Letter

U-spin puzzle in B decays

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We impose U-spin symmetry $[SU(2)_{\rm Uspin}]$ on the Hamiltonian for B decays. As expected, we find the equality of amplitudes related by the exchange $d \leftrightarrow s$. We also find that the amplitudes for the $\Delta S = 0$ processes $B^0 \to \pi^+\pi^-$, $B_s^0 \to \pi^+ K^-$, and $B^0 \to K^+K^-$ form a U-spin triangle relation. The amplitudes for $B_s^0 \to K^+K^-$, $B^0 \to \pi^- K^+$, and $B_s^0 \to \pi^+\pi^-$ form a similar $\Delta S = 1$ triangle relation. And these two triangles are related to one another by $d \leftrightarrow s$. We perform fits to the observables for these six decays. If perfect U spin is assumed, then the fit is very poor. If U-spin-breaking contributions are added, then we find many scenarios that can explain the data. However, in all cases, 100% U-spin breaking is required, considerably larger than the naive expectation of ~20%. This is the U-spin puzzle; it may be strongly hinting at the presence of new physics.

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In Ref. [1], a method was proposed to extract the angle γ of the unitarity triangle from measurements of $B^0 \to \pi^+\pi^-$ and $B_s^0 \to K^+K^-$, two decays related by the exchange $d \leftrightarrow s$. This is usually referred to as U spin symmetry.

Now, U spin is more than just a $d \leftrightarrow s$ symmetry. It is based on the group $SU(2)_{\rm Uspin}$. Here we examine the consequences of imposing this symmetry group on the Hamiltonian for B decays. We find that the $d \leftrightarrow s$ symmetry is reproduced as expected, but other useful relations also appear. In particular, the amplitudes for the $\Delta S = 0$ processes $B^0 \to \pi^+\pi^-$, $B_s^0 \to \pi^+K^-$ and $B^0 \to K^+K^-$ form a U-spin triangle relation similar to the isospin $B \to \pi\pi$ triangle relation [2]. Similarly, the amplitudes for $B_s^0 \to K^+K^-$, $B^0 \to \pi^-K^+$, and $B_s^0 \to \pi^+\pi^-$ form a $\Delta S = 1$ U-spin triangle relation. And these two triangles are related to one another by the $d \leftrightarrow s$ symmetry. All six decays have been measured. In this paper, we show that a simultaneous analysis of their observables has some puzzling results.

Under $SU(2)_{\rm Uspin}$, (d, s) is a doublet and $(\bar{s}, -\bar{d})$ is its conjugate. The mesons that are eigenstates of U spin are then

$$
\begin{aligned}
&K^0 = d\bar{s}, \quad U^0 = \frac{1}{\sqrt{2}}(s\bar{s} - d\bar{d}), \quad \bar{K}^0 = -s\bar{d},\\
&K^+ = u\bar{s}, \quad \pi^+ = -u\bar{d}, \quad \pi^- = d\bar{u}, \quad K^- = s\bar{u},\\
&U_8 = \frac{1}{\sqrt{6}}(2u\bar{u} - d\bar{d} - s\bar{s}). \qquad (1)
\end{aligned}
$$

Thus, under $SU(2)_{\rm Uspin}$, $(K^0, U^0, \bar{K}^0)$ form a triplet, (π^-, K^-) and (K^+, π^+) are doublets and U_8 is a singlet.

Consider $B \to PP$ decays in the U-spin basis (P is a pseudoscalar meson). Here the initial state is either a doublet $[(B^0, B_s^0)]$ or a singlet $[B^+]$, and the final state is one of TT, DD, TD, TS, DS or SS, where T, D, and S refer to a U-spin triplet, doublet and singlet, respectively. The weak Hamiltonian involves $b \to qu\bar{u}$ and $b \to q$ ($q = d$ or s), which has $U = \frac{1}{2}$ for both $q = d$ and $q = s$. However, note that $\Delta S = 0$ decays ($q = d$) and $\Delta S = 1$ decays ($q = s$) involve different Cabibbo-Kobayashi-Maskawa (CKM) matrix elements.

It is now straightforward to compute the $B \to PP$ decay amplitudes in terms of the $SU(2)_{\rm Uspin}$ reduced matrix elements (RMEs). As the U^0 and U_8 are both linear combinations of the π^0, η, and η' mesons, but with unknown relative strong phases, decays involving these particles are not very useful. It is more interesting to consider instead decays whose final states involve only $\pi^\pm$, $K^\pm$, K^0, and $\bar{K}^0$.

In this paper, we focus specifically on $B \to$ DD decays. There are three decays with $\Delta S = 0$ and three with $\Delta S = 1$. In terms of U-spin RMEs, the amplitudes are

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$$\Delta S = 0: \quad A(B^0 \to \pi^+\pi^-) = M^{\frac{1}{2}}_{1d} + M^{\frac{1}{2}}_{0d},$$
$$A(B^0 \to K^+K^-) = M^{\frac{1}{2}}_{1d} - M^{\frac{1}{2}}_{0d},$$
$$A(B^0_s \to \pi^+K^-) = 2M^{\frac{1}{2}}_{1d}, \tag{2}$$

$$\Delta S = 1: \quad A(B^0_s \to K^+K^-) = M^{\frac{1}{2}}_{1s} + M^{\frac{1}{2}}_{0s},$$
$$A(B^0_s \to \pi^+\pi^-) = M^{\frac{1}{2}}_{1s} - M^{\frac{1}{2}}_{0s},$$
$$A(B^0 \to \pi^-K^+) = 2M^{\frac{1}{2}}_{1s}, \tag{3}$$

where the subscripts d and s refer, respectively, to the transitions $b \to d$ ($\Delta S = 0$) and $b \to s$ ($\Delta S = 1$), and $M^{\frac{1}{2}}_{1q} \equiv \langle 1||H^{\frac{1}{2}}||\frac{1}{2}\rangle_q$, $M^{\frac{1}{2}}_{0q} \equiv \langle 0||H^{\frac{1}{2}}||\frac{1}{2}\rangle_q$, $q = d, s$. (Note that we have absorbed the magnitudes of the Clebsch-Gordan coefficients into the RMEs.)

The amplitudes $A(B^0 \to \pi^+\pi^-)$ and $A(B^0_s \to K^+K^-)$ are related to one another by the exchange $d \leftrightarrow s$, as are the pairs $(A(B^0 \to K^+K^-), A(B^0_s \to \pi^+\pi^-))$ and $(A(B^0_s \to \pi^+K^-), A(B^0 \to \pi^-K^+))$. But there is more: under $SU(2)_{\rm Uspin}$ symmetry, there are two triangle relations, each involving only $\Delta S = 0$ or $\Delta S = 1$ amplitudes[1]:

$$A(B^0 \to \pi^+\pi^-) + A(B^0 \to K^+K^-) = A(B^0_s \to \pi^+K^-),$$
$$A(B^0_s \to K^+K^-) + A(B^0_s \to \pi^+\pi^-) = A(B^0 \to \pi^-K^+). \tag{4}$$

As we will see below, these can be used to extract a great deal of information, including the CP phase γ.

The $B \to$ DD amplitudes are given in terms of U-spin RMEs in Eqs. (2) and (3). Now,

$$M^{\frac{1}{2}}_{0d} = V^*_{ub}V_{ud}\left\langle 0\middle\|H^{\frac{1}{2}}\middle\|\frac{1}{2}\right\rangle^u + V^*_{cb}V_{cd}\left\langle 0\middle\|H^{\frac{1}{2}}\middle\|\frac{1}{2}\right\rangle^c + V^*_{tb}V_{td}\left\langle 0\middle\|H^{\frac{1}{2}}\middle\|\frac{1}{2}\right\rangle^t,$$
$$\equiv \lambda^u_{bd}T^0_d + \lambda^c_{bd}P^0_d, \tag{5}$$

where $\lambda^i_{bj} = V^*_{ib}V_{ij}$ ($i = u, c$, $j = d, s$), and we have used the unitarity of the CKM matrix in passing from the first line to the second. (Note that, despite the notation, T^0_d and P^0_d do not necessarily correspond only to tree and penguin contributions, respectively.) Similarly,

$$M^{\frac{1}{2}}_{1d} = \lambda^u_{bd}T^1_d + \lambda^c_{bd}P^1_d,$$
$$M^{\frac{1}{2}}_{0s} = \lambda^u_{bs}T^0_s + \lambda^c_{bs}P^0_s, \qquad M^{\frac{1}{2}}_{1s} = \lambda^u_{bs}T^1_s + \lambda^c_{bs}P^1_s. \tag{6}$$

With this, the six $B \to$ DD decay amplitudes are given by

$$\Delta S = 0:$$
$$A(B^0 \to \pi^+\pi^-) = \lambda^u_{bd}T^1_d + \lambda^c_{bd}P^1_d + \lambda^u_{bd}T^0_d + \lambda^c_{bd}P^0_d,$$
$$A(B^0 \to K^+K^-) = \lambda^u_{bd}T^1_d + \lambda^c_{bd}P^1_d - \lambda^u_{bd}T^0_d - \lambda^c_{bd}P^0_d,$$
$$A(B^0_s \to \pi^+K^-) = 2\lambda^u_{bd}T^1_d + 2\lambda^c_{bd}P^1_d, \tag{7}$$

$$\Delta S = 1:$$
$$A(B^0_s \to K^+K^-) = \lambda^u_{bs}T^1_s + \lambda^c_{bs}P^1_s + \lambda^u_{bs}T^0_s + \lambda^c_{bs}P^0_s,$$
$$A(B^0_s \to \pi^+\pi^-) = \lambda^u_{bs}T^1_s + \lambda^c_{bs}P^1_s - \lambda^u_{bs}T^0_s - \lambda^c_{bs}P^0_s,$$
$$A(B^0 \to \pi^-K^+) = 2\lambda^u_{bs}T^1_s + 2\lambda^c_{bs}P^1_s. \tag{8}$$

These decays have all been measured, yielding a number of observables. The results of the present experimental measurements are shown in Table I. For each of the four decays $B^0 \to \pi^+\pi^-$, $B^0_s \to \pi^+K^-$, $B^0_s \to K^+K^-$, and $B^0 \to \pi^-K^+$, the branching ratio and direct and indirect (where applicable) CP asymmetries have been measured. For the rarer decays $B^0 \to K^+K^-$ and $B^0_s \to \pi^+\pi^-$, we have only the branching ratios.

The three $\Delta S = 0$ amplitudes [Eq. (7)] and three $\Delta S = 1$ amplitudes [Eq. (8)] each involve seven unknown hadronic parameters: the four magnitudes $|T^1_q|$, $|P^1_q|$, $|T^0_q|$, and $|P^0_q|$

TABLE I. Experimental values of $B \to$ DD observables. Here, $\mathcal{B}$, A^{CP} and S^{CP} refer to the branching ratio, direct CP asymmetry and indirect CP asymmetry, respectively. The average values given above are taken from Ref. [4]. These average values are generally dominated by a few measurements, whose references are given in the Table I.

Decay	Observable
	$\Delta S = 0$
$B^0 \to \pi^+\pi^-$	$\mathcal{B} = (5.15 \pm 0.19) \times 10^{-6}$ [5,6] $A^{CP} = 0.311 \pm 0.030$ [7] $S^{CP} = -0.666 \pm 0.029$ [7]
$B^0 \to K^+K^-$	$\mathcal{B} = (8.0 \pm 1.5) \times 10^{-8}$ [8]
$B^0_s \to \pi^+K^-$	$\mathcal{B} = (5.9 \pm 0.9) \times 10^{-6}$ [6,9] $A^{CP} = 0.225 \pm 0.012$ [7]
	$\Delta S = 1$
$B^0_s \to K^+K^-$	$\mathcal{B} = (2.66 \pm 0.32) \times 10^{-5}$ [8,10] $A^{CP} = -0.17 \pm 0.03$ [7] $S^{CP} = 0.14 \pm 0.03$ [7]
$B^0_s \to \pi^+\pi^-$	$\mathcal{B} = (7.2 \pm 1.1) \times 10^{-7}$ [8]
$B^0 \to \pi^-K^+$	$\mathcal{B} = (1.95 \pm 0.05) \times 10^{-5}$ [5,11] $A^{CP} = -0.0836 \pm 0.0032$ [7]

[1]Similar relations have been derived in Ref. [3] but in a different context.

TABLE II. Results of a fit to the observables of the six $B \to DD$ decays in the U-spin limit. Amplitudes are given in keV and phases (apart from γ) are given in radians.

Parameter	Best fit value
$\lvert T_d^1\rvert$	3.85 ± 0.22
$\lvert P_d^1\rvert$	0.56 ± 0.02
$\lvert T_d^0\rvert$	3.27 ± 0.24
$\lvert P_d^0\rvert$	0.71 ± 0.13
$\delta_{P_d^1}$	0.33 ± 0.01
$\delta_{T_d^0}$	0.14 ± 0.09
$\delta_{P_d^0}$	0.59 ± 0.20
γ	$(67.6 \pm 3.4)^\circ$

($q = d, s$), along with three relative strong phases. (We take the magnitudes of the CKM matrix elements from independent measurements [12].) The weak phase γ in λ_{bq}^u is present in all amplitudes; it can be allowed to vary or be constrained by its independently measured value of $(65.9^{+3.3}_{-3.5})^\circ$ [4,12]. In the U-spin limit, the $\Delta S = 0$ and $\Delta S = 1$ hadronic parameters are equal, so that all six amplitudes involve the same eight theoretical parameters. With 12 observables in Table I, one can perform a fit to the data. In this way, we can determine the preferred sizes of these parameters, allowing us to extract γ and/or ascertain how well the hypothesis of U-spin symmetry holds up.

In this fit, we allow γ to be a free parameter and, without loss of generality, we take $\delta_{T_d^1} = 0$. The fit is performed using the program MINUIT [13–15]; the results are shown in Table II. Although the best-fit value of γ is very close to its present value, this is unimportant, as the fit is very poor: it has $\chi^2_{\rm min}/{\rm d.o.f.} = 17.8/4$, for a p-value of 0.001.

It is instructive to search for the reason(s) for this poor fit. We find that there are two ingredients. The first is the "U-spin relation." In the U-spin limit, the observables associated with pairs of decays related by $d \leftrightarrow s$ obey the following relation [16]:

$$-\frac{A_s^{CP}}{A_d^{CP}}\frac{\tau(B_d)\mathcal{B}_s}{\tau(B_s)\mathcal{B}_d} = 1. \tag{9}$$

Here, B_d is the decaying B meson in the $\Delta S = 0$ process, $\tau(B_d)$ is its lifetime, A_d^{CP} is the direct CP asymmetry in the decay, and $\mathcal{B}_d$ is the branching ratio. The analogous quantities for the $\Delta S = 1$ process are indicated by the subscript s. The extent to which the U-spin relation is violated gives a handle on the size of U-spin breaking.

The second ingredient is more subtle. From Table I, we see that the branching ratio of $B^0 \to K^+K^-$ is much smaller than those of the other $\Delta S = 0$ decays. Similarly, $B_s^0 \to \pi^+\pi^-$ has by far the smallest branching ratio of the $\Delta S = 1$ decays.

Consider the limit in which these branching ratios are set to zero. This approximation is equivalent to setting the $B^0 \to K^+K^-$ and $B_s^0 \to \pi^+\pi^-$ amplitudes to zero. (In the diagrammatic language of Ref. [3], this corresponds to neglecting the subdominant diagrams E and PA.) At the RME level, this implies $T_q^1 = T_q^0$ and $P_q^1 = P_q^0$, $q = d, s$ [see Eqs. (7) and (8)]. In this limit, we have

$$\begin{aligned}
\Delta S = 0:\quad & A(B^0 \to \pi^+\pi^-) = A(B_s^0 \to \pi^+K^-) \\
& \qquad = 2\lambda_{bd}^u T_d^1 + 2\lambda_{bd}^c P_d^1, \\
\Delta S = 1:\quad & A(B_s^0 \to K^+K^-) = A(B^0 \to \pi^-K^+) \\
& \qquad = 2\lambda_{bs}^u T_s^1 + 2\lambda_{bs}^c P_s^1.
\end{aligned} \tag{10}$$

First, this implies that each of the $\Delta S = 0$ amplitudes is related to each of the $\Delta S = 1$ amplitudes by the exchange $d \leftrightarrow s$. That is, the U-spin relation [Eq. (9)] applies to four pairs of decays. For all four pairs, in Table III we present the values of the U-spin relation obtained from the experimental data.

These values are to be compared with the "prediction" of 1 for this quantity. (For two pairs, the prediction is approximate, as it results from setting the small branching ratios to zero.) We see that the two entries involving $B_s^0 \to K^+K^-$ are in disagreement with the prediction. On the other hand, the two entries with $B^0 \to \pi^-K^+$ are in good agreement.

Second, the triangle relations of Eq. (4) become simple amplitude equalities:

$$\begin{aligned}
A(B^0 \to \pi^+\pi^-) &= A(B_s^0 \to \pi^+K^-), \\
A(B_s^0 \to K^+K^-) &= A(B^0 \to \pi^-K^+).
\end{aligned} \tag{11}$$

These equalities apply to the CP-conjugate amplitudes as well. Using the measured values of the branching ratios and direct CP asymmetries for the decays (Table I), one can extract the magnitudes of A and $\bar{A}$ for each decay. We find

$$\begin{aligned}
& A_1 = A(B_s^0 \to \pi^+K^-), && A_2 = A(B^0 \to \pi^+\pi^-) \\
& \left|\frac{A_1}{A_2}\right| = 1.05 \pm 0.08, && \left|\frac{\bar{A}_1}{\bar{A}_2}\right| = 1.15 \pm 0.09, \\
& A_3 = A(B^0 \to \pi^-K^+), && A_4 = A(B_s^0 \to K^+K^-) \\
& \left|\frac{A_3}{A_4}\right| = 0.89 \pm 0.06, && \left|\frac{\bar{A}_3}{\bar{A}_4}\right| = 0.81 \pm 0.05.
\end{aligned} \tag{12}$$

TABLE III. Values of the U-spin relation for different pairs of decays. Entries marked with $(*)$ correspond to pairs related only when the small branching ratios are set to zero [Eq. (10)].

$\Delta S = 0$ decay	$\Delta S = 1$ decay	U-spin relation
$B^0 \to \pi^+\pi^-$	$B_s^0 \to K^+K^-$	2.78 ± 0.66
$B^0 \to \pi^+\pi^-$	$B^0 \to \pi^-K^+$	1.02 ± 0.12 (*)
$B_s^0 \to \pi^+K^-$	$B_s^0 \to K^+K^-$	3.41 ± 0.91 (*)
$B_s^0 \to \pi^+K^-$	$B^0 \to \pi^-K^+$	1.25 ± 0.21

Given the error implicit in the "prediction" of 1 for these quantities, these results show no obvious disagreements. Still, it is interesting to note that, once again, it is the ratios involving $B_s^0 \to K^+K^-$ that exhibit the largest differences from 1.

We have therefore identified certain tensions in the data that may contribute to the poor fit of Table II. The only possible way to improve the fit is to include U-spin-breaking contributions. But this may be somewhat delicate: since not all relations predicted by U spin (and/or the neglect of the small branching ratios) are broken, adding U-spin-breaking effects to correct one problem may create another problem where none existed before.

In addition to the seven hadronic parameters of Table II, there are nine U-spin-breaking parameters. We define[2]

$$T_s^0 = T_d^0(1 + t_0 e^{i\delta_{t_0}}), \qquad T_s^1 = T_d^1(1 + t_1 e^{i\delta_{t_1}}),$$
$$P_s^0 = P_d^0(1 + p_0 e^{i\delta_{p_0}}), \qquad P_s^1 = P_d^1(1 + p_1 e^{i\delta_{p_1}}),$$
$$A(B^0 \to \pi^+\pi^-) + A(B^0 \to K^+K^-) = (1+X)A(B_s^0 \to \pi^+K^-),$$
$$A(B_s^0 \to K^+K^-) + A(B_s^0 \to \pi^+\pi^-) = (1+X)A(B^0 \to \pi^-K^+). \qquad (13)$$

In the fits, we generally constrain the phase γ by its measured value. However, this could be incorrect in the presence of new physics. In light of this possibility, we occasionally allow γ in $\Delta S = 1$ decays (γ_1) to be a free parameter.

With 12 observables, there is room for five additional unknown parameters in the fit. Since there are an infinite number of possibilities for these five (linear combinations of) parameters, we cannot draw a definitive conclusion. However, we have examined many sets of five parameters, and certain patterns have emerged.

With the addition of these parameters, we have twelve equations in 12 unknowns. We search for a solution by doing a fit. If $\chi^2_{\rm min} = 0$ is found, then this corresponds to an exact solution. We make the following observations:

(i) If t_0 is not included, then we find no solutions.
(ii) If t_0 is included, but is real (i.e., $\delta_{t_0} = 0$), then we find no solutions.
(iii) If t_0 and δ_{t_0} are included, but t_0 is combined with another parameter (e.g., $p_0 = t_0$, $p_1 = t_0$ or $t_1 = t_0$), then we find no solutions.
(iv) We find a number of solutions with t_0 and δ_{t_0} nonzero; in all of them, the other magnitudes of U-spin-breaking parameters (t_1, p_0, p_1, X) are small.
(v) It is not necessary that γ_1 be included in order to find a solution. However, if it is included, there are solutions, and in all cases γ_1 is different from its measured value.

TABLE IV. Results of fits to the observables of the six $B \to DD$ decays in which some U-spin-breaking parameters have been included. Amplitudes are given in keV and phases are given in radians (δ_{t_0}) or degrees (γ_1).

Parameter	$\chi^2_{\rm min}$/d.o.f.	p-value
$t_0 = 0.5 \pm 0.4$	16.4/4	0.003
$t_0 = 1.25 \pm 0.35$ $\delta_{t_0} = -1.27 \pm 0.33$	6.1/3	0.11
$t_0 = 1.15 \pm 0.34$ $\delta_{t_0} = -1.22 \pm 0.34$ $p_1 = 0.28 \pm 0.15$	1.1/2	0.58
$t_0 = 1.02 \pm 0.31$ $\delta_{t_0} = -1.5 \pm 0.4$ $\gamma_1 = (91.1 \pm 14.9)^\circ$	1.7/2	0.43

These properties can be seen even in fits with fewer than five U-spin-breaking parameters, see Table IV. When only t_0 is included, the fit is poor. But it becomes passable when δ_{t_0} is added, and good with one more parameter. A good fit can be found if γ_1 is included and allowed to vary, though this is not absolutely necessary. When γ_1 is included, its best-fit value is found to be different from the measured value of γ.

But the key point is that, in all the fits that account for the data reasonably well, $t_0 = O(1)$. That is, 100% U-spin breaking is required, specifically in the T_q^0 RME. This U-spin breaking is considerably larger than the naive expectation of $f_K/f_\pi - 1 = \sim 20\%$. This is the U-spin puzzle in B decays. (Interestingly, large U-spin breaking has also been observed in D decays, see Refs. [18,19].)

What can be the explanation for this U-spin breaking? There are no known mechanisms in the standard model that can generate U-spin breaking this large; this may be strongly hinting at new physics. New physics contributions to U-spin breaking have been explored in Ref. [20].

At present, there are other hints of new physics in $b \to s$ transitions: in certain observables involving the transition $b \to s\mu^+\mu^-$ [21] and in $B \to \pi K$ decays [22]. The result of this paper can be added to that list.

To date, only the branching ratios of the decays $B^0 \to K^+K^-$ and $B_s^0 \to \pi^+\pi^-$ have been measured. If/when the direct and indirect CP asymmetries of these decays are measured, this will give us additional information that may help shed light on the U-spin puzzle.

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[2]Note that U-spin breaking can also be defined at the level of observables, see, for example, Ref. [17].

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