. 1. An example of partitioning.

For type i calls, we assume its corresponding partition has a size D_i . Therefore we have:

$$\sum_i D_i = W. \quad (1)$$

Then we define $d_i = \left\lfloor \frac{D_i}{k_i+1} \right\rfloor$ as the number of cells for type i calls in its corresponding partition, also called as the capacity of the partition. Note that one guard band is required for each call and therefore the actual number of FSs reserved for the call is $k_i + 1$. We show an example in Fig.1 where the link is divided into 3 partitions for the calls with demand size 1, 2, 4 FSs. The dashed FSs denote guard bands. For every connection, a cell is uniformly randomly assigned among the available cells in its respective partition. If no cells are available in its partition, the call is blocked even if there may be spectrum available in the other partitions. The analysis for this scheme is straightforward and follows the analysis for single-wavelength requests in fixed-grid networks, but we present it here briefly for completeness. For a single link, we have the stationary distribution of the number of type i calls as:

$$\pi(n_i) = \frac{\frac{\rho_i^{n_i}}{n_i!}}{\sum_{\alpha=0}^{d_i} \frac{\rho_i^\alpha}{\alpha!}} \quad (2)$$

where $\rho_i = \frac{\lambda_i}{\mu}$ is the offered load for type i calls. Blocking happens only when the partition is full and therefore we have:

$$P_i = \pi(d_i). \quad (3)$$

B. Full Slicing Assignment

Next, we analyze the full slicing assignment scheme. Here, a type i call requesting k_i FSs is sliced into k_i one-slot “sub-requests” and each is assigned spectrum independently. A connection request is successful only if *all* these k_i sub-requests can be assigned spectrum successfully. Since the sub-requests are assigned spectrum independently, there is a guard band needed for each, and thus, two FSs are needed for each sub-request. We therefore divide the spectrum into $C = \lfloor \frac{W}{2} \rfloor$ cells, and each sub-request is assigned an available cell randomly. We show an example in Fig. 2, where a 3-slot call is sliced into three 1-slot calls and assigned spectrum randomly. The black FSs are FSs occupied by existing connections and guard bands are shaded.

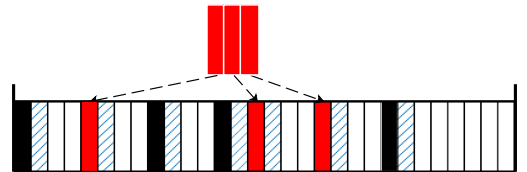


Fig. 2. An example of full slicing.

Then we have the stationary distribution for the number of

ongoing calls of type i as:

$$\pi(n_1, n_2, n_3) = \frac{\frac{\rho_1^{n_1}}{n_1!} \frac{\rho_2^{n_2}}{n_2!} \frac{\rho_3^{n_3}}{n_3!}}{\sum_{\alpha=0}^{\lfloor \frac{C}{k_1} \rfloor} \sum_{\beta=0}^{\lfloor \frac{C-\alpha k_2}{k_2} \rfloor} \sum_{\gamma=0}^{\lfloor \frac{C-\alpha k_1-\beta k_2}{k_3} \rfloor} \frac{\rho_1^\alpha}{\alpha!} \frac{\rho_2^\beta}{\beta!} \frac{\rho_3^\gamma}{\gamma!}} \quad (4)$$

If $B = \sum_i k_i n_i$ denotes the number of busy cells on the link, then its distribution is obtained as:

$$F(B) = \sum_{(n_1, n_2, n_3) | B = \sum_i k_i n_i} \pi(n_1, n_2, n_3). \quad (5)$$

Blocking for type i calls happens when $B > C - k_i$, and therefore its probability P_i is given by: $P_i = \sum_{B=C-k_i+1}^W F(B)$.

C. Partition with Slicing Assignment

The partition scheme eliminates fragmentation by not mixing up requests of different sizes, whereas the slicing scheme does so by breaking down every request into equal-sized sub-requests (at the expense of a guard band for each sub-request). We propose the partition with slicing assignment scheme as a hybrid of the above two schemes. Here, some call types (typically small request sizes) have their own partition, whereas the remaining call types (large request sizes) are sliced into sub-requests of sizes equal to one or more of the small request sizes. Recalling that we assumed 3 call types, let $k_3 > \max(k_1, k_2)$ and $k_3 = x \cdot k_1 + y \cdot k_2$. The spectrum is divided into two partitions for type 1 and type 2 calls, while type 3 calls are sliced into x calls of size k_1 and y calls of size k_2 . We show an example in Fig.3, where a 3-slot call is sliced into one 1-slot and one 2-slot call. Random assignment of cells is done within each partition. The black FSs are those occupied by existing connections and dashed FSs are guard bands. We now have the stationary distribution for the number

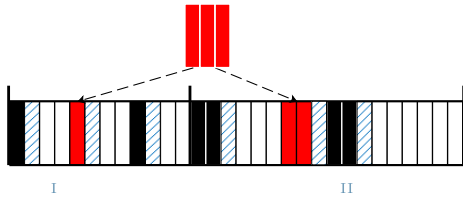


Fig. 3. An example of partitioning with slicing.

of type i calls:

$$\pi(n_1, n_2, n_3) = \frac{\frac{\rho_1^{n_1}}{n_1!} \frac{\rho_2^{n_2}}{n_2!} \frac{\rho_3^{n_3}}{n_3!}}{\sum_{\alpha=0}^{d_1} \sum_{\beta=0}^{d_2} \sum_{\gamma=0}^{\min(\lfloor \frac{d_1-\alpha}{x} \rfloor, \lfloor \frac{d_2-\beta}{y} \rfloor)} \frac{\rho_1^\alpha}{\alpha!} \frac{\rho_2^\beta}{\beta!} \frac{\rho_3^\gamma}{\gamma!}} \quad (6)$$

Blocking happens for type 1 calls when partition 1 is full, i.e., $n_1 + xn_3 = d_1$, and blocking happens for type 2 calls when partition 2 is full, i.e., $n_2 + yn_3 = d_2$. Type 3 calls are blocked when either partition 1 or partition 2 has not enough cells for type 3 calls. Therefore we have:

$$P_1 = \sum_{n_3=0}^{N_3} \sum_{n_2=0}^{d_2-yn_3} \pi(d_1 - xn_3, n_2, n_3)$$

$$P_2 = \sum_{n_3=0}^{N_3} \sum_{n_1=0}^{d_1-xn_3} \pi(n_1, d_2 - yn_3, n_3)$$

$$P_3 = 1 - \sum_{n_3=0}^{N_3} \sum_{n_1=0}^{d_1-xn_3-x+1} \sum_{n_2=0}^{d_2-yn_3-y+1} \pi(n_1, n_2, n_3) \quad (7)$$

where $N_3 = \min(\lfloor \frac{d_1}{x} \rfloor, \lfloor \frac{d_2}{y} \rfloor)$.

IV. TWO-HOP SEGMENT ANALYSIS

We now consider a two-hop segment of a path. Here, there are three paths: the path using link 1 only, the path using link 2 only, and the path using both links. We use subscript $i, 1$ for type i calls using only link 1; $i, 2$ for type i calls using only link 2; and i, c for the type i calls continuing from link 1 to link 2. The blocking probabilities for the calls only on link 1 or link 2 can be derived as in Section III. Here, we show how the blocking probability of continuing calls is computed for the three SA schemes.

A. Partition Assignment

In the two-hop segment, we can obtain the stationary distribution of the number of calls in the i th partition by solving a 3-D Markov chain:

$$\pi(n_{i,1}, n_{i,2}, n_{i,c}) = \frac{\frac{\rho_{i,1}^{n_{i,1}}}{n_{i,1}!} \frac{\rho_{i,2}^{n_{i,2}}}{n_{i,2}!} \frac{\rho_{i,c}^{n_{i,c}}}{n_{i,c}!}}{\sum_{\alpha=0}^{d_i} \sum_{\beta=0}^{d_i} \sum_{\gamma=0}^{\min(d_i-\alpha, d_i-\beta)} \frac{\rho_{i,1}^\alpha}{\alpha!} \frac{\rho_{i,2}^\beta}{\beta!} \frac{\rho_{i,3}^\gamma}{\gamma!}} \quad (8)$$

Let $\bar{B}_{i,c}$ denote the number of cells that are available on *both* links in the i th partition. Then, the conditional probability of having $\bar{B}_{i,c}$ cells is:

$$R(\bar{B}_{i,c} | n_{i,1}, n_{i,2}, n_{i,c}) = \frac{\binom{d_i - n_{i,1} - n_{i,c}}{\bar{B}_{i,c}} \binom{n_{i,1}}{d_i - n_{i,2} - n_{i,c} - \bar{B}_{i,c}}}{\binom{d_i - n_{i,c}}{d_i - n_{i,2} - n_{i,c}}} \quad (9)$$

where $\max(0, d_i - n_{i,1} - n_{i,c}) \leq \bar{B}_{i,c} \leq \min(d_i - n_{i,1} - n_{i,c}, d_i - n_{i,2} - n_{i,c})$. If $\bar{B}_{i,c} = 0$, a type i call is blocked in its partition over the two-link segment, and the corresponding blocking probability is:

$$P_{i,c} = \sum_{n_{i,1}=0}^{d_i} \sum_{n_{i,2}=0}^{d_i} \sum_{n_{i,c}=0}^{\min(d_i - n_{i,1}, d_i - n_{i,2})} \left(R(0 | n_{i,1}, n_{i,2}, n_{i,c}) \pi(n_{i,1}, n_{i,2}, n_{i,c}) \right) \quad (10)$$

B. Full Slicing Assignment

To analyze the full slicing assignment in a two-hop segment, we first assume that the links are independent of each other, and use the sum of arrival rates of all calls that traverse a link as the arrival rate for this single link. Therefore the offered loads on these two links are:

$$\tilde{\rho}_{i,1} = \rho_{i,1} + \rho_{i,c}$$

$$\tilde{\rho}_{i,2} = \rho_{i,2} + \rho_{i,c}.$$

Then with equations (4) and (5), we can compute the probability $F(B_1)$ and $F(B_2)$ of having B_1 and B_2 busy cells on the two links, respectively. Completely ignoring the fact that continuing calls use the same slots on both links would produce vastly inaccurate results. Here, we rely on a tractable approximation that yields sufficiently accurate results as verified by simulations later. We assume that the number of busy cells from the continuing sliced calls B_c follows a binomial distribution, as follows:

$$G(B_c|B_1, B_2) = \binom{\min(B_1, B_2)}{B_c} p_c^{B_c} (1 - p_c)^{\min(B_1, B_2) - B_c} \quad (11)$$

where

$$p_c = \min\{p_{1,c}, p_{2,c}, p_{3,c}\}$$

is the probability of continuation for the busy cells from the sliced calls on the two-hop segment, and $p_{i,c} = \frac{\rho_{i,c}}{\max(\bar{\rho}_{i,1}, \bar{\rho}_{i,2})}$ is the probability of continuation for i th type calls. Denoting by $\bar{B}_c$ the number of free cells continuing on the two-hop segment, we have:

$$H(\bar{B}_c|B_1, B_2, B_c) = \frac{\binom{C-B_1}{\bar{B}_c} \binom{B_1-B_c}{C-B_2-\bar{B}_c}}{\binom{C-B_c}{C-B_2}}. \quad (12)$$

If $\bar{B}_c < k_i$, type i calls are blocked on the two-link segment, and so the blocking probability is:

$$P_{i,c} = \sum_{\bar{B}_c=0}^{k_i-1} \sum_{B_1=0}^C \sum_{B_2=0}^C \sum_{B_c=0}^{\min(C-B_1, C-B_2)} \left(H(\bar{B}_c|B_1, B_2, B_c) G(B_c|B_1, B_2) F(B_1) F(B_2) \right). \quad (13)$$

C. Partition with Slicing Assignment

Here again, we use the link load independence assumption and then model the number of continuing calls using a binomial distribution as above. Then, using equation (6), we have the stationary distribution $\pi(n_{1,l}, n_{2,l}, n_{3,l})$ for the number of calls of different types on one link. We further assume that the partitions are independent of each other. (Note that they are actually not independent because the sliced calls are accommodated on both partitions.) Then the distribution of calls in partition 1 is given by:

$$I(n_{1,l}, n_{3,l}) = \sum_{y=0}^{d_2 - y n_{3,l}} \pi(n_{1,l}, y, n_{3,l}). \quad (14)$$

The distribution of calls in partition 2 is similarly obtained. Following the binomial assumption, the number of continuing type i calls, $n_{i,c}$ is distributed as:

$$J(n_{i,c}|n_{i,1}, n_{i,2}) = \binom{N_{i,c}}{n_{i,c}} (p_{i,c})^{n_{i,c}} (1 - p_{i,c})^{N_{i,c} - n_{i,c}}$$

where $N_{i,c} = \min(n_{i,1}, n_{i,2})$ and $p_{i,c} = \frac{\rho_{i,c}}{\max(\bar{\rho}_{i,1}, \bar{\rho}_{i,2})}$ is the probability of continuation for type i calls. Therefore, in partition 1, we have the number of busy cells as $B_1 = n_{1,1} + x n_{3,1}$,

$B_2 = n_{1,2} + x \cdot n_{3,2}$, and $B_c = n_{1,c} + x \cdot n_{3,c}$. We can obtain using equation (12), the probability that there are $\bar{B}_c$ free cells continuing on the two-hop segment given the number of busy cells B_1, B_2 and B_c . Then the blocking probability for continuing type 1 calls in its partition is:

$$P_{1,c} = \sum_{n_{3,1}=0}^{N_3} \sum_{n_{1,1}=0}^{d_1 - x n_{3,1}} \sum_{n_{3,2}=0}^{N_3} \sum_{n_{1,2}=0}^{d_1 - x n_{3,2}} \sum_{n_{1,c}=0}^{N_{1,c}} \sum_{n_{3,c}=0}^{N_{3,c}} \left(H(0|B_1, B_2, B_c) J(n_{1,c}|n_{1,1}, n_{1,2}) J(n_{3,c}|n_{3,1}, n_{3,2}) I(n_{1,1}, n_{3,1}) I(n_{1,2}, n_{3,2}) \right). \quad (15)$$

With condition $\bar{B}_c < x$, the blocking probability for continuing type 3 calls in partition 1 is:

$$P_{3,c_1} = \sum_{B_c=0}^{a-1} \sum_{n_{3,1}=0}^{N_3} \sum_{n_{1,1}=0}^{d_1 - x n_{3,1}} \sum_{n_{3,2}=0}^{N_3} \sum_{n_{1,2}=0}^{d_1 - x n_{3,2}} \sum_{n_{1,c}=0}^{N_{1,c}} \sum_{n_{3,c}=0}^{N_{3,c}} \left(H(\bar{B}_c|B_1, B_2, B_c) J(n_{1,c}|n_{1,1}, n_{1,2}) J(n_{3,c}|n_{3,1}, n_{3,2}) I(n_{1,1}, n_{3,1}) I(n_{1,2}, n_{3,2}) \right).$$

We can get the blocking probability for continuing type 2 calls and type 3 calls in partition 2 following similar steps. Note that the type 3 calls are blocked when either partition 1 or partition 2 has not enough cells for type 3 calls. Therefore we have: $P_{3,c} = 1 - (1 - P_{3,c_1})(1 - P_{3,c_2})$.

V. MULTI-HOP NETWORK ANALYSIS

We now extend our analysis to a multi-hop network.

A. Partition Assignment

For an l -hop path, we divide the path into $(l - 1)$ independent two-hop segments for analysis. With (8), we have the probability for the number of calls in the i th partition in a two-hop segment with link $l - 1$ and link l as $\pi(n_{i,l-1}, n_{i,l}, n_{i,c})$. We use $Q(\bar{B}_{l-1})$ to denote the probability there are $\bar{B}_{l-1}$ free cells in partition i on link $(l - 1)$. It is given by (the subscript i is dropped for simplicity):

$$Q(\bar{B}_{l-1}) = \sum_{n_{i,c}=0}^{d_i - \bar{B}_{l-1}} \sum_{n_{i,l}=0}^{d_i - n_{i,c}} \pi(d_i - \bar{B}_{l-1} - n_{i,c}, n_{i,l}, n_{i,c}). \quad (16)$$

Let $S(\bar{B}_l|\bar{B}_{l-1})$ denote the conditional probability that there are $\bar{B}_l$ free cells on link l when there are $\bar{B}_{l-1}$ free cells on link $(l - 1)$. Then:

$$S(\bar{B}_l|\bar{B}_{l-1}) = \frac{\sum_{n_{i,c}=0}^{Z_c} \pi(d_i - \bar{B}_{l-1} - n_{i,c}, d_i - \bar{B}_l - n_{i,c}, n_{i,c})}{\sum_{n_{i,c}=0}^{d_i - \bar{B}_{l-1}} \sum_{n_{i,l}=0}^{d_i - n_{i,c}} \pi(d_i - \bar{B}_{l-1} - n_{i,c}, n_{i,l}, n_{i,c})}, \quad (17)$$

where $Z_c = \min(d_i - \bar{B}_{l-1}, d_i - \bar{B}_l)$. Finally, we use $U(z_c|\bar{B}_l, \bar{B}_{l-1})$ to denote the conditional probability that there are z_c continuing calls from link $l-1$ to link l when there are $\bar{B}_{l-1}$ free cells on link $(l-1)$ and $\bar{B}_l$ free cells on link l . Then:

$$U(z_c|\bar{B}_l, \bar{B}_{l-1}) = \frac{\pi(d_i - \bar{B}_{l-1} - z_c, d_i - \bar{B}_l - z_c, z_c)}{\sum_{n_{i,c}=0}^{Z_c} \pi(d_i - \bar{B}_{l-1} - n_{i,c}, d_i - \bar{B}_l - n_{i,c}, n_{i,c})}. \quad (18)$$

We model load correlation among links by assuming that the distribution on the l th link depends only on that on link $l-1$; therefore, if we know the joint probability $T^{(l-1)}(\bar{B}^{(l-1)}, \bar{B}_{l-1})$ that in i th partition, there are $\bar{B}^{(l-1)}$ free cells on the $(l-1)$ -hop path and $\bar{B}_{l-1}$ free cells on link $(l-1)$, we have:

$$T^{(l)}(\bar{B}^{(l)}, \bar{B}_l) = \sum_{\bar{B}_{l-1}=0}^{d_i} \sum_{\bar{B}^{(l-1)}=0}^{\bar{B}_{l-1}} \sum_{z_c=0}^{Z_c} \left(R(\bar{B}^{(l)}|d_i - \bar{B}_{l-1} - z_c, d_i - \bar{B}_l - z_c, z_c) U(z_c|\bar{B}_l, \bar{B}_{l-1}) S(\bar{B}_l|\bar{B}_{l-1}) T^{(l-1)}(\bar{B}^{(l-1)}, \bar{B}_{l-1}) \right). \quad (19)$$

Clearly, the blocking probability for calls on the l -hop path in its partition is $P_i^{(l)} = \sum_{\bar{B}_l=0}^{d_i} T^{(l)}(0, \bar{B}_l)$.

B. Full Slicing Assignment

For an l -hop path, we give the joint probability $T^{(l)}(\bar{B}^{(l)}, \bar{B}_l)$ which denotes the probability that there are $\bar{B}^{(l)}$ free cells on the entire l -hop path and $\bar{B}_l$ free cells on link l as:

$$T^{(l)}(\bar{B}^{(l)}, \bar{B}_l) = \sum_{\bar{B}_{l-1}=0}^C \sum_{\bar{B}^{(l-1)}=0}^{\bar{B}_{l-1}} \sum_{B_c=0}^{\min(C-\bar{B}_{l-1}, C-\bar{B}_l)} \left(H(\bar{B}^{(l)}|B_{l-1}, B_l, B_c) G(B_c|B_{l-1}, B_l) T^{(l-1)}(\bar{B}^{(l-1)}, \bar{B}_{l-1}) F(B_l) \right). \quad (20)$$

Note that on link l , we have $B_l + \bar{B}_l = C$; so with equations (5), (11) and (12), we can obtain $T^{(l)}(\bar{B}^{(l)}, \bar{B}_l)$. When $\bar{B}^{(l)} < k_i$, the i th type calls are blocked on the l -hop path. Therefore we have:

$$P_i^{(l)} = \sum_{\bar{B}^{(l)}=0}^{k_i-1} \sum_{\bar{B}_l=0}^C T^{(l)}(\bar{B}^{(l)}, \bar{B}_l). \quad (21)$$

C. Partition with Slicing Assignment

For the partition with slicing scheme, we present the analysis in partition 1 as an example. Let $T^{(l)}(\bar{B}^{(l)}, n_{1,l}, n_{3,l})$ denote the joint probability that there are $\bar{B}^{(l)}$ free cells on the l -hop path and $n_{1,l}$ busy calls for type 1, $n_{3,l}$ busy calls

for type 3 on link l . Then:

$$T^{(l)}(\bar{B}^{(l)}, n_{1,l}, n_{3,l}) = \sum_{n_{3,l-1}=0}^{N_3} \sum_{n_{1,l-1}=0}^{d_1 - x n_{3,l-1}} \sum_{B^{(l-1)}=0}^{d_i - n_{1,l-1} - x n_{3,l-1}} \sum_{n_{1,c}=0}^{N_{1,c}} \sum_{n_{3,c}=0}^{N_{3,c}} \left(H(\bar{B}^{(l)}|B_{l-1}, B_l, B_c) J(n_{1,c}|n_{1,l-1}, n_{1,l}) J(n_{3,c}|n_{3,l-1}, n_{3,l}) T^{(l-1)}(\bar{B}^{(l-1)}, n_{1,l-1}, n_{3,l-1}) I(n_{1,l}, n_{3,l}) \right). \quad (22)$$

Note that in partition 1, we have the number of busy cells as $B_l = n_{1,l} + x \cdot n_{3,l}$, and $B_c = n_{1,c} + x \cdot n_{3,c}$. Thus the blocking probability in partition 1 for type 1 calls on the l -hop path is:

$$P_1^{(l)} = \sum_{n_{3,l}=0}^{N_3} \sum_{n_{1,l}=0}^{d_1 - n_{3,l}} T^{(l)}(0, n_{1,l}, n_{3,l}).$$

D. Reduced Load Approximation

Reduced load approximation is a well-known technique to improve the accuracy of blocking analyses in circuit-switched networks (e.g., see [14]). First, we calculate the initial blocking probability $B^{(l)}(0)$ for a path with offered load $\rho^{(l)}$. Then, for every iteration i , we use $\rho^{(l)}(i) = \rho^{(l)}(1 - B^{(l)}(i-1))$ and calculate the new blocking value $B^{(l)}(i)$ with the updated value of the load. The procedure is iterated until the blocking probability converges. It typically takes a fraction of a second to compute the BP for a path.

VI. NUMERICAL RESULTS

We now present numerical results comparing the three SA policies. We also performed call-based simulations for 10^7 calls and use the average results with 95% confidence interval over 10 trials in order to verify the analytical results. We use the NSFNET topology as shown in [4] with 320 FSs on each link. QPSK modulation is applied with demand size $\{2, 4, 16\}$ FSs (excluding guard band) corresponding to data rates $\{40, 100, 400\}$ Gbps [15]. The network traffic load is denoted by Γ , and we assume the following distribution for the three types of calls: 40% type 1 with 2 FSs, 40% type 2 calls with 4 FSs, and 20% type 3 with 16 FSs. The source and destination of calls are random (uniform distribution) and a fixed shortest path is used for each node pair. The bandwidth blocking probability (BBP) is reported in all the results. For the two partition-based schemes, we calculate the partition size based on the traffic load and request sizes as below:

$$d_i = \left\lfloor \frac{\rho_i(k_i + 1)}{\sum_i \rho_i(k_i + 1)} \right\rfloor.$$

Fig. 4 shows the BBP for the three policies as a function of Γ . Note that for partitioning with slicing, we have $k_3 = x \cdot k_1 + y \cdot k_2$ and here we use $x = 2$, $y = 3$ which means that a 16-slot call is sliced into two 2-slot and three 4-slot sub-requests. We observe that our analytical results match the simulation results very well. Full slicing

scheme performs the worst compared to the other policies since it doubles the spectrum usage with extra guard band for each 1-slot sub-request. Partitioning with slicing performs the best as it provides more flexibility compared to the pure partitioning scheme while reducing spectrum usage compared to full slicing.

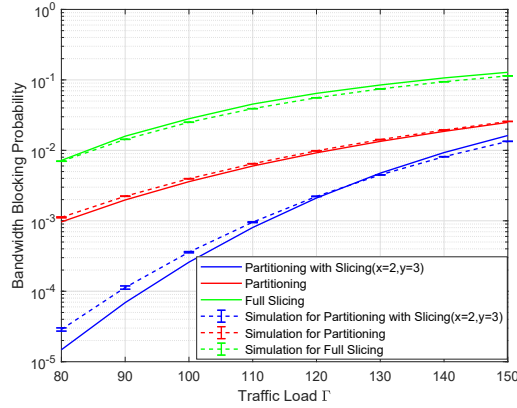


Fig. 4. BBP vs. Γ for the SA policies.

Next in Fig. 5, we show a comparison of the partitioning with slicing scheme for five different slicing settings (only analytical results shown). We observe that as the load increases, $x = 0, y = 4$ becomes better than $x = 2, y = 3$ which indicates a trade-off between extra spectrum usage from guard bands (more slicing) and the flexibility in spectrum assignment. As more guard bands are added to the sliced calls, the blocking performance becomes worse as it increases the spectrum usage on the link.

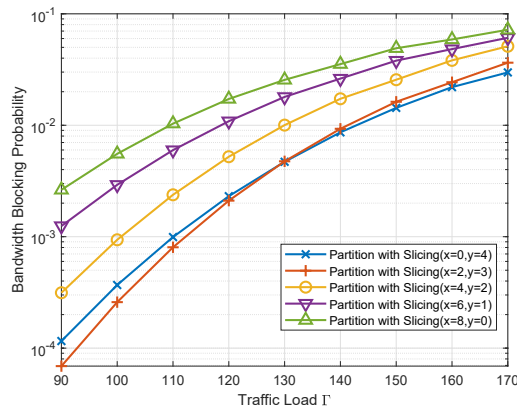


Fig. 5. BBP vs. Γ for partitioning with slicing scheme with different slicing strategies.

VII. CONCLUSION

In this paper, we present new blocking probability models for partition and slicing-based spectrum assignment schemes in elastic optical networks. In the full slicing policy, every request is sliced into multiple one-slot calls. In the partitioning

policy, each request size is allocated a dedicated spectrum partition, and each request can be only assigned spectrum from its corresponding partition. The partitioning with slicing policy is a hybrid of the above two. Results show that the models are quite accurate, and that the partition with slicing policy provides a good balance between reducing fragmentation and wasted spectrum.

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