

# Precise stellarator quasi-symmetry can be achieved with electromagnetic coils

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1 Magnetic fields with quasi-symmetry are known to provide good  
 2 confinement of charged particles and plasmas, but the extent to  
 3 which quasi-symmetry can be achieved in practice has remained an  
 4 open question. Recent work (Landreman & Paul, Phys. Rev. Lett. 128,  
 5 035001, 2022) reports the discovery of toroidal magnetic fields that  
 6 are quasi-symmetric to orders-of-magnitude higher precision than  
 7 previously known fields. We show that these fields can be accurately  
 8 produced using electromagnetic coils of only moderate engineering  
 9 complexity, i.e., coils that have low curvature and that are sufficiently  
 10 separated from each other. Our results demonstrate that these new  
 11 quasi-symmetric fields are relevant for applications requiring the  
 12 confinement of energetic charged particles for long time scales, such  
 13 as nuclear fusion. The coils' length plays an important role for how  
 14 well the quasi-symmetric fields can be approximated. For the longest  
 15 coil set considered and a mean field strength of 1 Tesla, the departure  
 16 from quasi-symmetry is of the order of the earth's magnetic field.  
 17 Additionally, we find that magnetic surfaces extend far outside the  
 18 plasma boundary used by Landreman & Paul, providing confinement  
 19 far from the core. Simulations confirm that the magnetic fields generated  
 20 by the new coils confine particles with high kinetic energy  
 21 substantially longer than previously known coil configurations. In  
 22 particular, when scaled to a reactor, the best found configuration loses  
 23 only 0.04% of energetic particles born at mid-radius when following  
 24 guiding center trajectories for 200 ms.

fusion | quasi-symmetry | confinement | magnetic coils

1 Controlled nuclear fusion is a promising candidate to satisfy ris-  
 2 ing electricity needs while avoiding carbon emission into the  
 3 atmosphere. To generate electricity from fusion, a plasma needs to  
 4 be maintained at extremely high temperatures over long time scales,  
 5 which requires excellent particle confinement. This is typically  
 6 achieved using powerful magnets. Tokamaks rely on a toroidally  
 7 axisymmetric system of magnetic coils to achieve good confinement,  
 8 at the cost of requiring a plasma current to generate a significant  
 9 fraction of the magnetic field. This plasma current is challenging to  
 10 drive in steady-state operation (1), and can be the source of disruptive  
 11 instabilities (2, 3). In contrast, the non-axisymmetric coil systems of  
 12 stellarators can generate a confining magnetic field in the absence  
 13 of plasma currents, relieving many of the challenges for continuous,  
 14 disruption free operation. However, the lack of axisymmetry implies  
 15 that neither nested magnetic flux surfaces nor particle confinement  
 16 is guaranteed (4). This motivates the need for a generalization of  
 17 axisymmetry to the stellarator context, called quasi-symmetry.

18 A magnetic field is said to satisfy quasi-symmetry if there exists  
 19 an invariant direction for the field strength  $B = |\mathbf{B}|$  in a certain  
 20 coordinate system (5). This condition leads to the conservation of  
 21 canonical angular momentum, which in turn implies the remarkable  
 22 property that such fields are guaranteed to confine charged  
 23 particles without requiring plasma currents. However, it remains

24 an open question whether three-dimensional magnetic fields that  
 25 are perfectly quasi-symmetric over a volumetric region exist. Re-  
 26 cently, using numerical optimization, Landreman and Paul (6) found  
 27 vacuum magnetic fields that satisfy the quasi-symmetry property in  
 28 toroidal geometries to a very high precision. Magnetic fields that  
 29 are quasi-symmetric to such a high degree have not been discovered  
 30 before. Simulations confirm excellent confinement properties even  
 31 for particles with a large kinetic energy. Naturally, the question  
 32 arises if these magnetic fields can be accurately produced by a set  
 33 of practical electromagnets, which would be a first step towards a  
 34 new generation of fusion experiments. In this report, we show that  
 35 such magnets in fact exist. Specifically, we find coils producing fields  
 36 whose deviation from perfect quasi-symmetry is more than four  
 37 orders of magnitude smaller than the mean field strength. For a 1 T  
 38 mean field, comparable to many stellarator experiments, this error  
 39 can be on the order of the earth's magnetic field. Simulations confirm  
 40 particle confinement comparable to the ideal fields discovered in (6).  
 41 This shifts the discovery of these highly quasi-symmetric fields from  
 42 one of theoretical interest to one of practical relevance for fusion  
 43 experiments and other confinement applications (7).

## Results & Discussion

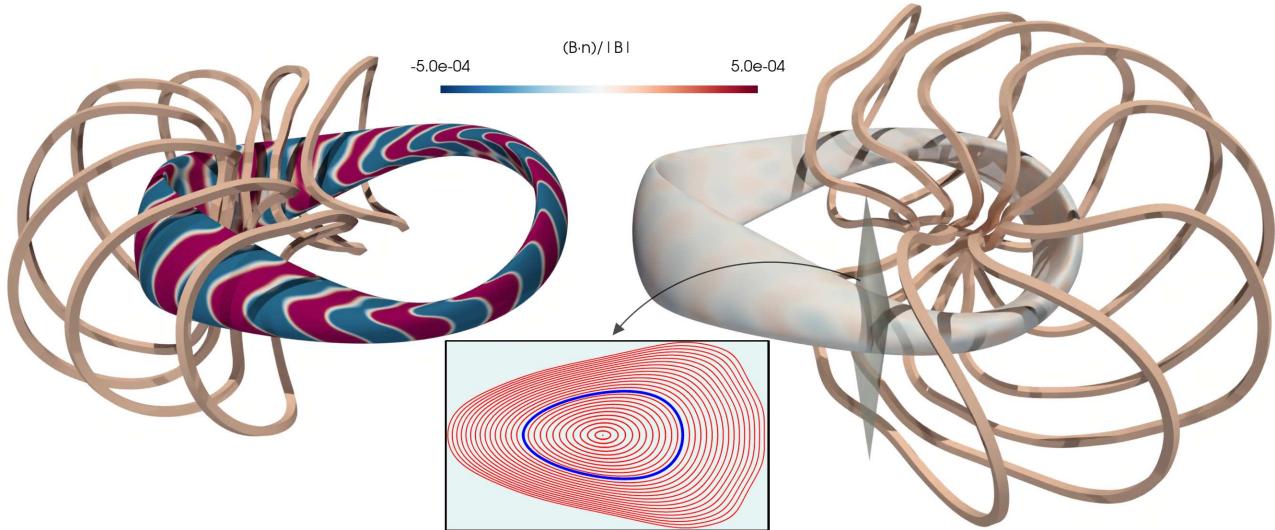
44 We solve a constrained optimization problem to obtain coils that  
 45 approximate the two quasi-axisymmetric fields of (6), which we refer  
 46 to by 'QA-LP' and 'QA+Well-LP', reflecting the absence or presence  
 47 of magnetic well. The design space consists of four distinct modular  
 48 coils, which results in 16 coils in total after applying symmetries. The  
 49 length of coils impacts the quality of the magnetic field approximation  
 50 strongly, and hence we compute coil sets of different coil length  
 51  $L_{\max}$  and compare their performance, e.g., 'QA+Well[20]' refers to  
 52 the coil set which approximates the 'QA+Well-LP' configuration and  
 53 for which the four modular coils have combined length 20 m. For  
 54 both 'QA-LP' and 'QA+Well-LP', as we allow longer coils, the field  
 55 induced by the coils becomes a better approximation of the target  
 56 field. Values of  $L_{\max}$  are relative to an average major radius of 1 m.  
 57

58 Figure 1 shows coils obtained by choosing  $L_{\max} = 18$  m and  
 59  $L_{\max} = 24$  m in the approximation to QA-LP, the relative normal  
 60 magnetic field  $\mathbf{B} \cdot \mathbf{n}/|\mathbf{B}|$  on the surface  $S$ , and a Poincaré plot for  
 61  $L_{\max} = 24$  m. Here  $\mathbf{B}$  is computed using the Biot-Savart law, and if  
 62 the normal magnetic field is exactly zero, then the field matches that  
 63 discovered in (6) everywhere in the volume contained by the surface.  
 64 For the shortest coils, we observe the largest normal magnetic field on  
 65 the surface, reaching values of up to  $3.3 \times 10^{-3}$ . Its oscillatory nature  
 66 is caused by the discrete nature of the electromagnetic coils and their

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**Fig. 1.** Top: Half of the coils for the QA[18] (left) and QA[24] (right) configurations and resulting relative normal component of the magnetic field on the target surface. The full set of 16 coils satisfies two-fold rotational symmetry. Bottom: Poincaré plot for the QA[24] configuration, obtained by numerically tracing magnetic field lines and recording their intersection with the shown cross section. Good flux surfaces extend far outside the boundary from (6) (highlighted in blue). We note that the rectangular cross section of the coils is for visualization only; in all computations for this manuscript the coils are each approximated by a single wire of infinitesimal thickness. We conducted numerical experiments in which we approximated finite-thickness coils using multiple filaments, and obtained very similar results, not shown here.

close proximity to the surface. Longer coils enable a larger distance between the surface and the magnets, thereby reducing these discrete effects, and more accurately reproducing the target magnetic field, with a relative normal magnetic field of at most  $1.6 \times 10^{-4}$  for the QA[24] configuration.

The Poincaré plot demonstrates the existence of nested magnetic surfaces in the entire volume. This feature is necessary (but not sufficient) for confinement, since charged particle trajectories are tangent to these surfaces in the limit of low energy.

Quasi-axisymmetric fields are characterized by the following property: when parametrized using Boozer coordinates  $\phi, \theta$ , the field strength on each magnetic surface only depends on the angle  $\theta$  (5, 8). When this property is satisfied exactly, even highly energetic collisionless particles are confined over long time scales, and collisional transport is minimized. In Figure 2-III), we compare the magnetic field strengths of three earlier stellarator configurations with the QA-LP field as well as our QA[24] and QA+Well[24] fields. The QA[24] field strength is visually indistinguishable from that of the QA-LP configuration; the same holds true for QA+Well-LP and QA+Well[24].

We quantify this statement by performing a Fourier transformation of  $|\mathbf{B}|$  and then studying the magnitude of those Fourier coefficients that break quasi-symmetry, i.e. we write  $|\mathbf{B}(s, \theta, \phi)| = \sum_{m,n} B_{m,n}(s) \cos(m\theta - n\phi)$  and then consider terms with  $n \neq 0$ . Here  $s$  indexes each surface by the normalized toroidal magnetic flux it encloses and we plot the largest symmetry-breaking Fourier mode on each surface in Figure 2-I). As coil length is increased, the symmetry-breaking error is reduced significantly, and approximates those of the fields discovered in (6). In fact, we are able to achieve errors smaller than the earth's magnetic field for the longest set of coils when the mean field strength is 1 T. For comparison, gray curves show the symmetry-breaking amplitudes for the eight previous quasi-symmetric configurations in Figure 1 of (6).

Finally, scaling the configurations to the mean field and minor radius of the ARIES-CS reactor (9), we compute collisionless guiding center trajectories for alpha particles initialized with a kinetic energy

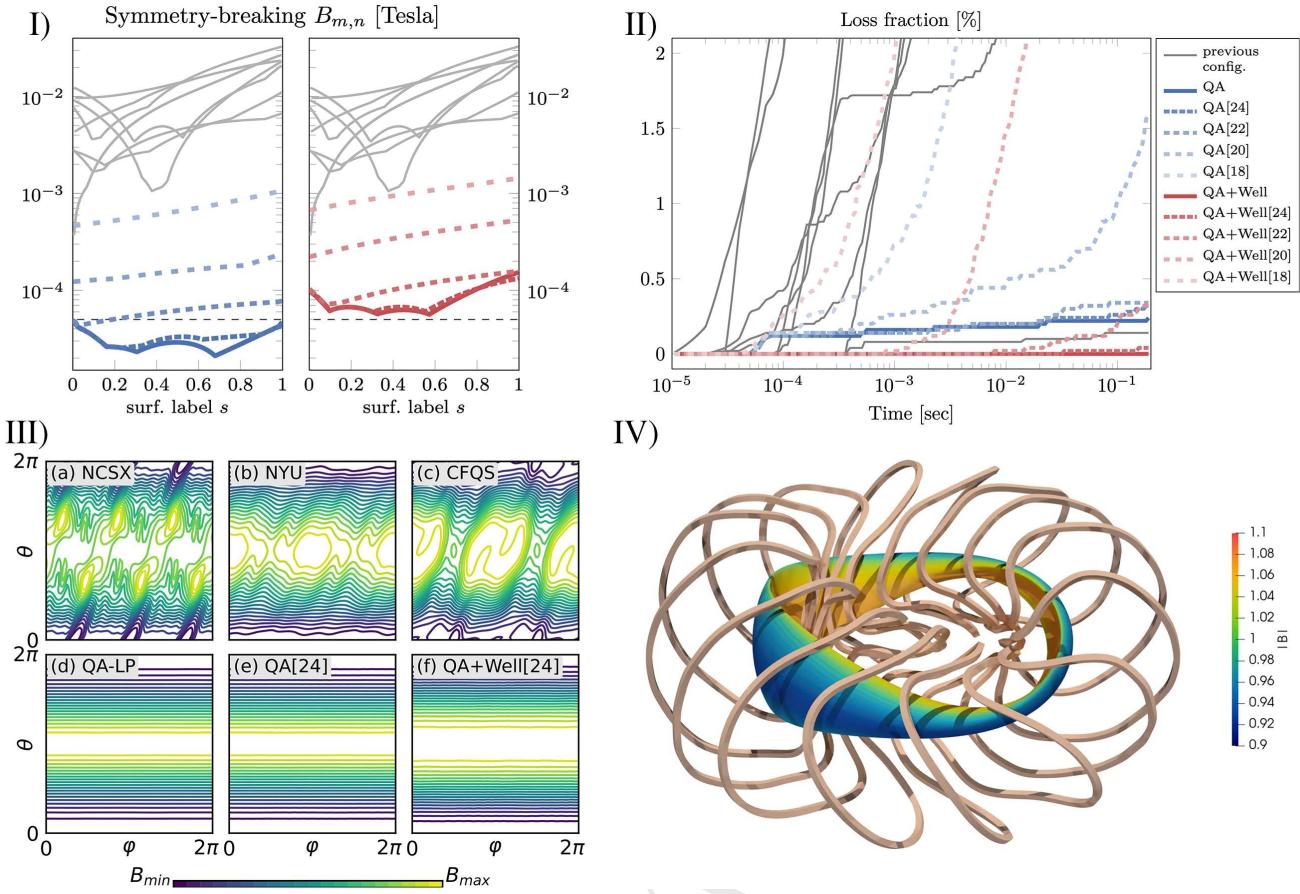
of 3.5 MeV, as by-products of a deuterium-tritium fusion reaction, on the surface with normalized toroidal flux  $s = 0.25$  (half radius). Particle losses are shown in Figure 2-II). For  $L_{\max} = 24$  m, the performance is nearly indistinguishable from the target equilibrium, and for the QA+Well[24] configuration less than 0.04% of particles are lost after 0.2 sec, a typical time for the alphas to thermalize with the main plasma. This coil set is shown in Figure 2-IV). Performance is only slightly worse for  $L_{\max} = 22$  m, but poor for the coils with  $L_{\max} = 18$  m. For comparison, grey curves show calculations for the nine previous stellarator configurations of figure 6a in (6), similarly scaled. We note that the thermal collisional transport magnitude  $\epsilon_{\text{eff}}^{3/2}$  (10) is less than  $2 \times 10^{-7}$  for coil lengths of  $L_{\max} = 22$  m and  $L_{\max} = 24$  m for both the QA and the QA+Well configurations, and  $< 6 \times 10^{-8}$  for QA[24]. These values are orders of magnitude below the values for other optimized stellarators (e.g.  $\sim 10^{-3}$  for Wendelstein 7-X (4)), and so small that collisional fluxes would be negligible compared to turbulent transport. This is the standard situation for tokamaks, but unusual and desirable for stellarators, since collisional and turbulent losses are additive (4).

In conclusion, we have shown that the magnetic fields of (6) can be produced very accurately using coils, making these fields practically relevant for stellarators. As a result, exceptionally good confinement of particle trajectories and remarkably small thermal collisional transport can be achieved. While longer coils are required for optimal performance, these coils are not particularly complex as measured in terms of curvature and coil-to-coil separation.

## Materials and Methods

The electromagnetic coils were optimized using the SIMSOPT software (11). Similar to the approach in (12), each coil is modelled as a closed, smooth curve in  $\mathbb{R}^3$  and represented using a Fourier series, truncated at order 16. Given a magnetic surface  $S$  (obtained from (6)), the objective that we minimize is given by

$$f_B = \int_S \left( \frac{\mathbf{B} \cdot \mathbf{n}}{|\mathbf{B}|} \right)^2 ds, \quad [1]$$



**Fig. 2.** I) Magnitude of the largest symmetry-breaking Fourier modes of  $|\mathbf{B}|$  for surfaces with normalized toroidal flux  $s$  ( $s = 1$  corresponds to the target surface). All configurations are scaled to a mean field strength of 1 T. The dashed line indicates  $50 \mu\text{T}$ , which is approximately the strength of the earth's magnetic field. II) Losses of alpha particles spawned on the  $s = 0.25$  surface. Apart from the Wistell-A configuration, all previous configurations loose 10% or more of energetic particles within 0.2 sec. The QA+Well[24] configuration gets extremely close to the perfect confinement of the QA+Well-LP configuration, with only 0.04% particles being lost. III) Comparison between magnetic field strength on the boundary surface of previous magnetic field configurations designed to approximate quasi-symmetry (top row), and the precise quasi-symmetric designs with coils presented in this paper (bottom row). Shown in (a) is the National Compact Stellarator eXperiment (NCSX), (b) a QA developed at New York University (NYU), (c) the Chinese First Quasiaxisymmetric Stellarator (CFQS). Shown in (d) are the field lines from the design without coils from (6), in (e) & (f) those obtained by using coils of length 24 to approximate the QA-LP and the QA+Well-LP configurations. IV) The coils corresponding to the QA+Well[24] configuration. The color indicates the magnetic field strength  $|\mathbf{B}|$ .

136 where  $\mathbf{B}$  is the field induced by the coils. If  $f_B = 0$ , then the induced field is  
137 exactly equal to the target field up to a scaling factor.

138 Finding coils that minimize this objective is an ill-posed problem, so we  
139 require additional regularization. In practice, it is desirable to have coils that  
140 are not too long, avoid high curvature, and are well separated. In this work  
141 we enforce constraints on the curvature ( $\kappa_{\max} \leq 5 \text{ m}^{-1}$ ), the mean squared  
142 curvature ( $\kappa_{\text{msc}} \leq 5 \text{ m}^{-2}$ ), the distance between coils ( $d_{\min} \geq 0.1 \text{ m}$ ), and  
143 we vary the constraint of the total length of the magnetic coils ( $L_{\max} \in \{18 \text{ m}, 20 \text{ m}, 22 \text{ m}, 24 \text{ m}\}$ ). The units quoted in the above assume a major  
144 radius scaled to 1 m. We refer to the support material for more detail on the  
145 exact implementation of these constraints.

146 The optimization objective has 399 parameters for the coils and is highly  
147 non-convex. The minimization uses the L-BFGS-B algorithm with analytic  
148 gradients. To remedy the possibility of the optimizer being stuck in a local  
149 minimum, we start the optimization from eight different initial coil sets and  
150 choose the best minimizer as measured by the objective. The code is highly  
151 optimized and parallelized and is publicly available at

152 <https://github.com/florianwechsung/CoilsForPreciseQS>.

153 Using 8 cores of an Intel Xeon Platinum 8268 CPU, solving an individual  
154 optimisation problem takes approximately 30 minutes.

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