**Proceedings of the ASME 2022
International Design Engineering Technical Conferences and
Computers & Information in Engineering Conference
IDETC/CIE 2022
August 14-17, 2022, St. Louis, Missouri**

DETC2022-91039

MEMETIC OPTIMIZER FOR STRUCTURAL DAMAGE IDENTIFICATION USING ELECTROMECHANICAL ADMITTANCE

Yang Zhang
Department of Mechanical Engineering
University of Connecticut
Storrs, CT 06269
yang.3.zhang@uconn.edu

Kai Zhou
Department of Mechanical Engineering-Engineering Mechanics
Michigan Technological University
Houghton, MI 49931
kzhou@mtu.edu

J. Tang
Department of Mechanical Engineering
University of Connecticut
Storrs, CT 06269
jiong.tang@uconn.edu

ABSTRACT

Electromechanical impedance-based (EMI) techniques using piezoelectric transducers are promising for structural damage identification. They can be implemented in high frequency range with small characteristic wavelengths, leading to high detection sensitivity. The impedance measured is the outcome of harmonic and stationary excitation, which makes it easier to conduct inverse analysis for damage localization and quantification. Nevertheless, the EMI data measurement points are usually limited, thus oftentimes resulting in an under-determined problem. To address this issue, damage identification process can be converted into a multi-objective optimization formulation which naturally yields multiple solutions. While this setup fits the nature of damage identification that a number of possibilities may exist under given observations/measurements, existing algorithms may suffer from premature convergence and entrapment in local extremes. Consequently, the solutions found may not cover the true damage scenario. To tackle these challenges, in this research, a series of local search strategies are tailored to enhance the global searching ability and incorporated into particle swarm-based optimization. The Q-table is utilized to help the algorithm select proper local search strategy based on the maximum Q-table values. Case studies are carried out for verification, and the results show that the proposed memetic algorithm achieves good performance in damage identification.

Keywords: Electromechanical impedance, particle swarm optimization, Q-table, memetic optimizer

1. INTRODUCTION

Structural Health Monitoring (SHM) provides practical means to assess and predict the structural performance under operational conditions. It is usually conducted by acquiring the measurement of the critical responses of a structure to track and evaluate the symptoms of deterioration and damage accumulation, etc [1-3]. The electromechanical impedance (EMI)-based technique has shown promising aspects in SHM due to its high sensitivity to structural damage. This technique works by integrating the host structure with a piezoelectric transducer, which has two-way electromechanical coupling and serves as both the actuator and sensor. The electromechanical impedance measured by frequency-sweeping harmonic excitation through the piezoelectric transducer can be used to monitor structural conditions [4-8].

In EMI-based techniques, inverse sensitivity method has been employed to reduce the computational cost in traditional FE model updating [9]. In such an approach, we typically divide the structure into a number of small segments and assume that each segment in the model is susceptible to damage occurrence [4-5,10-11]. The inverse analysis uses the changes of impedances/admittances as inputs, and a linearized sensitivity matrix is derived to link the damage index vector with the response measurement change vector. It is worth noting that measurement information available is usually limited since the impedance changes due to damage are noticeable only around the resonant peaks. Consequently, an inverse sensitivity method is oftentimes under-determined. To address this, recent investigations have started resorting to optimization approaches that aim at matching measurements with model prediction in the parametric space of structural damage.

Several global optimization techniques such as stochastic algorithms [12-13] or deterministic algorithm [5] have been attempted in SHM. Both single objective optimization [10-11] and multi-objective optimization [5] problems have been formulated. When a multi-objective optimization is employed, multi-modal objectives can be incorporated together, and we can generally obtain multiple solutions. Most of these algorithms assume a very small number of unknowns in problem formulation, i.e., a small number of segments are involved. This may lead to relatively large sized segments in the baseline model and the possibility of overlooking small-size damage. Besides, the algorithms may get trapped in local minima, resulting in 'untrue' solutions or misidentified segments/elements in damage identification. Premature convergence and lack of diversity in multi-objective optimization solutions are other possible limitations.

To address these challenges, in this research we formulate a new inverse identification algorithm built upon multi-objective particle swarm optimization (MOPSO) that is enhanced through a suite of local search strategies. Such strategies can help the particle jump out of the local extremes in a multi-modal function. Moreover, the Q-Table is combined with MOPSO so that the particle can select a proper local search strategy at each iteration based on the maximum Q-table values. The Q-table values reflect the performance of each local search strategy and will be updated by the reward or penalty the particle gets for the selected strategy. The rest of the paper is organized as follows. In Section 2, the EMI-based damage identification method is introduced, and the multi-objective optimization problem for inverse analysis is formulated. Section 3 elaborates the procedures of interaction between the MOPSO algorithm and Q-table. In Section 4, case studies with admittance/impedance data are used to verify the proposed algorithm. Section 5 summarizes the current research.

2. DAMAGE IDENTIFICATION PROBLEM FORMULATION

In this section, the first principle-based modelling of electromechanical admittance signature in piezoelectric damage identification is outlined first. Then, a multi-objective optimization problem is formulated to inversely locate and quantify the damage.

2.1 Admittance signature modelling

Without loss of generality, here we use the piezoelectric admittance (the reciprocal of the electric impedance) as the response of interest. The admittance signatures of a coupled system including host structure and PZT transducer can be derived as [4-5]

$$y_d^c(\omega) = \frac{\dot{Q}}{V_{in}} = \frac{j\omega}{j\omega R + k_c + \boldsymbol{K}_{12}^T\left(\boldsymbol{K}_d + j\omega\boldsymbol{C} - \omega^2\boldsymbol{M}\right)\boldsymbol{K}_{12}} \quad (1)$$

where j is the imaginary unit, k_c is the inverse of the capacitance of the piezoelectric transducer; $\boldsymbol{K}_{12}$ is the electromechanical coupling vector, and $\boldsymbol{K}$, $\boldsymbol{C}$, $\boldsymbol{M}$ are the stiffness, damping and mass matrices, respectively, Q is the electrical change on the surface of PZT patch. $\boldsymbol{K}_d$ is damaged stiffness matrix which is expressed as $\boldsymbol{K}_d = \sum_i \boldsymbol{K}_h^i\left(1-\alpha_i\right)$, $i = 1, \ldots, n$, $\boldsymbol{K}_h^i$ indicates the stiffness matrix of the ith segment or element under healthy status. $\alpha_i \in \left[0,1\right]$ is referred to as the damage index indicating the stiffness loss of the ith segment or element, which is the unknown to be identified. We can use Taylor series expansion to express the admittance expression in terms of the damage index, in which the higher terms are ignored here since the small damage is assumed. The vector of admittance change can be obtained as

$$\Delta {y^c}_{m\times 1} = \begin{bmatrix} \Delta Y(\omega_1) \\ \vdots \\ \Delta Y(\omega_m) \end{bmatrix} = \mathbf{S}_{m\times n}\boldsymbol{\alpha}_{n\times 1} \quad (2)$$

where $\mathbf{S}$ is the sensitivity matrix. We can experimentally acquire a set of piezoelectric measurements at the excitation frequencies $\boldsymbol{\omega}$ under healthy and damaged conditions. Then the admittance changes can be written as Δy^p .

2.2 Multi-objective optimization formulation

Damage in a structure usually causes the change of admittance curve only around the resonant frequencies, so limited information can be used for damage identification. This makes structural damage identification an underdetermined problem. To solve this problem more effectively, we convert it into an optimization problem. By minimizing the difference between the experimental response of the structure and the predicted data from the numerical model in the parametric space, we can inversely identify the location and severity of the damage. Moreover, as damage occurs only in a small region of the structure, the damage indicator vector is generally sparse. Therefore, we build another objective function to minimize the number of damaged locations. The multi-objective optimization model is expressed as

$$\begin{cases} \text{find} & \boldsymbol{\alpha} \in \boldsymbol{E}^n \\ \min & \left\|\Delta y^c - \Delta y^p\right\|_2 \\ \min & \left\|\boldsymbol{\alpha}\right\|_0 \\ \text{s.t.} & \alpha_l \le \alpha_i \le \alpha_u \end{cases} \quad (3)$$

where $\left\|\cdot\right\|_p$ denotes the l_p norm. Here we select l_0 norm of $\boldsymbol{\alpha}$ aiming at dealing with the sparsity of the damage index vector.

3. MEMETIC OPTIMIZER

Here we adopt the particle swarm optimization (PSO) as to solve the optimization problem presented in Section 2.2. In the case like damage identification, there are many elements/segments to be identified, i.e., many unknowns. Besides, the resulting multi-modal objective function has many local extremes so that the PSO algorithm may get stuck in the local minima, thus leading to incorrect identified solutions. To address the challenges, a series of local search strategies are proposed to enhance the global searching ability of PSO algorithm in this section. Furthermore, Q-table is employed to select the proper strategy for the particle at each iteration to prevent the particle from entrapment of local extreme.

3.1 Framework of proposed memetic optimizer

Particle swarm optimization (PSO) algorithm is a stochastic optimization technique based on swarm, which was originally proposed by Eberhard and Kennedy [14]. PSO is based on extrinsic behavior of population. In standard PSO the particles are manipulated by the following Equations (4) and (5). In PSO algorithms, each solution is like a 'bird', and each bird 'flies' around in the multidimensional problem space with an acceleration.

$$v_i(t+1) = sv_i(t) + c_1 r_1 \left(p_{\text{best}i}(t) - x_i(t)\right) + c_2 r_2 \left(g_{best}(t) - x_i(t)\right) \tag{4}$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \tag{5}$$

The coefficient s is the inertia weight, c_1 is coefficient for cognitive component which indicates that each particle learns from its experiences, and c_2 is coefficient for social component from which each particle will learn. v is velocity of particle, x is position of particle or solution for the optimization problem, r_1 and r_2 are random numbers, t is the current iteration, p_{best} is the personal best, and g_{best} is the global best.

PSO or Multi-objective Particle Swarm Optimization (MOPSO) [15] algorithm often falls into entrapment of swarm within local minima of search space. Moreover, it may achieve a premature convergence though it has several advantages such as its effectiveness, robustness, simplistic implementation. Besides, proper control should be exerted for exploration and exploitation. The algorithm may get trapped in the local minima for the functions with multi-modal characteristics. This problem will become more severe with the dimensional increase of the optimization problem. For example, in damage identification using EMI, large number of finite elements and large number of segments are employed to ensure satisfying accuracy of the finite element model and the capability of identifying small-sized damage more elements in finite element analysis. To tackle these challenges, a series of local search strategies are developed. The strategies in this research are Exploration, Convergence, High/Low Jump. Exploration and Convergence are two similar updating strategies with different cognitive and social coefficients used to achieve a dynamic updating process, in which the search starts with exploration and converges at the end of search process. High/Low Jumpy conducts a mutation for one of the dimensions of personal best to make the particle take a large/small walk to see if it can jump out of current personal best.

The suggested range for inertia weight is $s \in [0.4, 0.9]$ [14]. It is noteworthy that s must be high in the exploration and low in convergence [16]. For coefficients c_1 and c_2, they control the balance between p_{best} and g_{best}. Therefore, c_1 must be higher than c_2 in exploration and the opposite in the convergence [16-17]. For the 'Exploration', we now set coefficients in Equation (4) as $s = 0.8$, $c_1 = 2.5$, $c_2 = 0.5$, as used in literature [18-19]. In the 'Convergence' state, the algorithm will converge to the global best quickly, therefore, the social part now is given a larger value, so the coefficients are 0.4, 0.5, 2.5 respectively for w, c_1 and c_2. For the High/Low Jump, one of the dimensions of the personal best is randomly selected and perturbed using Gaussian distribution. The only difference between High and Low Jump is the standard deviation is larger for High Jump. Here we use 0.9 for High Jump and 0.1 for Low Jump. The mathematic expression can be denoted as [20]

$$x_i = P_{besti} + R_{norm}\left(V_{upper} - V_{lower}\right) \tag{6}$$

where R_{norm} is a random number sampled from Gaussian distribution $N \sim (\mu, \sigma^2)$. The mean values for both High and Low Jump are zero.

Since the particle cannot execute all local search strategies at one time (iteration), Q-table here is combined to help particle select the proper local strategy based on the maximum Q-table value, as shown in Figure 1. To elaborate on the memetic optimizer, we summarize the algorithm as the following

Step 1: Initialize all the necessary parameters, such as # of objectives, # of population, # of variables (dimension), searching bounds, maximum iterations, etc.

Step 2: Initialize population using random number within searching space. Initialize Q-table all zeros with dimension 4 by 4. Initialize personal best, global best. Initialize local strategy as 'Exploration'.

Step 3: Main loop for MOPSO algorithm until maximum iteration is reached: Executing 'Exploration' and using Non-dominated Sorting algorithm to determine if the updated solution is kept or ignored. If the newly generated solution is kept, an immediate reward is given, or a penalty will be given. The reward or penalty will be used to update the Q-table value (will be discussed in following subsection). Executing other local strategies based on maximum Q-table values until maximum iteration is reached.

Step 4: Updating personal best if newly obtained solution is kept or do nothing. Updating global best and Pareto solution repository.

Step 5: Solution visualization and interpretation.

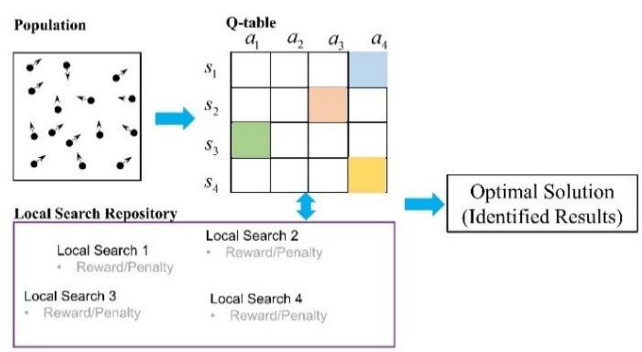


FIGURE 1: Q-TABLE INTERACTION WITH LOCAL STRATEGIES

3.2 Local search strategy selection

Local searching strategies are proposed here to help particles jump out of local minima to achieve a global search. However, all the strategies cannot be executed at the same time. Thus, Q-table here is selected to achieve the dynamic selection goal. Q-table is usually used in Q-learning, in which the learner performs an action causing a state transition in the environment it resides and receives a reward or penalty for the action executed to reach a definite goal [19, 21-22]. Here the particle is regarded as agent and the local strategies are actions.

The current state is set as Exploration because the particle is hoped to explore more searching space at the beginning of the algorithm. In the main iteration part, the particle will select the action which has the maximum value in the Q-table and execute the action. After executing the current action, the agent can get to the new state using observing function $o(s,a)$ which makes a transition from current state to another. The transition here is that current action index is the next state index. Here a diagram is drawn to show how to switch from current state to next state. As shown in the Fig. 2, the agent will select an action based on the maximum table values given the current state as S_1 . For example, if Action 3 (a_3) has a maximum value and then the next state will be set as S_3 , which has the same index to the index of current action a_3 . This process can be written mathematically as

$$a_t = index\left\{\max\left(Q\left(s_t, actions\right)\right)\right\} \tag{7}$$

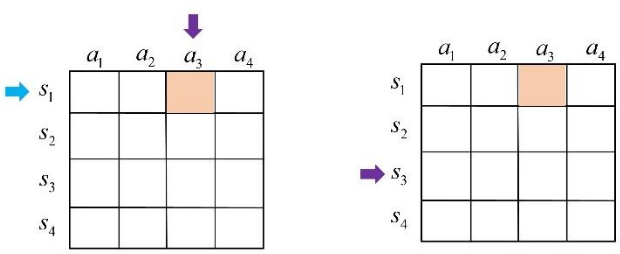


FIGURE 2: STATE TRANSITION PROCESS

After Action 3 is executed, the rewards obtained currently can be passed into Equation (8) to update the entry for the current state and current action. Then the personal best and the global best are updated, and the external optimal repository will be updated accordingly. All the steps in the main loop will be performed until maximum runs are reached.

There are 4 states and their corresponding actions. Therefore, we can create a Q-table with size 4 by 4 and initialize all the entries as zeros. Usually, the current state is randomly chosen from the state repository. However, here we initialize the current state as 'Exploration' since we hope the algorithm starts with exploring more searching space. Then the best action will be selected and executed based on the maximum Q-table value given the current state.

After executing the selected action, an immediate reward or penalty will be obtained. Reward or penalty is determined by the solution dominating algorithm. If the new solution dominates the old solution, then a positive value 1 is given as a reward, otherwise, a negative reward -1 is given as penalty. And then observe the maximum future reward under the next state (next state index is same to the current action index, referring to the Fig. 2) and this reward will be passed into the updating function to update the Q-table entry $Q(s_t, a_t)$. The updating function is

$$Q_{new}\left(s_t, a_t\right) = \left(1-\alpha\right)Q\left(s_t, a_t\right) + \alpha\left\lfloor r_{t+1} + \gamma \max_a Q\left(s_{t+1}, a_{all}\right)\right\rfloor \tag{8}$$

where, α is called the learning rate, which is defined as how much the new value vs the old value is accepted. γ is a discount factor. It is used to balance immediate and future reward. *r* is the value after completing a certain action at a given state. Max is operation to take the maximum of the future reward and apply it to the reward for the current state. Finally, the current state will be updated as current state

4. CASE STUDY

In this section, we conduct case study to examine the proposed algorithm improvements in multi-objective optimization for damage identification. A cantilevered aluminum plate structure is integrated with a single piezoelectric transducer, as shown in Figure 3. The dimension of the beam is specified as length 561 mm, width 19.05 mm and thickness 4.763 mm. The density and Young's modulus are 2700 kgm^{-3} and 68.9 GPa, respectively. The piezoelectric transducer is placed at 180 mm from the left end, with length 15 mm, width 19.05 mm, and thickness 4.763 mm. The Young's moduli of piezoelectric transducer are Y_{11} = 86 GPa and Y_{33} = 73 GPa, and the density is 9500 kgm^{-3}. The piezoelectric constant and dielectric constant are $h_{31} = -1.0288\times10^9\,\text{Vm}^{-1}$ and $\beta_{33} = 1.3832\times10^8\,\text{mF}^{-1}$, respectively.

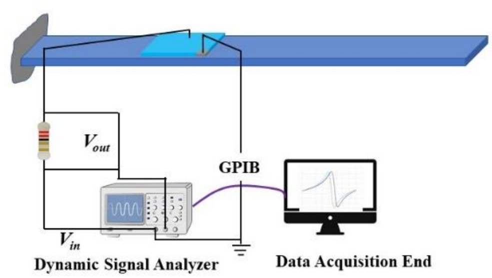


FIGURE 3: SYSTEM SETUP

Without loss of generality, admittance changes are acquired around 14th (1893.58 Hz) and 21st (3704.05) natural frequencies. Two frequency ranges are selected from 1891.69Hz to 1895.47Hz and from 3700.35Hz to 3707.75Hz with 100 sweeping points for each range. Two damage cases are considered. The first case is dividing the plate into 225 segments, each corresponding to a damage index to be identified, as shown in Figure 4 (a) and the damage is assumed at segment 75 with 13.2% stiffness reduction. For the second case, the segment is much finer, resulting in 1125 segments (Figure 4 (b)) and the damage is located at segment 800 with 9.5% stiffness reduction.

TABLE 1: DAMAGE IDENTIFICATION RESULTS

Case #	Solutions	Obj-fun 1	Obj-fun 2
Case 1	1	8.864434e-6	1
	2	8.864416e-6	2
Case 2	1	5.623299e-6	2
	2	2.197487e-7	2

The total number of admittance measurement data points are 200 for both cases. We can see that the damage identification for both cases are under-determined or the information for damage identification is limited. It should be mentioned that damage usually occurs at a small region of the structure, thus resulting in a sparse damage index vector. We use the sparse initialization algorithm that can generate the sparse population [23]. Then it will be passed into the main optimization loop. By solving the optimization model formulated in Section 2 using the proposed algorithms, the identified results are obtained for both cases, as plotted in Figures 5 and 6. The figures are arranged based on the non-zero identified segments (the second objective function.). The obtained solutions for both cases are summarized in Table 1.

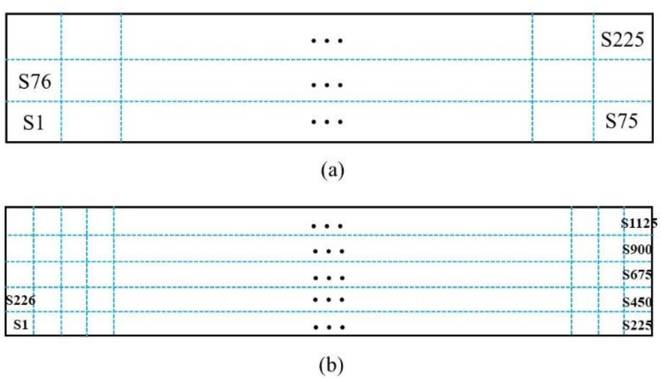


FIGURE 4: SEGMENT DIVISION (A) COARSE DIVISION (CASE 1); (B) FINE DIVISION (CASE 2)

For Case 1, the two optimal solutions obtained accurately pinpoint the true damage location and severity. The two solutions manifest the characteristic of underdetermined problem that has multiple solutions. Though there is a misidentified damage location in solution 2 at segment 26, as plotted in Figure 5, the misidentified damage has a negligible severity that reflects the modeling errors in finite element analysis. Besides, we use the Tylor series expansion to obtain the linear relationship between damage index and admittance change. Since we do not know the information about the damage, both solutions obtained, therefore, can be regarded as possible candidates for engineering practice.

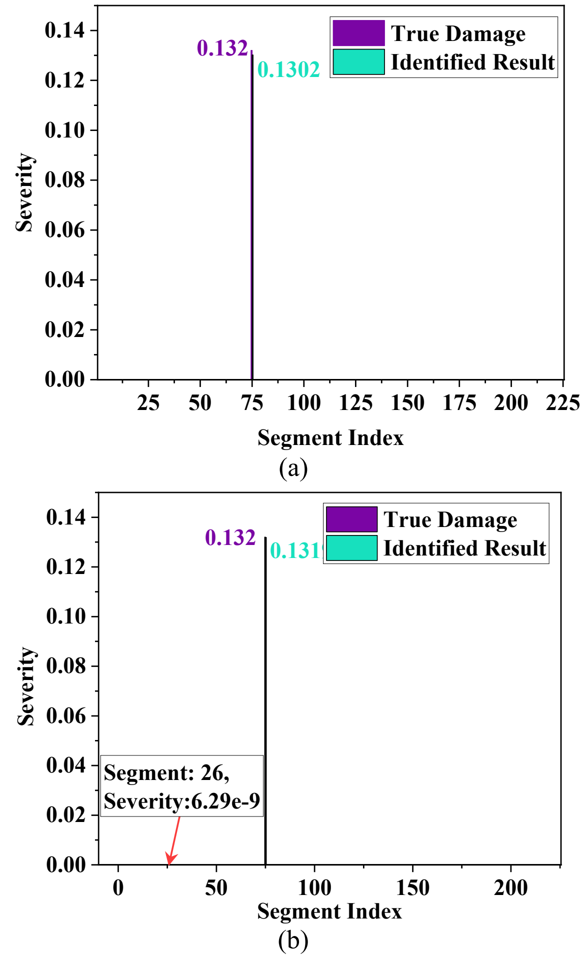
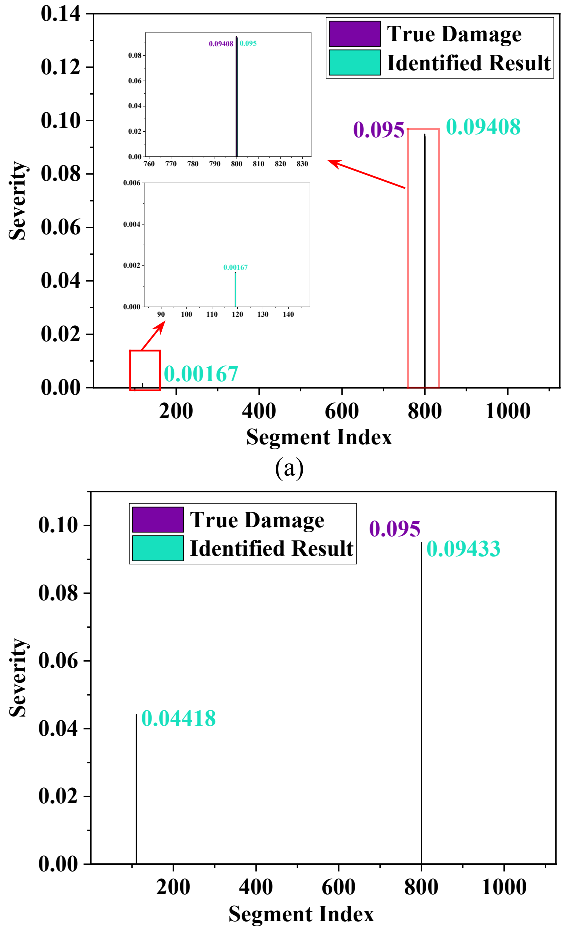


FIGURE 5: IDENTIFIED RESULTS FOR CASE 1 WITH 225 SEGMENTS IN PLATE STRUCTURE

As mentioned, modeling error and linear approximation may lead to misidentified damage locations, which is also reflected in the two solutions of Case 2, as shown in Figure 6. Each solution has two identified damage locations. One precisely captures the true damage scenario, and the other is the misclassified damage location. Within the multi-objective optimization setup, multiple solutions can be obtained for underdetermined problem and there is always one solution that captures the true damage scenario. In this case, both solutions can be regarded as possible candidate in practical problems.

In this research, case studies demonstrate the global searching ability of proposed memetic optimizer, which performs well in damage identification problems with underdetermined characteristics.



(a)

(b)

FIGURE 6: IDENTIFIED RESULTS FOR CASE 2 WITH 1125 SEGMENTS IN PLATE STRUCTURE

4. Conclusions

In this research, a memetic optimizer based on MOPSO algorithm is presented for damage identification using electromechanical admittance measurements. The inverse damage identification process is converted into an optimization problem. Given that the algorithm solving optimization model usually falls into the local extremes and get a premature convergence, a series of local search strategies are proposed aiming at enhancing the global searching ability when dealing with multi-modal objective function. The Q-table is utilized here to help the particle to select the proper local search based on maximum Q-table values. Case demonstrations verify the validity of the algorithm.

ACKNOWLEDGEMENTS

This research is supported in part by a Space Technology Research Institutes grant (number 80NSSC19K1076) from NASA's Space Technology Research Grants Program and in part by NSF under grant CMMI-1825324.

REFERENCES

[1] Zhu, W. D., and K. He. Detection of damage in space frame structures with L-shaped beams and bolted joints using changes in natural frequencies. J. Vib. Acoust., 135.5 (2013).
[2] He, K., and W. D. Zhu. Detecting loosening of bolted connections in a pipeline using changes in natural frequencies. J. Vib. Acoust., 136.3 (2014).
[3] Zhou, K., and J. Tang. Highly efficient probabilistic finite element model updating using intelligent inference with incomplete modal information. J. Vib. Acoust., 138.5 (2016).
[4] Shuai, Q., Zhou, K., Zhou, S. and Tang, J., Fault identification using piezoelectric impedance measurement and model-based intelligent inference with pre-screening, Smart Mater. Struct., 26.4 (2017): 045007.
[5] Cao, Pei, Shuai Qi, and Jiong Tang, Structural damage identification using piezoelectric impedance measurement with sparse inverse analysis, Smart Mater. Struct., 27. 3 (2018): 035020.
[6] Dziendzikowski, M., Kurnyta, A., Dragan, K., Klysz, S. and Leski, A., In situ Barely Visible Impact Damage detection and localization for composite structures using surface mounted and embedded PZT transducers: A comparative study, Mech Syst Signal Process, 78 (2016): 91-106.
[7] Li, H., Ai, D., Zhu, H. and Luo, H., An Orthogonal Matching Pursuit based signal compression and reconstruction approach for electromechanical admittance based structural health monitoring, Mech Syst Signal Process, 133 (2019): 106276.
[8] Saravanan, T.J. and Chauhan, S.S., Study on pre-damage diagnosis and analysis of adhesively bonded smart PZT sensors using EMI technique, Measurement, (2021): 110411.
[9] Zhou, K. and Tang, J., Structural model updating using adaptive multi-response Gaussian process meta-modeling, Mech Syst Signal Process, 147 (2021): 107121.
[10] Fan, Xingyu, Jun Li, Hong Hao, and Shenglan Ma, Identification of minor structural damage based on electromechanical impedance sensitivity and sparse regularization, J Aerosp Eng 31.5 (2018): 04018061.
[11] Fan, X. and Li, J., Damage Identification in Plate Structures Using Sparse Regularization Based Electromechanical Impedance Technique, J. Sens., 20.24 (2021): 7069.
[12] Minh, H.L., Khatir, S., Wahab, M.A. and Cuong-Le, T., An Enhancing Particle Swarm Optimization Algorithm (EHVPSO) for damage identification in 3D transmission tower, Eng. Struct., 242 (2021): 112412.
[13] Ding, Z., Li, J. and Hao, H., Structural damage identification using improved Jaya algorithm based on sparse regularization and Bayesian inference, Mech Syst Signal Process, 132 (2019): 211-231.

[14] Kennedy, J. and Eberhart, R., November. Particle swarm optimization. In Proceedings of ICNN'95-international conference on neural networks (4 (1995): 1942-1948). IEEE.
[15] Coello, C.A.C., Pulido, G.T. and Lechuga, M.S., Handling multiple objectives with particle swarm optimization, IEEE Trans Evol Comput, 8.3 (2004): .256-279.
[16] Zhan, Z.H., Zhang, J., Li, Y. and Chung, H.S.H, Adaptive particle swarm optimization, IEEE Trans. Syst. Man Cybern, 39. 6 (2009): 1362-1381.
[17] Wang, Y., Li, B., Weise, T., Wang, J., Yuan, B. and Tian, Q, Self-adaptive learning based particle swarm optimization, Inf. Sci, 181.20 (2011): 4515-4538.
[18] Ratnaweera, Asanga, Saman K. Halgamuge, and Harry C. Watson, Self-organizing hierarchical particle swarm optimizer with time-varying acceleration coefficients, IEEE Trans. Evol. Comput, 8.3 (2004): 240-255.
[19] Samma, Hussein, Chee Peng Lim, and Junita Mohamad Saleh, A new reinforcement learning-based memetic particle swarm optimizer, Appl. Soft Comput, 43 (2016): 276-297.
[20] Lim, Wei Hong, and Nor Ashidi Mat Isa, Two-layer particle swarm optimization with intelligent division of labor, Eng. Appl. Artif. Intell., 26.10 (2013): 2327-2348.
[21] Meerza, Syed Irfan Ali, Moinul Islam, and Md Mohiuddin Uzzal, Q-learning based particle swarm optimization algorithm for optimal path planning of swarm of mobile robots, 2019 1st International Conference on Advances in Science, Engineering and Robotics Technology (ICASERT). IEEE.
[22] Watkins, Christopher JCH, and Peter Dayan, Q-learning, Mach. Learn., 8.3 (1992): 279-292.
[23] Zhang, Yang, K. Zhou, and J. Tang. Structural damage identification using inverse analysis through optimization with sparsity. Sensors and Smart Structures Technologies for Civil, Mechanical, and Aerospace Systems 2022. Vol. 12046. SPIE, 2022.