Robust Dynamic TEP with an $N - c$ Security Criterion: A Computationally Efficient Model

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***Abstract*—This paper presents a hybrid column-and-constraint-generation augmented-Lagrangian algorithm to efficiently solve the robust security-constrained dynamic transmission expansion planning (TEP) problem. The column-and-constraint generation algorithm separates the TEP problem into a master problem and a set of subproblems decomposable by time period. Additionally, the computationally expensive master problem is decomposed into three computationally efficient sub-master problems: an upper-master quadratic problem, a middle-master quadratic unconstrained binary problem, and a lower-master quadratic unconstrained problem. A set of auxiliary variables are used to relax as real ones the binary variables corresponding to the status of candidate transmission lines enabling the master problem decomposition. The solutions of the three sub-master problems are coordinated using a three-block alternating direction method of multipliers algorithm to enforce binary variables to be binary. An initialization strategy based on load shedding is used to enhance the performance of the proposed algorithm. Simulation results on the IEEE 118-bus test system show the efficient performance of the proposed algorithm for solving security-constrained dynamic TEP problems.**

***Index Terms*—Robust optimization, transmission expansion planning, three-block decomposition, distributed optimization.**

I. Introduction

THE transmission expansion planning (TEP) problem aims to identify how to expand or reinforce existing transmission networks to supply the demand in a reliable and cost-effective manner [1]. Uncertainties associated with generation availability, demand growth, and equipment failures make TEP a challenging decision-making problem. Robust optimization is a suitable approach to deal with the uncertainties in TEP problems [2]–[9]. A state-of-the-art robust TEP model is presented in [4], [10], [11]. This model, which is based on adaptive robust optimization (ARO), is cast as a three-level optimization problem [4]. The first-level problem seeks to minimize investment cost. The worst realization of the uncertainties that maximizes operating cost is identified in the second-level problem for given investment decisions. The third-level problem represents the system operator's optimal response seeking to minimize operating cost, anticipating investment decisions and the worst-case realization of uncertainties.

Recent approaches for solving robust TEP are reported in [3], [4], [10], [12]–[14]. These approaches convert the original three-level problem into a two-level problem and then solve it using decomposition methods such as Benders' or column-and-constrain generation (CCG) [15]. In [3], the dual problem of the third-level problem is merged with the second-level problem to form a single-level bilinear maximization problem. The resulting two-level problem is solved using Benders' decomposition. The dual information of the subproblem is used to add new Benders' cut to the master problem. The main disadvantage of this method is its slow convergence [10]. In [4], a primal Benders (i.e., column-and-constrain generation) approach is used to coordinate the master problem and the subproblem. However, the subproblem is computationally challenging since the number of binary variables and constraints is high.

The solution methods of [3] and [4] are compared in [10]. Moreover, the method presented in [3] is used to derive a two-level problem, and primal Benders is then applied. The bilinear terms of the subproblem are linearized using a simplified poly-

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nomial uncertainty set to improve computational performance. The method proposed in [10] is used in [11] to solve a robust dynamic TEP problem. Additionally, [11] discussed the advantages of a robust dynamic model for TEP over static and sequential methods. The bilinear subproblem is solved in [12], taking advantage of Konno's mountain climbing algorithm [16], which is a heuristic technique previously used in [17] to solve a robust security-constraint unit commitment problem. A CCG algorithm is used in [13] to solve a dynamic, robust security-constrained TEP. In [14], a heuristic block coordinate descent method is used to avoid bilinear terms in a robust TEP. Linear decision rules are used in [18] to reformulate a dynamic robust TEP as a tractable problem, solvable by commercial solvers. A robust TEP problem considering contingencies and uncertainty on load and renewable generation is investigated in [19]. In [20], a disjunctive model is proposed to solve a dynamic security-constrained TEP. In [21], the potential of a continuously variable series reactor, a flexible AC transmission system device, is studied within the framework of a dynamic security-constrained TEP. A multi-objective method is presented in [22] to solve a dynamic TEP considering scenario-based uncertainties.

We note that previous studies have primarily addressed the advantages of robust optimization when faced with uncertainties. The computational challenges have not been sufficiently investigated when considering a robust dynamic decision-making and enforcing security constraints. As discussed in [4] and [11], the master problem of a robust static TEP model does not entail a considerable computational burden. However, this is not the case for a robust dynamic TEP problem with security constraints. Although Konno's algorithm reduces the computational requirement of the CCG subproblem [12], [17], the CCG master problem is still computationally expensive. This degrades the scalability and tractability of robust security-constrained dynamic TEP models.

This paper presents a nested CCG-augmented Lagrangian decomposition algorithm to reduce the computational burden of robust security-constrained dynamic TEP and enhance its tractability and scalability. Specifically, a three-level robust security-constrained dynamic TEP problem is converted into a two-level problem, with a master problem and a subproblem. The subproblem is decomposable by time period, and an approximate convexification algorithm is used to solve it. The key contribution of this paper is to decompose the computationally demanding master problem into three equivalent computationally inexpensive sub-master problems, namely, an upper-master quadratic problem, a middle-master unconstrained quadratic pure binary problem, and a lower-master unconstrained quadratic problem. Binary variables corresponding to candidate lines and auxiliary variables couple these three sub-master problems. A three-block alternating direction method of multipliers (ADMM) algorithm coordinates the solution of the decomposed sub-master problems. Although the ADMM was originally developed for solving convex problems, recent studies [23]–[27] have developed heuristic ADMM algorithms that can be applied to a variety of non-convex problems. Additionally, we propose a strategy to initialize binary variables corresponding to candidate lines based on a load shedding minimization heuristic rule. This strategy enhances the performance of the master problem. On the other hand, CCG is used to coordinate the master problem and the subproblem and obtain the optimal solution. Simulation studies show the efficient performance of the proposed algorithm for reducing the computational cost of the CCG master problem.

The rest of this paper is organized as follows. Section II provides a compact formulation. Section III presents the proposed decomposition approach. Numerical results are presented and discussed in Section IV. Concluding remarks are provided in Section V.

II. Dynamic Robust TEP: ARO Compact Formulation

A compact formulation of the three-level robust security-constrained dynamic TEP is presented in (1) below. In the first level, the system operator pursues the best expansion decisions to minimize the investment cost over the planning horizon. In the second level, the worst-case realization of uncertainties is derived. Generation levels and demand uncertainties are modeled using polyhedral uncertainty sets [1]. In the third level, given transmission expansion decisions and the worst-case uncertainty realization, operation cost is minimized endorsing a $N-1$ security criterion. More details about the three-level robust TEP model are provided in [1], [11], [12].

$$\min_{x} \left(c^T x + \max_{d \in D} \min_{y \in \Omega(x,d)} b^T y \right) \tag{1a}$$

s.t.

$$c^T x \leq \Pi \tag{1b}$$

$$x \in \{0, 1\} \tag{1c}$$

The sequential structure of the three-level dynamic robust model is reflected in objective function (1a), which is to minimize investment cost $c^T x$ and operation cost $b^T y$ over the planning horizon. Vector $x = \{x_{t_1}, ..., x_{t_n}\}$ contains transmission expansion variables. Vector $d = \{d_{t_1}, ..., d_{t_n}\}$ refers to variables associated with demand and generation. D defines the uncertainty set. The continuous operation variables are gathered in vector $y = \{y_{t_1}, y_{t_1}^{c1}, y_{t_1}^{c2}, ..., y_{t_n}, y_{t_n}^{c1}, y_{t_n}^{c2}, ...\}$. $y_{t_n}^{c_1}$ denotes variables pertaining to operation under contingency c_1 at planning time period t_n. Parameter Π is the maximum investment budget. To ensure operation feasibility of variables y for each x and d, constraint set $\Omega(x, d)$ is defined as:

$$\Omega(x,d) = \left\{ \begin{array}{lll} Ax & +By & = Ed : \lambda \\ Fx & +Gy & \leq Kd : \mu. \end{array} \right. \tag{1d}$$

Constraints (1d) is derived using a DC power flow representation. Equality constraints include transmission line flows, nodal power balances, and enforcing the reference node voltage angle. Line flow and voltage angle limits are inequality constraints. Corrective $N-1$ security constraints are also included in (1d). Coefficient matrices A, B, F, and G, and vectors E and K are constant. Dual variable vectors associated with equality and inequality constraints are λ and μ, respectively.

III. Proposed Hybrid CCG-ADMM Solution Approach

The second- and third-level problems are combined to convert (1) into a two-level problem. The first-level problem, the master problem, is to find the optimal investment decisions x. The second-level problem determines the worst-case uncertainty realization under fixed investment decisions. Since simulation results reveal that CCG generally performs computationally better than Benders [10], we use CCG to develop the proposed solution approach.

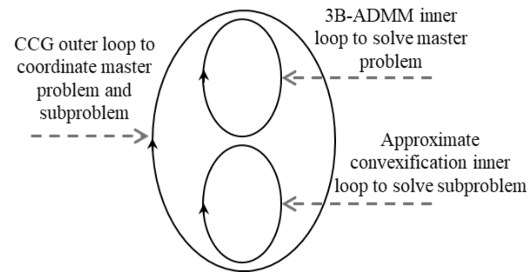

Fig. 1. Scheme of proposed three-loop approach.

As shown in Fig. 1, three loops are considered: an inner loop to solve the subproblem, an ADMM inner loop to solve the master problem, and a CCG outer loop for coordinating the master problem and the subproblem.

A. Master Problem

The master problem is a mixed-integer linear programming (MIP) problem:

$$\min_{x, y_i, \alpha} Z^M = c^T x + \alpha \tag{2a}$$

s.t. (1b), (1c), and

$$\alpha \quad \geq b^T y_i; \quad \forall i \leq k \tag{2b}$$

$$Ax + By_i \quad = Ed_i; \quad \forall i \leq k \tag{2c}$$

$$Fx + Gy_i \quad \leq Kd_i; \quad \forall i \leq k, \tag{2d}$$

where k is the CCG iteration counter and $i = 1, \cdots, k$. Auxiliary variable α reconstructs the objective function (1a) gradually with increasing accuracy. Variable d, which represents the uncertainty, is obtained from the subproblem solution. A new set of equations (2c)-(2d) is added to the master problem at each iteration. It is relevant to note that the master problem represents the complete three-level problem.

B. Subproblem

Combining the dual problem of the third-level problem of (1) and the second-level problem results in the following maximization problem, which is decomposable per time period [12], [17]:

$$\max_{d, \lambda, \mu, \phi} Z^S = (Ed - Ax^*)^T \lambda + (Kd - Fx^*)^T \mu \tag{3a}$$

s.t.

$$-B^T \lambda - G^T \mu = b \tag{3b}$$

$$\mu \geq 0, \lambda \text{ free}, d \in D. \tag{3c}$$

The objective function (3a) is the dual of the third-level problem. Dual constraints (3b) are obtained from differentiating the Lagrangian of (1d). Given the investment decisions, the solution of model (3) provides values for the variables that represent the uncertainty to be meant to the master problem at each iteration.

C. Proposed Solution Approach

1) Algorithm To Solve the Master Problem: The master problem is not computational challenging for static TEP problems [4]. However, the number of primal cuts and binary variables in the master problem is multiplied by the number of time periods in dynamic TEP. Moreover, the number of operational constraints is multiplied by the number of $N-1$ security constraints. Therefore, the dimension of the master problem dramatically increases in terms of variables and constraints over the course of the CCG iterations.

We rewrite the master problem (2) in the following compact form.

$$\min_{\{x,y_i,\alpha\}} f(x,\alpha) \tag{4a}$$

s.t.

$$\{x, y_i, \alpha\} \in \chi \tag{4b}$$

where set χ is:

$$\chi = \{(x, y_i, \alpha)|(1b)(1c)(2b)(2c)(2d)\}. \tag{4c}$$

We next decompose this computationally expensive master problem into three sub-master problems that can be solved efficiently. To this end, we reformulate (4) into: a scalable and computationally efficient quadratic problem, a computationally cheap quadratic unconstrained binary optimization, and a computationally cheap unconstrained quadratic problem.

a) Key Reformulations: We relax (1c) and allow binary variables x to be real, $0 \leq x \leq 1$. Then, we define a set of auxiliary binary variables z and a set of continuous variables r. We also formulate three sets of new constraints (5b), (5d), and (5e) to enforce the binary nature of x. We then reformulate (4) as (5):

$$\min_{\{x,y_i,\alpha,z,r\}} f(x,\alpha) \tag{5a}$$

s.t.

$$A_0x + A_1z + A_2r = 0 : \gamma \tag{5b}$$

$$\{x, y_i, \alpha\} \in \chi \backslash (1c) \tag{5c}$$

$$z \in \{0, 1\} \tag{5d}$$

$$r = 0. \tag{5e}$$

where A_0, A_1, and A_2 are identity matrices related as $A_0 = -A_1 = -A_2 = I_{n^+}$. Auxiliary continuous variables r need to be zero at the optimal solution so that x become binary. We note that problem (5) is still an MIP. We move implicit constraint (5c) and (5d) to the objective function using indicator functions $\iota_\chi(x, y_i, \alpha)$ and $\iota_{\{0,1\}}(z)$, the latter enforcing the binary nature of z.

$$\begin{aligned}\min_{\{x,y_i,\alpha,z,r\}} & F(x, y_i, \alpha, z) := f(x, \alpha) \\ & + \iota_\chi(x, y_i, \alpha) + \iota_{\{0,1\}}(z)\end{aligned} \tag{6a}$$

s.t.

$$A_0x + A_1z + A_2r = 0 : \gamma \tag{6b}$$

$$r = 0. \tag{6c}$$

We then split the optimization variables into three sets, namely, $\Delta = \{x, y_i, \alpha\}$, z, and r. The problem now has a separable structure with respect to these sets except for constraint (6b). We dualize this constraint using an augmented Lagrangian.

$$\begin{aligned}&\mathcal{L}_\rho(x, y_i, \alpha, z, r, \gamma) = F(x, y_i, \alpha, z) + \frac{\beta}{2}||r||_2^2 \\ &+ \gamma(A_0x + A_1z + A_2r) + \frac{\rho}{2}||A_0x + A_1z + A_2r||_2^2.\end{aligned} \tag{7}$$

We then use the ADMM (3B-ADMM) shared in Algorithm 1 to solve (7) by decomposition in three separate blocks. The first block, called upper-master problem, is a quadratic problem with respect to (x, y_i, α) given z and r. If this upper-master problem, which is computationally much cheaper than the original master problem, is infeasible, then so is the original TEP master problem (2), and Algorithm 1 terminates. The second block, called middle-master problem, is a quadratic unconstrained binary optimization problem with respect to auxiliary binary variables z given (x, y_i, α) and r. The third block, called lower-master problem, is a computationally cheap quadratic unconstrained optimization with respect to auxiliary continuous variables r given (x, y_i, α) and z. Since the second block problem is non-convex, 3B-ADMM is, in general, a heuristic. However, the sequence generated by ADMM converges under some conditions for sufficiently large $\rho > \beta$.

b) Convergence of the Mixed-Binary ADMM: The assumptions for which Algorithm 1 is proven to converge to a stationary point of the augmented Lagrangian $\mathcal{L}_\rho$ are as follows [23], [27]:

1) (Coercivity). The objective function F is coercive over the constraints.
2) (Feasibility). Im(A_T) $\subseteq$ Im(A_2), where $A_T = [A_0, A_1]$.
3) (Lipschitz subminimization paths). It is possible to find a positive constant M at any iteration such that

$$||x^{m-1} - x^m|| \leq M||A_0x^{m-1} - A_0x^m|| \quad \text{(8a)}$$

$$||z^{m-1} - z^m|| \leq M||A_1z^{m-1} - A_1z^m|| \quad \text{(8b)}$$

$$||r^{m-1} - r^m|| \leq M||A_2r^{m-1} - A_2r^m|| \quad \text{(8c)}$$

4) (Objective regularity). F is a lower semi-continuous function.

Moreover, if $\mathcal{L}_\rho$ is a Kurdyka–Łojasiewicz (KŁ) function [28], [29], Algorithm 1 converges globally. We below show that the above conditions are valid for (7).

1) (Coercivity). Coercivity holds for x and z, since they are bounded. Term $\frac{\beta}{2}||r||_2^2$ is quadratic, then it is coercive.
2) (Feasibility). Direct computing of Im(A_T) and Im(A_2), where $A_T = [A_0, A_1]$, shows that Im(A_T) $\subseteq$ Im(A_2), so feasibility holds.
3) (Lipschitz subminimization paths). This condition holds for every set of variables with $M = 1$ trivially.
4) (Objective F-regularity). since $f(x, \alpha) + \iota_\chi(x, y_i, \alpha) + \iota_{\{0,1\}}(z)$ is the sum of a convex function and the indicator function of a convex set, it is restricted prox-regular [23]. Term $\frac{\beta}{2}||r||_2^2$ is Lipschitz differentiable with a constant β.

Also, since function (7) is a semi-algebraic function, it is a KŁ function. Thus, Algorithm 1 converges to a stationary point of the augmented Lagrangian $\mathcal{L}_\rho$, which is a soft-constrained version of the dynamic TEP mixed-binary master problem (4). Starting from any initial point, this algorithm converges subsequently for any sufficiently large $\rho > \beta$. However, a good initialization leads to fewer 3B-ADMM iterations.

Algorithm 1 3B-ADMM algorithm

1: Initialize: $m = 1$, $\gamma^{(0)}$, $\rho > \beta > 0$, $z^{(0)}, r^{(0)}, \epsilon > 0$.
2: **for** $m = 1, 2, \dots,$ **do**
3: First block update:
4: $\Delta^{(m)} \leftarrow \underset{x, y_i, \alpha}{\operatorname{argmin}} \mathcal{L}_\rho(\Delta, z^{(m-1)}, r^{(m-1)}, \gamma^{(m-1)})$.
5: Second block update:
6: $z^{(m)} \leftarrow \underset{z}{\operatorname{argmin}} \mathcal{L}_\rho(\Delta^m, z, r^{(m-1)}, \gamma^{(m-1)})$.
7: Third block update:
8: $r^{(m)} \leftarrow \underset{r}{\operatorname{argmin}} \mathcal{L}_\rho(\Delta^m, z^m, r, \gamma^{(m-1)})$.
9: Dual variable update:
10: $\gamma^{(m)} \leftarrow \frac{\rho}{2}(A_0x^{(m)} + A_1z^{(m)} + A_2r^{(m)}) + \gamma^{(m-1)}$.
11: **if** $||A_0x^{(m)} + A_1z^{(m)} + A_2r^{(m)}|| \leq \epsilon$ **then** Stop.
12: **else**
13: $m \leftarrow m + 1$.
14: **end if**
15: **end for**
16: Return (x, y_i, α, z, r).

c) Discussions on the Number of Blocks: We note that a two-block implementation fixing $r = 0$ and skipping the third block update is possible. However, incorporating variable r has two advantages. First, linear constraint (5b) requires a three-block structure, and the last block is an identity matrix, whose image is the entire space. This feature is required to ensure feasibility; that is, for any fixed x and z, there always exists r such that (5b) is satisfied. Second, constraint (5e) can be processed independently of (5b) so that it can penalized and incorporated in the objective function as $\frac{\beta}{2}||r||_2^2$. This term is not only convex but also Lipschitz differentiable. The multiplier updating in step 10 of Algorithm 1 proceed smoothly as a result of the convexification effect that the penalty term of the augmented Lagrangian confers to the actual Lagrangian [30].

d) Convergence Improvement: Careful parameter selection and variable initialization strategies generally enhance the convergence speed of Algorithm 1. We use two rules.

1) Candidate transmission lines that facilitate supplying loads with low investment cost are more likely to be installed. To select a starting

solution wisely, ratio ζ is defined.

$$\zeta_l = \frac{\Delta load}{I_l}, \quad (9)$$

where $\Delta load$ is the difference of total supplied load before and after installing candidate line l in the target year. Parameter I_l is the annualized investment cost of candidate line l. We rank ratios ζ_l in descent order; then, assign $z = 1$ to candidate lines with high ζ_l. In this step, the summation of investment costs is not bounded.

2) We update penalty factor β according to the strategy described in [24]. Specifically, for given parameters $0 < \mu < 1$ and $\omega > 1$, $\beta^m = \omega\beta^{m-1}$ if $|r^m| > \mu|r^{m-1}|$, and $\beta^m = \beta^{m-1}$ otherwise. The ADMM penalty increases if residual $|r|$ does not decrease adequately. Also, we assign $2.5\beta^m$ to ρ^m.

We note that the second rule affects ADMM's iteration number and penalizes the auxiliary variable r, whereas the first one only affects ADMM's iteration number.

2) Algorithm to Solve the Subproblem: Because of bilinear terms $d^T\lambda$ and $d^T\mu$ in the dual objective function (3a), the subproblem is generally difficult to solve. We use an approximate convexification method to solve this subproblem [31], [32]. We write the term $d \cdot \lambda$ in the following algebraic form:

$$d \cdot \lambda = \frac{1}{4}(d+\lambda)^2 - \frac{1}{4}(d-\lambda)^2. \quad (10)$$

The right-hand side of (10), which is the subtraction of two convex functions, is non-convex. However, we exploit the fact that the subtraction of a convex function and a linear one is convex, and thus linearize the second term in (10) around $(d^{(j)}, \lambda^{(j)})$, using a first-order Taylor series approximation [33].

$$\frac{1}{4}(d-\lambda)^2 \approx (a^{(j)})^2 + a^{(j)}(d-d^{(j)}) - a^{(j)}(\lambda-\lambda^{(j)}) \quad (11)$$

where $a^{(j)} = \frac{1}{2}(d^{(j)} - \lambda^{(j)})$. We then replace $d \cdot \lambda$ in (3a) with (10) and (11). The same method is used to linearize $d \cdot \mu$. The approximately convexified subproblem (3) is then solved iteratively using Algorithm 2.

Three variable vectors, d, λ, and μ, need to be properly initialized. We have reasonable initial estimates for uncertain variables of demand and generation, i.e., $d^{(0)}$. The dual variables are unlikely to be known in advance. We obtain an estimate of $\lambda^{(0)}$ and $\mu^{(0)}$ by substituting $d^{(0)}$ into subproblem (3). Then, the original non-convex problem (3) is approximated by a sequence of convex problems iteratively solved until convergence is achieved. The accuracy of the approximation iteratively increases as variables $d^{(j)}$, $\lambda^{(j)}$ and $\mu^{(j)}$ get closer to their optimal values d^*, λ^* and μ^*. The performance of this approximation method is investigated in [31], [32].

Algorithm 2 Proposed Solution Algorithm

1: Initialize: $j = 1$, ϵ, $d^{(0)} \in D$, $x = x_k$, set $(\lambda^{(0)}, \mu^{(0)})$ to the solution of problem (3) with fixed $d^{(0)} \in D$.
2: **for** $j = 1, 2, \ldots,$ **do**
3: Solve (3) with $(d^{(j)}, \lambda^{(j)}, \mu^{(j)})$ and set $(d^{(j+1)}, \lambda^{(j+1)}, \mu^{(j+1)})$ to the solution.
4: **if** $|d^{(j+1)} - d^{(j+1)}| + |\lambda^{(j+1)} - \lambda^{(j)}| + |\mu^{(j+1)} - \mu^{(j)}| \leq \epsilon$ **then** Stop
5: **end if**
6: $j \leftarrow j+1$.
7: **end for**
8: Return (d, λ, μ).

The merit of convexification (11) as compared to methods using big-M constants and binary variables to model bilinear terms (e.g., [3], [10]) is its computational efficiency. Particularly, if the number of $N-1$ security constraints in the lower level increases, the number of binary variables in big-M methods significantly increases, resulting in potential intractability. Konno's algorithm [12], [16] is also another approach to tackle the subproblem bilinear terms. However, this algorithm is prone to stick to local minima and should be re-started with various initial points to obtain a reliable result. Also, block coordinate descent is a heuristic approach to tackle bilinear terms that does not guarantee convergence to the global optimum [14].

3) Outer Level Column-and-constraint-generation: The outer loop CCG of Fig. 1 is described in Fig. 2. The master problem passes to the subproblem investment decisions x, and the subproblem provides the master problem with uncertain variable values d representing uncertainty parameters. The algorithm proceeds as follows:

Step 0: Set $k = 0$ as the CCG outer loop iteration counter. Set the outer loop bounds to $LB^{(0)} = -\infty$ and $UB^{(0)} = \infty$. Solve the master problem (2a)

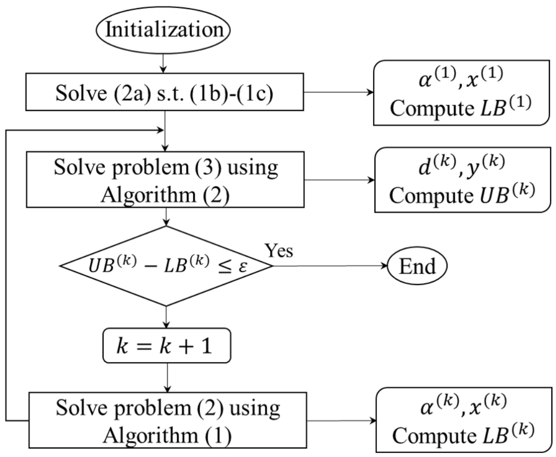

Fig. 2. Flowchart of outer level algorithm to solve the two-level problem.

subject to (1b)-(1c) to obtain $x^{(1)}$ and $\alpha^{(1)}$, then let $LB^{(1)} = Z^{M,(1)}$.

Step 1: For $k \geq 1$, solve master problem (2) using Algorithm 1 to obtain $x^{(k)}$ and $\alpha^{(k)}$, then let $LB^{(k)} = Z^{M,(k)}$.

Step 2: Fix investment decision variables $x^{(k)}$ to the values obtained in *Step 1*, solve subproblem (3) using Algorithm 2 to obtain uncertain variables d^k and let $UB^{(k)} = Z^{S,(k)}$.

Step 3: If $UB^{(k)} - LB^{(k)} \leq \epsilon$, then stop and return $x^{(k)}$ as optimal investment decisions. Otherwise, set $k \leftarrow k+1$ and go to *Step 1*.

The computational efficiency of this column-and-constraint generation is discussed in [10].

D. Computational Complexity Analysis

We focus below on the computational complexity of the master problem since our main contribution pertains to efficiently solve this problem. We compare the complexity of a conventional non-decomposable algorithm and the proposed three-block distributed algorithm. The size of each formulation is characterized with respect to the number of buses (n_b), loads (n_d), generators (n_g), lines (n_l), years (n_t), security-constrains (n_c), and the number of iterations (n_k).

1) Non-decomposable master solution: The numbers of variables and constraints are:

- $n_{l^+} \times n_t$ binary variables. Note that here n_{l^+} is the number of candidate lines.
- $(n_t \times (n_l+1)) + (n_g+n_d+n_l+n_b)(n_c+1)n_k \times n_t$ continuous variables.
- $\{((2n_b-1)+3n_l+n_d)(1+n_c)+n_g(1+2n_c)\} \times n_t \times n_k + (n_l+1)n_t + (2n_t+1)n_{l^+}$ constraints.

2) Proposed master solution: The master problem is decomposed in three blocks. The first block includes no binary variables, the second block has neither continuous variables nor constraints, and the third block contains neither binary variables nor constraints. The numbers of continuous variables and constraints in the first block problem are:

- $(n_l \times n_t) + (n_g + n_d + n_l + n_b)(n_c+1)n_k + 1 + 2(n_{l^+} \times n_t)$ continuous variables.
- $\{((2n_b-1)+3n_l+n_d)(1+n_c)+n_g(1+2n_c)\} \times n_t \times n_k + (n_l+1)n_t + (3n_t+1)n_{l^+}$ constraints.

The number of binary/continuous variables for both the second and third blocks is:

- $n_{l^+} \times n_t$ binary variables.

The non-decomposable master problem is a large-scale MIP and suffers from the curse of dimensionality. However, in our approach, we solve a quadratic optimization problem, a quadratic unconstrained binary optimization, and a quadratic unconstrained optimization problem that are scalable, and can be solved efficiently.

IV. Numerical Results

The proposed approach is tested on the Garver system [34] and the IEEE 118-bus system. Simulations are carried out on a personal computer with an Intel(R) Core(TM) i7-10850H CPU clocking at 2.7 GHz and 16 GB of RAM. GAMS 28.2.0 and CPLEX solver are used to solve the optimization problems. The converging tolerance for all simulations is set to 10^{-6}. TEP is solved using the following strategies:

S1: Classical Approach: column-and-constraint-generation using Algorithm 2 to solve the subproblem (i.e., Fig. 1 without 3B-ADMM inner loop).

S2: Proposed Approach: column-and-constraint-generation using Algorithm 1 to solve the master problem and Algorithm 2 to solve the subproblem.

A. Garver System

This system includes six buses, six existing lines, 45 candidate lines (three lines between each pair of buses), three generating units, and five demands. The maximum investment budget is \$48 million. Candidate line data and bus data are given in [35] and [36], respectively. The peak demand and generation capacity in the first year are 760 MW and 1110

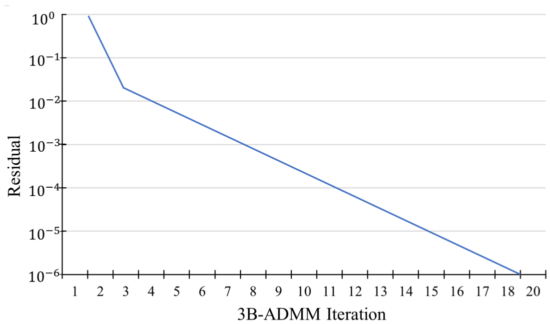

Fig. 3. 3B-ADMM residual for solving the master problem at the last CCG outer iteration (Garver system).

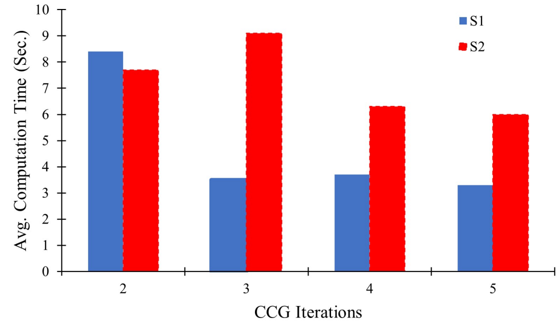

Fig. 4. Computational time of the master problem in *S1* and *S2* at every CCG outer iteration (Garver system).

TABLE I
ζ INDEX FOR CANDIDATE LINES OF GARVER TEST SYSTEM

corridor	2-6	4-6	5-6	1-6	Rest
ζ	14.38	14.38	5.51	4.44	0

TABLE II
NUMBER OF VARIABLES AND CONSTRAINTS FOR IEEE 118-BUS SYSTEM

	#cont. var.	#bin. var.	#const.
Master problem (last iteration)	$543290n_k$ + 1870	300	$1199780n_k$ + 2500
Subproblem	1125880	-	645320

MW, respectively. The time horizon includes ten one-year time periods. The yearly demand growth is 5%. $N-1$ security constraints are considered. The maximum level of demand d can deviate up to 20% of its expected value at each year. The installed generation capacity can deviate up to 20% below its expected value at each year. Uncertainty budgets are $\Gamma_t^G = 0.2$ and $\Gamma_t^D = 0.5$. We have initialized $\beta = 10^3$ with 10^8 as its upper bound. Initial points for z are selected based on the list of ζ_l ratios given in Table I. Auxiliary variables r are initialized to 0. The built candidate lines are 2-3, 2-6, 3-5, 4-6, and 4-6 in year one, 3-5 in year three, 2-3 in year six, and 4-6 in year eight. Strategies *S1* and *S2* provide the same investment results. The total investment cost for constructing new lines is \$46.33 million.

The outer loop of the CCG algorithm in *S1* and *S2* converges after five iterations. The average number of 3B-ADMM iterations over the four CCG iterations is 17.75. Fig. 3 illustrates the convergence residual of Algorithm 1 at the last iteration of the CCG outer loop. At every CCG iteration, the 3B-ADMM residual, defined as $r' = ||A_0x + A_1z + A_2r||$, goes to zero upon convergence. Fig. 4 shows the computational time of the master problem at every outer iteration. Total master problem solution time for *S1* and *S2* is 19.7 and 29.9 seconds, respectively. Since the Garver system is small, solving the master problem using *S1* is more efficient.

B. IEEE 118-Bus System

This is a relatively large test system for TEP studies. The system information for existing components and candidate lines is given in [37]. The peak demand and generation capacity in the first year are 5567 MW and 7470 MW, respectively. The maximum annual investment budget is \$150 million. The time horizon is ten years, with ten one-year-long time periods. One hundred and twenty line contingencies are considered. Demand and generation uncertainties are similar to those in Garver's case. Load-shedding is allowed up to 3% of the maximum demand.

We select initial values for z based on the list of ζ ratios calculated using (9). We initialize $\beta = 10^3$ with 10^8 as its upper bound and set $r = 0$. Table II shows the number of variables and constraints at the n_k^{th} CCG outer loop iteration. Strategies *S1* and *S2* obtain the same result with an investment cost of \$381.97 million. Candidate line sets {1, 5, 7, 15, 24}, {3, 13, 17, 29}, {6}, and {14} are to be installed, respectively, in years one, two, three, and seven.

The outer loop of the algorithm converges in 12 iterations. The average number of 3B-ADMM iterations per CCG iteration is 54.3. Fig. 5 shows the computation time for the master problem at every outer iteration. Cumulative solution times are

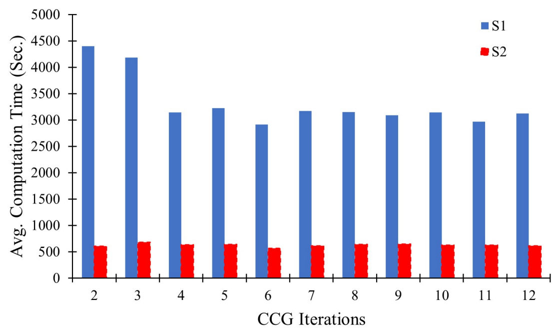

Fig. 5. Computational time of master problem in *S1* and *S2* at every CCG outer iteration (118-bus system).

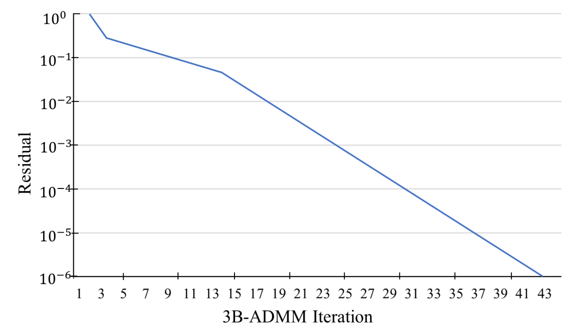

Fig. 7. 3B-ADMM residual for solving the master problem at the last CCG outer iteration (118-bus system).

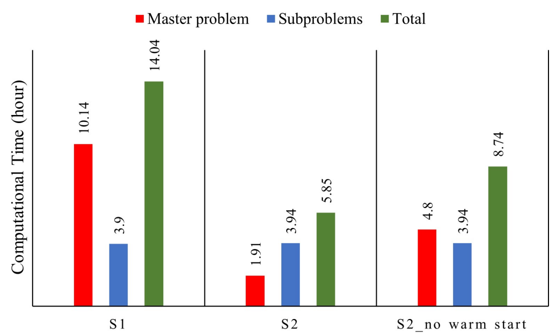

Fig. 6. Total time consumed by each algorithm at each strategy (118-bus system).

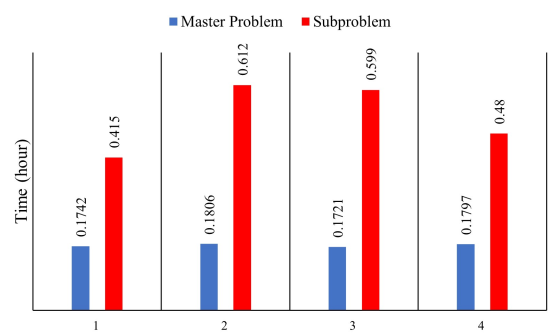

Fig. 8. Average computation time of Algorithm 1 and 2 per CCG outer iterations.

reported in Fig. 6. The proposed approach *S2* takes 1.91 hours to solve the master problem while the conventional approach *S1* takes 10.14 hours. The solution time for the whole TEP problem in *S1* is 14.04 hours, meaning that the master problem takes almost 72% of the total solution time. In *S2* the total solution time is 5.85 hours, with the master problem taking 32.62% of it. The master problem and the total TEP solution times reduce by 81% and 58% in *S2* as compared to *S1*. Fig. 7 represents the convergence residual of Algorithm 1 at the last iteration of the CCG outer loop.

Impact of Initialization Strategy: We repeat *S2* without the proposed initialization strategy. Fig. 6 shows the solution times. The total solution time is 8.74 hours, with the master problem taking 54.9% of it. Still, the master problem and the total TEP solution times are reduced by 52.2% and 37.7% as compared to *S1*. The 3B-ADMM converges after 210 iterations in the last iteration of the CCG outer loop, while it converges after 43 iterations if using *S2* with the initialization strategy. Results illustrated on Fig. 6 shows that the suggested load shedding-based initialization strategy reduces the computational time of *S2* by 60.2% to solve the master problem.

Impact of Uncertainty Budgets: As depicted in Fig. 8, Algorithm 1 is not affected by uncertainty budgets. These parameters mainly affect the sub-problem solution time and the number of CCG iterations. Regarding Fig. 8 the numbers of CCG outer loops in simulations one to four are 12, 17, 18, and 12, respectively.

C. $N-c$ *Security-Constrained Analysis*

The analysis below shows how the proposed approach alleviates the solution time and memory requirement under different $N-c$ security conditions with $c \in \{0,1,2,3,4\}$. The IEEE 118-bus system is used. Table III summarizes the solution time and the system cost. Table IV provides the size of the master problem. As the number of security constraints increases, the problem size, the solution time, and the cost increase. The numbers of lines built for c equal to 0, 1, and 2 are, respectively, nine, 11, and 24. This increasing trend changes for $c \geq 3$, becoming the load shedding cost the dominant part of the objective function. Nine new

TABLE III
RUN-TIME AND SYSTEM COST OF IEEE 118-BUS SYSTEM CONSIDERING $N-c$ SECURITY CRITERION

c	*S1*		*S2*	
	Time(h)	Cost($)	Time(h)	Cost($)
0	0.8	8.80×10^{10}	1.2	8.80×10^{10}
1	14.04	8.97×10^{10}	5.85	8.97×10^{10}
2	24*	$2.87\times10^{11*}$	9.9	2.80×10^{11}
3	Out of Memory	Out of Memory	12.1	1.07×10^{14}
4	Out of Memory	Out of Memory	23.5	1.81×10^{14}

$*$Unfinished (Optimality gap $\approx$ 1%)

TABLE IV
SIZE OF THE MASTER PROBLEM CONSIDERING AN $N-c$ SECURITY CRITERION FOR THE IEEE 118-BUS SYSTEM

c	#cont. var.	#bin. var.	#constraints
0	4490n_k+1870	300	9380n_k+2500
1	543290n_k+1870	300	1199780n_k+2500
2	22454490n_k+1870	300	49609380n_k+2500
3	67354490n_k+1870	300	148809380n_k+2500
4	112254490n_k+1870	300	248009380n_k+2500

lines are built for $c = 3$ and seven for $c = 4$. Moreover, for c equal to 3 and 4, the value of the objective function dramatically increases due to the high load shedding cost. As c increases, the solution time of strategy *S1* increases drastically, which is not the case for strategy $S2$. Changing from $c = 0$ to $c = 1$, the solution time increases from 0.8 hours to 14.04 hours for strategy *S1* and from 1.2 hours to 5.58 hours for strategy *S2*. For $N-2$ instances, we stopped strategy *S1* after 24 hours. The considered optimality gap was 1%, and the total cost 2.87×10^{11}. Strategy *S2* converges to the optimal solution with a 0% gap after 9.9 hours. The total cost provided by strategy *S2* is 7×10^{9} smaller than that provided by strategy *S1*. Due to the large size of the problem representing either the $N-3$ or $N-4$ security criterion, given the available computer, strategy *S1* could not solve these instances. However, *S2* provides optimal solutions for these cases after 12.1 and 23.5 hours.

V. CONCLUSIONS

Including security constraints in the dynamic TEP problem drastically increases its computational burden, particularly for adaptive robust formulations. This paper presents a hybrid CCG augmented-Lagrangian approach to efficiently solve this problem. The master problem, which is computationally expensive, is reformulated as a combination of a quadratic optimization sub-master problem, a quadratic unconstrained binary sub-master problem, and a quadratic unconstrained optimization sub-master problem. A three-block ADMM algorithm is used to coordinate the solution of these sub-master problems and find the master problem optimal solution at each CCG iteration. A load shedding-based initialization strategy is suggested.

Simulation results show that the proposed approach outperforms significantly the classical CCG approach. For instance, for the IEEE 118-bus system, the proposed hybrid decomposition approach reduces the solution time by roughly 58% (8.19 hours). Also, for the $N-2$ security condition, the proposed approach provides optimal TEP results after 9.9 hours while the classical approach reaches an optimality gap of 1% after 24 hours. The classical approach does not solve $N-3$ and $N-4$ instances due to memory requirements. However, the proposed approach finds optimal results in less than a day.

REFERENCES

[1] A. J. Conejo, L. Baringo, S. J. Kazempour, and A. S. Siddiqui, "Investment in electricity generation and transmission," *Cham Zug, Switzerland: Springer International Publishing*, vol. 106, 2016.

[2] L. El Ghaoui, F. Oustry, and H. Lebret, "Robust solutions to uncertain semidefinite programs," *SIAM Journal on Optimization*, vol. 9, no. 1, pp. 33–52, 1998.

[3] R. A. Jabr, "Robust transmission network expansion planning with uncertain renewable generation and loads," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4558–4567, 2013.

[4] C. Ruiz and A. J. Conejo, "Robust transmission expansion planning," *European Journal of Operational Research*, vol. 242, no. 2, pp. 390–401, 2015.

[5] P. Wu, H. Cheng, and J. Xing, "The interval minimum load cutting problem in the process of transmission network expansion planning considering uncertainty in demand," *IEEE Trans. Power Syst.*, vol. 23, no. 3, pp. 1497–1506, 2008.

[6] M. D. Filabadi and S. P. Azad, "Robust optimisation framework for sced problem in mixed ac-hvdc power systems with wind uncertainty," *IET Ren. Power Gen.*, vol. 14, no. 14, pp. 2563–2572, 2020.

[7] M. D. Filabadi and H. Mahmoudzadeh, "Effective budget of uncertainty for classes of robust optimization," *arXiv preprint arXiv:1907.02917*, 2019.

[8] H. Yu, C. Chung, and K. Wong, "Robust transmission network expansion planning method with taguchi's orthogonal array testing," *IEEE Trans. Power Syst.*, vol. 26, no. 3, pp. 1573–1580, 2010.

[9] B. Chen, J. Wang, L. Wang, Y. He, and Z. Wang, "Robust optimization for transmission expansion planning: Minimax cost vs. minimax regret," *IEEE Trans. Power Syst.*, vol. 29, no. 6, pp. 3069–3077, 2014.

[10] R. Mínguez and R. García-Bertrand, “Robust transmission network expansion planning in energy systems: Improving computational performance,” *European Journal of Operational Research*, vol. 248, no. 1, pp. 21–32, 2016.

[11] R. García-Bertrand and R. Mínguez, “Dynamic robust transmission expansion planning,” *IEEE Trans. Power Syst.*, vol. 32, no. 4, pp. 2618–2628, 2016.

[12] X. Zhang and A. J. Conejo, “Robust transmission expansion planning representing long-and short-term uncertainty,” *IEEE Trans. Power Syst.*, vol. 33, no. 2, pp. 1329–1338, 2017.

[13] A. Moreira, A. Street, and J. M. Arroyo, “An adjustable robust optimization approach for contingency-constrained transmission expansion planning,” *IEEE Trans. Power Syst.*, vol. 30, no. 4, pp. 2013–2022, 2014.

[14] R. Minguez, R. García-Bertrand, J. M. Arroyo, and N. Alguacil, “On the solution of large-scale robust transmission network expansion planning under uncertain demand and generation capacity,” *IEEE Trans. Power Syst.*, vol. 33, no. 2, pp. 1242–1251, 2017.

[15] B. Zeng and L. Zhao, “Solving two-stage robust optimization problems using a column-and-constraint generation method,” *Operations Research Letters*, vol. 41, no. 5, pp. 457–461, 2013.

[16] H. Konno, “A cutting plane algorithm for solving bilinear programs,” *Mathematical Programming*, vol. 11, no. 1, pp. 14–27, 1976.

[17] D. Bertsimas, E. Litvinov, X. A. Sun, J. Zhao, and T. Zheng, “Adaptive robust optimization for the security constrained unit commitment problem,” *IEEE Trans. power syst.*, vol. 28, no. 1, pp. 52–63, 2012.

[18] S. Dehghan, N. Amjady, and A. J. Conejo, “A multistage robust transmission expansion planning model based on mixed binary linear decision rules—part i,” *IEEE Trans. Power Syst.*, vol. 33, no. 5, pp. 5341–5350, 2018.

[19] Z. Wu, Y. Liu, W. Gu, Y. Wang, and C. Chen, “Contingency-constrained robust transmission expansion planning under uncertainty,” *International Journal of Electrical Power & Energy Systems*, vol. 101, pp. 331–338, 2018.

[20] J. Qiu, Z. Y. Dong, J. Zhao, Y. Xu, F. Luo, and J. Yang, “A risk-based approach to multi-stage probabilistic transmission network planning,” *IEEE Trans. Power Syst.*, vol. 31, no. 6, pp. 4867–4876, 2016.

[21] X. Zhang, K. Tomsovic, and A. Dimitrovski, “Security constrained multi-stage transmission expansion planning considering a continuously variable series reactor,” *IEEE Trans. Power Syst.*, vol. 32, no. 6, pp. 4442–4450, 2017.

[22] P. Maghouli, S. H. Hosseini, M. O. Buygi, and M. Shahidehpour, “A scenario-based multi-objective model for multi-stage transmission expansion planning,” *IEEE Trans. Power Syst.*, vol. 26, no. 1, pp. 470–478, 2010.

[23] C. Gambella and A. Simonetto, “Multiblock admm heuristics for mixed-binary optimization on classical and quantum computers,” *IEEE Trans. Quantum Eng.*, vol. 1, pp. 1–22, 2020.

[24] K. Sun and X. A. Sun, “A two-level distributed algorithm for general constrained non-convex optimization with global convergence,” *arXiv preprint arXiv:1902.07654*, 2019.

[25] S. Diamond, R. Takapoui, and S. Boyd, “A general system for heuristic minimization of convex functions over non-convex sets,” *Optimization Methods and Software*, vol. 33, no. 1, pp. 165–193, 2018.

[26] A. Alavian and M. C. Rotkowitz, “Improving admm-based optimization of mixed integer objectives,” in *2017 51st Annual Conf. Inf. Sci. Syst. (CISS)*, pp. 1–6, IEEE, 2017.

[27] Y. Wang, W. Yin, and J. Zeng, “Global convergence of admm in nonconvex nonsmooth optimization,” *Journal of Scientific Computing*, vol. 78, no. 1, pp. 29–63, 2019.

[28] J. Bolte, A. Daniilidis, and A. Lewis, “The łojasiewicz inequality for nonsmooth subanalytic functions with applications to subgradient dynamical systems,” *SIAM Journal on Optimization*, vol. 17, no. 4, pp. 1205–1223, 2007.

[29] H. Attouch, J. Bolte, and B. F. Svaiter, “Convergence of descent methods for semi-algebraic and tame problems: proximal algorithms, forward–backward splitting, and regularized gauss–seidel methods,” *Mathematical Programming*, vol. 137, no. 1, pp. 91–129, 2013.

[30] D. P. Bertsekas, *Constrained optimization and Lagrange multiplier methods*. Academic press, 2014.

[31] S. Boyd, S. P. Boyd, and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.

[32] G. E. Constanteflores, A. J. Conejo, and F. Qiu, “AC network-constrained unit commitment via relaxation and decomposition,” *IEEE Trans. Power Syst.*, 2021.

[33] A. L. Yuille and A. Rangarajan, “The concave-convex procedure,” *Neural computation*, vol. 15, no. 4, pp. 915–936, 2003.

[34] R. Romero, A. Monticelli, A. Garcia, and S. Haffner, “Test systems and mathematical models for transmission network expansion planning,” *IEE Proceedings-Gen., Trans. and Dist.*, vol. 149, no. 1, pp. 27–36, 2002.

[35] L. P. Garcés, A. J. Conejo, R. García-Bertrand, and R. Romero, “A bilevel approach to transmission expansion planning within a market environment,” *IEEE Trans. Power Syst.*, vol. 24, no. 3, pp. 1513–1522, 2009.

[36] M. Mehrtash, A. Kargarian, and M. Rahmani, “Security-constrained transmission expansion planning using linear sensitivity factors,” *IET Gen., Trans. & Dist.*, vol. 14, no. 2, pp. 200–210, 2019.

[37] A. Kargarian, “Available: https://sites.google.com/site/ aminkargarian/,” *IEEE*, 118-bus.

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