

# A Fast Cascading Method for Predicting the Coupling from External Plane Waves to PCBs

Shengxuan Xia<sup>#1</sup>, James Hunter<sup>#2</sup>, Aaron Harmon<sup>#3</sup>, Mohamed Z. M. Hamdalla<sup>\*4</sup>,  
Ahmed M. Hassan<sup>\*5</sup>, Chulsoon Hwang<sup>#6</sup>, Victor Khilkevich<sup>#7</sup>, Daryl G. Beetner<sup>#8</sup>  
<sup>#</sup>EMC Laboratory, Missouri University of Science and Technology, Rolla, MO, USA  
<sup>\*</sup>University of Missouri Kansas City, Kansas City, MO, USA  
<sup>#1</sup>sx7c3@mst.edu, <sup>#8</sup>daryl@mst.edu

**Abstract**— The radio frequency (RF) coupling to electronic devices impacts their EMC performance. The functionalities of a working electronic device may be disrupted when the electromagnetic (EM) coupling reaches a certain level. Studies of the EM coupling to printed circuit boards (PCBs) are therefore essential for RF susceptibility and EMC purposes. For decades, researchers focused on the analytical modeling of EM coupling to transmission lines. However, when it comes to more realistic PCBs the analysis usually still relies heavily on full-wave simulations because of the complexity of the structures and the lack of analytical solutions. Using a traditional full-wave modeling approach, however, could take hours to investigate the EM coupling from the external plane wave to the structure for one incident angle of arrival and polarization. In this paper, we present a methodology using reciprocity that allows for rapid estimation of the voltage induced in the terminations for multiple incident angles of the incoming plane wave and load values based on just one full-wave simulation. This reciprocity-based method is combined with a segmentation technique to enable the capability of studying the coupling to more realistic PCBs. For the cases studied here, estimates could be found in minutes using this approach rather than hours using a full-wave simulation. Estimates were within 2-3 dB of estimates using full-wave simulations for a simple trace structure. Accuracy was not as good for individual angles of arrival of an incident RF wave to a complicated structure including two integrated circuit (IC) packages connected by a trace, but statistical estimates of coupling were within 2-3 dB.

**Keywords**—plane wave, radio frequency, coupling, PCB, transmission line, trace, IC package.

## I. INTRODUCTION

External electromagnetic (EM) waves can couple to traces and IC packages on a printed circuit board (PCB) and interrupt the normal functions of the device if there is sufficient coupling. EM coupling to PCBs has been studied for decades. The simplest scenario is a plane wave coupling to a microstrip trace. Detailed analytical derivations are given in [1][2] for coupling to a single microstrip line from an external wave. Additional work was also done to consider coupling to slots [3], connectors and vias [4]. These studies of very simple structures, however, are difficult to apply to many realistic applications, especially as modern electronic devices become more and more complicated. As a result, the analysis of coupling to electronic devices relies heavily on full-wave simulations.

The incident angle of the incoming wave, the PCB board size, trace length, trace orientation angle, load terminations, and more, can all be important to coupling and could vary depending on the design and the application scenario. To better understand how EM energy couples directly to components on the PCB, i.e. to traces or IC lead frames, it is worthwhile to investigate coupling for multiple incident angles and polarizations. If a design is not fixed, or the design parameters are unknown, then many PCB configurations may need to be studied. Doing so requires a large number of simulations. Although numerical computational electromagnetic modeling tools can handle very complicated geometries, the calculation time will be substantial. A full-wave simulation for a single angle of incidence could take hours, making analysis of multiple geometries and incident angles virtually impossible. A faster method is essential for statistical studies of electromagnetic coupling to PCBs.

The EM coupling to a structure from an external plane wave can be estimated for multiple angles of incidence in just one simulation using the reciprocity theorem [5], which greatly reduces the total analysis time. Furthermore, for most PCBs, because the trace routings and components of interest for coupling analysis are placed above the ground plane at roughly the same height, a complicated trace routing can be segmented into several identical pieces like straight trace segments and trace bends. The authors of [6] adopted a similar idea to analyze the EM coupling to segments of a transmission lines analytically and then connect the segments in a circuit simulator. With a consideration of geometry variations on the victim structure, statistical analysis is used to find out the coupling characteristics [7]. Finding the coupling that results from discontinuities or transitions in the transmission lines and are responsible for a significant portion of the non-TEM fields cannot, however, be easily found with transmission line equations. Coupling to the packages of integrated circuits (ICs) also does not fit the standard transmission line approach. Based on the study in [8], the analysis of the EM coupling to the segments can be converted into an extended S-parameter modeling. As discussed in [9], while the trace is part of the coupling structure, one ultimately wants to find the voltage across the IC pins. The authors of [10] provide a way for using the extended S-parameter method to predict radiated emissions by segmenting the whole structure into several parts. These concepts are combined in our study in the sense that we first segment the complicated PCB structure into pieces and then the segmented

---

Supported by the Office of Naval Research (ONR).

parts are re-assembled as a complete structure. In the process, the extended S-parameters of the cascaded model can still predict the EM coupling to the structure. Thus, when the segmentation and cascading methods are applied together with the reciprocity theorem, the estimation of EM coupling becomes relatively efficient. Substantial improvement over conventional simulation techniques can be achieved when the coupling characteristics of commonly occurring structures on the PCBs can be reused in multiple simulations, such as straight trace segments, trace bends, IC packages, etc.

This paper proposes a segmentation and cascading method based on far-field reciprocity to study RF coupling to PCBs. A complicated IC-trace-IC structure can be first decomposed into small and simple pieces. Each segment is then simulated in a full-wave tool to generate the extended S-parameters representing coupling between the two ends of the segment as well as coupling from an incident field to the two ends. These S parameter representations will then be cascaded mathematically to estimate the coupling at the end of the trace or at the IC. This approach requires only several minutes (after each segment is analyzed in a full-wave solver) to estimate coupling with reasonably good accuracy to a complicated PCB routing, while a full-wave simulation of the entire structure requires hours.

A brief introduction to the theory and limitations of the method is given in Section II. The performance of the method when used to predict EM coupling to PCBs is shown in Section III, followed by conclusions in Section IV.

## II. 3-PORT S-PARAMETER MODELING BASED ON FAR-FIELD RECIPROCITY

The following sections introduce the concept of far field reciprocity, the development of S-parameter like representations of parts of a PCB design to determine coupling to each piece, and the assembly of the extended S-parameters for each piece to predict coupling to the overall structure.

### A. Far-field Reciprocity

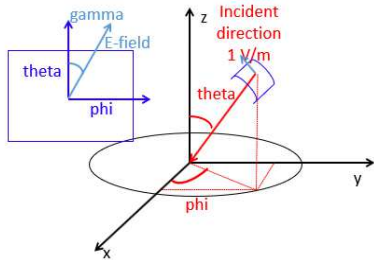


Fig. 1. Definition of incident and polarization angles of arrival.

To find the coupling to a structure, a victim port in the structure is monitored while the structure is illuminated with an external plane wave. As shown in Fig. 1, the angle of incidence of a plane wave can be defined by the theta and phi angles of the angle of arrival (in the spherical coordinate system), and a gamma angle to describe the linear polarization of the incoming plane wave. The voltage induced at the victim port for a given angle and polarization of arrival is given by:

$$V = f(\theta, \varphi, \gamma). \quad (1)$$

As mentioned in Section I, analysis of multiple incident angles and polarizations could require thousands of simulations using a traditional simulation of an incoming plane wave. Far-field reciprocity can greatly reduce the simulation time.

Each victim structure is essentially treated as an “antenna” [11]. The antenna factor (AF) and antenna gain (G) are then used to quantify the characteristics of these “antennas”. For a given angle of arrival, the antenna factor is defined as the ratio of incident E field strength over the voltage response at the port:

$$AF = \frac{E_{inc}}{V} \quad (2)$$

where  $E_{inc}$  is the amplitude of the incident E-field of the external plane wave and  $V$  is the coupled voltage at the port of interest.

The antenna gain is defined as the ratio of the radiated power over the power radiated by a hypothetical isotropic antenna [12]:

$$G = \frac{4\pi R^2 E_{rad}^2}{2\eta P_{in}} \quad (3)$$

where,  $P_{in}$  is the excitation power at the input port,  $R$  is the radius of the observation spherical surface,  $E_{rad}$  is the radiated E-field on the observation sphere, and  $\eta$  is the impedance of free space.

If the antenna gain is known for a particular angle of arrival, the antenna factor (AF) can be written in terms of  $G$ , as:

$$AF = \sqrt{4\pi\eta / \lambda^2 G Z_L}, \quad (4)$$

where  $Z_L$  is the load impedance at the antenna port, and  $\lambda$  is the wavelength of the incoming wave.

Combining (2), (3), and (4) the coupled voltage across the observation port can be found as:

$$V = \frac{E_{inc}}{AF} = \frac{\lambda R}{\eta} \sqrt{\frac{Z_L}{2P_{in}}} E_{inc} E_{rad}. \quad (5)$$

Considering that the  $\theta$  and  $\varphi$  components of the field are orthogonal, the total voltage can be calculated as a sum of their contributions:

$$V = \frac{\lambda R}{\eta} \sqrt{\frac{Z_L}{2P_{in}}} (E_{\theta inc} E_{\theta rad} + E_{\varphi inc} E_{\varphi rad}). \quad (6)$$

Equation (6) can be used to calculate the EM coupling to a device from a plane wave with any incident angle and polarization using on just one simulation for the far-field patterns  $E_{\theta rad}(\theta, \varphi)$  and  $E_{\varphi rad}(\theta, \varphi)$ .

### B. S-parameter Generalization of Far-field Reciprocity

Although far-field reciprocity (6) allows one to calculate EM coupling to a device from all incident angles and polarizations, it is still limited to a fixed impedance at the port location. To further generalize the application of the far-field reciprocity to arbitrary loading conditions, (6) can be reformulated in terms of S-parameters.

Consider a two-port structure (e.g. a trace segment) that is illuminated by a plane wave. The S-parameter representation of the segment would have two standard ports at the two ends of

the segment that could be connected to arbitrary loads, but it should also have a third port (port 3) representing coupling from an incoming plane wave as shown in Fig. 2.

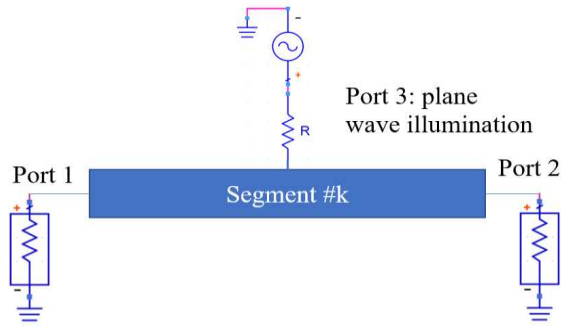


Fig. 2. The 3-port model for a segmented part under illumination from an external plane wave.

By definition, the transmission coefficient from port 3 to port 1 is the ratio of the wave outgoing from port 1 to the wave incoming to port 3 when ports 1 and 2 are terminated by matched loads. That is:

$$S_{13} = V_1^- / V_3^+ |_{V_1^+ = 0, V_2^+ = 0} \quad (6)$$

By assigning the incoming wave in port 3 equal to the amplitude of the incoming plane wave (for a certain incidence)  $V_3^+ = E_{inc}$  within a certainly polarization direction, it is possible to write:

$$S_{13} = V_1^- / E_{inc} |_{V_1^+ = 0, V_2^+ = 0} \quad (7)$$

The outgoing wave  $V_1^-$  is equal to the total voltage on the load of port 1 when port 1 and port 2 loads are matched to the port impedance (i.e.  $V_1^+ = 0$ , and  $V_2^+ = 0$ ). This approach implicitly assumes port 3 is matched. The voltage  $V_1^-$  can be calculated using (5) by setting  $Z_L$  equal to the characteristic impedance of port 1. Thus, the outgoing wave at port 1 can be calculated as

$$V_1^- = \frac{\lambda R}{\eta} \sqrt{\frac{Z_0}{2P_{in}}} (E_{\theta rad,1} E_{\theta inc} + E_{\phi rad,1} E_{\phi inc}) \quad (8)$$

where  $Z_0$  is the impedance of port 1 and  $E_{\theta rad,1}$ ,  $E_{\phi rad,1}$  are the components of the far field produced with the excitation at port 1 by a matched source and with a matched load connected to port 2.

The components of the incoming plane wave are related to the polarization angle  $\gamma$  as

$$\begin{aligned} E_{\theta inc} &= E_{inc} \cos \gamma \\ E_{\phi inc} &= E_{inc} \sin \gamma \end{aligned} \quad (9)$$

By combining (7), (8) and (9) the equation for the transmission coefficient takes the form:

$$S_{13}(\theta, \varphi, \gamma) = \frac{\lambda R}{\eta} \sqrt{\frac{Z_0}{2P_{in}}} (E_{\theta rad,1}(\theta, \varphi) \cdot \cos \gamma + E_{\phi rad,1}(\theta, \varphi) \cdot \sin \gamma) \quad (10)$$

A similar expression can be written for the transmission coefficient between port 3 and port 2.

The reflection coefficient from port 3 (or the scattering of the plane wave by the device) is not of interest for a typical susceptibility study and can be set equal to 0. This gives the

following structure for the S-parameter matrix (reciprocal properties considered)

$$\mathbf{S} = \begin{bmatrix} S_{11} & S_{21} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} \quad (11)$$

The elements  $S_{11}$ ,  $S_{21}$ ,  $S_{22}$  of the matrix are obtained from analysis of the structure. The elements  $S_{13}$  and  $S_{23}$  are calculated from (8) using the far-field patterns due to the excitations at port 1 and 2, respectively (obtained by the full-wave simulation in most cases). Notice that the elements  $S_{13}$  and  $S_{23}$  are only valid for a given incident angle and need to be recalculated for any new combination of the angles.

### C. Segmentation Approach

Calculation of coupling to a complicated structure can be achieved by decomposing the structure into several smaller pieces as illustrated in Fig. 3. Each piece can be represented as an  $(N+1)$ -port S-parameter block, where  $N$  is the number of physical ports, and one additional port is added to account for coupling from an incident plane wave. For a transmission line, each segment would therefore have three ports. Once the S-parameters of the block are determined, multiple S-parameter blocks can be connected together or they can be connected to arbitrary loads in order to re-assemble the original structure and calculate the plane wave coupling.

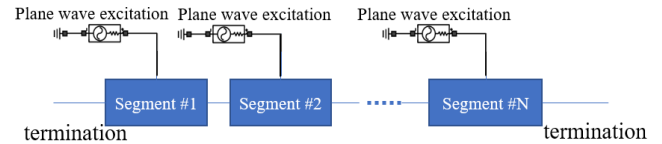


Fig. 3. Segmentation and cascading method.

This segmentation method can be extended to more complicated applications where both common and differential modes are considered [13]. When applying the segmentation concept to the PCBs, for example as shown in Fig. 3, the trace can be segmented into several pieces and each of them will be modeled as a 3-port block.

## III. APPLICATION OF SEGMENTATION APPROACH TO ESTIMATE EM COUPLING

To demonstrate the capabilities of the segmentation approach, coupling was estimated to a simple (multi-segment) trace and to a trace connecting to integrated circuits (ICs).

### A. Trace Only

The approach was first applied to the simple two-segment trace shown in Fig. 4. The two ends of the L-shaped trace were terminated with a 100-ohm resistor and with a 20-pF capacitor. Following the procedures in Section II, the L-shaped trace was broken into two parts (i.e. segments) and 3-port S-parameter blocks were found for each segment. The two segments were then cascaded and terminated with the appropriate loads as indicated in Fig. 4 (b) to estimate coupling to the loads from an

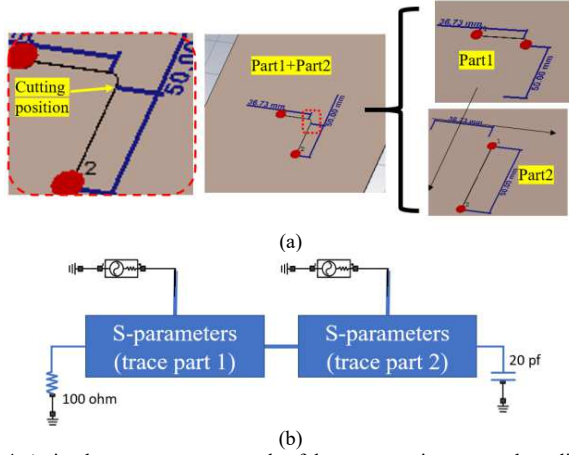


Fig. 4. A simple two-segment example of the segmentation approach applied to a trace: (a) segmentation; (b) equivalent schematic.

incident plane wave at different angles of incidence. CST simulations of coupling were also performed by illuminating the complete structure with a plane wave at several incident angles of arrival. A comparison of the EM coupling calculated with the cascaded model and with CST simulations of the full structure are shown in Fig. 5.

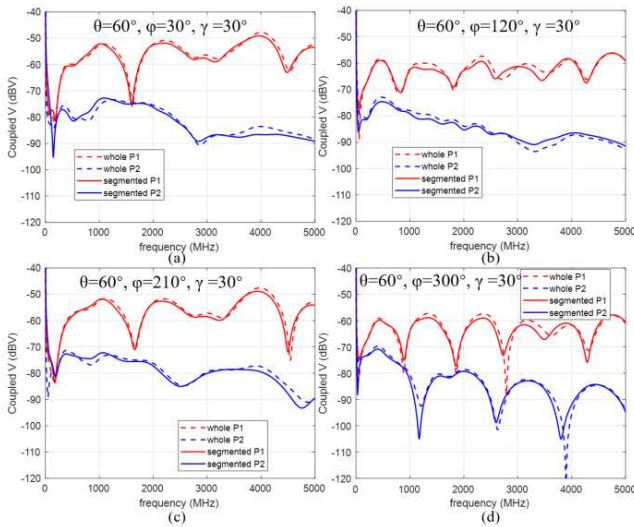


Fig. 5. Estimated coupling to a trace from a plane wave at given angles of incidence when using the segmentation approach and using full-wave models. P1 indicates port 1 loaded by a resistor and P2 indicates port 2 loaded by a capacitor.

The results in Fig. 5 demonstrate that the segmentation approach can do a good job of predicting the EM coupling to the trace. The segmentation approach found coupled voltage levels within 2-3 dB of the CST simulation for all four of the selected cases shown.

### B. Two IC Packages Connected with a Trace

To investigate the performance of the method under a more realistic scenario, a case was studied where two ICs were connected by a routed trace. Because of the size of the IC packages and their height above the return plane, coupling is expected to be larger to the packages than to the trace,

especially at high frequencies (above several GHz). Full-wave models of the IC packages were created and simulated to determine the S-parameters. The 3D model of the IC package studied is shown in Fig. 6. The PEC pad in the center of the package model in Fig. 6 represents the die. The IC has two lumped ports: the external port (port 1) to connect the IC to the PCB trace (defined between the IC pin and the PCB ground) and the internal port (port 2) to connect the circuit model of the input impedance of the IC (defined at the location of the bonding wire between the IC pin and the die). The die is also connected by a bonding wire to one of the pins, which is itself connected to the PCB ground to model a connection to Vss.

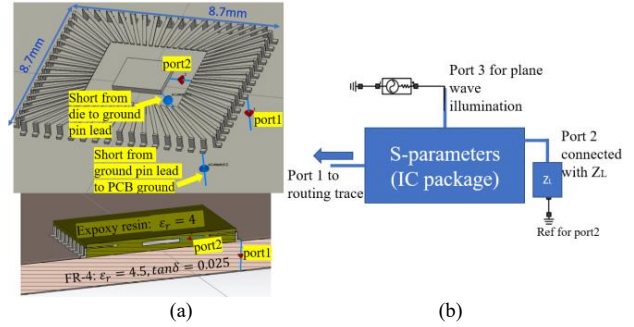


Fig. 6. Full-wave and S-parameter models for an IC package: (a) Full-wave model including port locations; (b) Use of S-parameter block.

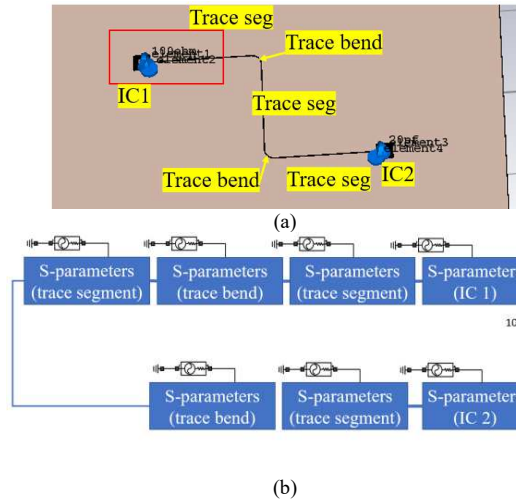


Fig. 7. Segmentation of an IC-trace-IC structure: (a) CST model for the complete geometry (package shown in Fig. 6); (b) equivalent schematic using segmentation approach.

Fig. 7(a) shows the IC-trace-IC structure investigated. It contains two identical IC packages connected by a curved trace. This structure was broken into 7 segments which represent the ICs, the straight portions of the trace, and the trace bends. The segments were each simulated in CST to generate the extended S-parameters for each segment. The segments were then combined to recreate the complete structure as shown in Fig. 7(b). 100 Ohm and 20 pF loads were placed on the inside of the two ICs.

The voltages coupled to the two loads by an incident plane wave were calculated using the proposed segmentation approach and were compared to voltages calculated from a full-



wave simulation of the complete structure in Fig. 7(a) using reciprocity. The results are shown in Fig. 8 for four angles of incidence. With the higher complexity of the structures and the increased number of cascaded segments, the method is not able to predict the coupling as well as for the simple trace (Fig. 5), but it is able to capture the main characteristics of the coupled voltage responses. For a statistical analysis, particularly of structures whose characteristics are poorly defined, these errors at individual angles are acceptable as long as the statistics of the coupling are well predicted.

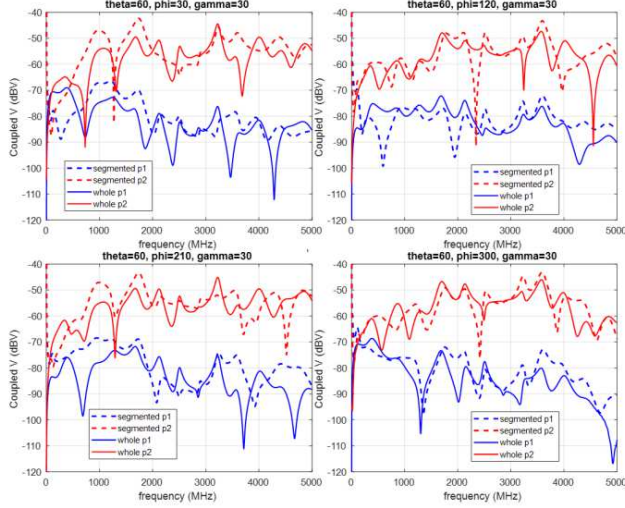


Fig. 8. Comparison of coupling to the IC-trace-IC case found using the segmentation approach and found using full-wave models of the complete structure. P1 indicates port 1 loaded by a resistor and P2 indicates port 2 loaded by a capacitor.

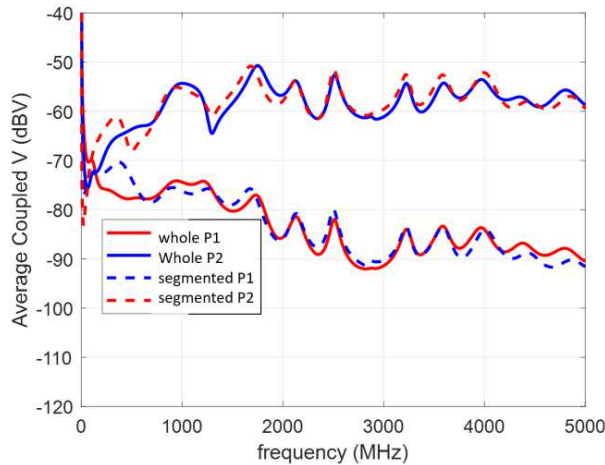


Fig. 9. Average coupled voltages at two port locations (segmentation and cascading method vs complete structure)

To consider the statistical case, an average was taken over all sampled incident angles of arrival over the entire sphere. The average coupling of these angles was defined as

$$V_{avg,1,2}(f) = \frac{\sum_{\gamma=0-90^\circ} \sum_{\varphi=0-360^\circ} \sum_{\theta=0-180^\circ} |V_{1,2}(\theta, \varphi, \gamma, f)|}{N_\gamma N_\varphi N_\theta} \quad (12)$$

where  $V_{1,2}$  are the voltages coupled to the loads (either 100 Ohm or 20 pF, respectively) and  $N_\theta = 13$ ,  $N_\varphi = 25$ , and  $N_\gamma = 10$  are the numbers of the angles analyzed (every 15 degree for the elevation/azimuth angles and every 10 degree for the polarization angle). The averaged voltages are shown in Fig. 9. The average coupled voltages at the loads are calculated with good accuracy (2-3 dB above 1 GHz in this case).

#### IV. CONCLUSION

This paper demonstrates the feasibility of estimating the EM coupling to complicated PCB structures by breaking the structure into small pieces, characterizing the coupling to each piece to find an equivalent S-parameter block to represent the EM characteristics of the block (including coupling), and then calculating coupling from the overall structure by cascading several blocks together. Far-field reciprocity is used to define the coupling to each segment. The far-field patterns are used to generate an extended 3-port S-parameter model of each segment. These blocks can be re-assembled to rapidly estimate coupling to the entire structure. More importantly, for a variety of similar geometries on PCBs with IC packages interconnected by trace routings, the complicated geometries can always be decomposed to a handful of the pre-simulated segments. Thus, EM coupling to a new geometry can be calculated quickly by chaining pre-computed segments together. The proposed segmentation approach was applied to two scenarios to demonstrate the accuracy of the method. While estimates of coupling to the simple trace found using this technique were within 2-3 dB of a reference full-wave model when estimating coupling from RF waves at individual angles of incidence, the segmentation approach did not perform as well when estimating coupling to a much more complicated IC-trace-IC structure. Preliminary results suggest that the accuracy of this coupling can be improved substantially through a better estimate of coupling to the trace bends, as will be explored in a future publication. Estimates of the average coupling to this complicated structure were within 2-3 dB of estimates found using full-wave models, indicating this approach can accurately predict the statistical characteristics of coupling to PCBs. Limitations of the segmentation approach will be fully investigated in a future study. Since the segmentation approach enables fast predictions of EM couplings to PCBs (e.g. a couple of minutes to determine coupling for all angles of arrival for the IC-trace-IC case studied here compared to several hours for full-wave analysis), this approach enables statistical analysis of coupling to many different geometries, for example to study variations in trace routing, packages, loading conditions and other features.

#### REFERENCES

- [1] M. Leone and H. L. Singer, "On the coupling of an external electromagnetic field to a printed circuit board trace," *IEEE Transactions on Electromagnetic Compatibility*, vol. 41, no. 4, pp. 418-424, Nov. 1999, doi: 10.1109/15.809842.
- [2] P. Bernardi and R. Cicchetti, "Response of a planar microstrip line excited by an external electromagnetic field," *IEEE Transactions on Electromagnetic Compatibility*, vol. 32, no. 2, pp. 98-105, May 1990, doi: 10.1109/15.52405.
- [3] M. Leone and V. Navratil, "On the external inductive coupling of differential signalling on printed circuit boards," *IEEE Transactions on Electromagnetic Compatibility*, vol. 46, no. 1, pp. 54-61, Feb. 2004, doi: 10.1109/TEM.2004.823611.

- [4] S. Park, Y. Kami and Y. Nahm, "Coupling Analysis of Complex-Layout Traces Using a Circuit-Concept Approach," *IEEE Transactions on Electromagnetic Compatibility*, vol. 56, no. 1, pp. 208-220, Feb. 2014, doi: 10.1109/TEM.2013.2273078.
- [5] F. Vanhee, D. Pissort, J. Catrysse, G. A. E. Vandenbosch and G. G. E. Gielen, "Efficient Reciprocity-Based Algorithm to Predict Worst Case Induced Disturbances on Multiconductor Transmission Lines due to Incoming Plane Waves," *IEEE Transactions on Electromagnetic Compatibility*, vol. 55, no. 1, pp. 208-216, Feb. 2013, doi: 10.1109/TEM.2012.2208754.
- [6] H. Zhao, X. Gao, B. Wang, E. Li and E. K. Chua, "Efficient analysis of radiated immunity of printed circuit boards using SPICE," *2012 IEEE International Symposium on Electromagnetic Compatibility*, 2012, pp. 289-293, doi: 10.1109/ISEMC.2012.6351768.
- [7] J. Hunter, S. Xia, A. Harmon, A. Hassan, V. Khilkevich and D. Beetner, "Modeling and Statistical Characterization of Electromagnetic Coupling to Electronic Devices," *2021 United States National Committee of URSI National Radio Science Meeting (USNC-URSI NRS)*, 2021, pp. 14-15, doi: 10.23919/USNC-URSINRSM51531.2021.9336496.
- [8] D. Drogoudis, J. Van Hese, B. Boesman and D. Pissort, "Combined circuit/full-wave simulations for electromagnetic immunity studies based on an extended S-parameter formulation," *2014 IEEE 18th Workshop on Signal and Power Integrity (SPI)*, 2014, pp. 1-4, doi: 10.1109/SaPIW.2014.6844551.
- [9] F. V. Zonouz, N. Masoumi and M. Mehri, "Effect of IC package on radiated susceptibility of board level interconnection," *2015 International Conference on Synthesis, Modeling, Analysis and Simulation Methods and Applications to Circuit Design (SMACD)*, 2015, pp. 1-4, doi: 10.1109/SMACD.2015.7301676.
- [10] V. S. Reddy, P. Kralicek and J. Hansen, "A Novel Segmentation Approach for Modeling of Radiated Emission and Immunity Test Setups," *IEEE Transactions on Electromagnetic Compatibility*, vol. 59, no. 6, pp. 1781-1790, Dec. 2017, doi: 10.1109/TEM.2017.2699480.
- [11] A. D. Wunsch, "The Receiving Antenna: A Classroom Presentation," *IEEE Antennas and Propagation Magazine*, vol. 53, no. 4, pp. 179-187, Aug. 2011, doi: 10.1109/MAP.2011.6097320.
- [12] Stutzman, W. L., & Thiele, G. A. (2013). *Antenna Theory and design*. Wiley.
- [13] J. Hunter, S. Xia, A. Harmon, M. Hamdalla, A. Hassan, V. Khilkevich and D. Beetner, "Segmentation Strategy for Structures with Common Mode Coupling," submitted to *2022 IEEE International Symposium on Electromagnetic Compatibility, Signal & Power Integrity*.