Structural Damage Identification using Inverse Analysis through Optimization with Sparsity

Yang Zhang[a], K. Zhou[b], J. Tang[a]
a: Department of Mechanical Engineering, University of Connecticut, Storrs, CT 06269, USA
b: Department of Mechanical Engineering-Engineering Mechanics, Michigan Technological University, Houghton, MI 4993, USA

ABSTRACT

Structural damage identification using piezoelectric impedance/admittance measurements of a piezoelectric transducer can be converted into an optimization problem that minimizes the difference between experimental measurements and prediction in the parametric space where damage locations and severities are treated as unknown variables. However, the number of unknowns is large. Meanwhile, in practical situations the location of damage occurrence is usually limited. In this research, we propose a multi-objective particle swarm optimization algorithm featuring a sparse population generation enhancement to tackle the challenge. The main idea is to design a masking procedure, so the damage location identified is sparse that fits the nature of damage identification. This approach is implemented to experimental testing for demonstration and validation.

Keywords: damage identification, sparsity, piezoelectric transducer, multi-objective optimization

1. INTRODUCTION

Structural health monitoring (SHM) and damage identification have received much attention due to their significance and broad applications in civil, mechanical, and aerospace engineering communities. The electromechanical impedance (EMI)-based method utilizing piezoelectric transducer is promising due to its high sensitivity to small-size structural damage. In this approach, the host structure is integrated with a piezoelectric transducer which has two-way electromechanical coupling effects and serves as both the actuator and sensor. By applying a harmonic voltage sweeping, the piezoelectric transducer can excite the structure and, at the same time, generate electromechanical signatures. These signatures, known as electromechanical impedance or admittance, can be used to monitor structural conditions [1-4]. In the identification process, we divide the structure into a number of segments and assume that each segment is susceptible to damage occurrence [5-7]. The inverse analysis uses the changes of impedances/admittances as inputs. As the impedance changes are more pronounced only around resonant peaks, the measurement information may be limited. As such, the inverse analysis is usually under-determined, and direct inversion based on linearized sensitivity matrix may not yield satisfying result. To address this issue, inverse analysis through optimization appears to be effective. The optimization aims to minimize the difference between the experimental measurements and model prediction in the parametric space.

There indeed exist a variety of optimization algorithms to facilitate damage identification, such as particle swarm [8], Jaya algorithm [9], and deterministic algorithms [3]. There are two challenges in damage identification using optimization formulation. First, the number of unknowns is large because damage may occur at any locations of a structure. Second, damage may only affect a small area of the structure, so the damage index vector is generally a sparse vector. While several investigations have been conducted on damage identification using the EMI information through single-objective optimization [4, 7] or multi-objective optimization [3], the aforementioned two challenges are not adequately addressed. In particular, the sparsity of damage index vector is peculiar to damage identification using optimization. One may actually take advantage of this feature to develop inverse identification through optimization. In this research, we include the sparsity as one objective function in a particle swarm multi-objective optimization and propose a masking technique to ensure the generation of sparse population in the solution procedure. The rest of the paper is organized as follows: In Section 2, EMI-based damage identification technique is introduced, and the optimization model is formulated for damage identification. Section 3 elaborates the sparse generation procedure combined with damage identification problem. In Section 4, case study is reported where the proposed algorithm is applied to experimental data. Concluding remarks are given in Section 5.

2. FINITE ELEMENT MODELING OF PIEZOELECTRIC EMI AND INVERSE ANALYSIS FORMULATION

2.1 Admittance signature modelling

Here we use the piezoelectric admittance, i.e., the reciprocal of the electric impedance, as the response of interest. The equations of motion of the host structure integrated with piezoelectric transducer can be derived as [2-3]

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} + \mathbf{K}_{12}Q = \mathbf{0} \tag{1a}$$

$$R\dot{Q} + K_c Q + \mathbf{K}_{12}^T\mathbf{q} = V_{in} \tag{1b}$$

where $\mathbf{q}$ is the displacement vector, k_c is the inverse of the capacitance of the piezoelectric transducer; $\mathbf{K}_{12}$ is the electromechanical coupling vector, $\mathbf{K}$, $\mathbf{C}$, $\mathbf{M}$ are the stiffness, damping and mass matrices, respectively, and Q is the electrical change on the surface of piezoelectric patch. In this research, we define damage as percentage change of stiffness in the segment. The stiffness matrix with damage occurrence in structure, $\mathbf{K}_d$, can be then expressed as $\mathbf{K}_d = \sum_{i=1}^{n} \mathbf{K}_h^i \left(1-\alpha_i\right)$. $\mathbf{K}_h^i$ is the stiffness matrix of ith segment under the healthy state. $\alpha_i \in [0,1]$ is the damage index indicating the stiffness loss of the ith segment, which is the unknow to be identified. n is the total number of segments. Thus, the piezoelectric admittance when damage occurs can be written as

$$y_d^c\left(\omega\right) = \frac{\dot{Q}}{V_{in}} = \frac{j\omega}{j\omega R + k_c + \mathbf{K}_{12}^T\left(\mathbf{K}_d + j\omega\mathbf{C} - \omega^2\mathbf{M}\right)\mathbf{K}_{12}} \tag{2}$$

where j refers to the imaginary unit. Under the assumption of linear relationship between admittance variation, we can use Taylor series expansion to expand the admittance in terms of the damage index, in which the higher terms are ignored here since small damage is assumed. The vector of admittance change can be obtained at the set of excitation frequencies, $\boldsymbol{\omega} = \{\omega_1, \ \ \omega_2, \ \ \ldots, \ \ \omega_m\}$, in matrix form as

$$\Delta {y^c}_{m\times 1} = \begin{bmatrix} \Delta Y(\omega_1) \\ \vdots \\ \Delta Y(\omega_m) \end{bmatrix} = \mathbf{S}_{m\times n}\boldsymbol{\alpha}_{n\times 1} \tag{3}$$

where $\mathbf{S}$ is the sensitivity matrix. Similarly, we denote $y^p\left(\boldsymbol{\omega}\right) = \left[y^p\left(\omega_1\right), \ \ \ldots, \ \ y^p\left(\omega_m\right)\right]^T$ as the piezoelectric admittance measurements.

2.2 Multi-objective optimization formulation

As we divide the structure into segments to facilitate damage identification, the structural property of each segment remains to be identified because each segment is susceptible of fault occurrence, which yields many unknowns. On the other hand, structural faults generally manifest themselves around the peaks of the piezoelectric impedance/admittance curves only, which means the input measurement information is usually limited in practice. Moreover, it is mathematically difficult to select frequency points to ensure the full rank of the sensitivity matrix even if the number of frequency points is large. Therefore, the inverse identification formulation typically is under-determined. In this research we cast the inverse identification problem into an optimization framework. Certainly, we need to minimize the difference between the measurements and model prediction in the parametric space. In addition, a true damage scenario in practical situation usually affects only small number of segments. Here we introduce the sparse regularization by enforcing a sparse constraint. We then have the following multi-objective optimization problem:

$$\begin{aligned} &\text{find} \quad \boldsymbol{\alpha} \in \mathbf{E}^n \\ &\min \quad \left\|\Delta y^c - \Delta y^p\right\|_2, \ \min \quad \left\|\boldsymbol{\alpha}\right\|_0 \\ &\text{s.t.} \ \alpha_l \le \alpha_i \le \alpha_u \end{aligned} \tag{4}$$

where $\|\cdot\|_p$ denotes the l_p norm. Conventional regularization methods of $\boldsymbol{\alpha}$, such as l_2 and l_1 norms, have some limitations. The former could spread out the error more evenly among variables and return a non-sparse with many non-zero elements and the latter can introduce a mismatch between the goal of itself and that of l_0 norm. One can refer to the literature for more discussion on regularization [7-8]. Here we select l_0 norm of $\boldsymbol{\alpha}$ aiming at dealing with the sparsity of index vector.

3. SPARSE GENERATION PROCEDURE

It is worth mentioning that the damage occurrence affects a small number of segments in a finite element model of the structure. Mathematically, the unknown vector to be updated in damage identification is sparse, i.e., many zero values. Besides, small element size in the finite element model for the piezoelectric system is usually used to ensure model fidelity under high frequency excitation, thus resulting in a large number of unknowns to be identified. To solve the problems, we introduce a sparse generation algorithm inspired from literature [10-11]. The algorithm can generate sparse population for the population-based optimization algorithms.

```
Algorithm: Initialization
   Input: number of population N, number of design variables n, lower
          bound of design variable vMin, upper bound of design variable
          vMax
   Output: Population
 1 Initialize a matrix randomly for severity, S = rand_{n×n}
 2 Initialize an identity matrix for occurrence of damage B = eye_{n×n}
 3 for i = 1 : length(S) do
 4     temp.Pop = S ∘ B
 5     temp.Cost = CostFunction(temp.Pop)
 6 rank number F ← NonDominatedSorting(temp)
 7 Initialize design variable matrix as
     S = (vMax − vMin) × unifrnd(0, 1, [N, n])
 8 Initialize a binary matrix as B = zeros(N, n)
 9 for j = 1 : N do
10     Ta = randperm(n, 2)
11     a = Ta(1)
12     b = Ta(2)
13     if F_a < F_b then
14         B(j, a) ← 1
15     else
16         B(j, b) ← 1
17 Population = S ∘ B
18 Return Population;
```

Figure 1. Pseudo code for sparse population initialization algorithm.

In damage identification, if damage index has a value greater than 0, it can show severity and location simultaneously. For example, $\alpha_1 = 0.10$ means the damage occurs at 1st segment and its damage severity is 10% stiffness reduction. In fact, it also indicates the occurrence of damage. Damage occurrence is a binary concept that can be described using 0 and 1. 0 means healthy and 1 indicates damage occurrence. As mentioned before, the damage only happens at a small number of segments. Hence we develop an initialization strategy to create a sparse population for the optimization process so that not all unknows will have values. In this research, we split the damage index into two parts: (1) fault location, $\boldsymbol{\theta}_1$; (2) a fault severity level $\boldsymbol{\theta}_2$. And then by using element-wise product, we can then have $\boldsymbol{\alpha} = \boldsymbol{\theta}_1 \circ \boldsymbol{\theta}_2$. Note that θ_{1i} is a binary vector with all elements denoted as 0 or 1 to indicate if there is damage or not, and θ_{2i} is a regular vector with randomly generated values to indicate the damage severity. The pseudo code for the population generation algorithm is shown in Figure 1. The generation algorithm mainly includes two parts, lines 1-6 try to get the rank for each variable and the rest part is generating sparse population based on the rank obtained. Then we can obtain a sparse population matrix, which will be passed to the main iteration process of optimization algorithm. In this research, we use the sparse initialization algorithm combined with multi-objective particle warm optimization algorithm (MOPSO) [12] to solve the model formulated for damage identification in Section 2.

4. CASE STUDY

In this section, case study is investigated to verify the proposed optimization algorithm for fault identification. We analyze a plate-like structure, which is divided into 225 segments, as shown in Figure 2. The dimension of the plate [2] is specified as length 561 mm, width 19.05 mm and thickness 4.763 mm. The density and Young's modulus are 2700 kg/m^3 and 68.9 GPa, respectively. The piezoelectric transducer is placed at 180 mm from the left end, with length 15 mm, width 19.05 mm, and thickness 4.763 mm. The Young's moduli of piezoelectric transducer are Y_{11} = 86 GPa and Y_{33} = 73 GPa, and the density is 9500 kgm^{-3}. The piezoelectric constant and dielectric constant are $h_{31} = -1.0288\times10^9\,\mathrm{Vm}^{-1}$ and $\beta_{33} = 1.3832\times10^8\,\mathrm{mF}^{-1}$, respectively. For verification purposes, we randomly choose two segments, No.5 and No.124 as examples. For Case 1, the damage is assumed to be in segment No.5 with severity of 2.4 % stiffness reduction. For Case 2, the damage locates at segment No.124 with severity of 2.5 % stiffness reduction.

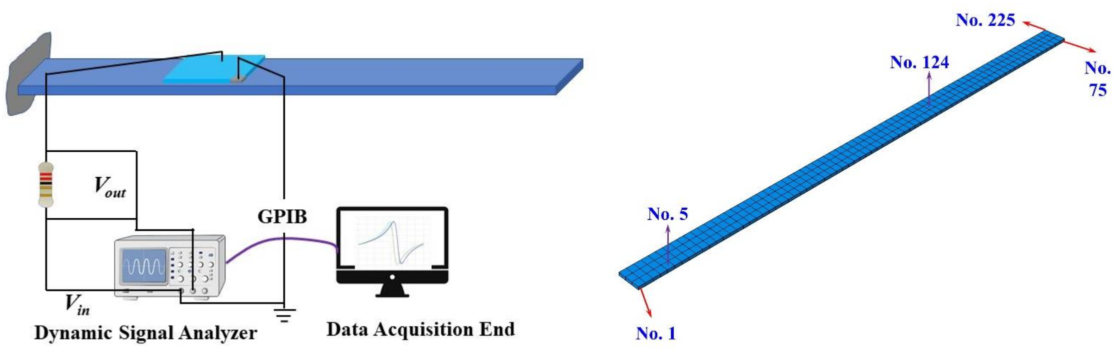

Figure 2. Experimental setup (left) and segment division (right).

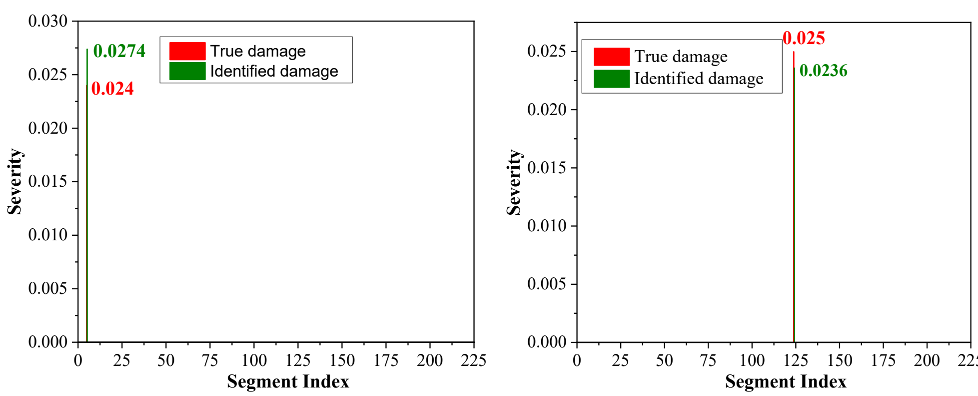

Figure 3. Identified results, (left) Case 1 and (right) Case 2.

By using sparse generation algorithm combined with MOPSO, we can finally identify the damage location and severity for both of cases, as shown in Figure 3. One can find that the algorithm can locate and quantify the damage with high precision. There is small difference between the assumed damage severity and identified severity, which is caused by modeling errors in finite element analysis. Besides, we use the Tylor series expansion to obtain the linear relationship between damage index and admittance change. This will also introduce the errors. The identified results fit the sparse nature of damage identification since damage usually occurs at a small amount of area of structure. The results demonstrate the validity of the proposed algorithm in damage identification.

5. CONCLUSION

In this research, a sparse generation algorithm is introduced for structural damage identification using piezoelectric admittance. The algorithm can switch damage occurrence based on pareto front rank for each dimension. And by randomly comparing the two dimensions, we can then generate a sparse population by piecewise product using binary and random severity matrices. The obtained sparse population will pass into the main iteration of the optimization algorithm. Here for verification, one of population-based algorithms, MOPSO is utilized to conduct the damage identification. The admittance data are used to inversely quantify and locate the damage in beam structure. The identified results show that the sparse initialization algorithm has a good performance in large-scale damage identification problem with sparsity.

ACKNOWLEDGMENT

This research is supported in part by NSF under grant CMMI – 1825324 and in part by the DOT Transportation Infrastructure Durability Center at the University of Maine under grant 69A3551847101.

REFERENCES

[1] Park, G., Sohn, H., Farrar, C.R. and Inman, D.J., Overview of piezoelectric impedance-based health monitoring and path forward. Shock and vibration digest, 35(6), pp.451-464 (2003).
[2] Shuai, Q., Zhou, K., Zhou, S. and Tang, J., Fault identification using piezoelectric impedance measurement and model-based intelligent inference with pre-screening. Smart Materials and Structures, 26(4), p.045007 (2017).
[3] Cao, Pei, Shuai Qi, and Jiong Tang, Structural damage identification using piezoelectric impedance measurement with sparse inverse analysis. Smart Materials and Structures, 27(3): 035020 (2018).
[4] Fan, X., Li, J. and Hao, H., Impedance resonant frequency sensitivity based structural damage identification with sparse regularization: experimental studies. Smart Materials and Structures, 28(1), p.015003 (2018).
[5] Dziendzikowski, M., Kurnyta, A., Dragan, K., Klysz, S. and Leski, A., In situ Barely Visible Impact Damage detection and localization for composite structures using surface mounted and embedded PZT transducers: A comparative study. Mechanical Systems and Signal Processing, 78, pp.91-106 (2016).
[6] Saravanan, T.J. and Chauhan, S.S., Study on pre-damage diagnosis and analysis of adhesively bonded smart PZT sensors using EMI technique. Measurement, p.110411 (2021).
[7] Fan, X. and Li, J., Damage Identification in Plate Structures Using Sparse Regularization Based Electromechanical Impedance Technique. Sensors, 20(24), p.7069 (2020).
[8] Minh, H.L., Khatir, S., Wahab, M.A. and Cuong-Le, T., An Enhancing Particle Swarm Optimization Algorithm (EHVPSO) for damage identification in 3D transmission tower. Engineering Structures, 242, p.112412 (2021).
[9] Ding, Z., Li, J. and Hao, H., Structural damage identification using improved Jaya algorithm based on sparse regularization and Bayesian inference. Mechanical Systems and Signal Processing, 132, pp.211-231(2019).
[10] Tian, Y., Zhang, X., Wang, C. and Jin, Y., An evolutionary algorithm for large-scale sparse multi-objective optimization problems. IEEE Transactions on Evolutionary Computation, 24(2), pp.380-393 (2019).
[11] Kropp, I., Nejadhashemi, A.P. and Deb, K., Benefits of Sparse Population Sampling in Multi-objective Evolutionary Computing for Large-Scale Sparse Optimization Problems. Swarm and Evolutionary Computation, p.101025 (2021).
[12] Coello, C.A.C., Pulido, G.T. and Lechuga, M.S., Handling multiple objectives with particle swarm optimization. IEEE Transactions on evolutionary computation, 8(3), pp.256-279 (2004).