Sustainable Energy, Grids and Networks 32 (2022) 100890

Contents lists available at ScienceDirect

Sustainable Energy, Grids and Networks

journal homepage: www.elsevier.com/locate/segan

A cooperative game theory-based approach to formulation of distributed slack buses

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ARTICLE INFO

Article history:
Received 9 December 2021
Received in revised form 2 July 2022
Accepted 25 July 2022
Available online 30 July 2022

Keywords:
Cooperative game theory
Distributed slack buses
Shapley value

ABSTRACT

The conventional power flow formulation is based on a single slack bus model, which may not necessarily provide accurate values for minimum generation cost and minimum power losses. On the other hand, distributed slack buses, which are used to distribute the slack power (power mismatch) among different voltage-controlled buses, can overcome this problem. A cooperative game theory-based approach is proposed in this paper to calculate active power participation factors to distribute slack active power among different participating generators. In the first stage, the worth (or value) of individual participating generators and their coalitions are computed. In the second stage, the Shapley value, which is one of the solution concepts of cooperative game theory, is used to calculate the participation factors of individual participating generators. The participation factors are then used to distribute the mismatch power among different generators. The effectiveness of the proposed approach is demonstrated through case studies on the IEEE 14-bus, IEEE 30-bus, and IEEE 57-bus systems. The results show that the cost of generation and power losses are reduced in case of systems with distributed slack buses compared to that with a single slack bus.

1. Introduction

During the formulation of power flow problems in power systems, a slack bus, also referred to as a swing bus or reference bus, serves as a reference for the calculation of voltage angles and compensates the difference between the net power injected into the system at other buses and the total system load and losses [1]. In conventional power flow calculations, the slack bus has merely been regarded as a mathematical entity without linking it with the actual operation of the power system [2]. In real power system operation, the total power mismatch (or power loss) is compensated by several geographically dispersed generators instead of one generator that is regarded as a slack bus. Moreover, the integration of distributed energy resources and electric vehicles into power systems has led to the increased uncertainty in power system operation and load demand [3]. The unexpectedly increased load demands of residential and fleet of electric vehicles are also affecting the power system operations [4,5]. Therefore, a distributed slack bus has been proposed to appropriately represent the practical operation of the power system. Distributed slack buses, which are used to more accurately distribute power mismatch among different voltage-controlled buses, can help distribute the slack power among different generators, which results in producing more accurate results and reducing total generation cost and power losses. Also, distributed slack buses can help in accurately quantifying economic impacts of generators on power system operation [6–9].

In addition to the objective of treating the mismatch power more realistically through dividing it among distributed slack buses, distributed slack buses can also be regarded as an alternative for optimal power flow (OPF) in transmission level networks [10], which is explained as follows. OPF represents the problem of determining the optimal set-points of generators to meet the overall system demand including losses, with the objective of minimizing the overall operating cost. From the perspective of distributed slack buses, if participation factors are computed based on the operating cost of each generator, sharing of mismatch power can be allocated to each generator which can result in minimum operating cost. Furthermore, the design and development of operation and control schemes require the foundation of distributed slack buses to achieve the objective of accuracy in power flow results. These applications include active power management in active distribution systems and proper evaluation of locational marginal prices [11]. Therefore, the participation factors, which denote the amount of active power that needs to be contributed by each generator, should be calculated to distribute the slack power among the participating generators.

Different approaches have been presented in the literature to compute participation factors of distributed slack buses. Several

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https://doi.org/10.1016/j.segan.2022.100890
2352-4677/

Nomenclature

$\mathcal{N}$	Set of players of a cooperative game
V, W	Characteristic functions
S	A coalition that is subset of $\mathcal{N}$
$2^{\mathcal{N}}$	Possible set of coalitions
α	Payoff vector
$\{i\}$	Singleton or unit set of player j
$S\backslash\{i\}$	Coalition set without player i
ψ_i	Shapley value of player i
n	Number of players or generators
N	Number of buses
P_i	Active power injection at bus i
Q_i	Reactive power injection at bus i
$\tilde{V}_k$	Phasor voltage at bus k
$\tilde{V}_k^*$	Complex conjugate of the phasor voltage at bus k
V_i	Voltage magnitude at bus i
C	Total generation cost function
C_j	Generation cost function of generator j
P_{gj}	Active power of generator j
$\alpha_j, \beta_j, \gamma_j$	Cost coefficients of generator j
P_{loss}	Total active power loss of the system
θ_{ik}	Voltage angle difference between bus i and k
Y_{ik}	Element of the bus admittance matrix corresponding to the ith row and the jth column
G_{ik}	Real part of Y_{ik}
B_{ik}	Imaginary part of Y_{ik}
ψ_j^{eqv}	Equivalent Shapley value of jth generator
π_j	Participation factor of jth generator

factors such as machine inertia, droop characteristics of generator governors, and frequency control participation factors can influence the determination of the participation factors [2]. The computation of participation factors of substations and distributed generators (DGs) having sufficient adjustable power output based on the concept of generator domain has been presented in [12]. Also, the work presented in [12] discusses the integration of a participation factor model for distributing the slack bus onto unbalanced distribution power flow solvers. In [13], two different methods based on network sensitivities and the concept of generator domains have been presented to calculate the participation factors for a three-phase distribution power flow with a distributed slack bus model. Moreover, the work in [13] has concluded that participation factors computed based on the aforementioned methods may vary significantly and that the three-phase power flow model with distributed slack buses will impact several analyses at the distribution system level such as capacitor and DG placement, network reconfiguration, and economic analysis. In [14], a generation domain-based technique has been proposed to determine the contribution of each generator to the slack power and the losses in power systems. Additionally, authors of [14] have proposed that the method can solve difficult pricing and costing problems in the electricity supply industry in addition to ensuring transparency in the operation of transmission systems. In [15], the participation factors of distributed slack buses have been computed by performing perturbation analysis in classical economic load dispatch problems. The work in [15] also modifies the load-flow formulation to obtain a load-flow participation factor, which can be used to solve economic load dispatch problems. In [16], an approach that combines reliability and cost criteria has been proposed to compute participation factors of distributed slack buses.

Game theory-based approaches (both cooperative and non-cooperative) have been successfully applied in various fields of power systems. These applications include power system reliability enhancement, loss allocation, and transmission expansion planning [17]. In [18], a game theory-based comparative analysis has been presented and the valuation of demand aggregators has been examined in terms of aggregation of both flexibility and information. Also, a potential game has been developed in [18] to analyze the system under the scenarios of complete and incomplete information. The applications of cooperative game theory in power system expansion planning have been reviewed in [19]. Also, the work presented in [19] analyzes several prospects and challenges of cooperative game theory-based approach including scalability and non-convexity of cooperative games. In [20], a cooperative game theory-based approach has been proposed to determine optimal sizes and locations of distributed energy resources. A cooperative game theory-based approach for participation of active distribution systems in secondary frequency regulation has been proposed in [21]. In [22], a game-theoretic approach based on computing the locational marginal price at each bus has been proposed for reliability enhancement and power loss reduction in active distribution systems and microgrids, where each player of the game receives economic incentives when system reliability is improved and power loss is reduced. In [23], the gaming problem of incentive-based demand response program has been addressed using a probabilistic approach. A cooperative game theory-based approach has been proposed in [24] for under-frequency load shedding control. During the formulation of distributed slack buses model, the cooperative game-theoretic approaches based on the Shapley value ensure that the slack (or mismatch) power is distributed among different generators taking into account their marginal contributions. Therefore, this paper investigates the cooperative game-theoretic approach to compute participation factors of distributed slack buses.

This paper proposes a cooperative game-theoretic two-stage approach to compute participation factors that are used to distribute the slack active power among different participating generators. In the first stage, the worth or value of each coalition of participating generators is computed. For computing the worth of individual participation generators and their coalitions, the power flow analysis is performed, and the total generation cost of participating generators and active power loss of each coalition are calculated. In the second stage, participation factors of individual participating generators are computed based on the Shapley values and these factors are used to distribute the total slack (or mismatch) power among participating generators. The effectiveness of the proposed approach is demonstrated through case studies on several standard IEEE test systems.

The rest of the paper is organized as follows. Section 2 explains the concept of cooperative game theory along with the Shapley value. Section 3 presents the formulation of the cooperative game model, which is essential for the computation of participation factors of distributed slack buses. Section 4 describes the proposed approach of calculating participation factors of distributed slack buses. Section 5 provides the evaluation of the proposed approach through case studies on the IEEE 14-bus, 30-bus, and 57-bus systems. Finally, the conclusion is provided in Section 6.

2. Shapley value in cooperative game theory

A game can be categorized into a cooperative or non-cooperative game in game theory. There is no cooperation or coalition between players in non-cooperative games, whereas there is a cooperation or coalition between players in cooperative games. Each player in cooperative games can establish alliances with other players to maximize their rewards. Because players form coalitions to increase their individual incentives, a coalition must always result in incentives that are equal to or larger than the individual player's incentives. We employ cooperative games in this work with the purpose of increasing grid benefits; therefore, we will only discuss and use cooperative games.

Each cooperative game has three components, which are as follows [17]:

1. A finite set of players, denoted by $\mathcal{N}$.
2. A set of coalitions that a player can form.
3. Preference of each player over all possible coalitions.

Value or worth of each coalition in the cooperative game is represented using a characteristic function, where a characteristic function is the total utility of all members of the coalition. The characteristic function can be represented as $V(S)$, where S is a coalition. The characteristic function is a real-valued function (i.e., $V(S) : 2^{\mathcal{N}} \rightarrow \mathbb{R}$) with an empty set having zero value (i.e., $V(\phi) = 0$). The total payoff or incentive is distributed among the players using solution concepts including the Shapley value, the Nucleolus, and Nash-bargaining solution.

2.1. The core of a cooperative game

In game theory, the core refers to the set of feasible allocations that cannot be further improved through any other coalitions. Generally, outcomes of a cooperative game are specified as n-tuples of utility: $\alpha = \{\alpha^i : i \in \mathcal{N}\}$, called payoff vectors that are measured in a common unit of money [25]. The core of a game is defined as the set of payoff vectors that are feasible and coalitionally rational. In other words, the core is the set of imputations under which no coalition has a value greater than the sum of its members' payoffs. In other words, the core is the set of imputations under which all sets of coalition have values less than or equal to the sum of its members' payoffs. Thus, α is core if and only if [25],

$$\alpha.e^S \geq V(S), \forall S \subset \mathcal{N} \tag{1}$$

$$\alpha.e^{\mathcal{N}} = V(\mathcal{N}) \tag{2}$$

where e^S denotes the vector of size n with $e_i^S = 1$ if $i \in S$ and $e_i^S = 0$ if $i \in \mathcal{N} - S$.

2.2. The shapley value

Shapley value, which is one solution concept of cooperative game theory, assigns a unique payoff vector that is efficient, stable, symmetric, and satisfies monotonicity [26]. The Shapley value allocates the payoffs in such a way that is fair for cooperative solutions. The Shapley value of a cooperative game is given as follows [27].

$$\psi_i(V) = \sum_{S \in 2^{\mathcal{N}}, i \in S} \frac{(|S|-1)!(n-|S|)!}{n!}[V(S) - V(S \backslash \{i\})] \tag{3}$$

where $n = |\mathcal{N}|$ is the total number of players.

The Shapley value satisfies the following axioms [27]:

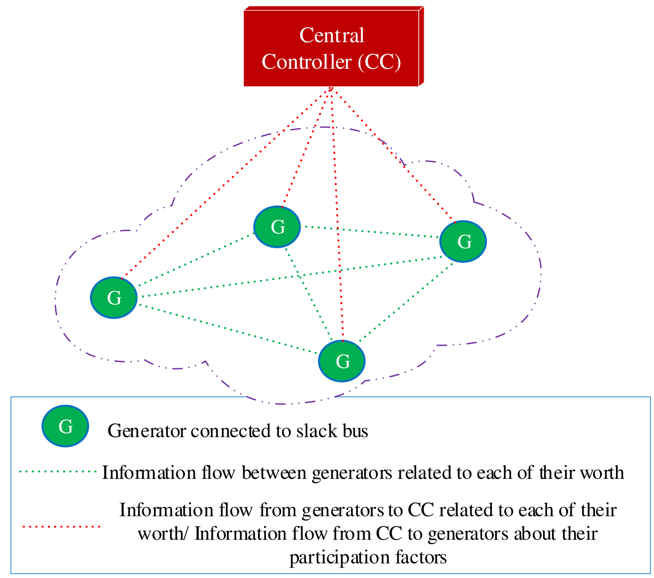

Fig. 1. Layout of the proposed cooperative game theoretic approach.

1. *Efficiency:* The efficiency axiom states that the sum of the Shapley values of all players is equal to the worth of grand coalition, so that the total gain is allocated among the players, i.e., $\sum_{i \in \mathcal{N}} \psi_i(V) = V(\mathcal{N})$.
2. *Individual Rationality:* This axiom states that the Shapley value of each player should be greater than or equal to its individual worth, i.e., $\psi_i(V) \geq V(\{i\}), \forall i \in \mathcal{N}$.
3. *Symmetry:* This axiom states that the players contributing the same amount in every coalition should have the same Shapley values. If j and k are such that $V(S \cup \{j\}) = V(S \cup \{k\})$ for every coalition S not containing j and k, then $\psi_j(V) = \psi_k(V)$.
4. *Dummy Axiom:* If j is such that $V(S) = V(S \cup \{j\})$ for every coalition S not containing j, then $\psi_j(V) = 0$.
5. *Additivity:* If V and W are characteristic functions, then $\psi(V + W) = \psi(V) + \psi(W)$.

3. Cooperative game model

A cooperative game model is formulated to compute the participation factors of distributed slack buses. Fig. 1 shows the layout of the proposed cooperative game-theoretic approach. In this paper, the task of computing participation factors of generators at distributed slack buses is regarded as a game. Since all generators should work in a coordinated manner for the determination of efficient participation factors, the game is a cooperative game. As explained in Section 2, a cooperative game is defined with a finite set of players and characteristic functions, which are essential to determine Shapley values of players. In this paper, two types of characteristic functions are implemented for the cooperative model. The first characteristic function is the total generation cost of participating generators, and the second characteristic function is the total active power loss. Using each of these characteristic functions, two Shapley values are computed for each participating generator using (3) and the equivalent Shapley values are determined by taking their average.

While determining the equivalent Shapley values, weighted average of the two Shapley values could be used. However, in the paper, equal weights are given to both Shapley values by taking the average to maximize benefit by lowering both generation cost and system power losses.

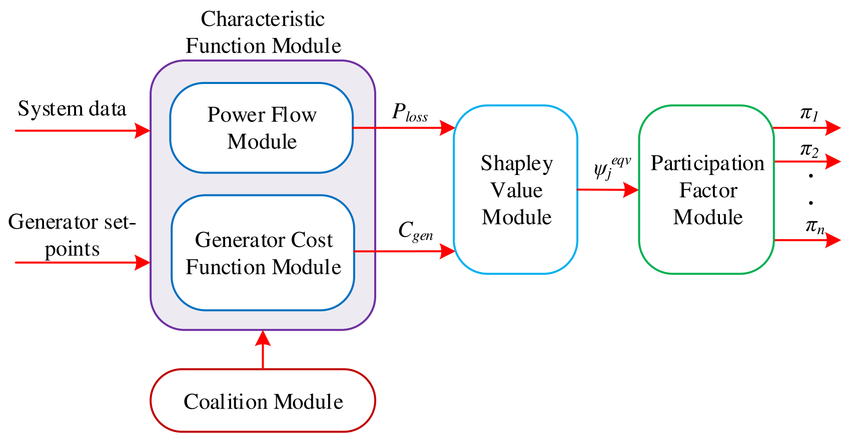

Fig. 2. Different modules of the proposed approach.

The cooperative model formulation for the proposed approach can be enumerated as follows.

1. Collect system data including generation data, transmission line data, load data, etc., which serve as input to the cooperative game model.
2. Generate the list of all possible coalitions of generators. For example, if three generators (G_1, G_2, and G_3) are participating in the process of computation of participation factors of distributed slack buses, the set of all possible coalitions, denoted by $2^{\mathcal{N}}$, is as follows. $2^{\mathcal{N}} = \{\phi, \{G_1\}, \{G_2\}, \{G_3\}, \{G_1, G_2\}, \{G_1, G_3\}, \{G_2, G_3\}, \{G_1, G_2, G_3\}\}$, where ϕ denotes an empty set.
3. For each participating generator and its possible coalitions, compute total cost of generation and total active power loss. These values serve as the worth of each generator and their coalitions.
4. Compute two Shapley values, $\psi_{1,i}$ and $\psi_{2,i}$, of each generator, G_i, using the characteristic functions determined in step 3 using (3).
5. Determine the equivalent Shapley value, ψ_i^{eqv}, of each participating generator, G_i, taking the average of two Shapley values computed in step 4.

The generator cooperative model, which is formulated using the procedures outlined above, is critical for establishing the participation factors of distributed slack buses.

4. The proposed approach

The proposed cooperative game theoretic approach is executed in the central controller shown in Fig. 1. The details of the modules in the central controller are shown in Fig. 2. The proposed approach to compute the participation factors of distributed slack buses is implemented in the following two steps [28]:

1. Computation of characteristic functions of the game which maps every coalition of players to a payoff.
2. Determination of participation factors on the basis of the Shapley values of each participating generator.

The inputs to the proposed cooperative game model are active power set-points of generators and the system data including line data, load data, etc. In the first stage (i.e., the computation of characteristic functions of the game), the power flow analysis is performed. The general form of the power flow equation can be expressed as follows:

$$P_i - jQ_i = \tilde{V}_i^* \sum_{k=1}^{N} Y_{ik}\tilde{V}_k. \tag{4}$$

The power flow equation given in (4) can be solved using an iterative method such as Gauss–Seidel or Newton–Raphson. After performing power flow analysis, the total cost of generation and active power loss are computed for each set of coalitions using (5) and (6), respectively.

$$\mathcal{V}_1 : C = \sum_{j=1}^{n} C_j(P_{gj}) = \sum_{j=1}^{n} \alpha_j P_{gj}^2 + \beta_j P_{gj} + \gamma_j, \tag{5}$$

$$\mathcal{V}_2 : P_{loss} = \sum_{i=1}^{N} P_i = \sum_{i=1}^{N}\sum_{k=1}^{N} V_i V_k (G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}), \tag{6}$$

where $\mathcal{V}_1$ is the first characteristic function and $\mathcal{V}_2$ is the second characteristic function.

Although generators' quadratic cost functions given in (5) are used to compute the first characteristic function (i.e., the total cost of generation), it is to be noted that the proposed cooperative game model is not affected by the order of the cost function or the method to compute the generation cost.

In the second stage, Shapley values are computed for each candidate location using (3), and the equivalent Shapley value ψ_j^{eqv} of the j^{th} generator is computed by taking the average of two types of Shapley values.

To account for load uncertainties, a scenario-based formulation is used to compute characteristic functions and Shapley values. In the scenario-based formulation, a certain number of loading scenarios are generated. To make the solution approach computationally tractable, the generated scenarios are reduced using the k-means method. Algorithm 1 describes the k-means method for scenario reduction [20].

The characteristic functions (5) and (6) are computed for all reduced scenarios. The expected value of the i^{th} characteristic function is then calculated as follows.

$$\mathbb{E}[\mathcal{V}_i] = \sum_{j=1}^{K} Pr(j) \times \mathcal{V}_i(j), \tag{7}$$

where $Pr(j)$ is the probability of the j^{th} reduced scenario; and $\mathcal{V}_i(j)$ is the i^{th} characteristic function of the j^{th} reduced scenario.

The expected Shapley value corresponding to the i^{th} characteristic function is calculated as follows [29].

$$\psi^{exp}(\mathcal{V}_i) = \sum_{S\in 2^{\mathcal{N}}, m\in S} \frac{(|S|-1)!(n-|S|)!}{n!} [\mathbb{E}[\mathcal{V}_i(S)] - \mathbb{E}[\mathcal{V}_i(S\backslash\{m\}])] \tag{8}$$

For the case of including load uncertainties, the equivalent Shapley value ψ_j^{eqv} of the j^{th} generator is calculated by taking the

Algorithm 1: *k*-means method for scenario reduction

```
Input : Input r data points x(i) ∈ ℝ
Initialize a set of K means μ_1, ...., μ_K ∈ ℝ
do
    for i ← 1 to r do
        C(i) = arg min_j ||x(i) − μ_j||^2
    for j ← 1 to K do
        sum = 0
        count(j) = 0
        for i ← 1 to r do
            if C(i) == j then
                sum = sum + x(i)
                count(j) = count(j) + 1
        μ_j = sum/count(j)
while means do not change;
for j ← 1 to K do
    Pr(j) = count(j)/r
Output: μ_j, Pr(j), ∀j = 1, ..., K
```

average of two expected Shapley values corresponding to each characteristic function.

The participation factor, π_j, of the j^{th} generator is determined using (9).

$$\pi_j = \psi_j^{eqv} \Big/ \sum_{j=1}^{n} \psi_j^{eqv}, \tag{9}$$

where ψ_j^{eqv} is the equivalent Shapley value of the j^{th} generator. Now, for the total mismatch power of ΔP, the change in active power of the j^{th} generator can be calculated as follows.

$$\Delta P_{gj} = \pi_j \times \Delta P. \tag{10}$$

The proposed approach (solution algorithm) to determine the generator participation factors of distributed slack buses can be summarized as follows.

1. Provide system data related to lines, loads, transformers, and generators.
2. Enumerate all possible coalitions of participating generators and compute their characteristic functions.
3. Compute the equivalent Shapley value of each generator and the respective participation factor using (9).

The flowchart of the proposed solution algorithm is shown in Fig. 3.

4.1. Application in the case of a liberalized energy market

In a liberalized energy market, the participation generators submit their offers to the Transmission System Operator (TSO)/Independent System Operator (ISO), the TSO/ISO analyzes offers from the generating companies along with the bids from the distribution companies, and the electricity price is determined after market clearance [30]. The market clearing prices and capacities are highly volatile in nature. Due to this reason, a day-ahead or hour-ahead forecast of market clearing prices and capacities can be used to calculate the first characteristic function based on generation cost. The second characteristic function of active power loss, calculated based on (6), is still applicable in case of the liberalized energy market. After computing characteristic functions, the equivalent Shapley values can be computed. Finally, the participation factors of distributed slack bus generators can be computed using (9).

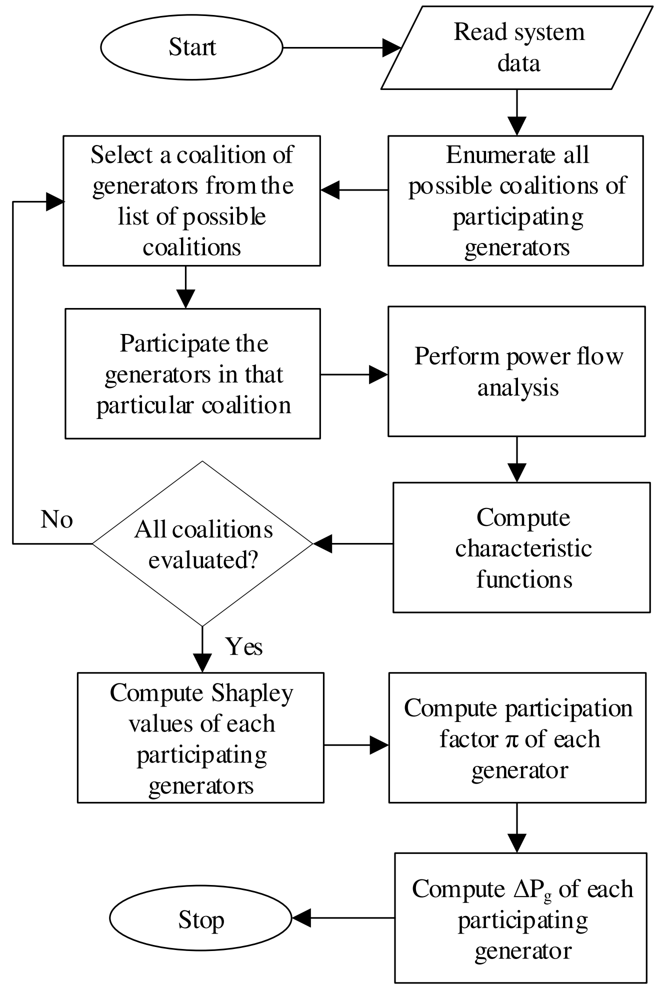

Fig. 3. Flowchart of the proposed approach.

5. Case studies and discussions

This section presents the evaluation of the proposed approach through case studies on the IEEE 14-bus, 30-bus, and 57-bus systems. Two test cases are conducted for each system.

5.1. IEEE 14-bus system

The IEEE 14-bus system consists of 14 buses, 5 generators, and 11 loads with a total generation capacity of 772.4 MW and total peak load of 259 MW [31]. The cost function and other generator parameters for this system are given in Table 1. The table also provides active power set-points of generators for two cases of the system.

The proposed approach starts by collecting system data including generation data, transmission line data, load data, etc., which serve as input to the cooperative game model. All possible sets of coalitions are then enumerated. Since all five generators of the IEEE 14-bus system are allowed to participate in the game (i.e., the task of computation of participation factors), 31 sets of coalitions, except an empty set, are formed, which are shown in the first column of Table 2. For each set of the coalitions, two types of characteristic functions, i.e., the generation cost due to participating generators and active power loss, are computed. The characteristic functions indicate the value or worth of each set of coalitions. The characteristic functions serve as motivations for calculating participation factors. Since slack (additional or

Table 1
Cost function and other parameters of generators for IEEE 14-bus System.

Generator number	Location (bus)	α [\$/MW^{2}h]	β [\$/MWh]	γ [\$/h]	Generator capacity [MW]	Active power set-points [MW] for Case-I	Active power set-points [MW] for Case-II
1	1	0.043	20	0	332.4	232.4	192.4
2	2	0.250	20	0	140	40	50
3	3	0.010	40	0	100	0	10
4	6	0.010	40	0	100	0	10
5	8	0.010	40	0	100	0	10

extra) power is being distributed among different generators, worthiness or value of each generator is indicated by the amount of available capacity of the generator. The slack power cannot be allocated to the generator that does not have excess available capacity. Therefore, the difference between generator capacity (sixth column of Table 1) and active power set-point (seventh column of Table 1 for Case-I) is utilized to compute P_{gj} (calculated based on (11)) and the generation cost, C.

$$P_{gj} = \rho \times (\text{Generation capacity} - \text{Active power set-point}), \quad (11)$$

where ρ is a factor that should be chosen in a way as to avoid non-convergence of power flow solutions. In this paper, $\rho = 0.5$ has been chosen.

For Case-I of the IEEE 14-bus system, if only the generator connected to bus 1 is allowed to participate in the game, generation cost of the system would be 1108 \$/h and the active power loss would be 15.92 MW. The sample generator cost calculation for the case wherein only the generator connected to bus 1 is allowed to participate is presented below.

$$P_{g1} = 0.5 \times (332.4 - 232.4) = 50 \text{ MW}$$
$$C_1 = \alpha_1 P_{g1}^2 + \beta_1 P_{g1} + \gamma_1 = 1108 \text{ \$/h}$$

If only the generator connected to bus 2 is allowed to participate in the game, generation cost of the system would be 1625 \$/h and the active power loss would be 12.9 MW. If the generators connected to buses 1 and 2 are allowed to participate in the game, its generation cost would be 2733 \$/h and the active power loss would be 12.89 MW. For the coalition of generators connected to buses 1 and 2, the generation cost is the sum of individual generation costs. However, the active power loss of the coalition is less than the sum of individual active power losses. Similarly, if the generators connected to buses 1 and 3 are allowed to participate in the game, its generation cost would be 3133 \$/h and the active power loss would be 9.76 MW. Similar to the coalition of generators connected to buses 1 and 2, the generation cost of the coalition of generators connected to buses 1 and 3 is the sum of individual generation costs. However, the active power loss of the coalition of generators connected to buses 1 and 3 is less than the individual active power losses.

Comparison of these two coalitions shows that the active power loss of the coalition 1 and 3 is less than that of the coalition 1 and 2. This indicates that the generator connected to bus 3 has more significant contribution to power loss reduction than the generator connected to bus 2 whenever they form coalitions with the generator connected to bus 1. In this way, the worthiness of each player (i.e., generator) of the game are utilized while computing Shapley values and participation factors of distributed slack bus generators. The characteristic functions of all sets of coalitions computed for Case-I are shown in Table 2. Similarly, the characteristic functions of all sets of coalitions for Case-II can be computed.

Based on the characteristic functions, Shapley values of each generator location can be computed using (3) and the equivalent Shapley values can be computed by taking their average. For Case-I of IEEE 14-bus system, the equivalent Shapley values obtained using the proposed approach for distributed slack bus locations 1, 2, 3, 6, and 8 are 555.4, 813.1, 1011.7, 1012.6, and 1012.2, respectively, which are shown in the third column of Table 3. For Case-II of IEEE 14-bus system, the equivalent Shapley values obtained using the proposed approach for distributed slack bus locations 1, 2, 3, 6, and 8 are 807.0, 703.7, 909.5, 910.3, and 909.9, respectively, which are shown in the fifth column of Table 3.

Table 2
Characteristic functions of possible coalitions for Case-I of IEEE 14-bus system.

Coalitions of generator locations (buses)	Generation cost [\$/h]	Active power loss [MW]
1	1108	15.92
2	1625	12.90
3	2025	9.79
6	2025	11.73
8	2025	10.71
1,2	2733	12.89
1,3	3133	9.76
1,6	3133	11.71
1,8	3133	10.69
2,3	3650	7.56
2,6	3650	9.31
2,8	3650	8.36
3,6	4050	6.50
3,8	4050	5.68
6,8	4050	7.43
1,2,3	4758	7.55
1,2,6	4758	9.30
1,2,8	4758	8.34
1,3,6	5158	6.49
1,3,8	5158	5.67
1,6,8	5158	7.41
2,3,6	5675	4.88
2,3,8	5675	4.12
2,6,8	5675	5.67
3,6,8	6075	3.29
1,2,3,6	6783	4.88
1,2,3,8	6783	4.11
1,2,6,8	6783	5.66
1,3,6,8	7183	3.28
2,3,6,8	7700	2.31
1,2,3,6,8	8808	2.30

The participation factors of the generators can then be calculated using (9). For Case-I of IEEE 14-bus system, participation factors obtained for distributed slack bus locations 1, 2, 3, 6, and 8 using the proposed approach are 0.1261, 0.1846, 0.2297, 0.2299, and 0.2298, respectively, which are shown in the fourth column of Table 3. For Case-II of IEEE 14-bus system, participation factors obtained for distributed slack bus locations 1, 2, 3, 6, and 8 using the proposed approach are 0.1903, 0.1660, 0.2145, 0.2147, and 0.2146, respectively, which are shown in the sixth column of Table 3.

For comparison purposes and to demonstrate the effectiveness of the proposed approach, the participation factors for distributed slack buses are computed based on generation capacity (a conventional approach) in addition to that based on the proposed approach. A power mismatch (i.e., increment in load) of 100 MW is simulated, and the cost of generation and active power loss are computed for single slack bus, distributed slack buses (based on the conventional approach), and distributed slack buses (based

Table 3
Shapley values and participation factors of generators for IEEE 14-bus system.

Generator locations (buses)	Participation factors (conventional)	Case-I		Case-II	
		Equivalent shapley values	Participation factors (proposed)	Equivalent shapley values	Participation factors (proposed)
1	0.4303	555.4	0.1261	807.0	0.1903
2	0.1813	813.1	0.1846	703.7	0.1660
3	0.1295	1011.7	0.2297	909.5	0.2145
6	0.1295	1012.6	0.2299	910.3	0.2147
8	0.1295	1012.2	0.2298	909.9	0.2146

Table 4
Cost of generation and active power loss in case of IEEE 14-bus system for power mismatch of +100 MW.

Approaches	Case-I		Case-II	
	Cost of generation	Active power loss	Cost of generation	Active power loss
Single slack bus	13,299 $/h	27.55 MW	12,739 $/h	21.84 MW
Distributed slack buses (conventional)	12,628 $/h	19.98 MW	12,466 $/h	15.39 MW
Distributed slack buses (proposed)	12,382 $/h	15.88 MW	12,363 $/h	12.51 MW

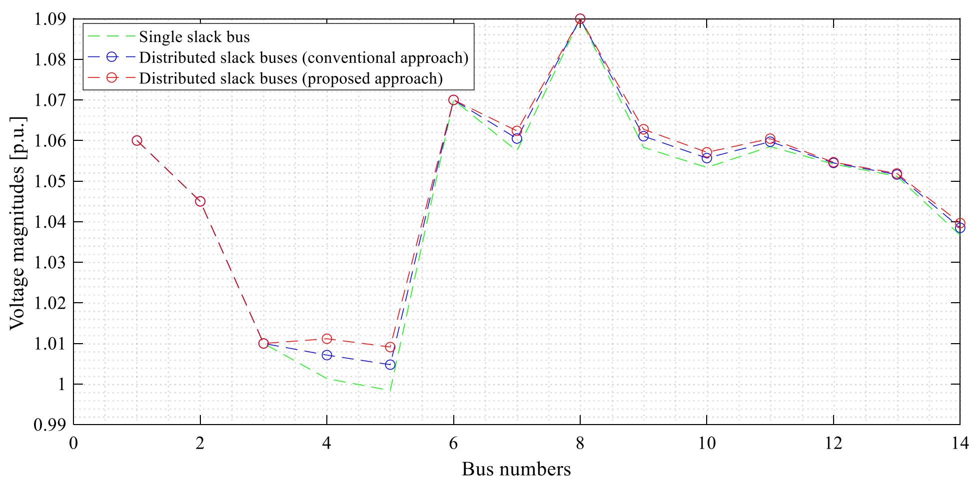

Fig. 4. Voltage profile for Case-I of IEEE-14 bus system.

on the proposed approach) for both cases of the IEEE 14-bus system. In the single slack bus model, the power mismatch is taken by a single generator at bus 1, whereas in distributed slack buses model (both conventional and proposed), the power mismatch is shared among all generators according to the respective participation factors.

For Case-I, the total generation cost obtained using a single slack bus, distributed slack buses (conventional), and distributed slack buses (proposed) are, respectively, 13299 $/h, 12628 $/h, and 12382 $/h. The total active power loss obtained using a single slack bus, distributed slack buses (conventional), and distributed slack buses (proposed) are, respectively, 27.55 MW, 19.98 MW, and 15.88 MW. For Case-II, the total generation cost obtained using a single slack bus, distributed slack buses (conventional), and distributed slack buses (proposed) are, respectively, 12739 $/h, 12466 $/h, and 12363 $/h. The total active power loss obtained using a single slack bus, distributed slack buses (conventional), and distributed slack buses (proposed) are, respectively, 21.84 MW, 15.39 MW, and 12.51 MW. These values for both cases are shown in Table 4. Bar plots showing comparison of total generation cost and total power loss for both cases are, respectively, shown in Figs. 6 and 7. Similarly, voltage profiles for the three approaches in both the cases are shown in Figs. 4 and 5.

The comparison of the results shows that the generation cost and the active power loss are reduced, and the voltage profile is improved in the distributed slack buses model where participation factors are computed using the proposed approach. It is to be noted that there are no changes in voltages of generator buses since they are modeled as voltage-controlled buses.

5.1.1. Computing participation factors of distributed slack bus generators considering load uncertainty

A scenario-based formulation is used to model the load uncertainty for Case-I of the IEEE 14-bus system. During scenario generation, 1000 normally distributed load scenarios (for 11 loads) are generated with 15% standard deviation from the base load. The k-means method is used to generate 20 reduced load scenarios. The two types of characteristic functions are computed for each reduced scenario using (5) and (6). The expected values of both characteristic functions and the expected Shapley values are then computed, respectively, using 7. Finally, the participation factors are computed using (9). After incorporating load uncertainty, the participation factors of distributed slack bus generators 1, 2, 3, 6, and 8 are 0.1261, 0.1846, 0.2297, 0.2299, and 0.2298, respectively. These results are identical to those obtained without considering load uncertainty. The results suggest that participation factors computed using the proposed approach are not affected by load uncertainty. This is because the calculation of participation factors employs the Shapley value, which accounts for the average marginal contributions of each generator.

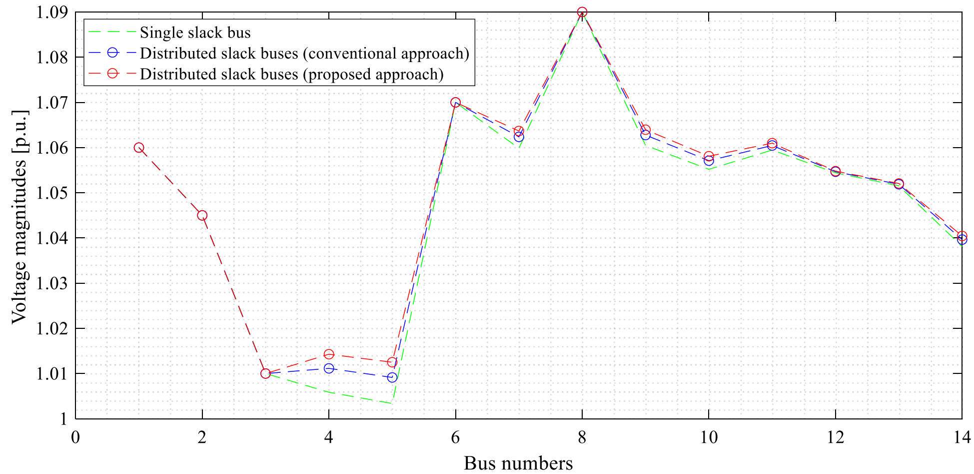

Fig. 5. Voltage profile for Case-II of IEEE-14 bus system.

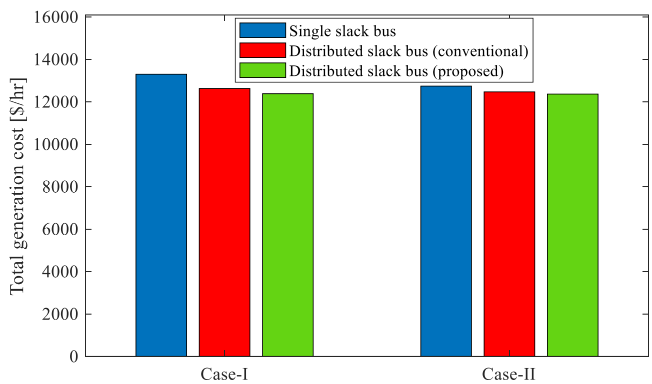

Fig. 6. Comparison of generation cost for both cases of IEEE 14-bus system.

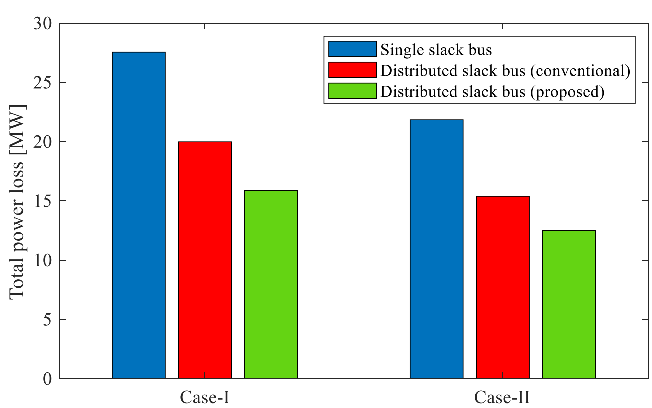

Fig. 7. Comparison of power loss for both cases of IEEE 14-bus system.

5.1.2. Computing participation factors of distributed slack bus generators considering hydroelectric power plant operated power system

The variable production cost of a hydroelectric power plant is negligible. When considering a power system operated with hydroelectric power plants, the proposed approach can be applied by replacing the first characteristic function of generation cost by the available capacity of hydroelectric power plants. The second characteristic function of active power losses is still applicable in case of the hydroelectric power plants dominated power system.

For Case-I of the IEEE 14-bus system, if only the generator connected to bus 1 is allowed to participate in the game, the available capacity would be 100 MW and the active power loss would be 15.91 MW. If only the generator connected to bus 2 is allowed to participate, the available capacity is 100 MW and the active power loss is 10.73 MW. However, if the generators connected to buses 1 and 2 are allowed to participate in the game, the available capacity would be 200 MW and the active power loss would be 10.70 MW. This result shows that the available capacity of the coalition is the sum of individual available capacities of generators. Also, the active power loss the system is reduced as a result of the coalition. In this way, we can compute characteristic functions for all possible sets of coalitions of generators.

Based on the obtained values of characteristic functions, Shapley values of each generator can be computed using (3) and the equivalent Shapley values can be obtained by taking the average of two different Shapley values of individual generators. The participation factors of the generators can then be calculated using (9). The participation factors obtained for distributed slack bus generators 1, 2, 3, 6, and 8 are respectively, 0.2045, 0.2019, 0.1957, 0.2007, and 0.1972. In this way, the participation factors can be obtained using the proposed approach in case of power system consisting of only hydroelectric power plants similar to Sitka grid in Alaska, USA.

5.2. IEEE 30-bus system

The IEEE 30-bus system consists of 30 buses, 6 generators, and 21 loads with a total generation capacity of 900.2 MW and total peak load of 283.4 MW [31]. The cost function and other parameters of generators for this system are shown in Table 5. The table also shows active power set-points of generators for two cases of the system.

All possible coalitions are listed. Since all six generators of the IEEE 30-bus system are allowed to participate in the game (i.e., the task of computation of participation factors), 63 sets of coalitions, except an empty set, are formed. For each set of the coalitions, two types of characteristic functions, i.e., the generation cost due to participating generators and active power loss are computed. The characteristic functions indicate the value or worth of each set of coalitions. The characteristic functions of all sets of coalitions for both cases of the IEEE 30-bus can be computed in the manner similar to that of the IEEE 14-bus system.

Based on the characteristic functions, Shapley values of each generator location can be computed using (3) and the equivalent Shapley values can be computed by taking their average. For each of the test cases of the IEEE 30-bus system, participation factors obtained using the conventional capacity-based approach,

Table 5
Cost function and other parameters of generators for IEEE 30-bus system.

Generator number	Location (bus)	α [\$/MW^{2}h]	β [\$/MWh]	γ [\$/h]	Generator capacity [MW]	Active power set-points [MW] for Case-I	Active power set-points [MW] for Case-II
1	1	0.0200	2.00	0	360.2	260.2	160.2
2	2	0.0175	1.75	0	140	40	40
3	5	0.0625	1.00	0	100	0	30
4	8	0.0083	3.25	0	100	0	20
5	11	0.0250	3.00	0	100	0	20
6	13	0.0250	3.00	0	100	0	20

Table 6
Shapley values and participation factors of generators for IEEE 30-bus system.

Generator locations (buses)	Participation factors (conventional)	Case-I		Case-II	
		Equivalent shapley values	Participation factors (proposed)	Equivalent shapley values	Participation factors (proposed)
1	0.4001	76.7	0.1396	201.7	0.3633
2	0.1555	66.3	0.1207	66.1	0.1191
5	0.1111	102.3	0.1862	55.4	0.0999
8	0.1111	91.5	0.1666	71.6	0.1290
11	0.1111	106.1	0.1931	80.0	0.1441
13	0.1111	106.5	0.1938	80.3	0.1446

Table 7
Cost of generation and active power loss in case of IEEE 30-bus system for power mismatch of +100 MW.

Approaches	Case-I		Case-II	
	Cost of generation	Active power loss	Cost of generation	Active power loss
Single slack bus	2764.8 \$/h	25.16 MW	1760.3 \$/h	12.84 MW
Distributed slack buses (conventional)	2372.3 \$/h	21.12 MW	1554.9 \$/h	10.30 MW
Distributed slack buses (proposed)	2214.5 \$/h	19.13 MW	1544.2 \$/h	10.07 MW

and equivalent Shapley values and participation factors obtained using the proposed approach are given in Table 6.

The participation factors for distributed slack buses are computed based on generation capacity (a conventional approach) in addition to that based on the proposed approach for comparison purposes and to illustrate the effectiveness of the proposed approach. A power mismatch (here, increment in load) of 100 MW is simulated, and the cost of generation and active power loss are computed for single slack bus, distributed slack buses (based on the conventional approach), and distributed slack buses (based on the proposed approach) for both cases of the IEEE 30-bus system. In case of single slack bus model, bus 1 is considered as slack bus.

The total generation cost and total active power loss obtained using single slack bus and distributed slack buses for both cases are shown in Table 7. Bar plots showing comparison of total generation cost and total power loss for both cases are, respectively, shown in Figs. 8 and 9. Similarly, voltage profile for the three approaches in Case-I are shown in Fig. 10.

Similar to that for the IEEE 14-bus system, the comparison of the results for the IEEE 30-bus system shows that the generation cost and the active power loss are reduced, and the voltage profile is improved in the distributed slack buses model where participation factors are computed using the proposed approach.

5.3. IEEE 57-bus system

The IEEE 57-bus system consists of 57 buses, 7 generators, and 42 loads with a total generation capacity of 1975.9 MW and total peak load of 1250.8 MW [31]. The cost function and other parameters of generators for this system are shown in Table 8. The table also shows active power set-points of generators for two cases of the system.

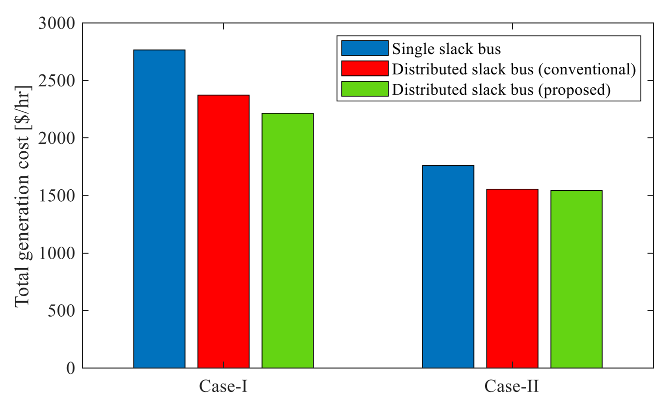

Fig. 8. Comparison of generation cost for both cases of IEEE 30-bus system.

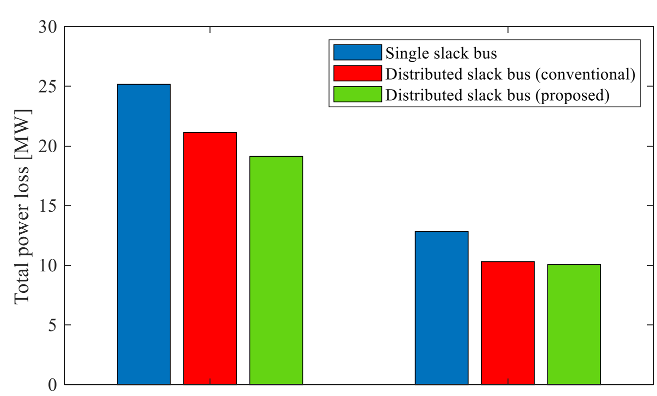

Fig. 9. Comparison of power loss for both cases of IEEE 30-bus system.

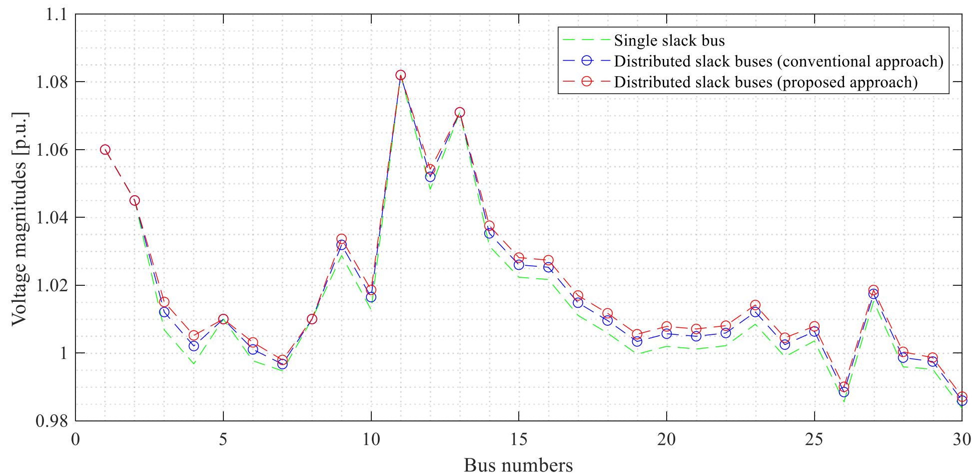

Fig. 10. Voltage profile for Case-I of IEEE-30 bus system.

Table 8
Cost function and other parameters of generators for IEEE 57-bus system.

Generator number	Location (bus)	α [\$/MW^{2}h]	β [\$/MWh]	γ [\$/h]	Generator capacity [MW]	Active power set-points [MW] for Case-I	Active power set-points [MW] for Case-II
1	1	0.077	20	0	575.88	428.9	528.9
2	2	0.010	40	0	100	0	0
3	3	0.250	20	0	140	40	40
4	6	0.010	40	0	100	0	0
5	8	0.022	20	0	550	150	150
6	9	0.010	40	0	100	0	0
7	12	0.032	20	0	410	310	210

Table 9
Shapley values and participation factors of generators for IEEE 57-bus system.

Generator locations (buses)	Participation factors (conventional)	Case-I		Case-II	
		Equivalent shapley values	Participation factors (proposed)	Equivalent shapley values	Participation factors (proposed)
1	0.2915	963.1	0.1231	275.0	0.0355
2	0.0506	1030.4	0.1317	1030.4	0.1329
3	0.0709	826.1	0.1055	826.2	0.1066
6	0.0506	1020.5	0.1304	1020.9	0.1317
8	0.2784	2417.8	0.3089	2419.3	0.3121
9	0.0506	1019.9	0.1303	1020.3	0.1316
12	0.2075	548.9	0.0701	1160.7	0.1497

All possible sets of coalitions are enumerated. Since all seven generators of the IEEE 57-bus system are allowed to participate in the game (i.e., the task of computation of participation factors), 127 sets of coalitions, except an empty set, are formed. For each set of the coalitions, two types of characteristic functions, i.e., the generation cost due to participating generators and active power loss are computed. The characteristic functions indicate the value or worth of each set of coalitions. The characteristic functions of all sets of coalitions for both cases of the IEEE 57-bus can be computed in the manner similar to that of the IEEE 14-bus system, which is not shown here.

Based on the characteristic functions, Shapley values of each generator location can be computed using (3) and the equivalent Shapley values can be computed by taking their average. For Case-I of the IEEE 57-bus system, the equivalent Shapley values obtained using the proposed approach are shown in the third column of Table 9. For Case-II of the IEEE 57-bus system, the equivalent Shapley values obtained using the proposed approach are shown in the fifth column of Table 9. The participation factors of the generators can then be calculated using (9). For Case-I of the IEEE 57-bus system, participation factors obtained using the proposed approach are shown in the fourth column of Table 9. For Case-II of the IEEE 57-bus system, participation factors obtained using the proposed approach are shown in the sixth column of Table 9.

Similar to the IEEE 14-bus and 30-bus systems, the participation factors for distributed slack buses in case of the IEEE 57-bus system are computed based on generation capacity (a conventional approach) in addition to that based on the proposed approach for comparison purposes and to illustrate the effectiveness of the proposed approach. A power mismatch (i.e., increment in load) of 100 MW is simulated, and the cost of generation and active power loss are computed for a single slack bus, distributed slack buses (based on the conventional approach), and distributed slack buses (based on the proposed approach) for both cases of the IEEE 57-bus system. In case of the single slack bus model, bus 1 is considered as a slack bus.

The total generation cost and total active power loss obtained using the single slack bus and distributed slack buses for both cases are shown in Table 10. Bar plots showing comparison of total generation cost and total power loss for both cases are, respectively, shown in Figs. 11 and 12.

Table 10
Cost of generation and active power loss in case of IEEE 57-bus system for power mismatch of +100 MW.

Approaches	Case-I		Case-II	
	Cost of generation	Active power loss	Cost of generation	Active power loss
Single slack bus	103,870 $/h	104.3 MW	122,990 $/h	132.3 MW
Distributed slack buses (conventional)	92,062 $/h	86.4 MW	108,710 $/h	111.0 MW
Distributed slack buses (proposed)	89,665 $/h	83.3 MW	104,400 $/h	105.5 MW

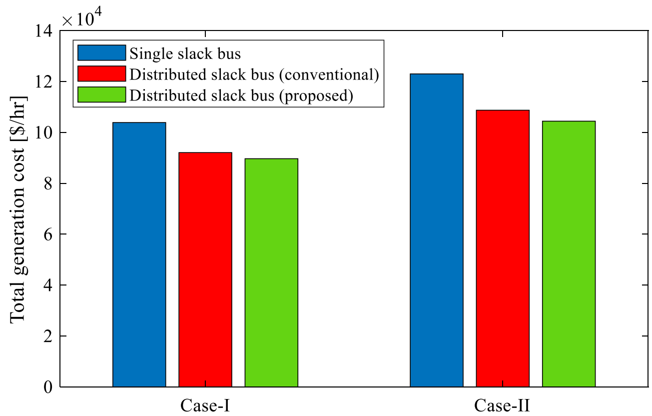

Fig. 11. Comparison of generation cost for both cases of IEEE 57-bus system.

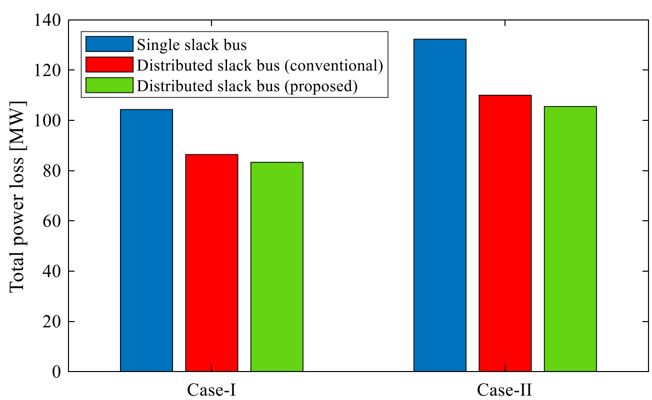

Fig. 12. Comparison of power loss for both cases of IEEE 57-bus system.

Similar to the IEEE 14-bus and 30-bus systems, the comparison of the results for the IEEE 57-bus system shows that the generation cost and the active power loss are reduced.

5.4. Limitation of the proposed approach

The computation of two distinct types of characteristic functions (generation cost and active power losses) serves as the foundation for the proposed cooperative game theoretic approach for the computation of participation factors. The characteristic function of generation cost is calculated based on the available capacities of distributed slack bus generators. Due to this reason, the proposed approach is not directly applicable to non-dispatchable generators (e.g., photovoltaic systems, wind generators, etc.) which produce real power with "must-take" paradigm.

6. Conclusion

In this paper, a cooperative game-theoretic two-stage approach has been proposed to calculate the participation factors of distributed slack buses. In the first stage, the generation cost and active power loss were calculated, which served as the characteristic functions of the cooperative game. In the second stage, the participation factors of distributed slack buses were calculated using the equivalent Shapley values. The proposed approach can calculate the participation factors of distributed slack buses taking into account the marginal contribution of each generator to distribute slack (or mismatch) power among different generators. The proposed approach was implemented in the IEEE 14-bus, 30-bus, and 57-bus systems. The case studies exhibit the effectiveness of the proposed approach to compute participation factors that help reduce generation cost and power loss.

CRediT authorship contribution statement

Mukesh Gautam: Conceptualization, Methodology, Data curation, Writing – original draft, Software, Investigation, Validation, Formal analysis. **Narayan Bhusal:** Conceptualization, Methodology, Writing – original draft, Data curation, Formal analysis. **Jitendra Thapa:** Methodology, Writing – original draft. **Mohammed Benidris:** Conceptualization, Methodology, Supervision, Validation, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

This work was supported by the U.S. National Science Foundation (NSF) under Grant NSF 1847578.

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