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An eavesdropping and jamming dilemma with sophisticated players

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Abstract

Wireless networks are susceptible to malicious attacks, especially those involving eavesdropping and jamming. In this paper, we consider a communication scenario involving a transmitter who wishes to communicate secretly and reliably with a receiver, while an adversary wants to obstruct this communication by means of either eavesdropping or jamming. The transmitter as well as the adversary wants to achieve its own goal in a manner that is as unpredictable as possible to its rival. We model this problem by a non-zero sum game. The expected throughput that is delivered secretly and non-jammed to the receiver is considered as the metric that reflects communication secrecy and reliability. The entropy of the player's strategies is considered as the metric to reflect the player's unpredictability. The equilibrium is found in closed form, and parameters of the transmitter's utility supporting both goals are optimized via a proportional fairness approach.

Keywords: Eavesdropping; Jamming; Entropy; Nash equilibrium

1. Introduction

The problem of establishing secret communication between a transmitter and a receiver is fundamental to building secure communication systems. Physical layer security problems have commonly been studied under the threat of passive eavesdroppers [1–3]. Some works have studied how an active eavesdropper with the dual capability of either eavesdropping passively or jamming any ongoing transmission can disrupt the security and reliability of wireless communications networks [4–9]. In all of these works, the anti-adversary strategy was focused on the basic goal of maximizing the expected throughput delivered secretly and non-jammed to a receiver. Meanwhile, the adversary strategy was focused on minimizing such a payoff.

In this paper, different from prior works, we design a new class of transmitter and adversary strategies, which we call *sophisticated* strategies. For a sophisticated adversary, beyond the basic goal to obstruct communication of the transmitter with a receiver by means of combined eavesdropping and jamming attack, the adversary also has a secondary goal to achieve such objective in the most unpredictable way for the transmitter. A sophisticated transmitter also, beyond the basic goal of communicating secretly and reliably, has a secondary goal to achieve such objective in the most unpredictable way to the adversary. The problem is modeled in the framework of a game-theoretical approach. The expected throughput delivered secretly and un-jammed to the receiver is considered as a metric that reflects the basic goal of the players. The entropy of the player's strategies is implemented as a metric to reflect secondary goals. In particular, proven uniqueness of designed anti-adversary strategy reflects its stability to combined eavesdropping and jamming attack.

2. Short overview of communication model

Our communication scenario involves a transmitter who wishes to communicate secretly and reliably with a receiver. The adversary aims to obstruct this communication by means of either eavesdropping or jamming. A wireless transmission with n subcarriers using orthogonal frequency-division multiplexing (OFDM) is considered as the basic example of such communication. These sub-carriers are affected by fading channel gains h_i, $i = 1, \ldots, n$. The sub-carriers from adversary to transmitter have corresponding eavesdropping channel gains h_{Ei}, where $h_{Ei}/\sigma_E^2 \leq h_i/\sigma^2$ for $i = 1, \ldots, n$, and σ_E^2 and σ^2 are the variances of additive white Gaussian noise processes at adversary's and receiver's receivers, respectively, while the fading channel coefficients from adversary to receiver are represented by g_i (Fig. 1). Let $\boldsymbol{P} = (P_1, \ldots, P_n)$ be a power allocation strategy for the transmitter, where P_i

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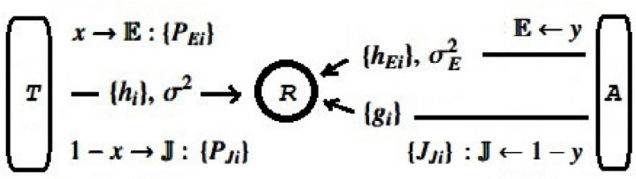

Fig. 1. Communication model between transmitter (T) and receiver (R) in presence of adversary (A).

is a power allocated by the transmitter to communicate with receiver through sub-carrier i, and $\sum_{i=1}^{n} P_i = P_{total}$ with P_{total} is the total transmission power. Let $\boldsymbol{J} = (J_1, \ldots, J_n)$ be a power allocation strategy for the adversary, where J_i is the jamming power employed by the adversary to jam sub-carrier i, and $\sum_{i=1}^{n} J_i = J_{total}$ with J_{total} is the total jamming power budget. Let $\mathcal{P}$ and $\mathcal{J}$ be sets of all feasible power allocation strategies of transmitter and adversary, respectively.

The adversary can implement one of two malicious attack (*modes*): *eavesdropping* or *jamming*.

Under an eavesdropping attack, the maximum achievable rate (secrecy rate) for transmission from transmitter to receiver is given by the secrecy capacity as follows: $u_{SC}(\boldsymbol{P}) \triangleq \max\{u(\boldsymbol{P}, 0) - u_E(\boldsymbol{P}), 0\}$, where $u(\boldsymbol{P}, \boldsymbol{J})$ is the capacity, or, more loosely, the throughput, of direct transmission between transmitter and receiver if transmitter and adversary apply strategies $\boldsymbol{P}$ and $\boldsymbol{J}$, respectively, and $u_E(\boldsymbol{P})$ is the capacity of adversary as a receiver in the eavesdropper mode.

In eavesdropping mode, the adversary eavesdrops, i.e., $\boldsymbol{J} = 0$. Then the optimal strategy for the transmitter is the one which maximizes its secrecy capacity, i.e.,

$$\boldsymbol{P}_E \triangleq \text{argmax}\{u_{SC}(\boldsymbol{P}) : \boldsymbol{P} \in \mathcal{P}\}. \tag{1}$$

In jamming mode, the adversary wants to implement a power allocation strategy that minimizes throughput $u(\boldsymbol{P}, \boldsymbol{J})$ at the receiver, while the transmitter wants to implement a power allocation strategy to maximize such throughput. Thus, the transmitter and adversary want to implement Nash equilibrium (NE) strategies in a *zero-sum game* with $u(\boldsymbol{P}, \boldsymbol{J})$ as the payoff and cost function for the transmitter and adversary, respectively, i.e., a pair of strategies $(\boldsymbol{P}_J, \boldsymbol{J}_J)$ for which each of them is the best response to the other, i.e. the following relations hold

$$\boldsymbol{P}_J \triangleq \text{argmax}\{u(\boldsymbol{P}, \boldsymbol{J}_J) : \boldsymbol{P} \in \mathcal{P}\}, \tag{2}$$

$$\boldsymbol{J}_J \triangleq \text{argmin}\{u(\boldsymbol{P}_J, \boldsymbol{J}) : \boldsymbol{J} \in \mathcal{J}\}. \tag{3}$$

In particular, for the basic OFDM scenario, we have

$$u_{SC}(\boldsymbol{P}) = \sum_{i=1}^{n} \ln\left(1 + h_i P_i/\sigma^2\right) - \ln\left(1 + h_{Ei} P_i/\sigma_E^2\right), \tag{4}$$

$$u_E(\boldsymbol{P}) = \sum_{i=1}^{n} \ln\left(1 + h_{Ei} P_i/\sigma_E^2\right), \tag{5}$$

$$u(\boldsymbol{P}, \boldsymbol{J}) = \sum_{i=1}^{n} \ln\left(1 + h_i P_i/(\sigma^2 + g_i J_i)\right), \tag{6}$$

and $\boldsymbol{P}_E$ and $(\boldsymbol{P}_J, \boldsymbol{J}_J)$ can be found via [10,11], respectively. Also, without loss of generality we can assume that $\boldsymbol{P}_E \neq \boldsymbol{P}_J$.

In the following lemma we establish relations between transmitter's payoffs in dependence on the modes the transmitter and adversary implement.

Proposition 1. *Let* $A \triangleq u_{SC}(\boldsymbol{P}_E)$, $a \triangleq u_{SC}(\boldsymbol{P}_J)$, $B \triangleq u(\boldsymbol{P}_J, \boldsymbol{J}_J)$ *and* $b \triangleq u(\boldsymbol{P}_E, \boldsymbol{J}_J)$. *Between these payoffs the following relations hold:*

$$A > a \text{ and } B > b. \tag{7}$$

Proof. Since $\boldsymbol{P}_E \neq \boldsymbol{P}_J$, relations (1)–(3) imply (7) . ■

In the following corollary we establish the relationship between the sums of the transmitter's payoffs if both players implement the same mode and if both players implement different modes.

Proposition 2. *The following relation holds*

$$D > 0, \tag{8}$$

where

$$D \triangleq A + B - a - b. \tag{9}$$

Proof. The result follows from (7). ■

3. Eavesdropping and jamming dilemma

Now suppose the adversary can choose whether to eavesdrop or jam, but it cannot tune its jamming power. The transmitter does not know whether the adversary is going to eavesdrop or jam, and it must choose whether to transmit as if it is being jammed or being eavesdropped upon. Thus, the adversary has two malicious strategies: to implement eavesdropping or jamming mode, denoted by $\mathbb{E}$ and $\mathbb{J}$, respectively. In jamming mode, the adversary implements the corresponding optimal power allocation strategy $\boldsymbol{J}_J$ for jamming.

The transmitter, on the other hand, chooses between two modes to communicate with the receiver: (a) mode $\mathbb{E}$, to implement power allocation strategy $\boldsymbol{P}_E$ which is optimal to deal with an eavesdropping attack, and (b) mode $\mathbb{J}$, to implement power allocation strategy $\boldsymbol{P}_J$ that is optimal to deal with a jamming attack. This scenario leads to the following payoff matrix M, where the rows and columns are the transmitter's and adversary's strategies, respectively:

$$M = \begin{matrix} \\ \mathbb{E} \\ \mathbb{J} \end{matrix} \begin{matrix} \begin{matrix} \mathbb{E} & \mathbb{J} \end{matrix} \\ \begin{pmatrix} A & b \\ a & B \end{pmatrix} \end{matrix}. \tag{10}$$

4. Sophisticated transmitter and adversary

Let the transmitter, with probabilities x and $1 - x$, implement transmission power allocation strategies $\boldsymbol{P}_E$ and $\boldsymbol{P}_J$ corresponding to transmission modes $\mathbb{E}$ and $\mathbb{J}$, respectively. Similarly, let the adversary, corresponding to adversary modes $\mathbb{E}$ and $\mathbb{J}$, eavesdrop or implement jamming power allocation strategy $\boldsymbol{J}_J$ with probabilities y and $1-y$, respectively (Fig. 1).

Let probability vectors $\boldsymbol{x} = (x, \overline{x})$ and $\boldsymbol{y} = (y, \overline{y})$[1] be strategies for transmitter and adversary, respectively. Then, by (10), the expected throughput secured and non-jammed delivered to receiver is given as follows:

$$\mathbb{T}(\boldsymbol{x}, \boldsymbol{y}) \triangleq Axy + bx\overline{y} + a\overline{x}y + B\overline{x}\,\overline{y}. \tag{11}$$

Traditionally, in secrecy communication problems the adversary and transmitter are antagonists. Specifically, the adversary wants to reduce the secret and non-jammed throughput delivered to the receiver by means of a combined eavesdropping and jamming attack, and the transmitter wants to increase it.

Meanwhile, the (sophisticated) transmitter wants to find a trade-off between two goals: (i) *the basic goal,* to increase the secret and non-jammed throughput delivered to the receiver, and (ii) *the secondary goal,* to achieve this increase in the most unpredictable way for the adversary. As a metric for the transmitter to confuse the adversary, we consider the (informational) entropy of its strategy, i.e.,

$$H(\boldsymbol{x}) \triangleq -x\ln(x) - \overline{x}\ln(\overline{x}). \tag{12}$$

The payoff for such a transmitter is taken as a weighted sum the expected throughput and secrecy rate, and the entropy of its strategy, i.e.,

$$v_T(\boldsymbol{x}, \boldsymbol{y}) = \overline{w_T}\,\mathbb{T}(\boldsymbol{x}, \boldsymbol{y}) + w_T H(\boldsymbol{x}), \tag{13}$$

where $\overline{w_T} = 1 - w_T$ and w_T are non-negative normalized weighting coefficients.

The (sophisticated) adversary wants to find a trade-off between two goals: (i) *the basic goal*, to reduce the secret and non-jammed throughput delivered to the receiver by means of a combined eavesdropping and jamming attack, and (ii) *the secondary goal*, to achieve this reduction in the most unpredictable way for the transmitter. The payoff for such an adversary is taken as difference between weighted entropy of its strategy and the expected throughput and secrecy rate delivered to a receiver, i.e.,

$$v_A(\boldsymbol{x}, \boldsymbol{y}) = -\overline{w_A}\,\mathbb{T}(\boldsymbol{x}, \boldsymbol{y}) + w_A H(\boldsymbol{y}), \tag{14}$$

where $\overline{w_A} = 1 - w_A$ and w_A are non-negative normalized weighting coefficients.

Thus, the transmitter and adversary want to implement NE strategies in a *non-zero-sum game* with $v_T(\boldsymbol{x}, \boldsymbol{y})$ and $v_A(\boldsymbol{x}, \boldsymbol{y})$ as payoffs to the transmitter and adversary, respectively, i.e., a pair of strategies $(\boldsymbol{x}, \boldsymbol{y})$ for which each of them is the best response to the other, i.e., the following relations hold

$$\boldsymbol{x} = \mathrm{BR}_T(\boldsymbol{y}) \triangleq \operatorname*{argmax}_{\tilde{\boldsymbol{x}}} v_T(\tilde{\boldsymbol{x}}, \boldsymbol{y}), \tag{15}$$

$$\boldsymbol{y} = \mathrm{BR}_A(\boldsymbol{x}) \triangleq \operatorname*{argmax}_{\tilde{\boldsymbol{y}}} v_A(\boldsymbol{x}, \tilde{\boldsymbol{y}}). \tag{16}$$

Denote this non-zero sum game by Γ.

Proposition 3. *In game Γ there exists at least one NE.*

Proof. By (11)–(13), for $w_T = 0$ we have that $v_T(\boldsymbol{x}, \boldsymbol{y})$ is linear on $\boldsymbol{x}$. Meanwhile for $w_T > 0$ we have that $\partial v_T^2(\boldsymbol{x}, \boldsymbol{y})/\partial x^2 = -w_T/(x(1-x)) < 0$. Thus, $v_T(\boldsymbol{x}, \boldsymbol{y})$ is concave in $\boldsymbol{x}$. Similarly, by (11), (12) and (14), for $w_A = 0$ we have that $v_A(\boldsymbol{x}, \boldsymbol{y})$ is linear on $\boldsymbol{y}$. Meanwhile, for $w_A > 0$ we have that $\partial v_A^2(\boldsymbol{x}, \boldsymbol{y})/\partial y^2 = -w_A/(y(1-y)) < 0$. Thus, $v_A(\boldsymbol{x}, \boldsymbol{y})$ is concave in $\boldsymbol{y}$, and the result follows from Nash's theorem [12] since the set of feasible strategies for each player is compact. ■

Further, we find equilibrium strategies in closed form using a constructive approach by solving the best response equations (15) and (16).

5. Best response strategies

In this section we derive in closed form the best response strategies for the players.

Note that here and throughout the rest part of the paper we will label the strategies of transmitter $\boldsymbol{x} = (x, \overline{x})$ and adversary $\boldsymbol{y} = (y, \overline{y})$ by their first components x and y, respectively, since they uniquely define probability vectors $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively.

Proposition 4. *(a) For a fixed $y \in [0, 1]$, the best response $x = BR_T(y)$ of transmitter is given as follows:*

$$BR_T(y) = \begin{cases} \dfrac{1}{1 + \exp((Y_0 - y)\,\eta_T)}, & w_T > 0, \\ \begin{cases} 1, & y > Y_0, \\ \in [0, 1], & y = Y_0, \\ 0, & y < Y_0, \end{cases} & w_T = 0, \end{cases} \tag{17}$$

where

$$Y_0 \triangleq (B - b)/D \text{ and } \eta_T \triangleq \overline{w_T}/(Dw_T). \tag{18}$$

(b) For a fixed $x \in [0, 1]$, the best response $y = BR_A(x)$ of adversary is given as follows:

$$BR_A(x) = \begin{cases} \dfrac{1}{1 + \exp\left((x - X_0)\,\eta_A\right)}, & w_A > 0, \\ \begin{cases} 1, & x < X_0, \\ \in [0, 1], & x = X_0, \\ 0, & x > X_0, \end{cases} & w_A = 0, \end{cases} \tag{19}$$

where

$$X_0 \triangleq (B - a)/D \text{ and } \eta_A \triangleq \overline{w_A}/(Dw_A). \tag{20}$$

Proof. By (11)- (13), we have that

$$\frac{\partial v_T(\boldsymbol{x}, \boldsymbol{y})}{\partial x} = \overline{w_T}(Dy + b - B) + w_T \ln\left(\overline{x}/x\right). \tag{21}$$

with D given by (9). Thus, for $w_T > 0$ we have that $\partial v_T(\boldsymbol{x}, \boldsymbol{y})/\partial x$ is decreasing from infinity for $x \downarrow 0$ to negative infinity for $x \uparrow 1$. Thus, for a fixed $y \in [0, 1]$, the best response x is given as the unique root of the following equation: $\overline{w_T}(Dy + b - B) + w_T \ln\left(\overline{x}/x\right) = 0$. Solving this equation by x implies the first row of (17).

For $w_T = 0$, by (21), we have that $\partial v_T(\boldsymbol{x}, \boldsymbol{y})/\partial x = Dy + b - B$, i.e., this derivative is a constant on $\boldsymbol{x}$, and the second row of (17) follows.

[1] In the paper we use the following notation: $\overline{\xi} \triangleq 1 - \xi$.

By (11), (12) and (14), we have that

$$\frac{\partial v_A(\boldsymbol{x},\boldsymbol{y})}{\partial x} = \overline{w_A}(B-a-Dx) + w_A \ln(\overline{y}/y)\,. \tag{22}$$

Thus, for $w_A > 0$ we have that $\partial v_A(\boldsymbol{x},\boldsymbol{y})/\partial y$ is decreasing from infinity for $y \downarrow 0$ to negative infinity for $y \uparrow 1$. This implies that for a fixed $x \in [0,1]$ the best response y is given as the unique root of equation: $\overline{w_A}(B-a-Dx)+w_A \ln(\overline{y}/y) = 0$ with D given by (9). Solving this equation by y implies the first row of (19). For $w_A = 0$, by (22), we have that $\partial v_A(\boldsymbol{x},\boldsymbol{y})/\partial y = B-a-Dx$, and the second row of (19) follows. ■

6. Nash Equilibrium

In this section we prove uniqueness of NE and derive it in closed form. In Theorems 1–4 we consider separately all cases which could arise depending on whether all weighting coefficients w_T and w_A are positive or at least one of them equals zero.

Theorem 1. *Let $w_T > 0$ and $w_A > 0$. Then NE is unique in game Γ, and it is equal to $(x_*, BR_A(x_*))$, where x_* is the unique root in $(0,1)$ of the equation*

$$F(x_*) = 0, \tag{23}$$

where

$$F(x) \triangleq x - BR_T(BR_A(x)). \tag{24}$$

This root x_* can be found via the bisection method.

Proof. Since $w_T > 0$, by (8), (17) and (18), we have that $\mathrm{BR}_T(y)$ is increasing in y. Since $w_A > 0$, by (8), (19) and (20), we have that $\mathrm{BR}_A(x)$ is decreasing in x. Thus, $\mathrm{BR}_T(\mathrm{BR}_A(x))$ is decreasing in x as a superposition of decreasing and increasing functions $\mathrm{BR}_A(\cdot)$ and $\mathrm{BR}_T(\cdot)$, respectively. Thus, function $F(x)$ given by (24) is strictly increasing in $[0,1]$. Moreover, by (17), (19) and (24), we have that $F(0) = -\mathrm{BR}_T(\mathrm{BR}_A(0)) < 0$ and $F(1) = 1 - \mathrm{BR}_T(\mathrm{BR}_A(1)) > 0$. Thus, the root x_* of (23) is the unique, and it can be found via the bisection method. This, jointly with (19) and (23), imply that $(x_*, \mathrm{BR}_A(x_*))$ is the unique NE of the game Γ. ■

Theorem 2. *Let $w_T > 0$ and $w_A = 0$. Then NE (x,y) is unique in game Γ, except of the only case in (26) where a continuum of equilibrium strategies arises, and it is given as follows:*

(a) if $w_T < 1$ then

$$(x,y) = \begin{cases} (1,1), & BR_T(1) \le X_0, \\ (X_0, BR_T^{-1}(X_0)), & BR_T(0) < X_0 < BR_T(1), \\ (0,0), & X_0 \le BR_T(0), \end{cases} \tag{25}$$

where $BR_T(\cdot)$ is given by the first row of (17), and, so, $BR_T^{-1}(X_0) = Y_0 - \ln(1/X_0 - 1)/\eta_T$,

(b) if $w_T = 1$ then $x = 1/2$ and

$$y \begin{cases} = 1, & 1/2 < X_0, \\ \in [0,1], & 1/2 = X_0, \\ = 0, & 1/2 > X_0. \end{cases} \tag{26}$$

Note that, in case (b), the transmitter's payoff is equal to $v_T(1/2, y) = -\ln(1/2) = 0.693$ independently to adversary's strategy y, which reflects stability in communication protocol even under the worst network parameters with $X_0 = 1/2$ where a continuum of adversary equilibrium strategies arises.

Proof of Theorem 2. $\mathrm{BR}_T(y)$, given by the first row of (17), is increasing in y. Then, substituting such $\mathrm{BR}_T(y)$ and (19) with $w_A = 0$ into (15) and (16), and solving by (x,y) implies (25). Finally, $x = 1/2$ for $w_T = 1$. Substituting such x and $w_A = 0$ into (19) implies (26). ■

Theorem 3. *Let $w_T = 0$ and $w_A > 0$. Then NE (x,y) is unique in game Γ, except for the only case in (28) where a continuum of equilibrium strategies arises, and it is given as follows:*

(a) if $w_A < 1$ then

$$(x,y) = \begin{cases} (0,0), & BR_A(0) \le Y_0, \\ (BR_A^{-1}(Y_0), Y_0), & BR_A(1) < Y_0 < BR_A(0), \\ (1,1), & Y_0 \le BR_A(1), \end{cases} \tag{27}$$

where $BR_A(\cdot)$ is given by the first row of (19), and, so, $BR_A^{-1}(Y_0) = X_0 + \ln(1/Y_0 - 1)/\eta_A$,

(b) if $w_A = 1$ then $y = 1/2$ and

$$x \begin{cases} = 1, & 1/2 > Y_0, \\ \in [0,1], & 1/2 = Y_0, \\ = 0, & 1/2 < Y_0. \end{cases} \tag{28}$$

Note that, here in case (b) with $Y_0 = 1/2$, $\mathbb{T}(x, 1/2) = (B+a)/2$ for all x which reflects that even the case of multiple equilibria cannot have an impact on the communication stability via the suggested protocol.

Proof of Theorem 3. Function $\mathrm{BR}_A(x)$, given by the first row of (19), is decreasing in x. Then, substituting such $\mathrm{BR}_A(x)$ and (17) with $w_T = 0$ into (15) and (16), and solving by (x,y) implies (27). Finally, $y = 1/2$ for $w_A = 1$. Substituting $y = 1/2$ and $w_T = 0$ into (17) implies (28). ■

Theorem 4. *Let $w_T = 0$ and $w_A = 0$. Then NE (x,y) is uniquely given as follows:*

$$(x,y) = \begin{cases} (0,0), & X_0 < 0, \\ (X_0, Y_0), & 0 < X_0 < 1, \\ (1,1), & 1 < X_0. \end{cases} \tag{29}$$

Proof. By (7)–(9) and (18), we have that $0 < Y_0 < 1$. Then, substituting (17) with $w_T = 0$ into (19) with $w_A = 0$ and solving by (x,y) implies (29). ■

7. Numerical illustration

To illustrate how the equilibrium strategies given by Theorems 1–4 depend on the weighting coefficients of the transmitter's payoff let us consider a communication example involving $n = 4$ sub-carries with main channel gains $h =$

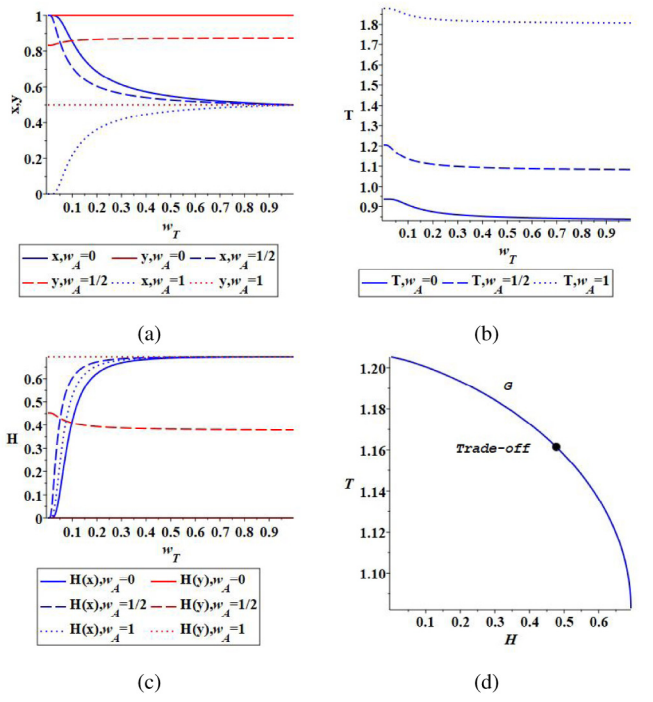

Fig. 2. (a) Strategies x and y, (b) expected throughput and secrecy rate $\mathbb{T}$, (c) entropy H as functions on w_T, and (d) parameterized set G in plane $(H, \mathbb{T})$ with $w_A = 1/2$.

(7.1, 5.9, 9.8, 10.6), jamming channel gains $g = (7.1, 5.9, 9.8, 10.6)$, and eavesdropping channel gains $h_E = (0.9, 2.85, 5.7, 3.53)$. Let also background noises be $\sigma^2 = \sigma_E^2 = 1$, total transmission and jamming powers be $P_{total} = J_{total} = 3$, and weighting coefficient of the adversary be $w_A = 0, 0.5, 1$. We can find the equilibrium power allocation strategy $\boldsymbol{P}_E = (0.316, 1.296, 0.682, 0.488, 0.217)$ via the waterfilling algorithm [10, Theorem 1]. Equilibrium power allocation strategies $\boldsymbol{P}_J = (0.147, 0.319, 0.670, 0.776, 1.086)$ and $\boldsymbol{J}_J = (0.067, 0.330, 0.717, 0.910, 0.974)$ can be found via the superposition of two bisection algorithms [11, Section 4 "Algorithm"]. This leads to entries of the matrix (10) as follows: $A = 0.936$, $a = 0.741$, $B = 3.014$ and $b = 2.536$. Thus, $Y_0 = 0.710$ and $X_0 = 3.377$, by (18) and (20), respectively. Note that the boundary case $w_T = 0$ and $w_A = 0$, i.e., both players are not sophisticated, corresponds to the classical 2×2 matrix game [12]. For this boundary case, the NE is (1,1) (see, Fig. 2(a)) which corresponds to the minimal entropy (0,0) for both players (see, Fig. 2(c)), and such a classical solution is the most predictable for both players. Fig. 2(a) illustrates that the transmitter's equilibrium strategy monotonically tends to $x = 1/2$ with an increase in w_T, which corresponds to the equilibrium strategy where the transmitter focuses only on the objective to maximize unpredictability in its communication.

Moreover, an increase in w_T leads to a decrease in the expected throughput and secrecy rate (see, Fig. 2(b)), and an increase in entropy of the transmitter's strategy (see, Fig. 2(c)). Thus, a question arises for the transmitter as to which weighting coefficient w_T is preferable to maintain both of its goals. We derive such weighting coefficient w_T by applying a proportional fairness criteria. Let us denote by x_{w_T} and y_{w_T}, the equilibrium strategies of the transmitter and adversary, respectively, parameterized by weighting coefficient w_T. Let G be the set of all possible pairs of transmitter payoffs for both its objectives (see, Fig. 2(d)), i.e., $G \triangleq \left\{(H(x_{w_T}), \mathbb{T}(x_{w_T}, y_{w_T})) : 0 \le w_T \le 1\right\}$. Then, the trade-off weighting coefficient w_T can be found by maximizing the proportional fairness utility given as follows

$$\varphi_{w_T} \triangleq \ln(\mathbb{T}(x_{w_T}, y_{w_T}) - \mathbb{T}(x_1, y_1)) + \ln(H(x_{w_T}) - H(x_0)), \quad (30)$$

where: (a) $\mathbb{T}(x_{w_T}, y_{w_T}) - \mathbb{T}(x_1, y_1)$ reflects an increase in the expected throughput and secrecy rate in comparison with its minimum achieved at $w_T = 1$, and (b) $H(x_{w_T}) - H(x_0)$ reflects an increase in the entropy of the strategy in comparison with its minimum achieved at $w_T = 0$. In the considered example, the trade-off value for the weighting coefficient is $w_T = 0.059$ with the expected throughput and secrecy rate and entropy being equal to 1.161 and 0.478, respectively.

8. Conclusions

In this paper, a problem involving the secret and reliable communication of a transmitter with a receiver in the presence of an adversary has been modeled in a game theoretical framework with the transmitter and adversary as players. We have introduced a new type of player called a sophisticated player. Specifically, such a sophisticated player has two goals: a basic goal and a secondary goal. Regarding the basic goal, the adversary and transmitter are antagonists. The adversary wants to reduce the secret and non-jammed throughput delivered to the receiver by means of a combined eavesdropping and jamming attack, meanwhile the transmitter wants to increase it. Regarding the secondary goal, each of the players wants to achieve its basic goal in a manner that is as unpredictable as possible to the other player. We consider the entropy of player's strategy as a metric for such unpredictability. The equilibrium has been found in closed form, and its proven uniqueness demonstrates the stability of the suggested transmission protocol. Finally, the parameters of the transmitter's utility supporting both its goals were optimized via proportional fairness approach. A goal of our future research is to generalize the eavesdropping and jamming dilemma to dynamic models described by stochastic games with sophisticated players.

CRediT authorship contribution statement

Andrey Garnaev: Conceptualization, Writing – original draft, Formal analysis. **Wade Trappe:** Writing – review & editing, Project administration.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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