Pre-positioning of Movable Energy Resources for Distribution System Resilience Enhancement

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***Abstract*—This paper proposes an approach based on graph theory and coalitional game theory for pre-positioning of movable energy resources (MERs) to improve the resilience of the electric power supply. By utilizing the weather forecasting and monitoring data, the proposed approach determines staggering locations of MERs in order to ensure the quickest possible response following an extreme event. The proposed approach starts by generating multiple line outage scenarios based on fragility curves of distribution lines, where the k-means method is used to create a set of reduced line outage scenarios. The distribution network is modeled as a graph and distribution network reconfiguration is performed for each reduced line outage scenario. The expected load curtailment (ELC) corresponding to each location is calculated using the amount of curtailed load and probability of each reduced scenario. The optimal route to reach each location and its distance is determined using Dijkstra's shortest path algorithm. The MER deployment cost function associated to each location is determined based on the ELC and the optimal distance. The MER deployment cost functions are used to determine candidate locations for MER pre-positioning. Finally, the Shapley value, a solution concept of coalitional game theory, is used to determine the sizes of MERs at each candidate location. The proposed approach for pre-positioning of MERs is validated through a case study performed on the 33-node distribution test system.**

***Index Terms*—Coalitional game, movable energy resources, network reconfiguration, resilience, spanning forest.**

NOMENCLATURE

$\mathcal{N}$	set of players of a coalitional game
V	characteristic function
S	a coalition that is subset of $\mathcal{N}$
$2^{\mathcal{N}}$	possible set of coalitions
ELC_i	expected load curtailment of location i
CDF_i	capacity distribution factor of location i
ψ_i	Shapley value of player i
K	the number of reduced scenario
$Pr(j)$	probability of the j^{th} reduced scenario
β_1, β_2	cost function weighting coefficients
ω_m	critical load factor at node m
$P_{MER-tot}$	total MER capacity
PDSR	effective power distribution service restoration
MER	movable energy resource

I. INTRODUCTION

A. Motivation and Background

Over the last decade, the frequency of extreme events, both natural (e.g., hurricanes, wildfires, ice or hail storms, and earthquakes) and man-made (e.g., cyber and physical attacks), has increased dramatically. For example, there were 20 weather related catastrophic events in the United States in 2021 alone, each with costs surpassing \$1 billion [1]. Such extreme events have resulted in severe damages to important power system equipment resulting in system-wide extended power outages. The electric companies' goal of delivering reliable and resilient electrical supply to its customers has been compromised by catastrophic weather events and subsequent outages. As a result, PDSR procedures must be established in order to reduce the impact of these incidents on end-user customers. PDSR's major goal is to reduce load curtailments and outage duration by making the best use of available resources. Smart grid technologies, such as microgrid formation, network reconfiguration, repair crew dispatch, distributed generation, energy storage, MERs, and combinations of these methods and techniques, have proven to be the most effective PDSR solutions.

MERs are mobile and versatile resources that can be redeployed quickly from staggering locations to fault locations. They are versatile in the notion that they can be built to variable size and quickly integrated into the distribution grid after a disaster. These resources can be designed to supply up to a few megawatts of load. When part of a distribution system is islanded due to equipment failures or damages, MERs can be deployed to supply local and isolated critical loads if no other resources are available.

B. Relevant Literature

Deployment of MERs for PDSR has gained significant momentum. A two-stage robust optimization framework has been developed in [2] for routing and scheduling MERs to enhance the resilience of distribution systems. A two-stage PDSR strategy based on mixed-integer linear programming (MILP) has been proposed in [3] to enhance seismic resilience of distribution systems with MERs. A mixed integer linear programming-based PDSR strategy has been proposed in [4] for an active distribution system, where routing and scheduling of mobile energy storage systems is performed for enhanced resilience. In [5], a two-stage optimization strategy has been proposed to enhance distribution system resilience with mobile energy storage units, where dynamic microgrid formation is also considered. The majority of the aforementioned studies primarily focus on coordinating and dispatching MERs with other PDSR techniques for service restoration, without considering MER pre-positioning.

C. Contributions and Organization

In our previous work [6], a distribution service restoration strategy has been proposed where the minimum sizes of MERs are determined for resilience enhancements. In this paper, we propose an approach based on graph theory and coalitional game theory for pre-positioning of MERs. High wind speed is taken as an example of weather-related extreme events. A set of line outage scenarios is generated based on forecasted wind speed. Generated scenarios are then reduced using the k-means method. The reduced scenarios are used to determine expected load curtailments when MERs are placed at each node. The MER deployment cost function of each node is determined using expected load curtailment and the optimal distance of MER deployment location calculated using Dijkstra's shortest path algorithm. A certain number of candidate locations of MERs is selected based on the MER deployment cost function. The candidate locations thus selected are treated as players of a game. Since the players are allowed to form coalitions among themselves to maximize the expected critical load recovery, the game is a coalitional game. Shapley value, one of the solution concepts of coalitional game theory, is then used to determine sizes of MERs at each candidate location. The proposed approach is validated through a case study on a distribution test system.

The remainder of the paper is laid out as follows. The mathematical modeling of the MER pre-positioning problem is explained in Section II. The proposed approach and solution algorithm are described in Section III. A case study on the 33-node system is used to validate the proposed work in Section IV. Section V provides some concluding remarks.

II. Mathematical Modeling

This section presents the graph theoretic modeling of distribution network and road network, and the coalitional game theoretic model of the MER pre-positioning problem under study for resilience enhancement of the distribution system.

A. Graph Theoretic Modeling of Distribution Network

Distribution systems are equipped with sectionalizing switches (normally closed) and tie-switches (normally open). When all the switches of a distribution network are closed, a meshed network is formed, and the meshed network thus formed can be represented by an undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is a set of nodes (or vertices) and $\mathcal{E}$ is a set of edges (or branches).

1) Spanning Tree: A spanning tree is defined as a subset of the undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ that has a minimal number of edges linking all vertices (or nodes). In a spanning tree, the number of edges is one less than the number of vertices. There are no cycles in a spanning tree, and all of the vertices are connected [7]. A linked graph can have many spanning trees, each of which has the same number of edges and vertices. Each of the undirected graph $\mathcal{G}$'s edges has a specific value (or weights). The edge weights vary depending on the problem. The sum of all edge weights of a spanning tree is minimized when establishing the minimum spanning tree. Fig 1(a) shows a spanning tree of a hypothetical 12-node system. The spanning tree shown in the figure consists of all system nodes (i.e., 12) and 11 closed branches (edges).

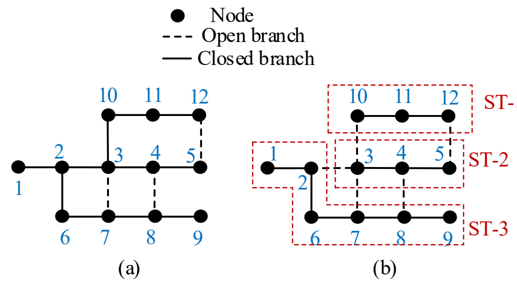

Fig. 1. (a) A spanning tree; and (b) a spanning forest of a hypothetical 12-node system

2) Spanning Forest: In graph theory, a forest is a disconnected union of trees. A spanning forest is a forest that covers all vertices of the undirected graph $\mathcal{G}$ and consists of a set of disconnected spanning trees [7]. When all spanning trees are connected, each vertex of the undirected graph $\mathcal{G}$ is included in one of the spanning trees [8]. On the other hand, when a disconnected graph has many connected components, a spanning forest is formed and it contains a spanning tree of each component [9]. Fig. 1(b) shows the spanning forest formed as a result of disconnection of two additional branches (2–3 and 3–10) in the spanning tree presented in Fig. 1(a). The spanning forest shown in Fig. 1(b) consists of three spanning trees (ST-1, ST-2, and ST-3).

In this paper, Kruskal's algorithm [10] is used to search for the optimal spanning forest. Kruskal's spanning forest search algorithm (KSFSA) starts by constructing a forest F with each graph vertex acting as a single tree based on the given undirected graph. Since KSFSA is a greedy algorithm, it goes on connecting the next least-weight edge that avoids loop or cycle to the forest F at each iteration. The resulting forest F after the last iteration is the optimal spanning forest. Fig. 2 shows the flowchart of KSFSA.

B. Graph Theoretic Modeling of Road Network

The meshed configuration of the road network is modeled as an undirected graph $\mathcal{G}_r = (\mathcal{N}_r, \mathcal{E}_r)$, where $\mathcal{N}_r$ is a set of nodes and $\mathcal{E}_r$ is a set of road edges. The weight of each road edge is determined by its length.

1) Dijkstra's Shortest Path Algorithm (DSPA): Since multiple routes from the initial location of MERs to the final location may be possible, determining the best route can significantly minimize the MER deployment cost function. In this work, DSPA is used to find the shortest (optimal) path between two different nodes of a road network graph. DSPA uses the least edge weight to calculate the shortest path from the initial location to the destination. DSPA can only be applied in case of the graph with non-negative edge weights [11]. DSPA is appropriate for this study since the length of each road edge (which is non-negative) is used to calculate edge weights.

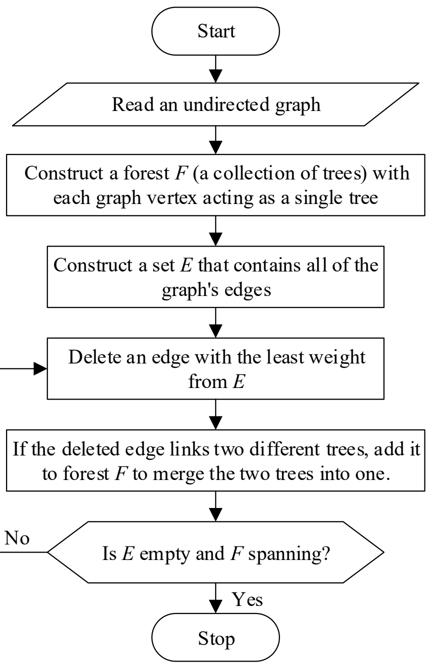

Fig. 2. Flowchart of Kruskal's spanning forest search algorithm

C. Coalitional Game Theory and Shapley Value

In game theory, coalitional game refers to the game where players can establish alliances or coalitions with one another to maximize coalitional and individual utilities. Since coalitions among players are formed to increase their individual incentives, a coalition must always result in equal or greater incentives than individual player's incentives [12]. A coalitional game is defined by assigning a value to each of the coalitions. The coalitional game is composed of the following two components:

- A finite players' set $\mathcal{N}$, known as the grand coalition.
- A characteristic function $V(S) : 2^{\mathcal{N}} \to \mathbb{R}$ that maps the set of all possible player coalitions to a set of coalitional values that satisfy $V(\phi) = 0$.

The characteristic function, representing the worth or value of each coalition, is defined in every coalitional game. The characteristic function of a coalition is the aggregated worth of all coalition members.

1) The Shapley Value: The Shapley value is a solution paradigm of coalitional game theory. The Shapley value is an approach to allocate the overall earnings to individual players when all participants participate in the game. The Shapley value of a coalitional game is expressed as follows [13].

$$\psi_j(V) = \sum_{S \in 2^{\mathcal{N}}, j \in S} \frac{(|S|-1)!(n-|S|)!}{n!}[V(S) - V(S\backslash\{j\})] \quad (1)$$

where $n = |\mathcal{N}|$ is the total number of players.

III. Pre-positioning of MERs

This section presents event modeling, scenario generation and reduction, and formulation of the coalitional game.

A. Extreme Event Modeling and Scenario Generation

In this work, the weather-related fragility curve is used to model extreme events and generate multiple line outage scenarios. A fragility curve is applied to characterize the performance and vulnerabilities of different system components confronting uncertain weather-related extreme events. The failure probabilities of each component are obtained by mapping the weather forecast and monitoring data to the fragility curve [14]. We have taken the multiple line outages caused by high wind speeds as an example of a weather-related extreme event in this study. Mathematically, the probability of line outages caused by high wind speeds can be represented as follows [15].

$$P_l(w) = \begin{cases} \overline{P_l}, & \text{if } w < w_{\text{crl}} \\ P_{l\_hw}(w), & \text{if } w_{\text{crl}} \leq w < w_{\text{cpse}} \\ 1, & \text{if } w \geq w_{\text{cpse}} \end{cases} \quad (2)$$

where P_l is the probability of line failure as a function of wind speed w; $\overline{P_l}$ is the failure probability at normal weather condition; P_{l_hw} is the probability of line failure at high wind; w_{crl} is the critical wind speed (i.e., the speed above which the distribution lines start experiencing failure); and w_{cpse} is the speed above which the distribution lines completely collapse.

B. Scenario Reduction Using k-means Method

The accuracy of an approach is always improved when a large number of line outage scenarios is used. However, solving the problem with a large number of scenarios takes a long time. The generated line outage scenarios are, therefore, reduced using the k-means method in this work to make the proposed approach computationally tractable. The k-means method is an iterative procedure that attempts to split a set of scenarios into a set of unique clusters. It attempts to minimize distance between scenarios in the same cluster while maximizing the distance between different clusters. In addition, when scenarios are assigned to a cluster, the distance between them and the cluster centers is kept to a minimum [16].

C. Selection of Candidate MER Locations

For the selection of candidate MER locations, the MER deployment cost function is used, which is calculated based on the expected load curtailment (ELC) of each location and the optimal distance of MER deployment location from the initial MER location. The ELC corresponding to the i^{th} location is determined using the amount of curtailed critical load for each reduced line outage scenario as follows.

$$ELC_i = \sum_{j=1}^{K} Pr(j) \times LC_i(j), \quad (3)$$

where K is the total number of reduced scenarios; $Pr(j)$ is the probability of the j^{th} reduced scenario; and $LC_i(j)$ is the critical load curtailment of the j^{th} reduced scenario for MER deployment location i, which is calculated as follows.

$$LC_i(j) = \sum_{m=1}^{N} \omega_m \Delta P_{mi}(j), \quad (4)$$

where $\Delta P_{mi}(j)$ is the load curtailment at node m of the j^{th} reduced scenario for MER deployment location i; ω_m is the critical load factor at node m; and N is the total number of nodes in the system.

While computing the critical load curtailment, the nodal power balance constraints and radiality constraint should always be satisfied, which are described below.

(a) Node power balance constraints: The power balance constraint at each node of the system can be expressed as follows.

$$\sum_{r\in\Omega_g(r)} P_{g,r} + \sum_{l\in\Omega_L(r)} P_{l,r} = P_{D,r} \tag{5}$$

where $\Omega_g(r)$ is the set of sources (including MER) connected to node r; $\Omega_L(r)$ is the set of lines connected to node r; $P_{g,r}$ is the power injected from source r; $P_{D,r}$ is the load at node r; and $P_{l,r}$ is the line power flow from node l to node r.

(b) Radiality constraint: A distribution system must always meet the radiality requirement. Therefore, each potential configuration should be radial (i.e., the radiality constraint should be met for each spanning tree of the network). Each spanning tree of the network is represented by a sub-graph $\mathcal{G}_s = (\mathcal{N}_s, \mathcal{E}_s)$, where $\mathcal{N}_s$ is a set of nodes (or vertices) and $\mathcal{E}_s$ is a set of edges (or branches) in the sub-graph. For the sub-graph, a node-branch incidence matrix should be constructed. If $n_s = |\mathcal{N}_s|$ denotes the number of nodes and $e_s = |\mathcal{E}_s|$ denotes the number of edges of a particular spanning tree, then the node-branch incidence matrix $A \in \mathbb{R}^{n_s\times e_s}$ is the matrix with element a_{ij} calculated based on (6). If the node-branch incidence matrix A is full ranked, then the radiality constraint is satisfied.

$$a_{ij} = \begin{cases} +1 & \text{if branch } j \text{ starts at node } i \\ -1 & \text{if branch } j \text{ ends at node } i \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

The second component of the MER deployment cost function is the optimal distance of MER deployment location from the initial MER location, which is determined using the DSPA. The MER deployment cost function of the i^{th} location is expressed as follows.

$$C_i = \beta_1 \times ELC_i + \beta_2 \times d_i, \tag{7}$$

where ELC_i is the expected load curtailment corresponding to the i^{th} location; d_i is the optimal distance of MER deployment location i from the initial MER location; and β_1 and β_2 are weighting coefficients which sum to unity.

A certain number of candidate MER locations is selected based on least MER deployment cost functions. Determination of the optimum number of candidate MER locations is beyond the scope of this work.

D. Computation of Characteristic Functions of the Coalitional Game Model

A coalitional game model is formulated considering candidate MER locations as players of the game. The list of all possible coalitions of candidate MER locations is generated. For example, if three candidate MER locations (L_1, L_2, and L_3) are selected, the set of all possible coalitions, denoted by $2^{\mathcal{N}}$, is as follows.

$2^{\mathcal{N}} = \{\phi, \{L_1\}, \{L_2\}, \{L_3\}, \{L_1, L_2\}, \{L_1, L_3\}, \{L_2, L_3\}, \{L_1, L_2, L_3\}\}$,

where ϕ denotes an empty set.

For each set of coalitions, the expected critical load recovery (ECLR) is computed by taking the difference of ELCs before and after MER placement. The ECLR serves as the characteristic function of each coalition.

E. Determination of MER Sizes at Candidate Locations

After computation of characteristic functions of all possible sets of coalitions, Shapley values of each candidate MER location are determined using (1). Based on the Shapley values, the capacity distribution factor (CDF) of the candidate MER location, i, is determined as follows.

$$CDF_i = \frac{\psi_i}{\sum_{k=1}^{n} \psi_k}, \tag{8}$$

where ψ_i is the Shapley value of the i^{th} location; and n is the number of candidate MER locations.

Now, the total size of MERs is distributed among different candidate MER locations based on CDF as follows.

$$P_{MER-i} = CDF_i \times P_{MER-tot} \tag{9}$$

where P_{MER-i} is the size of MER at the i^{th} candidate location; and $P_{MER-tot}$ is the total MER capacity.

The flowchart of the proposed approach for pre-positioning of MERs is shown in Fig. 3.

IV. Case Study and Discussion

A. System Description

To demonstrate the effectiveness of the proposed approach, the 33-node system is used for numerical simulations. The 33-node distribution test system is a radial distribution system with 33 nodes, 32 branches, and 5 tie-lines (37 branches) [17]. All branches (including tie-lines) are numbered from 1 to 37. The system's overall load is 3.71 MW. The locations and amounts of critical loads considered for the 33-node system are shown in Table I. A road network for the 33-node system is considered, which is shown in Fig. 4.

B. Implementation and Results

For the implementation of the proposed approach, multiple line outage scenarios are generated by considering a high wind speed event as an example of a weather-related extreme event. The critical wind speed of 30 m/s and the collapse speed of 55 m/s are assumed for the fragility model (2) under consideration [15]. The failure probability of 0.01 is considered at normal weather conditions. The wind fragility curve for distribution lines adopted in the work is as shown in Fig. 5. In this paper, 10,000 random outage scenarios are generated and the k-means method is used to reduce the generated scenarios into 200 reduced outage scenarios for wind speed of 38 m/s. The k-means method outputs 200 reduced line outage scenarios along with their probabilities.

TABLE I
LOCATIONS OF CRITICAL LOADS FOR THE 33-NODE SYSTEM

Nodes	4	5	6	7	8	9	10	11	18	19	20	21	22	23	26	27	28	29	30	33
Critical Loads (kW)	60	30	60	200	200	60	30	25	45	45	45	45	45	45	60	60	60	60	60	30

TABLE II
CHARACTERISTIC FUNCTIONS OF POSSIBLE COALITIONS FOR THE 33-NODE SYSTEM

Coalitions	7	8	9	21	7, 8	7, 9	7, 21	8, 9	8, 21	9, 21	7, 8, 9	7, 8, 21	7, 9, 21	8, 9, 21	7, 8, 9, 21
ECLR (kW)	108.6	110.1	114.3	110.1	218.7	222.9	218.7	215.7	211.4	215.7	313.9	309.7	313.9	296.8	366.4

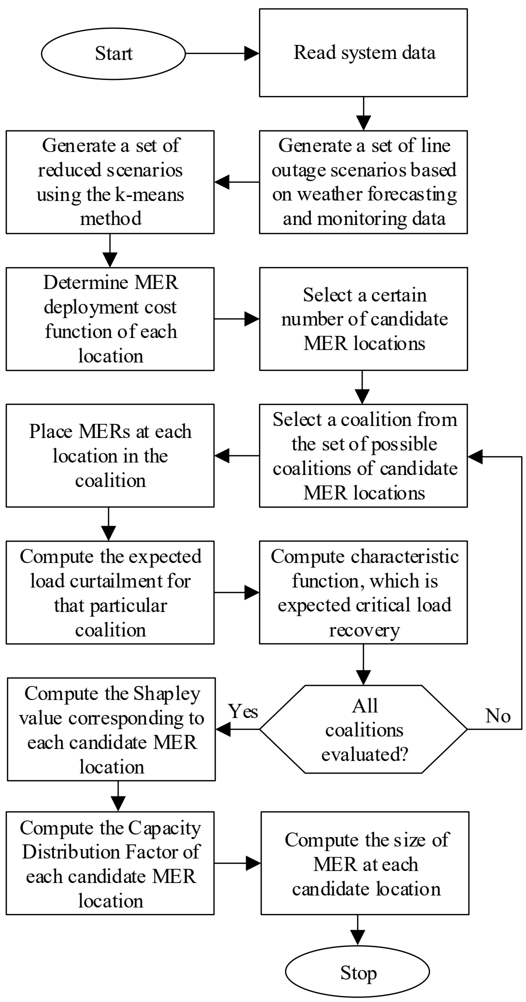

Fig. 3. Flowchart of the proposed approach for the pre-positioning of MERs

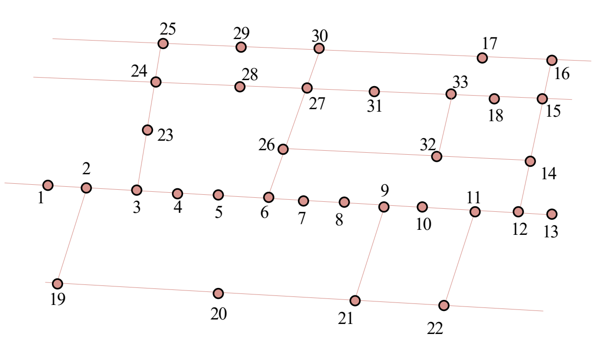

Fig. 4. Road network for 33-node distribution test system

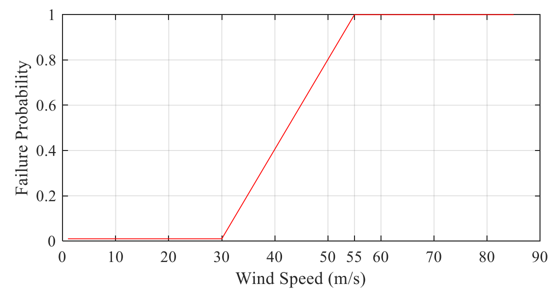

Fig. 5. Wind fragility curve for distribution lines

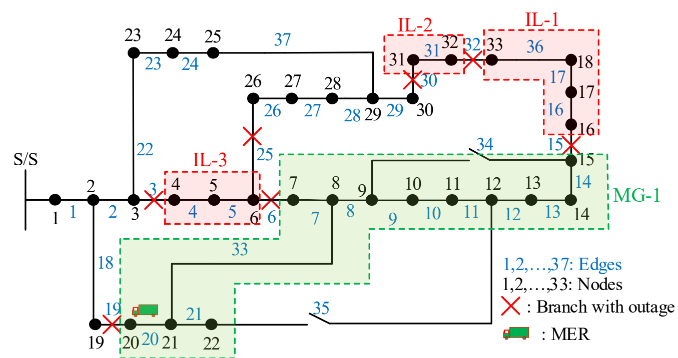

Fig. 6. 33-node distribution test system

For each reduced line outage scenario, the distribution network reconfiguration is performed using tie-switches present in the network and spanning forest is formed by deploying MER of capacity 1200 kW at a location (node). Fig. 6 shows the case for a reduced scenario where outages of lines 3, 6, 15, 19, 25, 30, and 32 occur. In this scenario, the distribution network is reconfigured by closing tie-switches 33, 36, and 37 using KSFSA. The tie-switches 34 and 35 are not closed to maintain radial configuration. When the MER is deployed at node 20, a microgrid (MG-1) and three isolates (IL-1, IL-2, and IL-3) are formed. These isolates are devoid of power supply. The total critical loads of IL-1, IL-2, and IL-3 are, respectively, 75 kW, 0 kW, and 150 kW. Therefore, when the MER is deployed at node 20, the total critical load curtailment for this reduced scenario is 225 kW. This process is repeated for all locations (nodes) and all reduced scenarios. The expected load curtailment (ELC) corresponding to each location then is determined based on load curtailment and probability of each reduced scenario.

MERs are assumed to be initially located at the substation node. The optimal path and distance of each node from the substation node is computed using DSPA. If only ELC is considered as the criterion for selecting candidate MER location, nodes 8, 9, 15, and 21 are obtained as candidate MER locations. Similarly, if only distance from the substation is considered as the criterion for selecting candidate MER location, nodes 2, 3, 4, and 19 are obtained as candidate MER locations since these nodes are closest to the substation. However, this work uses MER deployment cost based on both ELC and distance from the substation, which is computed using (7). The values of weighting coefficients β_1 and β_2 are taken as 0.75 and 0.25, respectively. The four locations (nodes 7, 8, 9, and 21) with least MER deployment costs are selected as candidate MER locations.

To compute the size of each MER, the four candidate MER locations are treated as players of the coalitional game. The characteristic function (here, the expected critical load recovery) is calculated for each set of possible coalitions. The expected critical load recovery (ECLR) is calculated by taking the difference of ELCs before and after MER placement. Before MER placement, the ELC of the system is 489.55 kW. The ECLR (or characteristic functions) for all sets of possible coalitions are shown in Table II. We can see from the table that the ECLR for the coalition of locations 7 and 8 is equal to the sum of ECLRs of individual locations. However, the ECLR for the coalition of locations 8 and 9 is less than the sum of ECLRs of individual locations. This indicates that some coalitions are worthier than others and this property is utilized to compute Shapley values of individual candidate MER locations. The Shapley values and sizes of MER of each candidate location are shown in Table III.

TABLE III
SHAPLEY VALUES AND SIZES OF MERS AT CANDIDATE LOCATIONS

Locations (nodes)	Shapley values	MER sizes (kW)
7	96.25	320
8	88.65	290
9	92.87	300
21	88.65	290

The proposed solution algorithm (shown in Fig. 3) takes approximately 30 seconds to execute on a PC with a 64-bit Intel i5 core processor running at 3.15 GHz, 8 GB RAM, and Windows OS.

V. CONCLUSION

This paper has proposed an approach based on graph theory and coalitional game theory for pre-positioning of movable energy resources (MERs) to improve resilience of the power supply. Multiple line outage scenarios were generated and the k-means method was used to reduce the generated scenarios. The proposed approach was implemented on a 33-node distribution test system. The results showed that the proposed approach can effectively determine the pre-positioning locations and sizes of MERs with the least expected critical load curtailments. Because to the use of the Shapley value, which takes into account the average marginal contribution of each location, the proposed approach's main benefit is a fair allocation of the overall MER size among different candidate locations. The use of more accurate and better algorithms (such as the fuzzy k-means algorithm) for reducing the generated multiple outage scenarios and the implementation with other types of weather-related extreme events (e.g., flooding) are some areas of possible extensions of the work proposed in this paper.

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