

Data Driven Learning of Constrained Measurement Matrices for Signal Reconstruction

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Abstract—Data-driven sparse signal reconstruction envisions a new dimension of sensor designing protocol via jointly learn the measurement matrices (MM) and signal reconstruction through deep neural networks (DNN) to outperform classical sparse reconstruction approaches. However, in many real-world applications, these data-driven approaches cannot be applied directly due to learning the MMs with arbitrary values thus ignoring the physical constraints of the measurements. To this end, we propose novel loss functions to incorporate the effects of constraining the measurements with the values like ± 1 or binary values to train the DNN for learning the constrained MMs. We also incorporate a mutual coherence term in the loss function to find a more optimized MMs in the constrained settings of the measurements. Results obtained over the CIFAR-10 image database show that the proposed deep learning architectures utilizing the constraint terms in the loss function provide improve reconstruction over the state-of-the-art compared techniques.

Index Terms—Constrained sensing, compressive sensing, learning measurement matrix, mutual coherence, reconstruction.

I. INTRODUCTION

To get optimized set of measurements in the sensing pipeline to reconstruct and use a given class of signals is very essential for many signal processing and data science applications. Compressed sensing (CS) [1]–[3] revolutionizes data acquisition aka sensing protocol to remove the computational complexity of classical sensing approaches. CS uses information from a dictionary $\Psi \in R^{N \times N}$ to deal with the sparsity of the given signal $\mathbf{x} = \Psi \mathbf{s}$, where $\|\mathbf{s}\|_0 = K$ and $K \ll N$. Typically, it acquires a fixed a fixed set of random linear measurements of the signal as $\mathbf{y} = \Phi \mathbf{x}$, with $\mathbf{y} \in R^{M \times 1}$ and $M \ll N$, while entries of the MM Φ can be randomly selected from Gaussian or Bernoulli distribution. A fixed random MM ensures recovery of sparse signals even with a lower number of measurements on a known basis Ψ [4]–[7], but in many real applications, the sparsity is not fully known and reconstruction is also computationally expensive.

Recently, DNN inspired approaches [8]–[10] have shown better performance to reconstruct images from a given measurements with a variety of signal/image reconstruction modules [11]–[17]. These approaches eliminate the need for a known sparsity basis to reconstruct a signal via learning a mapping from low dimensional measurement space to the original signal/image space with the aid a large existing dataset. They take the advantage of end-to-end training nature of DNNs to model the measurement process by a densely

connected unit to replicate the compressed measurements from a fixed random Gaussian MM in low dimension to generate more optimal sets of measurements to jointly learn the MMs and the signal reconstruction for different applications. A wide variety of works can be found in [13], [17]–[21] to generate and learn MMs for joint learning of MMs and reconstruction in a data-driven scheme. Considering each row of Φ as a filter, the works in [13], [18] use convolutional layers to mimic the CS measurement acquisition process. The approaches in [17], [19]–[21] introduce a densely connected layer to produce the measurements to mimic the data acquisition process. The sensing mechanism can be linear or non-linear depending on the activation functions used to excite the neurons of convolution or densely connected layer. To model the linear CS measurement process $\mathbf{y} = \Phi \mathbf{x}$, only linear activations are used in [17]. These DNN based approaches have a common point of view to learn an optimal MM from data regardless the variations in the structures. They use randomly initialized network parameters and update them via training to learn a MM that has the same form a random Gaussian MM. Once MM is learned, they use it for different analysis in terms of testing cases essentially giving a freedom to arbitrarily select the values of MM.

However, the arbitrary range of values in the learned MM can be a major source of hindrance if we try to configure those in a real sensing system where the measurements in the MM limited to a predefined sets of values due to hardware configurations or the application itself. Therefore, having a constraint MM in these applications bears the utmost importance. For instance, single-pixel camera [22] acquires measurements in a form of projections of image to useful binary structures. Hence, this well-known CS application deals only with binary values. In addition, most state-of-the-art methods exploit a subsampling scheme to acquire samples in time or space. Existing data-driven MM learning approaches [13], [17]–[21] don't provide the tools to directly learn such constrained MMs to deal with the requirement with such applications.

In this paper, a novel loss function is introduced to learn a constrained MM jointly with the signal reconstruction task, where the entries in MM only takes the form of finite set of values such as ± 1 or binary. Our simulation results show that using the learned constrained MM, we can achieve good image reconstruction i.e. higher peak signal to noise ratio (PSNR) values compared to random MMs. We also show the

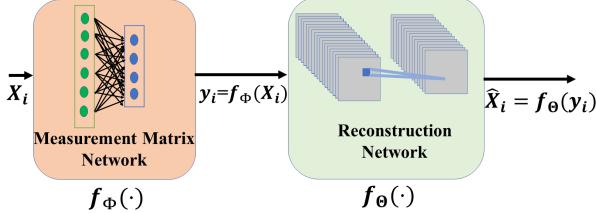


Fig. 1: General DNN structure parameterized by Φ and Θ for joint MM learning and signal reconstruction

importance of the initialization of the MM layer prior training the DNN to find that an orthogonal random initialization provides significantly enhanced performance. Classical CS approaches [23] tells us the mutual coherence of the sensing system plays an integral part to determine the required number of compressive measurements and we need a lower mutual coherence to reconstruct the image with good quality. To this end, we propose to include an additional loss term to represent the mutual coherency of the sensing system. Our results show that models learned through the proposed loss functions provide enhanced results compared to the state-of-the-art approaches.

The rest of the paper is organized as follows: The proposed method is detailed in Section II. The dataset, experimental settings, and training and testing results of the proposed method with the compared techniques have been presented in Section III. Finally, conclusions are drawn in Section IV.

II. PROPOSED METHOD

The general DNN-based framework to jointly learn the MM and signal reconstruction is illustrated in Fig. 1. To this end, several existing state-of-the-art approaches [17], [18], [21], [24]–[26] use different DNN modules for both the MM and reconstruction networks. The working principle of these modules are different, but the general structure is same for sensing system to derive optimal sets of measurement for image reconstruction task. First, we model the measurement acquisition process in the MM network which is parameterized by Φ to derive the encoded compressive measurements of the original signal \mathbf{X}_i as $\mathbf{y}_i = f_\Phi(\mathbf{X}_i)$. We employ convolutional or densely connected layers to initialize and parameterize the MM network. In the second stage, we use the obtained measurements \mathbf{y}_i to produce an estimate of the original signal as $\widehat{\mathbf{X}}_i = f_\Theta(\mathbf{y}_i)$, which is fed as the input to reconstruction module of framework. To optimize the parameters of this DNN-based general framework, existing approaches use the Euclidean loss between the original and reconstructed signals over a training dataset of T total number of training samples as

$$L_R(\Phi, \Theta) = \frac{1}{T} \sum_{i=1}^T \|f_\Theta(f_\Phi(\mathbf{X}_i)) - \mathbf{X}_i\|_F. \quad (1)$$

Prior training, all the model parameters are randomly initialized from a Gaussian distribution. The minimization of the loss function in (1) over a training dataset produce the learned estimate of unconstrained MM parameters Φ and the

reconstruction network parameters Θ . While in validation and testing case, we can detach the networks components and use the learned estimate of MM to sense the signals and use them in the reconstruction network. In this approach, the learned estimate of MM Φ can take any arbitrary value to describe the MM as unconstrained.

A. Learning a constrained measurement matrix

We propose to jointly optimize the parameters of MM and reconstruction networks in the DNN via the following loss function:

$$L_C(\Phi, \Theta) = L_R(\Phi, \Theta) + \alpha \sum_{k,n} (1 + \Phi_{k,n})^2 (1 - \Phi_{k,n})^2 \quad (2)$$

Using this loss function would give us specific values of ± 1 in the learned estimate of MM thus we are constraining the MM from arbitrary values to a specific set of values. We see that the additional loss term in (2) can force the elements of the MM $\Phi_{k,n}$ to be either $+1$ or -1 to minimize the loss term without incurring additional loss. α term in the loss function plays the role of a regularization parameter to control the amount of added constraint. In addition to constraining the MM in ± 1 format, we can also constraint the MM in the uni-polar format i.e. making the entry either 0 or 1 by the following manner:

$$L_C(\Phi, \Theta) = L_R(\Phi, \Theta) + \alpha \sum_{k,n} \Phi_{k,n}^2 (1 - \Phi_{k,n})^2 \quad (3)$$

In this work, we show our simulation results in terms of bipolar (± 1) MM form utilizing (2).

B. The mutual coherence constraint

The mutual coherence of a system with a given basis Ψ and MM Φ , $\mathbf{A} = \Phi\Psi$ plays an important role to provide sufficient condition [27] to recover a K -sparse signal as $\mu(\mathbf{A}) \leq 1/(2K - 1)$, where the mutual coherence $\mu(\mathbf{A})$ is defined as

$$\mu(\mathbf{A}) = \max_{k \neq l} \frac{|\mathbf{a}_k^T \mathbf{a}_l|}{\|\mathbf{a}_k\|_2 \|\mathbf{a}_l\|_2}. \quad (4)$$

The k -th column of \mathbf{A} is defined as \mathbf{a}_k . It tells us we can recover a better image reconstruction of the given signal for the same measurement number with a larger K by minimizing the mutual coherence $\mu(\mathbf{A})$. Classical CS approaches [4]–[6] focus to minimize the average mutual coherence under the assumption that the averaged metric will reflect an average signal recovery performance. Current state-of-the-art DNN based approaches have not employed the mutual coherence aspect in their optimization scheme to reflect a much better image reconstruction. In this paper, we also propose the inclusion of an additional term that incorporates the mutual coherence term in the loss function. This would impose the mutual coherence criterion in the joint learning of the MM and the reconstruction.

For successful image reconstruction regarding the mutual coherence, we need to have a prior knowledge of an sparsity basis Ψ . If we know the information about the basis that is available to the problem at our hand, we can include $\mu(\mathbf{A})$

in the loss function, otherwise we can only use (2) to learn the DNNs for constraining the MM with the reconstruction networks. Hence, we can modify the total loss function as follows:

$$L_T(\Phi, \Theta) = L_C(\Phi, \Theta) + \beta\mu(\Phi) \quad (5)$$

where β works as another regularization parameter controlling mutual coherence cost. The analysis of utilized proposed loss functions to learn constrained MMs have been detailed in Section III.

III. SIMULATION RESULTS

In this section, we provide simulation results to investigate the benefits of using the proposed loss functions in the different DNN settings to obtain and compare the performance of the constraint MMs under different scenarios.

A. Dataset, Evaluation metrics, and Learning Parameters

For the simulations, the publicly available image dataset, CIFAR-10 [28] is used. This dataset has 60,000 images from 10 classes with size of $32 \times 32 \times 3$ for each image. For training, validation, and testing purposes, we opt to use the grayscale versions of the images. Our testing set contains a total of 10,000 images with 1000 randomly selected images per class. The training and validation sets contain the remaining 50000 images. In our simulation set up, we select 80% of the 50000 images randomly for training, while the rest of them are used for validation. The image reconstruction performance is evaluated with the peak to signal noise (PSNR) ratio metric [29] to measure the image reconstruction quality based on the euclidean losses in between the ground truth and predicted images. In this work, we employ two state-of-the-art approaches ConvMMNet [30], and CSNET [31] that utilize DNN models for joint MM learning with sparse signal reconstruction. The MM is modelled with a linearly activated densely connected layer in ConvMMNet. The CSNET introduces the use of convolutional layer to model measurement acquisition process. The computation of the backpropagation of the DNNs has been performed by using mini-batch gradient descent routine. The updating of the parameters has been done via ADAM optimization for a batch size of 32 and epoch size of 500 with a varying learning rate from 0.1 to 0.001. All simulations are carried out using the open source deep learning framework, Tensorflow [32] in a deep learning machine with 3 NVIDIA Titan RTX GPUs.

B. Regularization parameters and initialization

Since, the loss function in (2) has a regularization parameter α to control the amount of added constrained in the resultant MM; therefore, we opt to simulate different ConvMMNet models over the training dataset for a set of α values from 0 to 0.8 where $\alpha = 0$ denotes an unconstrained MM is learned. Increasing values of α put more weights on constraining the MMs to either $+1$ or -1 rather than estimating a reconstructed image closer to the ground truth one. We use two performance metrics: average PSNR and rounding error (RE) of the MM to evaluate the performance of each tested value of α over the

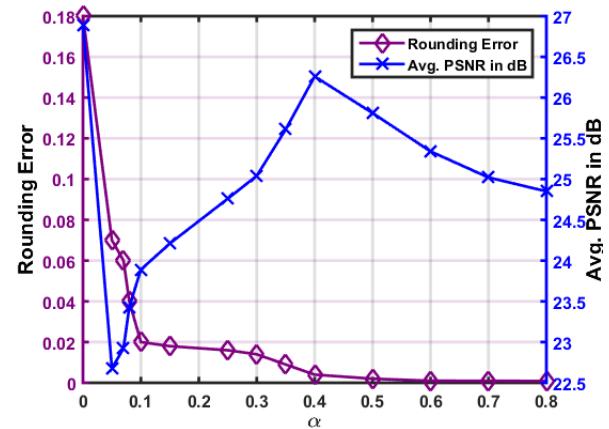


Fig. 2: Comparison of rounding error and average PSNR with respect to α at $M = 128$.

validation dataset. Since the learned MM Φ after training does not contain values of exact ± 1 ; hence, we round the values to the closest ± 1 value before calculating the round error as $RE = \|\Phi - \hat{\Phi}\|_F$, where $\hat{\Phi}$ is the ± 1 rounded version of Φ . The simulation result for different values of α for a fixed measurement number of $M = 128$ in terms of RE and average PSNR values obtained over validation dataset is illustrated in Fig. 2. Fig. 2 shows that the highest PSNR achieved with the highest RE at $\alpha = 0$, which is expected due to the unconstraint nature of MM happening at that point. As we increase the value of α , RE decreases and becomes almost zero at $\alpha > 0.6$ specifying the entries of learned MMs are only ± 1 . At smaller and larger values of α , we see that PSNR values are low and becomes maximum at $\alpha = 0.4$. We also find that same value of $\alpha = 0.4$ at almost all measurement ratios to find the maximum average PSNR values. Hence, we opt to use $\alpha = 0.4$ as the regularization parameter to be used over the test set.

We also observe the initialization aspect of the MM network parameters prior training. We analyzed the output PSNR performance of DNN models with different initialization settings. Usually, classical DNNs parameters are randomly drawn from a Gaussian distribution whereas in this approach, we opt to initialize the MM network with an orthogonalized Gaussian random distribution. For parameter orthogonalization, initially, a random Gaussian MM is generated where its rows are orthogonalized to be used as the initial parameters for the sensing part in the reconstruction network. We show the comparison of average PSNR values for random and unconstrained learned MMs with and without orthogonalization in fig. 3. We find that the initialization with orthogonalization provides approximately 2 – 3 dB more average PSNR values for all tested number of measurements. Hence, we use orthogonalized initialization in the MM part for simulations, if otherwise stated.

C. Constrained Measurement Matrix Learning

We mainly focus to the performance comparison of various MMs in terms of PSNR metric. Firstly, using the initialization

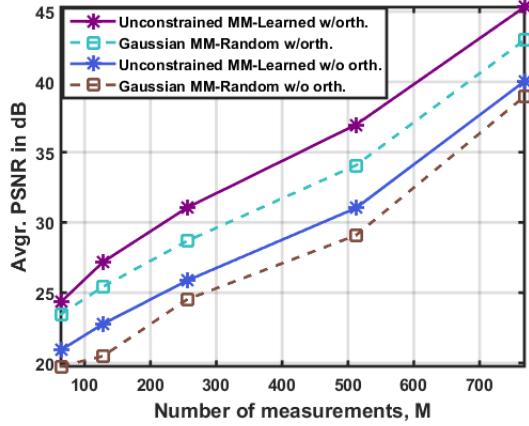


Fig. 3: Effect of initialization of MM parameters Φ with and without row orthogonalization.

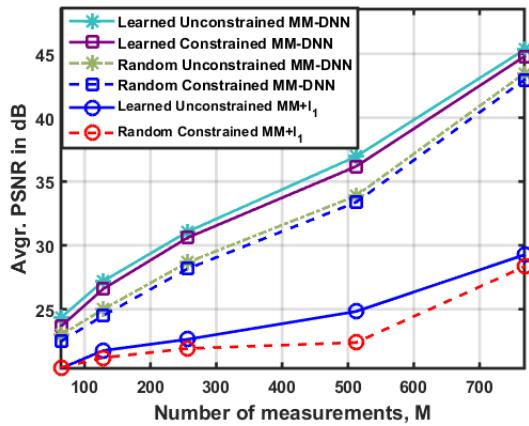


Fig. 4: Comparison of PSNR values at different measurements

scheme and regularized parameters as described in the previous subsection, we train the DNN models-ConvMMNet and CSNet for joint learning of MM with image reconstruction by adopting (2). Then we compare the performance of the tested DNNs with learned constrained and unconstrained MMs utilizing the loss function in (1), the uniformly random ± 1 MMs used with the reconstruction networks of the same techniques as well as the baseline CS solution, and ℓ_1 minimization based reconstruction. We show the comparison of compared cases in terms of average PSNR values as a function of measurements in fig.4. We see that the DNN based reconstruction networks outperform ℓ_1 minimization when they all use random or learned MMs in a significant margin. Since performance of ConvMMNet and CSNet is very similar in terms of image reconstruction; therefore, we opt to use the result of ConvMMNet in Fig. 4 and remaining simulations results. We see for all measurement cases, an increment of around 0.5-2dB in PSNR for learned constrained MMs compared to the random MM for all number of measurements using both DNN based techniques. It can also be seen that the learned constrained MM provides approximately 1 dB in average improved performance when it is independently used

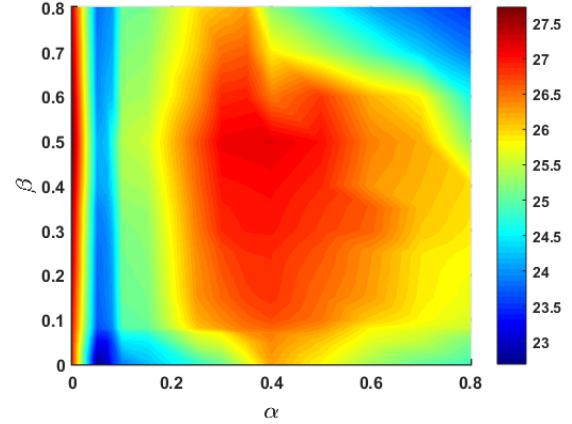


Fig. 5: Average PSNR values for different values of α and β at $M = 128$

with the ℓ_1 minimization. In case of learning an unconstrained MM i.e. without applying the constraint on the MM, the DNNs produce a slightly better image reconstruction than learning a constrained MM. Same performance difference is noticed if we employ unconstrained and ± 1 random MMs in the reconstruction networks. We round the values in the learned MM layer to the nearest ± 1 to get the learned constrained MMs and utilize these rounded MM in the testing stage. We also find that the utilized regularization parameter, $\alpha = 0.4$ in this case, the effect of rounding to the resultant PSNR is observed to be insignificant, less than 0.5 dB for all tested scenarios.

D. Learning a MM with mutual coherence constraint

In this work, we use (5) to allow DNN structures to include a mutual coherence term to guide learning a constrained MM along with a reconstruction network. We use the ConvMMNet structure for training with a various values of α and β regularization parameters, both in the range of 0 – 0.8, where β weights the mutual coherence term in (5). Fig. 5 shows the obtained PSNR values for the tested (α, β) combinations and it can be seen that the maximum PSNR is obtained for a combination of constraint settings at $\alpha = 0.4$ and $\beta = 0.5$, where moving from this optimal setting in both directions decreases the PSNR output. We also show the average PSNR and rounding error metrics in Fig. 6 as a function of regularization parameter α with loss term of mutual coherence β has values of 0 and 0.5. We see rounding error behaves similarly for both cases where the mutual coherence term increases the average PSNR value around 1 dB for the constrained MM case ($\alpha = 0.4$). We also see mutual coherence term also helps learning an unconstrained MM learning ($\alpha = 0$) with an increment of PSNR values of around 0.8 dB. We also use discrete cosine transform (DCT) as sparsity basis to calculate the mutual coherence in (4) with an increment of PSNR results no more than 0.2dB. While the mutual coherence of a learned constrained MM at $\alpha = 0.4$ and $\beta = 0.5$ is 0.41, the random ± 1 MM has a coherence of 0.73.

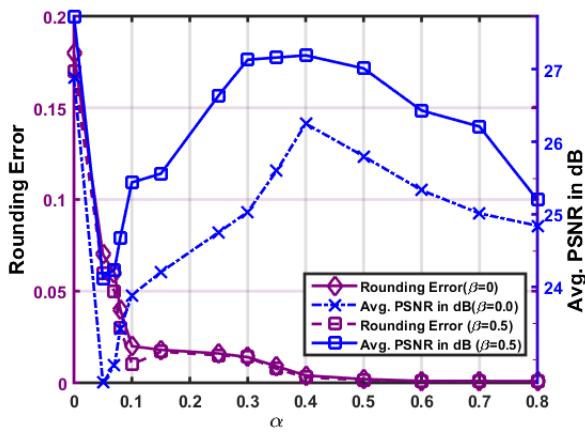


Fig. 6: Comparison of rounding errors and average PSNR with ($\beta = 0.5$) or without ($\beta = 0$) introducing the mutual coherence term in the loss function with respect to α at $M = 128$

IV. CONCLUSION

In this work, new loss functions are utilized to learn constrained MMs jointly with the reconstruction networks, where the elements of the MM belong to a finite set of values such as ± 1 or binary. Simulation results in case of ± 1 values show that learned constrained MMs leads to improved signal reconstruction compared to randomly generated constrained MMs, when they are utilized in either DNN based or classical sparse reconstruction approaches. In addition, inclusion of a mutual coherence term in the loss function is shown to learn enhanced MMs improving the reconstruction performance.

REFERENCES

- [1] E. Candès, J. Romberg, and T. Tao, "Stable signal recovery from incomplete and inaccurate measurements," *Comm. on Pure and Applied Math.*, vol. 59, no. 8, pp. 1207–1223, 2006.
- [2] E. J. Candès and M. B. Wakin, "An introduction to compressive sampling," *IEEE Signal Processing Magazine*, vol. 25, no. 2, pp. 21–30, 2008.
- [3] D. L. Donoho, A. Maleki, and A. Montanari, "Message-passing algorithms for compressed sensing," *Proceedings of the National Academy of Sciences*, vol. 106, no. 45, pp. 18914–18919, 2009.
- [4] M. Elad, "Optimized projections for compressed sensing," *IEEE Transactions on Signal Processing*, vol. 55, pp. 5695–5702, Dec 2007.
- [5] J. M. Duarte-Carvajalino and G. Sapiro, "Learning to sense sparse signals: Simultaneous sensing matrix and sparsifying dictionary optimization," *IEEE Transactions on Image Processing*, vol. 18, pp. 1395–1408, July 2009.
- [6] L. Zelnik-Manor, K. Rosenblum, and Y. C. Eldar, "Sensing matrix optimization for block-sparse decoding," *IEEE Transactions on Signal Processing*, vol. 59, pp. 4300–4312, Sept 2011.
- [7] K. Ardah, M. Pesavento, and M. Haardt, "A novel sensing matrix design for compressed sensing via mutual coherence minimization," in *2019 IEEE 8th International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP)*, pp. 66–70, 2019.
- [8] A. Lucas, M. Iliadis, R. Molina, and A. K. Katsaggelos, "Using deep neural networks for inverse problems in imaging: beyond analytical methods," *IEEE Signal Processing Magazine*, vol. 35, no. 1, pp. 20–36, 2018.
- [9] G. Ongie, A. Jalal, C. A. Metzler, R. G. Baraniuk, A. G. Dimakis, and R. Willett, "Deep learning techniques for inverse problems in imaging," *IEEE Journal on Selected Areas in Information Theory*, vol. 1, no. 1, pp. 39–56, 2020.
- [10] Y. Bai, W. Chen, J. Chen, and W. Guo, "Deep learning methods for solving linear inverse problems: Research directions and paradigms," *Signal Processing*, vol. 177, p. 107729, 2020.
- [11] A. Mousavi, A. B. Patel, and R. G. Baraniuk, "A deep learning approach to structured signal recovery," in *Communication, Control, and Computing (Allerton), 2015 53rd Annual Allerton Conference on*, pp. 1336–1343, IEEE, 2015.
- [12] S. Lohit, K. Kulkarni, R. Kerviche, P. Turaga, and A. Ashok, "Convolutional neural networks for noniterative reconstruction of compressively sensed images," *IEEE Transactions on Computational Imaging*, vol. 4, pp. 326–340, Sep. 2018.
- [13] A. Mousavi, G. Dasarathy, and R. G. Baraniuk, "A data-driven and distributed approach to sparse signal representation and recovery," in *International Conference on Learning Representations*, 2019.
- [14] J. Zhang and B. Ghanem, "Ista-net: Interpretable optimization-inspired deep network for image compressive sensing," in *2018 IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pp. 1828–1837, 2018.
- [15] M. Borgerding, P. Schniter, and S. Rangan, "Amp-inspired deep networks for sparse linear inverse problems," *IEEE Transactions on Signal Processing*, vol. 65, no. 16, pp. 4293–4308, 2017.
- [16] B. Zhu, J. Z. Liu, S. F. Cauley, B. R. Rosen, and M. S. Rosen, "Image reconstruction by domain-transform manifold learning," *Nature*, vol. 555, pp. 487–492, Mar. 2018.
- [17] R. MdRafi and A. C. Gurbuz, "Joint learning of measurement matrix and signal reconstruction via deep learning," *IEEE Transactions on Computational Imaging*, vol. 6, pp. 818–829, 2020.
- [18] W. Shi, F. Jiang, S. Zhang, and D. Zhao, "Deep networks for compressed image sensing," *CoRR*, vol. abs/1707.07119, 2017.
- [19] S. Li, W. Zhang, Y. Cui, H. V. Cheng, and W. Yu, "Joint design of measurement matrix and sparse support recovery method via deep auto-encoder," *IEEE Signal Processing Letters*, vol. 26, no. 12, pp. 1778–1782, 2019.
- [20] S. Wu, A. G. Dimakis, S. Sanghavi, F. X. Yu, D. Holtmann-Rice, D. Storcheus, A. Rostamizadeh, and S. Kumar, "Learning a compressed sensing measurement matrix via gradient unrolling," 2019.
- [21] S. Lohit, K. Kulkarni, R. Kerviche, P. Turaga, and A. Ashok, "Convolutional neural networks for noniterative reconstruction of compressively sensed images," *IEEE Transactions on Computational Imaging*, vol. 4, pp. 326–340, Sep. 2018.
- [22] M. F. Duarte, M. A. Davenport, D. Takhar, J. N. Laska, T. Sun, K. F. Kelly, and R. G. Baraniuk, "Single-pixel imaging via compressive sampling," *IEEE Signal Processing Magazine*, vol. 25, no. 2, pp. 83–91, 2008.
- [23] J. Tropp and A. Gilbert, "Signal recovery from random measurements via orthogonal matching pursuit," *IEEE Trans. Information Theory*, vol. 53, no. 12, pp. 4655–4666, Dec. 2007.
- [24] R. MdRafi and A. C. Gurbuz, "Learning to sense and reconstruct a class of signals," in *2019 IEEE Radar Conference (RadarConf)*, pp. 1–5, April 2019.
- [25] R. MdRafi and A. C. Gurbuz, "Data driven measurement matrix learning for sparse reconstruction," in *2019 IEEE Data Science Workshop (DSW)*, pp. 253–257, June 2019.
- [26] J. Zhang and B. Ghanem, "Ista-net: Iterative shrinkage-thresholding algorithm inspired deep network for image compressive sensing," *CoRR*, vol. abs/1706.07929, 2017.
- [27] J. Tropp, "Greed is good: Algorithmic results for sparse approximation," *IEEE Trans. Information Theory*, vol. 50, no. 10, pp. 2231–2242, Oct. 2004.
- [28] A. Krizhevsky, I. Sutskever, and G. E. Hinton, "Imagenet classification with deep convolutional neural networks," in *Advances in neural information processing systems*, pp. 1097–1105, 2012.
- [29] R. C. Gonzalez and R. E. Woods, *Digital image processing*. N.J.: Prentice Hall, 2008.
- [30] R. MdRafi and A. C. Gurbuz, "Joint learning of measurement matrix and signal reconstruction via deep learning," *IEEE Transactions on Computational Imaging*, 2020.
- [31] W. Shi, F. Jiang, S. Liu, and D. Zhao, "Image compressed sensing using convolutional neural network," *IEEE Transactions on Image Processing*, vol. 29, pp. 375–388, 2020.
- [32] M. Abadi, A. Agarwal, P. Barham, E. Brevdo, Z. Chen, C. Citro, G. S. Corrado, A. Davis, J. Dean, M. Devin, S. Ghemawat, I. Goodfellow, A. Harp, G. Irving, M. Isard, Y. Jia, R. Jozefowicz, L. Kaiser, M. Kudlur, J. Levenberg, D. Mané, R. Monga, S. Moore, D. Murray, C. Olah, M. Schuster, J. Shlens, B. Steiner, I. Sutskever, K. Talwar, P. Tucker, V. Vanhoucke, V. Vasudevan, F. Viégas, O. Vinyals, P. Warden, M. Wattenberg, M. Wicke, Y. Yu, and X. Zheng, "TensorFlow: Large-scale machine learning on heterogeneous systems," 2015. Software available from tensorflow.org.