Calculating the acoustic input impedance of a simplified brass instrument as an educational laboratory activity

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JASA ARTICLE

Calculating the acoustic input impedance of a simplified brass instrument as an educational laboratory activity[a)]

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ABSTRACT:
The concept of acoustic impedance is often difficult for students in introductory acoustics courses to make sense of, especially students without advanced mathematics backgrounds. This work summarizes a laboratory activity for students in a general education musical acoustics class where a simplified brass musical instrument is examined, focusing on how the geometry of the air column affects the input impedance of the instrument. Students are guided through making bore profile measurements for use in a computation of the input impedance. Options for making experimental measurements of the simplified instrument are explained. The laboratory activity was successfully used with students who reported their increased understanding of the acoustics of brass musical instruments.
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I. INTRODUCTION

For college and university departments offering a general education science credit with required laboratory experience, a musical acoustics course is an engaging option for a wide variety of students. The students typically enrolled in this course are not science or engineering majors. The students taking the course tend to be averse to applying mathematical thinking to their coursework. With this student population in mind, we developed an engaging laboratory activity to help make the concepts discussed in the classroom connect with the acoustics of wind instruments in our students' minds.

One of the concepts that is often more mathematically challenging to our students is acoustic input impedance of a wind instrument. Introductory musical acoustics textbooks often describe the idea of acoustic impedance by simply defining the impedance as the ratio of sound pressure, p, to volume velocity, U (Hall, 2002; Rossing *et al*., 2002). Input impedance is then described as the acoustic impedance at the mouthpiece end of the instrument. In our experience, we find that these descriptions, while accurate, are somewhat difficult for students to conceptualize and to make connections with the rest of the topics covered in an introductory musical acoustics class. Additionally, making comparisons to electrical impedances, while also accurate, are not meaningful for students who have never studied electronics in any form.

We have developed a laboratory activity where students measure the dimensions of simplified brass instruments and then use those measurements to calculate the acoustic input impedance of the instrument. The students in the lab then have the option to experimentally measure the input impedance of an instrument in the laboratory to compare to the predicted impedance curve from the calculation.

After doing this lab for the first time with students in two sections of the class, students were asked for feedback on what they thought worked well in the laboratory activity and where they thought there were opportunities for improvement.

II. MEASURING BORE PROFILES

The input impedance of a brass instrument depends on the profile of the air column of the instrument. Except for a few examples published in the literature (Braden, 2006; Myers, 1998; Worland, 2012) the geometry of a brass instrument from mouthpiece to bell is not generally known without making independent measurements.

In the undergraduate laboratory there are two basic methods of measuring the bore profile: via the interior of the instrument or by measuring the outside diameter of the instrument (Worland, 2012).

Measuring the interior profile of a brass instrument can be done by inserting disks of varying diameters into the bell end of the instrument until the disk makes contact with the bell. By measuring the depth to which each disk is inserted and using disks of varying diameter, the profile of the bell can be determined. This method has been previously shown to be useful for determining the profile of brass instrument mouthpieces (Myers, 1998). This method is tedious and is limited to measuring the profile of the instrument from either end only up to the first bend in the instrument.

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Additionally, there is a risk of scratching the inner surface of the bell with the rods inserted to measure the profile.

Measuring the exterior profile of the instrument can be accomplished in a variety of ways. For example, the circumference of the instrument bore at various locations can be measured by wrapping a string or strip of paper around the bore. From the circumference, the radius of the bore at each location can be calculated. Alternatively, a photo of the instrument could be used to extract the profile of the instrument with image processing software and a known reference length in the photo for calibration purposes. The advantage of measuring the exterior of the instrument bore is that the entire length of the instrument can be measured. However, to get the interior radius of the bore, the wall thickness of instrument must be subtracted from the measured radius. Accurately measuring the wall thickness along the length of the bore is not a simple process. The main disadvantage of this method of measuring the bore profile is a larger uncertainty in the radius measurements. Also, this method does not work well for the mouthpiece, as the interior profile of a mouthpiece is significantly different from its outside profile.

To avoid many of the difficulties described above, we chose in this lab to not make measurements on actual musical instruments. Instead, for this lab, we had students construct a simplified trumpet. In class, this instrument was often called a "hose-o-phone."

A. Measuring bore profile of a simplified brass instrument

The simplified trumpet consisted of a mouthpiece, a long cylindrical section, and a bell. The mouthpiece used was a three-dimensional- (3D-) printed mouthpiece (Robers, 2012). The model of this mouthpiece is available for anyone to download for 3D printing. The cylindrical section was made from a single piece of vinyl tubing cut to 1.40 m in length, to approximate the length of a trumpet bore. The vinyl tubing had an inner diameter of $3/8''$ (9.5 mm) and outer diameter of $1/2''$ (12.5 mm). The simplified trumpet's bell is a funnel [see Fig. 1(b)], specifically the Hopkins FloTool Multi-Purpose Funnel model #10701. These funnels were modified [see Fig. 1(c)] by sawing off an asymmetric part of the end of the funnel. The funnel was modified to make the bell of the hose-o-phone be axially symmetric to simplify measurements and calculations made by the students (Fig. 2).

The class was prompted to measure the interior profiles of the three parts of their hose-o-phone: the mouthpiece, the cylindrical bore, and the bell. A rough sketch of the instrument was provided to the class to identify all the dimensions to be measured. Students, working in lab groups of 3 or 4 members, measured the dimensions of the different parts of their instrument. Groups were instructed on how to use digital calipers for measuring the internal diameter of each of the parts of their instrument. For the mouthpiece, groups

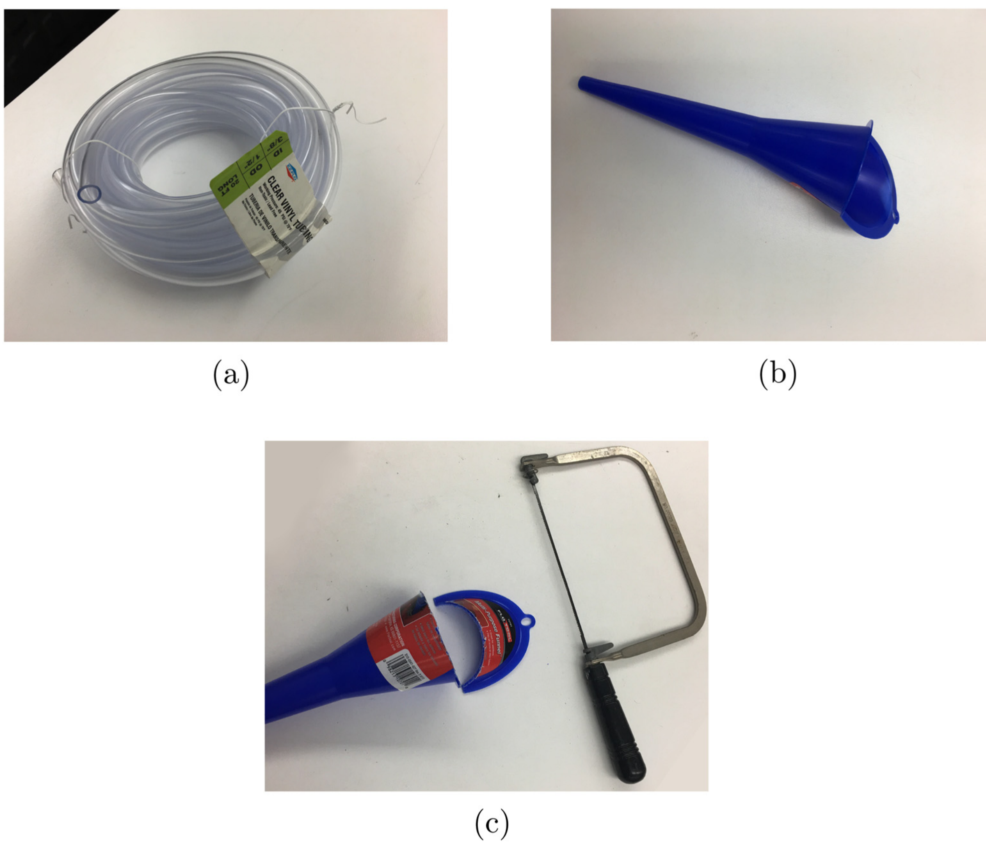

FIG. 1. (Color online) The main components of the simplified trumpet are (a) a section of vinyl tubing and (b) a funnel. (c) The funnel we used was modified by sawing off a section of the end of the funnel to make the bell be axially symmetric.

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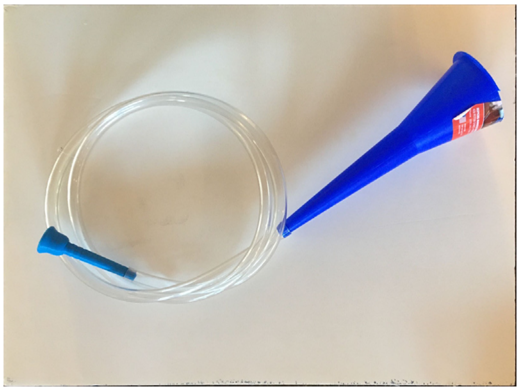

FIG. 2. (Color online) Students assemble the instrument as shown by connecting the vinyl tubing to the modified funnel and trumpet mouthpiece.

were instructed to measure the largest diameter of the mouthpiece cup, the overall length of the mouthpiece, and the inner radius at the end of the mouthpiece's backbore. These parts of the mouthpiece were the easiest parts for the students to measure; all other sections of the mouthpiece profile were provided to the groups by their instructor. The mouthpiece profile was based on a sample trumpet mouthpiece from an OPENWIND tutorial.

B. Example bore profile

An example bore profile is shown in Fig. 3. The length of the bore as shown in the figure has been shortened to more clearly illustrate the shape of the profile. The three main features of the simplified trumpet are shown. A complete description of the instrument's profile as used in the impedance calculations is given in Sec. III A.

III. CALCULATE IMPEDANCE CURVES

To make impedance calculations for the hose-o-phone instrument profiles, we use a library of PYTHON modules called OPENWIND (Chabassier *et al.*, 2021a). OPENWIND is an open source project consisting of several modules for doing computational acoustics calculations related to brass and woodwind musical instruments. The OPENWIND project is intended for use by the scientific research community as well as by musical instrument makers (Castera *et al.*, 2019; Chabassier *et al.*, 2020; Chabassier and Tournemenne, 2019; Ernoult *et al.*, 2021; Thibault and Chabassier, 2021; Tournemenne and Chabassier, 2019). Currently, the project has three modules: an impedance calculation module, a sound simulation module, and a module that calculates the instrument's bore profile from a measured impedance curve. For our lab activity, we used only the module for making impedance calculations. However, the other available modules would make for useful extensions of the lab activity.

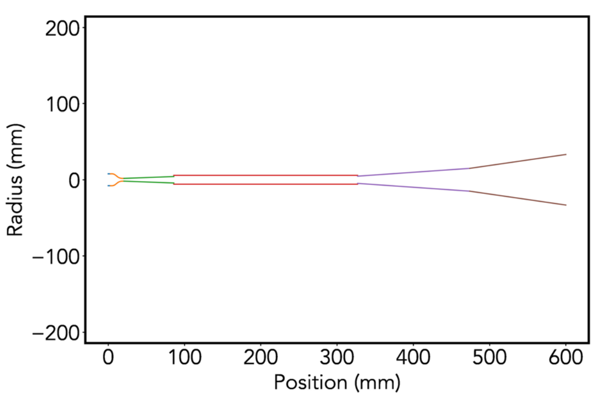

FIG. 3. (Color online) example profile for a simplified trumpet used in this laboratory activity. The cylindrical section of the bore has been truncated severely for illustrative purposes only.

A. How to use OPENWIND

Once OPENWIND is installed, making the impedance calculations for the simplified trumpet is straightforward. The measured profile of a musical instrument is formatted in sections based on the shape of the section of the bore. For modeling a real musical instrument, the profile of the air column is given without any bends in the instrument. For our simplified trumpet, the profile the students built for OPENWIND looked like their instrument when the vinyl tubing was unwound and held such that the parts of the instrument were all in a straight line.

Wind instrument bore profiles are constructed in OPENWIND by piecing the profile together using shapes defined by OPENWIND. The types of shapes that can be used by OPENWIND to describe the bore profile include: spline, Bessel, circle, and cone. The spline type is a C^2 spline curve in which specific coordinate points to pass through are defined. This type is useful for making curved sections of a profile that are not a part of a circle, such as the cup of the mouthpiece. The Bessel type draws a line based on a Bessel function where the power is specified as a parameter. The Bessel type is useful for modeling a real brass instrument bell. The circle type draws an arc of a circle of given radius. The cone type draws a straight line from an initial position coordinate, x_0, and initial radius, r_0, to a final position coordinate, x_0, and final radius, r_1. The cone type was used for specifying the majority of the simplified trumpet's profile. Cylindrical shapes can be defined using a cone type with the same initial and final radius.

The complete table of the parameters which specified the profile shown in Fig. 3 is given in Table I. As a reminder, this example profile shown in the figure and in the table has had the cylindrical section reduced for illustrative

TABLE I. The parameters for the profile shown in Fig. 3 in the OPENWIND format. All units are in meters.

x_0	x_1	r_0	r_1	Type	Parameters
0.0	0.003	0.00782	0.00782	Cone	
0.003	0.0207	0.00782	0.001825	Spline	0.008 0.013 0.0071 0.0034
0.0207	0.086	0.001825	0.00425	Cone	
0.086	0.327	0.00582	0.00582	Cone	
0.327	0.474	0.00475	0.015	Cone	
0.474	0.600	0.015	0.03325	Cone	

purposes. Students in the lab had instruments that were approximately double the length of the instrument shown in Fig. 3. All other figures are shown for full-sized instruments as used by students in the laboratory.

In the lab activity, students were given a blank table of the parameters to define the instrument's profile. The blank table students were given used the headings of the first four columns shown in Table I. Before the students started making measurements, there was a whole-class discussion to describe what each column in the table represented.

The first three lines of Table I specify the shape of the mouthpiece. The spline element forms the curved portion of the mouthpiece. In the lab, students measured only the initial radius of the cup part of the mouthpiece (r_0 in line 1 of Table I), the final radius of the backbore (r_1 in line 3 of Table I), and the overall length of the mouthpiece (x_1 in line 3 of Table I). The rest of the parameters in the first three lines of Table I) were based of measurements made by the instructor on the bore profile of the 3D-printed mouthpiece model. These measurements were added to the OPENWIND profile with the assistance of the lab instructor to check for the correct mouthpiece profile.

The fourth line of Table I specifies the cylindrical portion of the hose-o-phone. Note the initial and final radius on that line are the same, which make a cone into a cylinder. The last two lines of the table are the two parts of the funnel which have different cone angles and needed to be separated into the two parts. Students in lab were responsible for defining all the entries for the last three lines of this table.

B. Example calculations based on measured bore profiles

Once the instrument profile is completed, making the input impedance calculation in OPENWIND is done using only a few lines of PYTHON code. We followed tutorials in the OPENWIND project documentation (Chabassier *et al.*, 2021b) to prepare a Jupyter notebook for students to use as a template for making their own calculations of the input impedance of the simplified trumpets that they measured the geometries of. The example code given to students for making their impedance curves is freely available (Morrison, 2021). An example impedance curve calculation is shown in Fig. 4.

After making the impedance curve calculations, each group had a discussion with their lab instructor. The discussion focused on how to interpret the calculated impedance curves. The interpretation of the impedance curves could be more quantitative for more advanced students since the peak frequencies are readily accessible from the OPENWIND calculations. The effect of the mouthpiece was pointed out to students by having students note the increase in the acoustic impedance around 650 Hz. Students were asked to calculate the input impedance of the mouthpiece alone and plot the mouthpiece input impedance on the same graph with the input impedance of their instrument (Fig. 5).

Groups were also reminded that the peaks in the curve would appear at frequencies that would correspond to notes easily playable on their instrument. We discussed whether

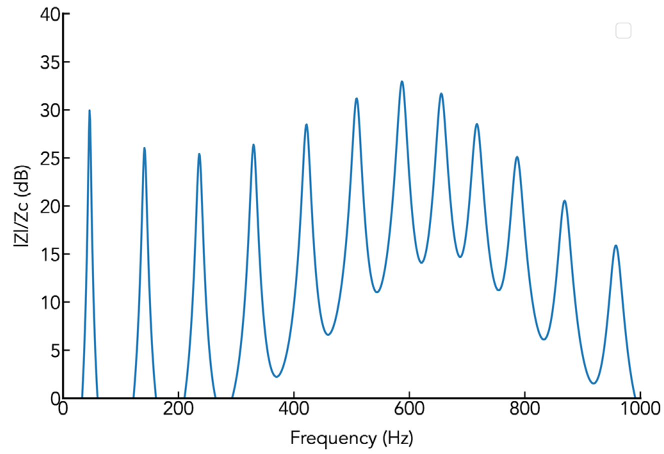

FIG. 4. (Color online) An example impedance curve calculation from OPENWIND based on a laboratory group's measurements of the geometry of their simplified trumpet. The vertical axis is input impedance (normalized to the characteristic impedance, Z_c shown on a dB scale) and the horizontal axis is frequency.

or not the peaks in their impedance curve were all evenly spaced in frequency or if they thought the first peak in the spectrum showed the anharmonic behavior that would be expected from the impedance curve of a real trumpet. Students were reminded that in a real trumpet the addition of the bell to a cylindrical pipe shifts the impedance peaks to be close to a harmonic series starting with the second peak in the impedance curve.

As more groups finished making their calculations, students compared impedance curves for simplified trumpets of different lengths. Some groups were given vinyl tubes with lengths equal to the length of a trumpet with one or more valves opened. By comparing impedance curves between groups, students were able to see the overall effect of increasing the length of the instrument.

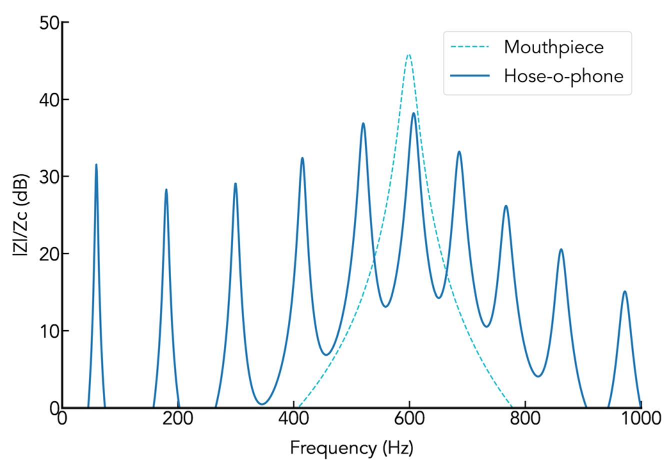

FIG. 5. (Color online) A graph showing the comparison of the calculated input impedance of just a trumpet mouthpiece to the calculated input impedance of the simplified trumpet. The vertical axis is input impedance (normalized to the characteristic impedance, Z_c shown on a dB scale) and the horizontal axis is frequency. By making this comparison students are able to see part of the effect of the mouthpiece on the instrument as a whole.

IV. COMPARING CALCULATIONS TO MEASUREMENTS

After making the impedance curve calculations some students in the lab were curious to know how accurate their measurements were. The simplified trumpets were played by their instructor and a spectrogram of was captured by the SpectrumView smartphone app (OxfordWaveResearch, 2021). A screenshot of the app is shown in Fig. 6.

Students were surprised to see that the majority (three of the five notes played) of the fundamental frequencies of the played notes on their instrument were closely matched to the corresponding peak in the calculated input impedance curve. For the spectrogram shown in Fig. 6, the highlighted peak in the sound spectrum was measured to be 271 Hz. The corresponding peak in the calculated input impedance curve was 272.5 Hz. There was not sufficient time in the laboratory period to discuss the relative closeness of the frequencies in terms of how many cents apart they were, such a discussion would make for a reasonable followup in class.

A. An advanced option—BIAS system

A possible extension to the basic laboratory activity is to use the Brass Instrument Analysis System (BIAS) (Widholm *et al.*, 2021) for directly measuring the input impedance of the simplified trumpet. BIAS is software and hardware system that is designed to measure the input impedance of brass instruments with an attached mouthpiece (Kausel, 1999, 2004; Widholm, 1995).

BIAS has been used widely by instrument makers and musical acoustics researchers, however, the cost of a BIAS (around €5000) tends to limit the use in a teaching lab. However, for the institutions with access to a BIAS, the advantage of the system is that it allows for quick and accurate measurements of the input impedance of nearly any brass instrument that has a mouthpiece attached to it. Alternatively, low-cost piezo-electric devices can be used to construct an impedance measurement system as described by Benade and Ibisi (1987).

Although implementing the BIAS measurements has not yet been done with students in a lab, we believe in the potential for using BIAS to enhance the laboratory experience for institutions with access to this system. The next section describes two possible uses for BIAS with this laboratory activity.

B. Sample BIAS measurements

One example of a useful BIAS measurement is shown in Fig. 7. The figure is a screenshot of the BIAS software showing the comparison of two related measurements. In blue is shown the input impedance of a Bach 7C trumpet mouthpiece alone. Overlaid on the same plot is the measured input impedance of a 140 cm length of tubing with the mouthpiece inserted into it. Comparing this figure to the plots shown in Fig. 5 there are obvious parallels which would be useful for students to examine.

One of the limitations of BIAS is that making an input impedance measurement must be done with a brass instrument mouthpiece coupled to BIAS. It is not convenient to measure the input impedance of the vinyl tubing by itself. However, BIAS can be used to show the effect of changing mouthpieces, tubing lengths, and funnels.

As an example of changing funnels on the simplified trumpet, two different funnels of similar shape, but different sizes, were connected to the same length of vinyl tubing and same mouthpiece. The input impedance of each simplified trumpet was measured with BIAS and the impedance curves were overlaid for comparison. The comparisons are shown in Fig. 8(a).

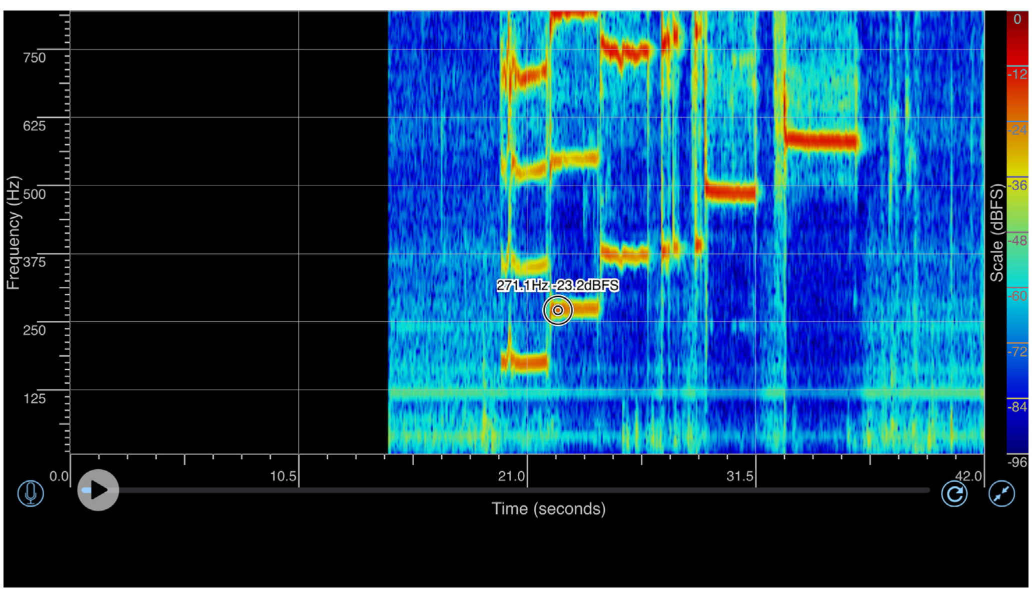

FIG. 6. (Color online) Spectrogram of the simplified trumpet being played as measured by the SpectrumView smartphone app. The highlighted fundamental frequency in the spectrogram corresponds to the third peak in the calculated input impedance curve.

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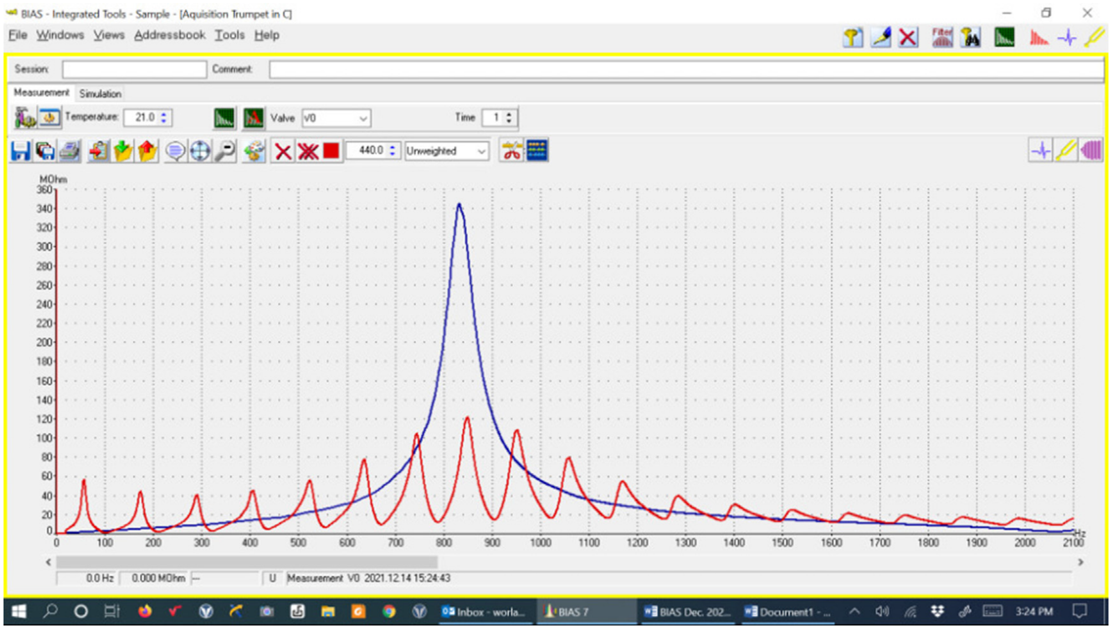

FIG. 7. (Color online) A screenshot from the BIAS software showing in red the input impedance of a 140 cm tube (3/8″ ID) with an attached Bach 7C mouthpiece. In blue is shown the input impedance of the mouthpiece alone. The vertical axis is in acoustic megohms; the scale is linear. The horizontal axis is frequency.

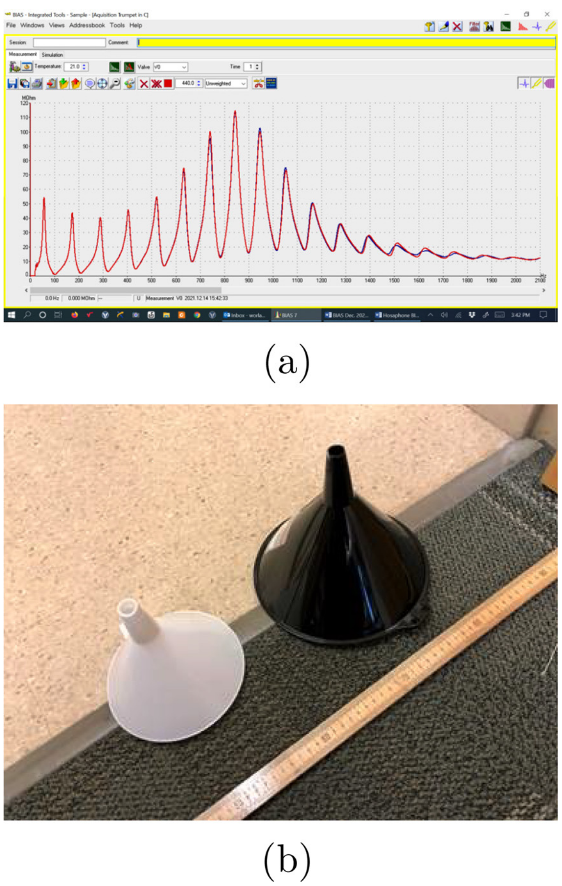

FIG. 8. (Color online) (a) A screenshot from the BIAS software showing the measurement of two input impedance curves for simplified trumpets using two different funnels. The vertical axis is in acoustic megohms; the scale is linear. The horizontal axis is frequency. The two funnels were similarly shaped although different in size. The two impedance curves are nearly identical. (b) A photo of the two differently sized but similarly shaped funnels.

The somewhat surprising result of the impedance measurements shown in the figure are that changing the funnel from a large size to a small size [as shown in Fig. 8(b)] did not significantly change the input impedance of the simplified trumpet. This should indicate to students that the profile of the bell of an instrument has a larger effect on the input impedance than the size of the bell, at least for moderate changes of the bell size.

Changing the funnel on the end of the hose-o-phone to differently shaped funnel, the measured input impedance as shown in Fig. 9 was changed significantly. The blue curve shown in the figure corresponds to the funnel shown in Fig. 2 whereas the red curve corresponds to the black funnel in Fig. 8(b).

A comparison of the impedance curves for a simplified trumpet with and without a funnel is shown in Fig. 10. The blue curve shows the input impedance of just the mouthpiece and the vinyl tubing. The red curve shows the input impedance of the whole instrument. By comparing these two curves, students can see some of the effects of the bell on a real instrument.

V. STUDENT COMMENTS AND CONCLUSION

After running this laboratory activity with two sections of a general-education musical acoustics class, each group discussed with their instructor what they thought worked well about the laboratory activity and what aspects could be improved.

Students commented that they felt the example measurements of the instrument geometry given at the start of the lab were helpful for understanding how OPENWIND would be expecting the instrument profile to be formatted.

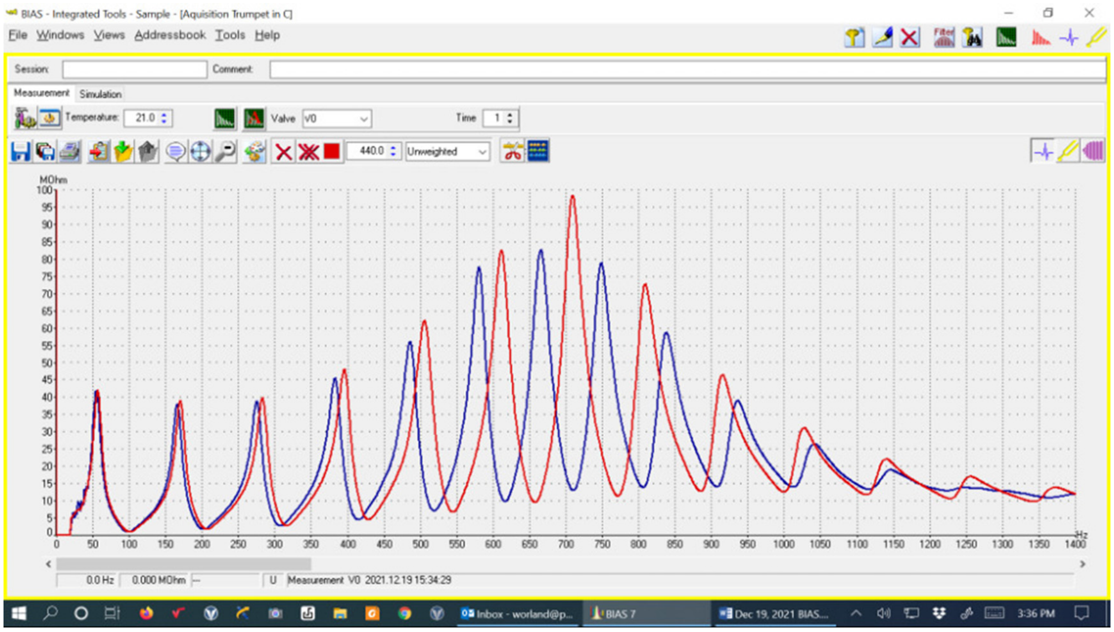

FIG. 9. (Color online) A screenshot from the BIAS software showing the measurements of two input impedance curves for hose-o-phones using two differently shaped funnels. The vertical axis is in acoustic megohms; the scale is linear. The horizontal axis is frequency.

Another group had a similar comment and added that having the blank table with column headings was also useful for their measurements of the dimensions of the instrument.

Lab groups in both sections commented that they felt this laboratory was more relatable to experiences from their lives than most of the other laboratory activities completed in the course. Although the students did not specify the exact comparisons they were making, it was understood by the instructor that they were comparing this activity to the more traditional physics lab activities such as the Hooke's law or vibrating string lab. The students felt that this lab had more direct connections to musical instruments and musicians than most of the other labs from the semester. They also commented that the lab was simple, conceptually, at least in terms of measurements they needed to make, but that it connected to previous laboratory investigations of resonances of pipes.

One student made the observation that "chunking the instrument into sections was valuable" for them to see how the instrument is constructed. As instructors, we hope that

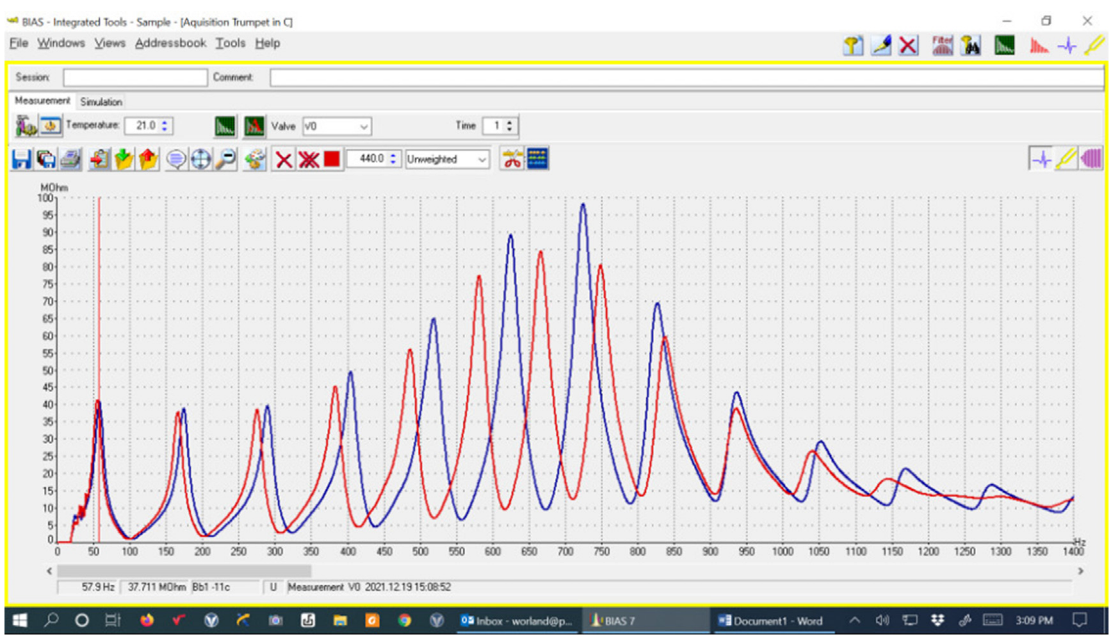

FIG. 10. (Color online) A screenshot from the BIAS software showing the measurement of a simplified trumpet with a funnel attached (curve shown in red) and without a funnel attached (curve shown in blue). The vertical axis is in acoustic megohms; the scale is linear. The horizontal axis is frequency.

students would also see the acoustical effects on the instrument as a whole that each section has on it.

One student suggested that making measurements of the interior of the funnel might have been easier if their group had an identical funnel that was cut in half through a diameter. A discussion was had as to whether a similarly prepared mouthpiece could be provided to groups so they could get more detailed measurements of the mouthpiece interior dimensions. Preparing a half of a mouthpiece should be possible by modifying the 3D-printed mouthpiece model and simply 3D printing half a mouthpiece.

One group thought the best part of the lab was the use of the SpectraView smartphone app. They suggested that the app should be used more for other labs.

The greatest strength of this lab is the ability for students to see brass instruments as a series of connected sections which each have a role in shaping the resonances of the instrument's air column. By simplifying the geometry and making the instrument easy to explore either computationally or experimentally, students gain a deeper conceptual understanding of how brass instruments work. We believe this laboratory activity is an excellent addition to our general education musical acoustics course.

ACKNOWLEDGMENTS

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