Optimal Correlated Equilibria in General-Sum Extensive-Form Games: Fixed-Parameter Algorithms, Hardness, and Two-Sided Column-Generation

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We study the problem of finding *optimal correlated equilibria* of various sorts: *normal-form coarse correlated equilibrium (NFCCE)*, *extensive-form coarse correlated equilibrium (EFCCE)*, and *extensive-form correlated equilibrium (EFCE)*. This is NP-hard in the general case and has been studied in special cases, most notably *triangle-free games* [2], which include all two-player games with public chance moves. However, the general case is not well understood, and algorithms usually scale poorly. In this paper, we make two primary contributions.

First, we introduce the *correlation DAG*, a representation of the space of correlated strategies whose structure and size are dependent on the specific solution concept desired. It extends the *team belief DAG* of Zhang et al. [3] to general-sum games. For each of the three solution concepts, its size depends exponentially only on a parameter related to the information structure of the game. We also prove a fundamental complexity gap: while our size bounds for NFCCE are similar to those achieved in the case of team games by Zhang et al. [3], this is impossible to achieve for the other two concepts under standard complexity assumptions.

Second, we propose a *two-sided column generation approach* to compute optimal correlated strategies in extensive-form games. Our algorithm improves upon the *one-sided* approach of Farina et al. [1] by means of a new decomposition of correlated strategies which allows players to re-optimize their sequence-form strategies with respect to correlation plans which were previously added to the support.

Experiments show that our techniques outperform the prior state of the art for computing optimal general-sum correlated equilibria, and that our two families of approaches have complementary strengths: the correlation DAG is fast when the parameter is small and the two-sided column generation approach is superior when the parameter is large. For team games, we show that the two-sided column generation approach vastly outperforms standard column generation approaches, making it the state of the art algorithm when the parameter is large. Along the way, we also introduce two new benchmark games: a *trick-taking game* that emulates the endgame phase of the card game *bridge*, and a *ride-sharing game*, where two drivers traversing a graph are competing to reach specific nodes and serve requests.

The full version is available at https://arxiv.org/abs/2203.07181

CCS Concepts: • **Theory of computation** → **Exact and approximate computation of equilibria**.

Additional Key Words and Phrases: extensive-form games, correlated equilibrium, public information decomposition, column generation

ACM Reference Format:
Brian Hu Zhang, Gabriele Farina, Andrea Celli, and Tuomas Sandholm. 2022. Optimal Correlated Equilibria in General-Sum Extensive-Form Games: Fixed-Parameter Algorithms, Hardness, and Two-Sided Column-Generation. In *Proceedings of the 23rd ACM Conference on Economics and Computation (EC '22), July 11–15, 2022, Boulder, CO, USA.* ACM, New York, NY, USA, 2 pages. https://doi.org/10.1145/3490486.3538330

EC '22, July 11–15, 2022, Boulder, CO, USA

ACM ISBN 978-1-4503-9150-4/22/07.
https://doi.org/10.1145/3490486.3538330

ACKNOWLEDGEMENTS

This material is based on work supported by the National Science Foundation under grants IIS-1901403 and CCF-1733556, and the ARO under award W911NF2010081.

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