A Non-Zero Sum Bandwidth Scanning Game with a Sophisticated Adversary

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2022 56th Annual Conference on Information Sciences and Systems (CISS) | 978-1-6654-1796-9/22/$31.00 ©2022 IEEE | DOI: 10.1109/CISS53076.2022.9751180

***Abstract*—Detecting malicious users (adversaries) or unlicensed activities is a crucial problem facing dynamic spectrum access. Traditionally, in such a problem, the adversary is considered to be one who wants to get achieve malicious goal undetected. In this paper we deal with a new type of adversary, called *sophisticated* adversary, who, besides the basic goal of being malicious and undetected, it also wants to achieve this in the most unpredictable way. As a metric for such unpredictability we consider the entropy of adversary strategy. We model this problem by a nonzero-sum two players resource allocation game. One of the players, called the Scanner, wants to detect the sophisticated adversary. The other player (adversary), called the Invader, wants to find a trade-off between two goals: to sneak bandwidth usage undetected and to achieve such sneaking in the most unpredictable way. The equilibrium is found in closed form, and its dependence on communication network parameters is illustrated. Finally, weighting coefficients for the basic and secondary goals of the Invader are optimized via Nash bargaining.**

***Index Terms*—Detection probability, Entropy, Non-zero sum game, Nash equilibrium, Nash bargaining**

I. Introduction

The open nature of the wireless medium, in spite of supporting many benefits given by the ability to access spectrum dynamically, also makes cognitive radios a powerful tool for conducting malicious activities or policy violations by secondary users. Therefore, detecting malicious users or unlicensed activities is a crucial problem facing dynamic spectrum access [1], and one of the challenges to enforcing the proper usage of spectrum is the development of an intrusion detection systems that can scan large amounts of spectrum and identify illegal activity [2]. Since, in such security problems, there are two agents with different goals (the adversary aims to sneak into bands undetected for their illegal usage, while the intrusion detection system intends to prevent illegal spectrum usage), *game theory* is an ideal tool to employ. As examples of applying game theory to detect an adversary to prevent malicious attack on networks, we mention [3]–[14]. In all of these papers the adversaries aimed to accomplish their malicious goals without being detected.

In contrast to these works, in this paper we consider a new type of adversary, called *sophisticated* adversary. Such an adversary besides the basic goal to get its malicious goal undetected, also has a secondary goal to do this in the most unpredictable way. As a metric for such unpredictability we employ the entropy of the adversary's strategy. We model this problem by a nonzero-sum two-player resource allocation game between an Scanner and the Invader (sophisticated adversary) where the Invader wants to find a trade-off between two goals: (i) the basic goal of using bandwidth without being detected and (ii) the secondary goal of achieving such sneaking in the most unpredictable way. The equilibrium is found in closed form, which allows one to design an algorithm for optimal spectrum scanning. Using Nash bargaining we show how to optimize the Invader's utility which supports its basic and secondary goals.

The organization of this paper is as follows. In Section II, the bandwidth scanning model is described. In Section III, the equilibrium for an Invader supporting only the basic goal to minimize detection probability is presented. In Section IV, the non zero-sum bandwidth scanning game with a (sophisticated) Invader supporting basic and secondary goals is formulated. In Section V, the equilibrium strategies are derived as parameterized functions in closed form. In Section VI, monotonicity properties for the equilibrium strategies in terms of their parameters are established. In Section VII, the uniqueness of the equilibrium is proven. In Section VIII, numerical illustration of the derived equilibrium is provided. In Section IX, a Nash bargaining approach is applied to optimize the Invader's trade-off utility. Finally, in Section X, conclusions are offered. All proofs are provided in the Appendix.

II. A basic scanning model

In this section, we give a basic model as an example of rational and indifferent behavior in bandwidth scanning/sneaking. The model involves a scenario where a *primary user* (Scanner) owns n frequency bands $\mathcal{N} \triangleq \{1, 2, \dots, n\}$. The Invader will attempt to "sneak" usage on *only* one of these bands. By assumption, the Scanner can only scan a single band at a time to detect such malicious activity. We assume that the Invader will be detected with probability γ_i, $\gamma_i \in (0, 1)$, if it sneaks in band i and the Scanner scans that band. If the Scanner does not scan the band that the Invader is using, then the Invader sneaks safely, i.e., its detection probability is zero.

Let $\boldsymbol{x} = (x_1, \dots, x_n)$ be a strategy for the Scanner, where x_i is the probability (reflecting the likelihood of revisiting that

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band when the game is repeated) that it scans band i. So,

$$\sum_{i\in\mathcal{N}} x_i = 1, \text{ and } x_i \geq 0 \text{ for } i \in \mathcal{N}. \tag{1}$$

Let $\boldsymbol{y} = (y_1, \dots, y_n)$ be a strategy for the Invader, where y_i is the probability that it sneaks in band i. Thus,

$$\sum_{i\in\mathcal{N}} y_i = 1, \text{ and } y_i \geq 0 \text{ for } i \in \mathcal{N}. \tag{2}$$

Then, the detection probability of the Invader when the players employ strategies $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively, is given as follows:

$$\mathbb{P}(\boldsymbol{x}, \boldsymbol{y}) = \sum_{i\in\mathcal{N}} \gamma_i x_i y_i. \tag{3}$$

III. Invader aims to minimize detection probability

Traditionally, in detection problems the Scanner wants to maximize the detection probability of the Invader, i.e., to maximize $\mathbb{P}(\boldsymbol{x}, \boldsymbol{y})$ on $\boldsymbol{x}$ for each fixed $\boldsymbol{y}$. Meanwhile, the Invader wants to minimize such probability. Thus, this is a zero sum game [15], with a diagonal payoff matrix and its equilibrium strategies are given in closed form as follows (see, for example [16]):

$$x_i = y_i = 1/\sum_{j\in\mathcal{N}} (\gamma_i/\gamma_j) \text{ for } i \in \mathcal{N}. \tag{4}$$

IV. Sophisticated Invader

The (Sophisticated) Invader wants to find a trade-off between two goals: (i) the basic one, to sneak usage in bandwidth undetected, and (ii) the secondary one, to achieve such sneaking in the most unpredictable way. As a metric for the Invader to confuse the Scanner we consider the entropy of its strategy, i.e.,

$$H(\boldsymbol{y}) = -\sum_{i\in\mathcal{N}} y_i \ln(y_i). \tag{5}$$

The payoff to such Invader is taken as a weighted sum of entropy of its strategy and negative of its detection probability, i.e.,

$$V_I(\boldsymbol{x}, \boldsymbol{y}) = -w_P \mathbb{P}(\boldsymbol{x}, \boldsymbol{y}) + w_E H(\boldsymbol{y}), \tag{6}$$

where w_P and w_E are non-negative weighting coefficients.

The Scanner wants to maximize the detection probability. Thus, its payoff is given as follows:

$$V_S(\boldsymbol{x}, \boldsymbol{y}) = \mathbb{P}(\boldsymbol{x}, \boldsymbol{y}). \tag{7}$$

We look for a Nash equilibrium. Recall that $(\boldsymbol{x}, \boldsymbol{y})$ is Nash equilibrium if and only if, for each pair of feasible strategies $(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{y}})$, the following inequalities hold:

$$V_S(\tilde{\boldsymbol{x}}, \boldsymbol{y}) \leq V_S(\boldsymbol{x}, \boldsymbol{y}), \tag{8}$$
$$V_I(\boldsymbol{x}, \tilde{\boldsymbol{y}}) \leq V_I(\boldsymbol{x}, \boldsymbol{y}). \tag{9}$$

Denote this non-zero sum game by Γ.

Proposition 1: In the game Γ there exists at least one equilibrium.

The proof can be found in Appendix XII-A.

Further, we find equilibrium strategies in closed form using a constructive approach via solving the best response equations. Recall that, by (6), $(\boldsymbol{x}, \boldsymbol{y})$ is a Nash equilibrium if and only if each of these strategies is the best response to the other, i.e., $(\boldsymbol{x}, \boldsymbol{y})$ is

$$\boldsymbol{x} = \operatorname{argmax}_{\boldsymbol{x}} V_S(\boldsymbol{x}, \boldsymbol{y}), \tag{10}$$
$$\boldsymbol{y} = \operatorname*{argmax}_{\boldsymbol{y}} V_I(\boldsymbol{x}, \boldsymbol{y}). \tag{11}$$

Note that (10) is Linear Programming (LP) problem, while (11) is Non-Linear Programming (NLP) problem.

We note that, generally, in resource allocation problems even if the payoffs are concave the game might have multiple equilibria (see, for example, [17]). In this paper we establish the uniqueness of the equilibrium as a side effect of solving the best response equations associated with (8) and (9).

V. Explicit form for the equilibrium strategies

In this section we find in closed form for all of the possible solutions of the best response equations, i.e., equilibrium strategies, as functions of the two auxiliary parameters ω and ν. Note that intuition for parameters ω and ν is the following: ω is the maximal coefficient for x_i in the Scanner's payoff, and ν is Lagrange multiplier for the NLP problem (11).

Proposition 2: In the game Γ, each pair of equilibrium strategies $\boldsymbol{x} = (x_1, \dots, x_n)$ and $\boldsymbol{y} = (y_1, \dots, y_n)$ for the Scanner and the Invader, respectively, must have the following form:

$$x_i = x_i(\omega, \nu) \triangleq \begin{cases} 0, & i \in I_0(\omega, \nu), \\ -\dfrac{w_E \ln(\omega/\gamma_i) + w_E + \nu}{w_P \gamma_i}, & i \in I(\omega, \nu) \end{cases} \tag{12}$$

and

$$y_i = y_i(\omega, \nu) \triangleq \begin{cases} \exp(-1 - \nu/w_E), & i \in I_0(\omega, \nu), \\ \omega/\gamma_i, & i \in I(\omega, \nu), \end{cases} \tag{13}$$

where

$$I_0(\omega, \nu) \triangleq \{i \in \mathcal{N} : w_E \ln(\gamma_i/(e\omega)) \leq \nu\}, \tag{14}$$
$$I(\omega, \nu) \triangleq \{i \in \mathcal{N} : \nu < w_E \ln(\gamma_i/(e\omega))\}. \tag{15}$$

Moreover, the parameters ω and ν are solution of the following equations

$$X(\omega, \nu) \triangleq \sum_{i\in\mathcal{N}} x_i(\omega, \nu) = 1, \tag{16}$$
$$Y(\omega, \nu) \triangleq \sum_{i\in\mathcal{N}} y_i(\omega, \nu) = 1, \tag{17}$$

such that the following inequalities hold

$$0 < \omega \leq \overline{\gamma}, \tag{18}$$
$$\nu \geq -w_P \overline{\gamma} - w_E \tag{19}$$

with

$$\overline{\gamma} \triangleq \max_{i\in\mathcal{N}} \gamma_i. \tag{20}$$

The proof can be found in Appendix XII-B.

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VI. Auxiliary results

In this section we establish auxiliary monotonicity properties of the functions $X(\omega,\nu)$ and $Y(\omega,\nu)$, which allows us to prove the uniqueness of the equilibrium in game Γ as well as to derive an algorithm to find this equilibrium.

Proposition 3: Functions $X(\omega,\nu)$ and $Y(\omega,\nu)$ have the following properties:

(a) Function $X(\omega,\nu)$ is continuous on both parameters ω and ν. Moreover, it is decreasing on both parameters while $X(\omega,\nu)$ is positive.

(b) Function $Y(\omega,\nu)$ is continuous on both parameters ω and ν. Moreover, it is increasing in ω and it is decreasing in ν.

(c) For each fixed ω there is the unique $\mathbb{N}(\omega)$ such that

$$X(\omega,\mathbb{N}(\omega)) = 1. \tag{21}$$

Such $\mathbb{N}(\omega)$ can be found via the bisection method.

(d) Function $\mathbb{N}(\omega)$ is continuous and decreasing on ω.

(e) Function $Y(\omega,\mathbb{N}(\omega))$ is continuous and increasing on ω.

(f) There is the unique root $\omega_* \in (0,\overline{\gamma})$ of equation

$$Y(\omega,\mathbb{N}(\omega)) = 1. \tag{22}$$

This root can be found via the bisection method.

The proof can be found in Appendix XII-C.

VII. Uniqueness of equilibrium

In this section we establish the uniqueness of the equilibrium and give it in closed form.

Theorem 1: In the game Γ, the Nash equilibrium is unique. Moreover, the unique equilibrium $(\boldsymbol{x},\boldsymbol{y})$ is given as follows:

$$\boldsymbol{x} = \boldsymbol{x}(\omega_*,\mathbb{N}(\omega_*)), \tag{23}$$

$$\boldsymbol{y} = \boldsymbol{y}(\omega_*,\mathbb{N}(\omega_*)), \tag{24}$$

where Vector-valued functions $\boldsymbol{x}(\omega,\nu)$ and $\boldsymbol{y}(\omega,\nu)$ are given by Proposition 2, meanwhile function $\mathbb{N}(\omega)$ and the unique value ω_* are given by Proposition 3.

Note that, by Proposition 3(c) and (f), the ω_* can be found as a superposition of two bisection methods.

Proof of Theorem 1 can be found in Appendix XII-D.

In the following corollary we establish the relation between the Invader's weighting coefficients when the Scanner cannot narrow its scanning efforts to a subset of bands.

Corollary 1: The Scanner scans each band, i.e., $x_i > 0$ for all $i \in \mathcal{N}$, if and only if the following relation holds:

$$\frac{w_P}{w_E} > \sum_{k\in\mathcal{N}} \frac{1}{\gamma_k} \ln\left(\frac{\gamma_k}{\min_{i\in\mathcal{N}}\gamma_i}\right). \tag{25}$$

Moreover, in this case equilibrium strategies $\boldsymbol{x} = (x_1,\dots,x_n)$ and $\boldsymbol{y} = (y_1,\dots,y_n)$ of Scanner and Invader, respectively, are given as follows:

$$x_i = \frac{w_P - w_E \sum_{k\in\mathcal{N}} \ln(\gamma_k/\gamma_i)/\gamma_k}{w_P \sum_{k\in\mathcal{N}} \gamma_i/\gamma_k}, \tag{26}$$

$$y_i = \frac{1/\gamma_i}{\sum_{k\in\mathcal{N}} 1/\gamma_k} \text{ for } i \in \mathcal{N}. \tag{27}$$

Proof can be found in Appendix XII-E.

VIII. Numerical illustration

To illustrate how the equilibrium strategies in Theorem 1 depend on the number of bands and weighting coefficients of the Invader's payoff let us consider an example involving spectrum consisting of $n = 3, 4, 5$ bands with detection probabilities distributed according to an exponential law $\gamma_i = A\exp(-\kappa i)$ with $A = 1$ and $\kappa = 0.2$, $w_P = 1$ and w_E varying from 0.1 to 2. Fig. 1 and Fig. 2 illustrate that the Scanner's strategy is more sensitive to varying the weighting coefficient w_E than the Invader's strategy. Smaller sensitivity of the Invader strategy is reflected by flat segments that arise for small weighting coefficient w_E (see, also, Corollary 1). An increase in the weighting coefficient w_E makes the Scanner focus its scanning efforts on band 1. Meanwhile, an increase in the weighting coefficient w_E makes the Invader reduce the difference in sneaking efforts between the different bands. Fig 2 illustrates that an increase in the number of bands leads to a decrease in the detection probability and an increase in the entropy of the Invader's strategy. Meanwhile, an increase in weighting coefficient w_E leads to an increase in detection probability as well as in the entropy of the Invader's strategy.

IX. Optimization of weighting coefficients via Nash bargaining

In the previous section it was shown that an increase in the weighting coefficient w_E leads to an increase in detection probability and entropy of the Invader's strategy. Thus, a question arises: which weighting coefficient $w_E \in [\underline{w}_E, \overline{w}_E]$ could be preferable for the Invader to maintain both of the Invader's goals: (a) to sneak undetected, and (b) to do it the most unpredictable way. Note that, without loss of generality, we can assume that w_P is fixed since the Invader's payoff (6) is a linear function of weighting coefficients (w_P, w_E).

In this paper, we show how Nash bargaining approach can be implemented to design such trade-off value for weighting coefficient w_E via parameterizing all feasible outcomes [18]. A survey of different bargaining concepts used in wireless communication can be found in [19].

First, let us denote by $\boldsymbol{x}_{w_E}$ and $\boldsymbol{y}_{w_E}$, the equilibrium strategies of the Scanner and the Invader, respectively, given by Theorem 1 and parameterized by weighting coefficient w_E. Let $P_{w_E} \triangleq \mathbb{P}(\boldsymbol{x}_{w_E}, \boldsymbol{y}_{w_E})$ and $H_{w_E} \triangleq H(\boldsymbol{y}_{w_E})$ be the detection probability when the players implement strategies

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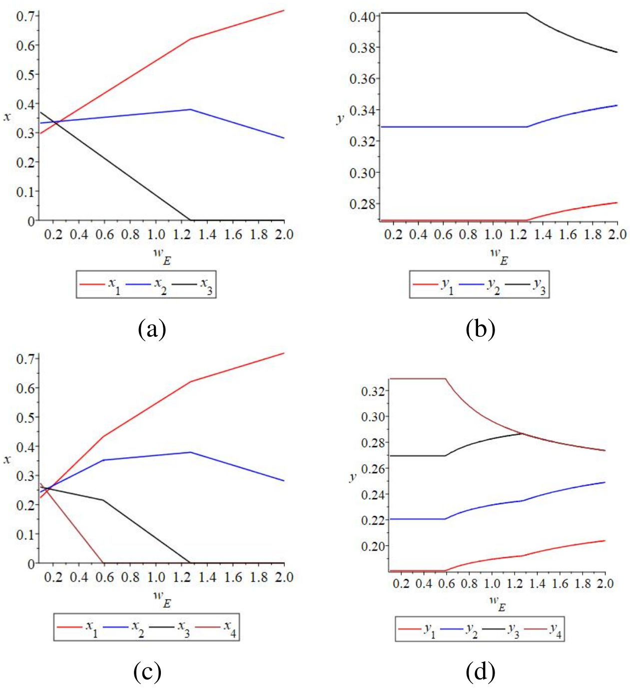

Fig. 1. (a) Scanner's strategies for $n = 3$, (b) Invader's strategies for $n = 3$, (c) Scanner's strategies for $n = 4$ and (d) Invader's strategies for $n = 4$ as functions on w_E.

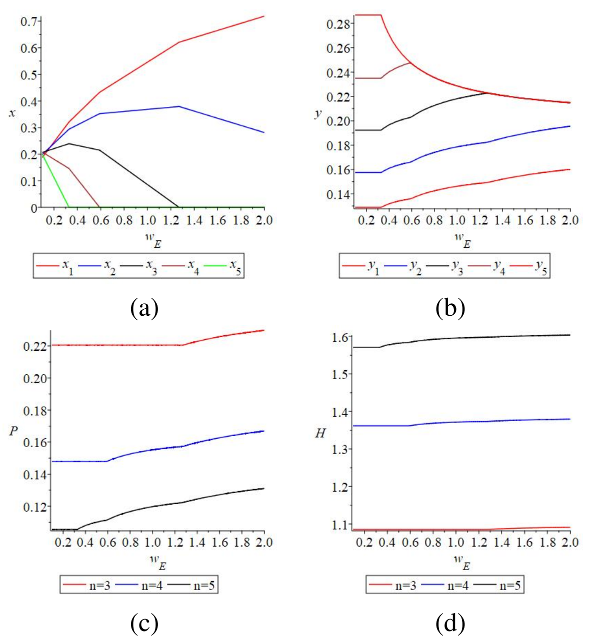

Fig. 2. (a) Scanner's strategies for $n = 5$, (b) Invader's strategies for $n = 5$, (c) Detection probability, i.e., scanner's payoff and (d) Entropy of Invader's strategies as functions on w_E.

$\boldsymbol{x}_{w_E}$ and $\boldsymbol{y}_{w_E}$ and entropy of the Invader's strategy $\boldsymbol{y}_{w_E}$, respectively.

Detection probability P_{w_E} is increasing on w_E (Fig 2). Thus, the basic objective of the Invader to be undetected can be modeled by the difference between the maximal and current detection probability, i.e., by the following outcome as function of w_E :

$$\Delta P_{w_E} = P_{\overline{w}_E} - P_{w_E}. \tag{28}$$

The secondary objective of the Invader to sneak unpredictable can be modeled by the entropy H_{w_E}, which reflects the second corresponding outcome.

Now we can introduce the set of all possible pair of Invader's outcomes, i.e.,

$$G \triangleq \{(\Delta P_{w_E}, H_{w_E}) : w_E \in [\underline{w}_E, \overline{w}_E]\}. \tag{29}$$

This (bargaining) set is illustrated on Fig. 3 by the example considered in the previous section with $n = 5$ bands. Then,

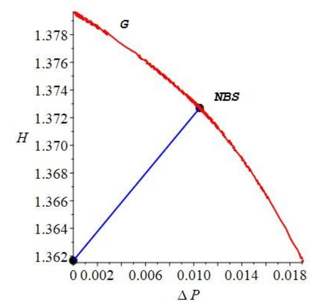

Fig. 3. Bargaining set and Nash bargaining solution for $n = 5$.

we define the Nash product [19]:

$$\begin{aligned}\mathrm{NP}_{w_E} &\triangleq \left(\Delta P_{w_E} - \Delta P_{\overline{w}_E}\right)\left(H_{w_E} - H_{\underline{w}_E}\right) \\ &= \Delta P_{w_E}\left(H_{w_E} - H_{\underline{w}_E}\right)\end{aligned} \tag{30}$$

with $(\Delta P_{\overline{w}_E}, H_{\underline{w}_E}) = (0, H_{\underline{w}_E})$ being the so-called disagreement point in Nash bargaining [19].

Finally, the Nash bargaining solution (NBS) can be found as the solution of the problem

$$\max\{\mathrm{NP}_{w_E} : w_E \in [\underline{w}_E, \overline{w}_E]\}. \tag{31}$$

Solution of this problem can be found via the Nelder-Mead simplex algorithm [20]. In the considered example, the Nash bargaining value for the weighting coefficient is $w_E = 0.765$ with detection probability and entropy are equal to 0.0149 and 0.0209, respectively.

X. Conclusions

In this paper, a new type of adversary has been modeled: specifically, a sophisticated adversary, who besides the basic goal of sneaking spectrum usage while being undetected, also has a secondary goal to do such sneaking in the most unpredictable way. This adversarial scenario has been modeled by a non-zero sum resource allocation game. Negative detection probability has been used as payoff to model the

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Invader's basic goal. Meanwhile, the entropy of the Invader's strategy has been employed as a payoff to model the secondary Invader's goal. The equilibrium has been found in closed form, and its uniqueness has been proven. A higher level of sensitivity of the Scanner's strategy on network parameters compared to the Invader's strategy has been established. The proven uniqueness of equilibrium demonstrates the stability of the suggested scanning algorithm. Finally, the parameters of the Invader's utility supporting its basic and secondary goals were optimized via Nash bargaining approach.

XI. Acknowledgement

This work was supported in part by the U.S. National Science Foundation under grants CNS-1909186 and ECCS-2128451.

XII. Appendix

A. Proof of Proposition 1

Note that $V_S(\boldsymbol{x},\boldsymbol{y})$ is linear on $\boldsymbol{x}$. Meanwhile $V_I(\boldsymbol{x},\boldsymbol{y})$ is additively separable function of y_i with $i \in \mathcal{N}$, and

$$\frac{\partial V_I^2(\boldsymbol{x},\boldsymbol{y})}{\partial y_i^2} = -\frac{w_E}{y_i} < 0. \tag{32}$$

Thus, $V_I(\boldsymbol{x},\boldsymbol{y})$ is concave in $\boldsymbol{y}$, and the result follows from the Nash's theorem [15] since set of feasible strategies for each player is compact. ■

B. Proof of Proposition 2

By (3) and (7), (10) is LP problem, and the Scanner's feasible strategy $\boldsymbol{x}$ is the best response to a fixed Invader's strategy $\boldsymbol{y}$ if and only if there is an ω such that

$$x_i \begin{cases} > 0, & \gamma_i y_i = \omega, \\ = 0, & \gamma_i y_i < \omega. \end{cases} \tag{33}$$

By (2) and (32), NLP problem (11) is concave. So, to find the Invader best response $\boldsymbol{y}$ to a fixed Scanner's strategy $\boldsymbol{x}$ we introduce Lagrangian $\mathcal{L}_\nu(\boldsymbol{y})$ with ν is a Lagrange multiplier as follows:

$$\mathcal{L}_\nu(\boldsymbol{y}) = V_I(\boldsymbol{x},\boldsymbol{y}) + \nu\left(1 - \sum_{i=1}^{n} y_i\right). \tag{34}$$

Then, the Invader's strategy $\boldsymbol{y}$ is the best response to the Scanner's strategy $\boldsymbol{x}$ if and only if the following condition holds:

$$\begin{aligned}\frac{\partial \mathcal{L}_\nu(\boldsymbol{y})}{\partial y_i} &= -w_P\gamma_i x_i - w_E - w_E\ln(y_i) - \nu \\ &\begin{cases} = 0, & y_i > 0, \\ \le 0, & y_i = 0. \end{cases}\end{aligned} \tag{35}$$

By (35), we have that

$$y_i > 0 \text{ for all } i. \tag{36}$$

This jointly with (35) imply that

$$-w_P\gamma_i x_i - w_E - w_E\ln(y_i) - \nu = 0 \text{ for all } i. \tag{37}$$

By (33), we have that ω has to be such that inequalities (18) hold with $\overline{\gamma}$ given by (20).

Meanwhile, by (37), we have that ν has to be such that inequality (19) holds.

Thus, only two cases arise to consider separately: (a) $x_i = 0$ and (b) $x_i > 0$.

(a) Let $x_i = 0$. Substituting such x_i into (33) and (37) imply, respectively, the following relations:

$$\gamma_i y_i \le \omega \tag{38}$$

and

$$-w_E - w_E\ln(y_i) - \nu = 0. \tag{39}$$

Solving (39) by y_i implies

$$y_i = \exp\left(-1 - \nu/w_E\right). \tag{40}$$

Substituting (40) into (38) implies

$$\gamma_i \exp\left(-1 - \nu/w_E\right) \le \omega. \tag{41}$$

Thus, the assumption that $x_i = 0$ jointly with (40) and (41) imply the first rows in (12) and (13) with $I_0(\omega,\nu)$ given by (14).

(b) Let $x_i > 0$. Substituting such x_i into (33) implies

$$\gamma_i y_i = \omega. \tag{42}$$

Meanwhile, substituting such x_i into (37) implies:

$$w_P\gamma_i x_i = -w_E - w_E\ln\left(\omega/\gamma_i\right) - \nu. \tag{43}$$

Substituting lower bound for x_i, i.e., $x_i = 0$, into (43) implies

$$\gamma_i > \omega\exp\left(1 + \nu/w_E\right). \tag{44}$$

Finally, (42), (43) and (44) imply the second rows in (12) and (13) with $I(\omega,\nu)$ given by (15). ■

C. Proof of Proposition 3

First note that (12) and (14)-(16) imply (a)

Similarly, (b) follows from (13)-(15) and (17).

Then, (c) follows from (a). Meanwhile, (d) follows from (a) and (c). Finally, (b) and (d) imply (e). Meanwhile, (f) follows from (c). ■

D. Proof: of Theorem 1

The result straightforward follows from Proposition 2 and Proposition 3. ■

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E. Proof of Corollary XII-E

Taking into account assumption that

$$x_i > 0 \text{ for all } i, \tag{45}$$

by (12), we have that

$$I_0(\omega, \nu) \text{ is empty set.} \tag{46}$$

This and (13) imply that

$$y_i = \frac{\omega}{\gamma_i} \text{ for all } i \in \mathcal{N}. \tag{47}$$

Summing up (47) by $i \in \mathcal{N}$ and taking into account condition (2) imply that

$$\omega = \frac{1}{\sum_{i\in\mathcal{N}} \frac{1}{\gamma_i}}. \tag{48}$$

Substituting such ω given by (48) into (47) implies (27).

Substituting (46) and (47) into (12) implies that

$$x_i = -\frac{w_E \ln\left(\frac{1}{\gamma_i\left(\sum_{k\in\mathcal{N}} \frac{1}{\gamma_k}\right)}\right) + w_E + \nu}{w_P \gamma_i} \text{ for } i \in \mathcal{N}. \tag{49}$$

Summing up (49) by $i \in \mathcal{N}$, by (1), we have that

$$\begin{aligned} 1 = &-\frac{w_E+\nu}{w_P}\sum_{k\in\mathcal{N}}\frac{1}{\gamma_k} - \frac{w_E}{w_P}\sum_{k\in\mathcal{N}}\frac{1}{\gamma_k}\ln\left(\frac{1}{\gamma_k}\right) \\ &+ \frac{w_E}{w_P}\sum_{k\in\mathcal{N}}\frac{1}{\gamma_k}\ln\left(\sum_{i\in\mathcal{N}}\frac{1}{\gamma_i}\right). \end{aligned} \tag{50}$$

Solving this equation on ν implies

$$\begin{aligned} \nu = &\, w_E \ln\left(\sum_{k\in\mathcal{N}}\frac{1}{\gamma_k}\right) - w_E \\ &- \frac{w_P + w_E\sum_{k\in\mathcal{N}}\frac{1}{\gamma_k}\ln\left(\frac{1}{\gamma_k}\right)}{\sum_{k\in\mathcal{N}}\frac{1}{\gamma_k}}. \end{aligned} \tag{51}$$

Substituting ν given by (51) into (49) implies (26). Finally, the assumption (45) and (26) imply (25). ■

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