

Generalized Theory of PT -Symmetric Radio-Frequency Systems With Divergent Exceptional Points

Maryam Sakhdari^{ID}, Zhilu Ye^{ID}, Mohamed Farhat^{ID}, and Pai-Yen Chen^{ID}, *Senior Member, IEEE*

Abstract—The emergence of exceptional point (EP) and its divergent form, so-called divergent EP (DEP), in parity-time (PT)-symmetric trimer opens a new pathway for making radio frequency (RF) telemetry sensor systems with unprecedented sensitivity. Here, we put forward a rigorous and formally exact analysis for PT -symmetric electronic multimers, shedding light on the lower bound of the inductive coupling coefficient required to achieve DEPs. One clear observation is that there exists a subtle compromise among the degree of diverging bifurcation, critical inductive coupling strength, and spectral noises arising from modal interferences. We conclude that an electronic PT -symmetric trimer, although not sufficient to work at the lowest inductive coupling strength, may be the simplest structure that enables the emergence of DEPs. Our findings may provide the theoretical underpinnings and open avenues for various RF telemetry and wireless sensing systems.

Index Terms—Exceptional points (EPs), non-Hermitian physics, parity-time (PT) symmetry, radio frequency (RF) telemetry, wireless sensors.

I. INTRODUCTION

IN THE past decade, notions taken from non-Hermitian systems and parity-time (PT)-symmetry have attracted considerable attention in different fields, such as electromagnetics and optics [1]–[15], acoustics [16]–[18], elastodynamics [19], and electronics and electromagnetics [20]–[30], of which systems based on coupled gain and loss components can be practically built and tested. These non-Hermitian systems share a common feature [1]–[10], that is, there are branch points at which Taylor series expansion fails to converge in the multi-valued complex eigenspectrum. Such peculiar singularities are known as exceptional points (EPs), where not only eigenvalues but also their corresponding eigenvectors become the same (i.e., degeneracy). While EPs had been previously studied in

the context of complex-valued Hamiltonian, it was not until discovery of PT -symmetric non-Hermitian Hamiltonian that the exotic degeneracy accompanied by the phase transition (i.e., exact- PT and broken- PT phases) aroused widespread interest. In particular, the introduction of PT -symmetry in optical and photonic systems has opened up many unforeseen opportunities for designing devices and structures with new functionalities (e.g., unidirectional reflectionless transmissions [5], [6], nonreciprocity [3], [31], negative refraction [32], and coherent perfect absorber laser [4], [33]–[38]) and for building the next-generation sensors that can leverage the bifurcation phenomena around EPs to accomplish an unprecedented sensitivity [22]–[25], [39].

It has been experimentally demonstrated that when the PT -symmetric optical microcavity system with the N th-order EP is subjected to a perturbation of strength ε , the resulting eigenvalue splitting can be proportional to $\varepsilon^{1/N}$. Such a result means that a sensor consisting of large arrays of microcavity resonators could have an unprecedentedly high sensitivity. Nevertheless, in practice, due to the simultaneous presence of multiple eigenvalues at the higher order EP [27], [40]–[43], the sensor will unavoidably suffer from serious phase and flicker noises [44], [45], which become more severe as the order increases. In a different vein, our recent theoretical and experimental works suggested that an exotic divergent EP (DEP), describing the situation where an EP coincides with a divergent singularity, can be found in the third-order PT -symmetric radio frequency (RF) telemetry system composed of inductively coupled $-RLC$, LC , and RLC oscillators [40], [46]. In this electronic PT -symmetric trimer structure, there exists a critical inductive coupling strength related to the DEP, which allows real eigenfrequencies to diverge rapidly and asymptotically in response to perturbations.

In this context, higher order RF telemetry systems comprising inductively coupled oscillators have been commonly used for wireless sensing and wireless power transfer [27], [40], [42], [46]. In these systems, the battery-free (passive) sensor that can be equivalent to a variable RLC tank is inductively interrogated by the active reader and intermediary(s). Short-range wireless sensors have influenced many aspects of our daily lives, as they enable the development of real-time environmental monitoring (e.g., temperature and humidity) [47], [48], health monitoring of civil structures [49], [50], biosensing and routine vital signal monitoring

Manuscript received 16 December 2021; revised 6 April 2022; accepted 7 May 2022. Date of publication 7 June 2022; date of current version 9 November 2022. This work was supported by the National Science Foundation (NSF) under Grant ECCS-1917678. (Maryam Sakhdari and Zhilu Ye contributed equally to this work.) (Corresponding author: Pai-Yen Chen.)

Maryam Sakhdari, Zhilu Ye, and Pai-Yen Chen are with the Department of Electrical and Computer Engineering, University of Illinois at Chicago, Chicago, IL 60607 USA (e-mail: pychen@uic.edu).

Mohamed Farhat is with the Computer, Electrical and Mathematical Sciences and Engineering Division, King Abdullah University of Science and Technology, Thuwal 23955, Saudi Arabia (e-mail: mohamed.farhat@kaust.edu.sa).

Color versions of one or more figures in this article are available at <https://doi.org/10.1109/TAP.2022.3179528>.

Digital Object Identifier 10.1109/TAP.2022.3179528

0018-926X © 2022 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission.

See <https://www.ieee.org/publications/rights/index.html> for more information.

in clinical practice [51]–[53], food quality monitoring [54], and the automated quality control in industry [55], [56], to name a few. In particular, in the era of the Internet of Things (IoT) [57]–[59], the growing demand for wireless data query requires the development of the low-cost, lightweight passive sensors and new efficient telemetry method enhancing wireless readout sensitivity. So far, there have been tremendous efforts put into the former task, i.e., development of new types of micromachined actuators and varactors [22], [23], [60], [61], piezoelectric transducers [28], [42], and chemiresistive sensors [48], [62]. To bring us one step closer to the desired pragmatic performance metrics, the later task is also of paramount importance, which, however, received limited attention in the last decades. In this regard, since the eigenvalue bifurcation in *PT* electronic systems can be boosted near the DEP, even a small impedance perturbation on the *RLC* sensor can cause a significant resonance frequency shift, thereby providing ultrahigh sensitivity and resolvability. Notwithstanding the experimental evidence of the DEP, the evolution of eigenspectrum and the lower bound on the critical coupling strength for higher order electronic *PT* systems are yet unknown. It is fundamentally important to develop the generalized formulation to understand the characteristics and physical limits of *PT*-symmetric electronic multimer of the *N*th order, as shown in Fig. 1. This will unveil the optimal design and performance of telemetry systems for RF sensors used in diverse fields.

II. GENERALIZED *N*TH-ORDER *PT*-SYMMETRIC ELECTRONIC SYSTEMS WITH SERIES-TYPE OSCILLATORS

Fig. 1 presents the generalized *N*th-order *PT*-symmetric electronic systems composed of multiple gain, loss, and neutral elements, respectively, represented by $-RLC$, RLC , and LC oscillators [25], [40] with a mutual inductance M existing between adjacent oscillators. The higher order electronic systems satisfying the *PT*-symmetry can be built upon: 1) even-numbered oscillators, i.e., $N/2$ pairs of $-RLC$ and RLC oscillators; 2) odd-numbered oscillators, i.e., $(N-1)/2$ pairs of $-RLC$ and RLC oscillators and one neutral LC oscillator in the middle of the system; and 3) a pair of $-RLC$ and RLC oscillators that are coupled via $N-2$ neutral intermediators, as shown in Fig. 1(a). These telemetry systems can be described by: $H_{\text{eff}}\Psi = id\Psi/d\tau$ [22], [23], [27], where H_{eff} is the effective non-Hermitian Hamiltonian of dimensionality $N \times N$, $\Psi \equiv (q_1, q_2, \dots, q_N, \dot{q}_1, \dot{q}_2, \dots, \dot{q}_N)^T$ is a modal column vector, q_k and $\dot{q}_k$ are the charge and current flow on the k th capacitor, respectively, $\tau \equiv \omega_0 t$, and the resonant angular frequency of an LC oscillator is $\omega_0 = 1/(LC)^{1/2}$. After the substitution of time-harmonic charge distributions in each oscillator, $q_k = A_k e^{i\omega\tau}$, and $2N$ eigenfrequencies (i.e., the system's resonance frequencies) can be obtained as roots of the polynomial equation $\det(H_{\text{eff}} - \omega_n \mathbf{I}) = 0$.

Let us first consider the even-order *PT*-symmetric electronic systems constituted by $N/2$ pairs of series $-RLC$ and RLC oscillators where N is an even integer [type I in Fig. 1(a)]. After some mathematical manipulation, we find that the system's n th eigenfrequencies ω_n in unit of ω_0 are zeros of a

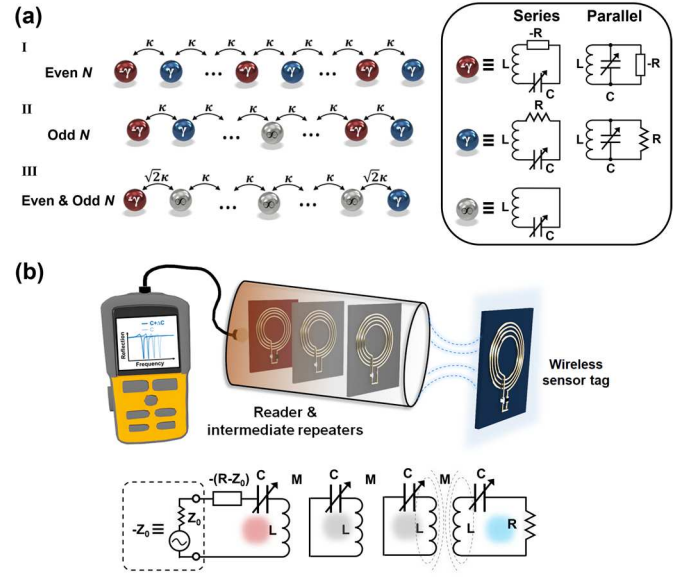


Fig. 1. (a) Schematics of the generalized *N*th-order *PT*-symmetric electronic multimers; here, gain, loss, and neutral elements in an electronic circuitry may be readily realized with magnetically coupled series/parallel $-RLC$, RLC , and LC oscillators. (b) Illustration of the generalized *PT*-symmetric wireless sensing and telemetry system, where the information of a passive sensor or transducer (e.g., capacitive pressure sensor or piezoresistive transducer) is commonly encoded in the reflection spectrum on the reader side. The frequency synthesizer as an external radio energy source is equivalent to a negative resistance, with its absolute value equal to that of the generator impedance Z_0 .

sequence of orthogonal polynomials U_α^{even} , with a recurrence relation given by

$$\begin{aligned} U_0^{\text{even}} &= 1 \\ U_1^{\text{even}} &= x - \kappa^2 \\ U_{\alpha+1}^{\text{even}} &= (x - 2\kappa^2)U_\alpha^{\text{even}} - \kappa^4 U_{\alpha-1}^{\text{even}} \end{aligned} \quad (1a)$$

or

$$U_\alpha^{\text{even}} = \prod_{\beta=0}^{\alpha-1} (x - (2\kappa A_{\alpha\beta})^2) \quad (1b)$$

where

$$x = \left(\frac{\omega^2 - 1}{\omega^2} \right)^2 + \left(\frac{1}{\gamma \omega} \right)^2 \quad (2)$$

$$A_{\alpha\beta} = \sin\left(\frac{\pi}{2} \frac{2\beta + 1}{2\alpha + 1}\right) \quad (3)$$

$\alpha = N/2, \beta = 0, 1, 2, \dots, \alpha - 1$, the coupling strength $\kappa = M/L$, and the gain-loss parameter (non-Hermiticity) $\gamma = R^{-1}(L/C)^{1/2}$. The system's eigenfrequencies that make $U_\alpha^{\text{even}} = 0$ can be expressed as

$$\omega_n^{\text{even}} = \pm \sqrt{\frac{2\gamma^2 - 1 \pm \sqrt{1 - 4\gamma^2 + 16\gamma^4 \kappa^2 A_{\alpha\beta}^2}}{2\gamma^2 (1 - 4\kappa^2 A_{\alpha\beta}^2)}}. \quad (4)$$

Throughout this article (unless otherwise stated), all frequencies are normalized with respect to ω_0 . We note that positive and negative eigenfrequencies are essentially identical, and the redundancy must be removed by considering only the positive

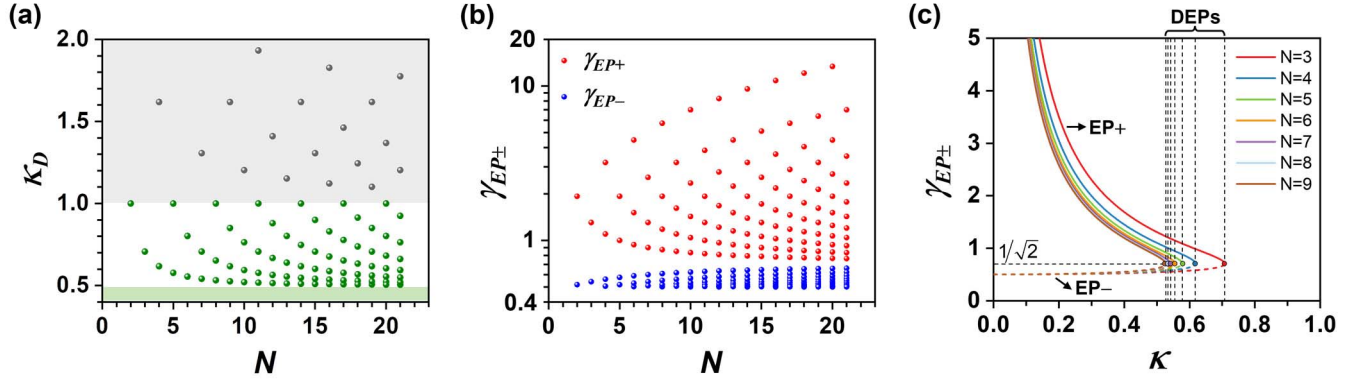


Fig. 2. (a) Critical coupling strength κ_D and (b) EPs γ_{EP+} and γ_{EP-} for the type-I and type-II PT -symmetric electronic systems in Fig. 1(a), which are formed by the series-type oscillators and the magnetic coupling strength between each of them $\kappa = 1/2$. From (a), we find that there exists a lower bound ($1/2$) and an upper bound (1 , which is physically infeasible) for critical coupling strength κ_D . (c) γ_{EP+} (solid lines) and γ_{EP-} (dashed lines) of the first eigenvalue branch as a function of κ for the types-I and -II structures with series-type oscillators.

signs. By inspecting (4), the expressions for N EPs and the critical coupling strengths are given by

$$\gamma_{EP\pm}^{even} = \frac{1}{2\kappa} \sqrt{\frac{1 \pm \sqrt{1 - 4\kappa^2 A_{\alpha\beta}^2}}{2A_{\alpha\beta}^2}} \quad (5)$$

and

$$\kappa_D^{even} = \frac{1}{2A_{\alpha\beta}}. \quad (6)$$

At the DEPs, the EPs degenerate, i.e., $\gamma_{EP+}^{even} = \gamma_{EP-}^{even} = 1/(2)^{1/2}$. We know from (4) that the bifurcation of eigenfrequencies is proportional to a factor of $1/(1 - 4\kappa^2 A_{\alpha\beta}^2)^{1/2}$. Thus, if the system operates close enough to the divergent singularities κ_D^{even} such that the denominator of (4) vanishes, the eigenvalue splitting around EPs could be drastic, as has been demonstrated experimentally with the electronic PT trimer [46].

In the same vein, for odd-order PT -symmetric electronic systems comprising $(N - 1)/2$ pairs of $-RLC$ and RLC oscillators and one neutral LC oscillator, where N is an odd integer [type II in Fig. 1(a)], eigenfrequencies are found to be roots of the transcendental equation $(\omega^2 - 1)U_{\alpha}^{odd} = 0$, where $\alpha = (N - 1)/2$ and U_{α}^{odd} is a sequence of orthogonal polynomials that can have a recurrence relation

$$\begin{aligned} U_0^{odd} &= 1 \\ U_1^{odd} &= x - 2\kappa^2 \\ U_{\alpha+1}^{odd} &= (x - 2\kappa^2)U_{\alpha}^{odd} - \kappa^4 U_{\alpha-1}^{odd} \end{aligned} \quad (7a)$$

or

$$U_{\alpha}^{odd} = \prod_{\beta=0}^{\alpha-1} x - (2\kappa B_{\alpha\beta})^2 \quad (7b)$$

where

$$B_{\alpha\beta} = \sin\left(\frac{\pi \beta + 1}{2 \alpha + 1}\right) \quad (8)$$

$\beta = 0, 1, 2, \dots, \alpha - 1$, and the expression for x can be found in (2). Subsequently, eigenfrequencies can

be derived as

$$\omega_n^{odd} = \pm 1, \pm \sqrt{\frac{2\gamma^2 - 1 \pm \sqrt{1 - 4\gamma^2 + 16\gamma^4 \kappa^2 B_{\alpha\beta}^2}}{2\gamma^2 (1 - 4\kappa^2 B_{\alpha\beta}^2)}}. \quad (9)$$

Equation (9) indicates that for odd-order PT multimers, there are $N - 1$ EPs

$$\gamma_{EP\pm}^{odd} = \frac{1}{2\kappa} \sqrt{\frac{1 \pm \sqrt{1 - 4\kappa^2 B_{\alpha\beta}^2}}{2B_{\alpha\beta}^2}} \quad (10)$$

and DEPs can be obtained when

$$\kappa_D^{odd} = \frac{1}{2B_{\alpha\beta}}. \quad (11)$$

Since $0 \leq A_{\alpha\beta}, B_{\alpha\beta} \leq 1$ and $0 \leq \kappa \leq 1$ (if $\kappa = 1$, the two coils are perfectly coupled), the lower and upper bounds of coupling strengths for DEPs are: $1/2 \leq \kappa_D^{even}(\kappa_D^{odd}) \leq 1$. The same conclusion is also valid for electronic PT multimers formed by the parallel-type $RLC/-RLC$ oscillators, which will be discussed in the following. The above results are important for obtaining the DEP in the short-range, inductively coupled RF telemetry systems.

Fig. 2(a) and (b) show the dependency of $\kappa_D^{even/odd}$ and $\gamma_{EP\pm}$ (here, $\kappa = 0.5$) on the order of the PT -symmetric electronic system discussed above (here, $2 \leq N \leq 21$). It is worthwhile mentioning that Fig. 2(a) explains why the DEP was never found in the standard PT dimer ($N = 2$) [22], [24], as it requires perfect magnetic coupling (i.e., $\kappa_D^{even}|_{N=2} = 1$), which is impractical from an experimental viewpoint. The critical coupling coefficient required for observing DEPs decreases with increasing the order of electronic PT system, as can be seen in Fig. 2(a). An experimental test of the effect of divergent bifurcation nearby the DEP becomes plausible when $N \geq 3$ because the critical inductive coupling strength is reduced below unity (e.g., $\kappa_D^{odd}|_{N=3} = 1/(2)^{1/2}$). In this sense, our recent experimental work [46] showed that an electronic PT trimer consisting of gain-neutral-loss oscillators can exhibit a huge eigenvalue bifurcation ever obtained before. One can anticipate that this feature can be exploited to build

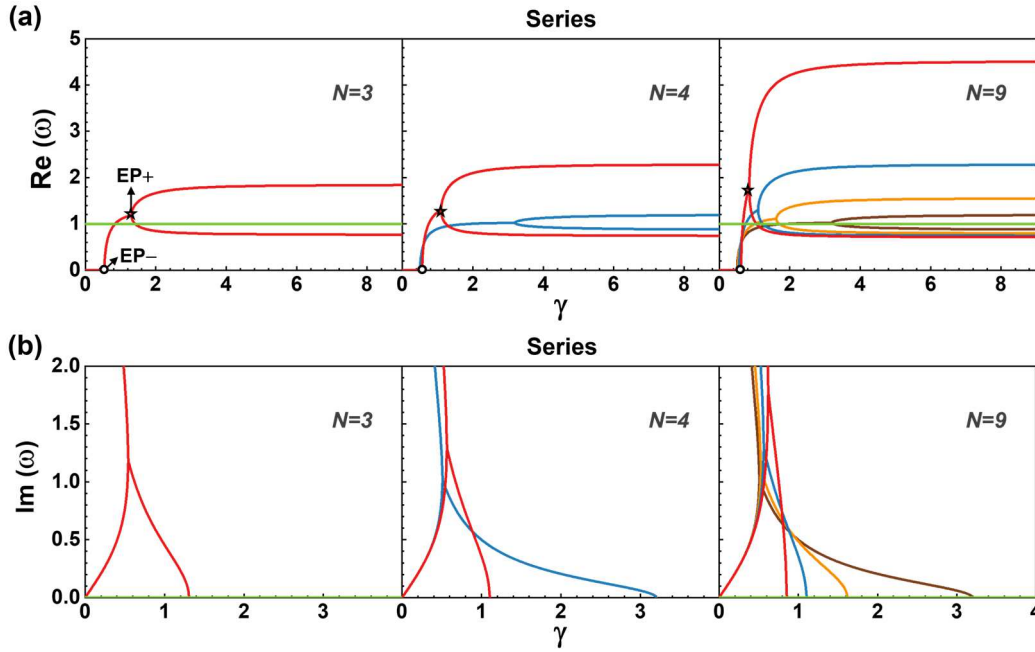


Fig. 3. (a) Real and (b) imaginary eigenspectra for the PT -symmetric systems with series-type oscillators of order $N = 3, 4$, and 9 ; here, κ is fixed to $1/2$. The eigenfrequencies with the largest variation are highlighted as stars in (a). There are three distinct regions: 1) when $\gamma \in [0, \gamma_{EP-}]$, the eigenvalues are imaginary and the system is in the overdamped broken- PT phase; 2) when $\gamma \in [\gamma_{EP-}, \gamma_{EP+}]$, the system is in the underdamped broken- PT phase; and 3) when $\gamma \in [\gamma_{EP+}, +\infty]$, the eigenvalues are purely real and the system is in the exact- PT phase.

ultraresponsive telemetric sensors beyond the current state of the art. As can be observed from Fig. 2(b), when $\kappa = 1/2$ and a large number of oscillators are used such that $\kappa_D^{\text{even/odd}} \sim 1/2$, γ_{EP+} and γ_{EP-} appearing in pairs would meet at one point, resulting in remarkable eigenfrequency splitting. Fig. 2(c) reports EPs as a function of κ for the first eigenvalue branch of the PT systems with series-type oscillators, which correspond to the largest eigenfrequency bifurcation. It can be seen from Fig. 2(c) that the critical magnetic coupling strength for observing DEP shifts to a lower value when the number of oscillators increases. Fig. 3(a) and (b), respectively, presents the real and imaginary parts of eigenfrequencies calculated based on (4) and (9) for PT -symmetric electronic systems with a different number of constituent elements (left to right: $N = 3, 4$, and 9) and a fixed coupling strength $\kappa = 1/2$. It is evident that the degree of bifurcation of a specific branch increases as N increases because κ_D found in Fig. 2(a) continues to decrease toward the lower bound [labeled by the green bar in Fig. 2(a)]. The corresponding reflection spectra for the electronic PT multimers with $N = 3$ and $N = 9$ are plotted in Fig. 4(a) and (b), respectively; here, a steady-state time-harmonic excitation source is connected to the active oscillator on the left [see Fig. 1(b)] and $\Delta\gamma = \gamma - \gamma_{EP+}^{(1)}$ is varied to observe changes in eigenfrequencies. Although there are N positive eigenfrequencies (resonance frequencies) signaled by resonant dips in the reflection spectrum [22], [27], only the one with the largest variation is considered for sensing applications, while others emerge as modal interferences and background noises. By comparing Fig. 4(a) and (b), we find that the shift in the specific resonance frequency [highlighted in Fig. 3(a)] can be enhanced by increasing the number of intermediate oscillators, which effectively reduces the critical coupling strength required to meet the DEP condition. Yet, it is intuitive that an electronic circuitry of very high order

could pose some practical problems, such as modal interferences that worsen spectral resolvability for sensor applications [see Fig. 4(b)], increased cost, measurement unreliability, and instability. It is also noteworthy that higher order PT electronic systems are rather robust to thermal and flicker noises [42]. Consequently, based on the above discussion, the optimum design for a specific application may be made by reaching a compromise among various aspects of the sensing system, including sensitivity, stability, physical dimension, and requisite coupling strength or wireless interrogation range.

To illustrate how the proposed concept can be applied to practical applications, we also performed a numerical study of a wireless temperature sensor interrogated using the DEP-enhanced telemetry technique, as sketched in Fig. 5 (insets). Here, we assume that the temperature sensor is formed by a negative temperature coefficient thermistors (NTC thermistors; NCP15XM221J03RC, Murata Electronics) [28] connected to an LC tank ($L = 4\mu H$ and $C = 113pF$), which is interrogated by the higher order PT -symmetric telemetry system. The resistance of the NTC thermistor $R = 220\ \Omega$ at $25\ ^\circ C$ (which corresponds to $\gamma = 0.855$) and drops to tens of ohms at $75\ ^\circ C$. For simplicity, we assume that the values of $|-R|$ and R on the reader and intermediators are the same as the resistance of the NTC thermistor. Fig. 5(a) and (b) shows the simulated reflection coefficients of the third- and ninth-order PT multimers with different ambient temperatures, respectively. The magnetic coupling strength is $\kappa = 0.95\kappa_D^{\text{odd}}$, where κ_D is $1/(2)^{1/2} (\approx 0.707)$ and $((5 - (5)^{1/2})/10)^{1/2} (\approx 0.526)$ for $N = 3$ and 9 , respectively; the eigenfrequency (resonance frequency) that corresponds to the greatest shift is highlighted with star and another eigenfrequency in the same eigenbranch is labeled with circle. It can be seen from Fig. 5 that near the DEPs, specific eigenfrequencies (resonance frequencies) drastically shift with increasing

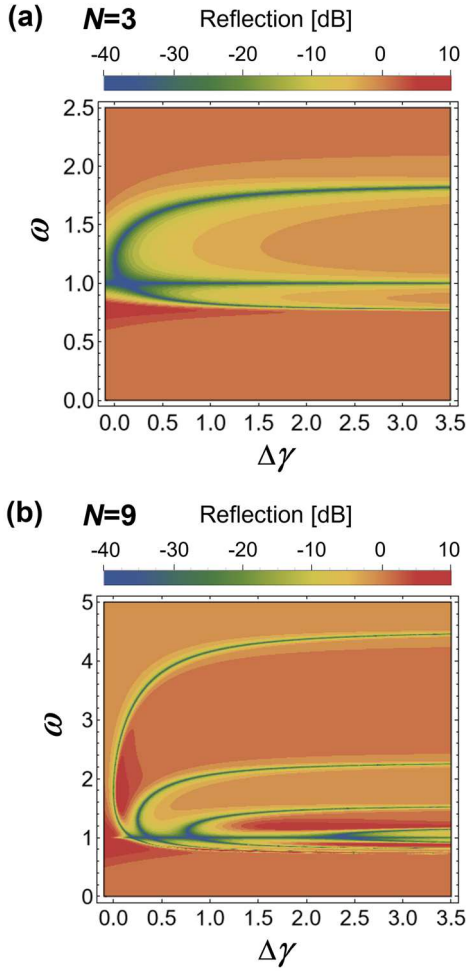


Fig. 4. Contours of reflection coefficients in logarithmic scale as functions of $\Delta\gamma$ ($\Delta\gamma = \gamma - \gamma_{EP+}^{(1)}$) and frequency ω for (a) third- and (b) ninth-order PT -symmetric electronic systems formed by series oscillators [type II in Fig. 1(a)]; here, $\gamma_{EP+}^{(1)}$ represents the first EPs found with the lowest value of γ (which also leads to the largest magnitude of bifurcation), the ratio of R or $| -R |$ to the generator impedance $\eta = R/Z_0 = 2$, and the coupling strength $\kappa = 1/2$.

the temperature, following the prediction in Figs. 3 and 4. We should note that the highlighted resonance frequencies in the third- and ninth-order PT systems exhibit the same frequency shift under temperature-dependent γ perturbations, implying that a PT trimer operating near the DEP is sufficient to achieve the optimum sensitivity. Although the critical coupling strength of the ninth-order PT multimer is reduced, it experiences more severe modal interferences, which may affect resolvability and data interpretation. The DEP-enhanced PT -symmetric telemetry can also be used to interrogate a reactive sensor and achieve similar performance because both R information and C information are simply encoded in the dimensionless γ [22]–[24], [63].

III. GENERALIZED NTH-ORDER PT -SYMMETRIC ELECTRONIC SYSTEMS WITH PARALLEL-TYPE OSCILLATORS

Here, we also examine the lower bound of DEP for electronic PT multimers constituted by parallel-type $-RLC$ and

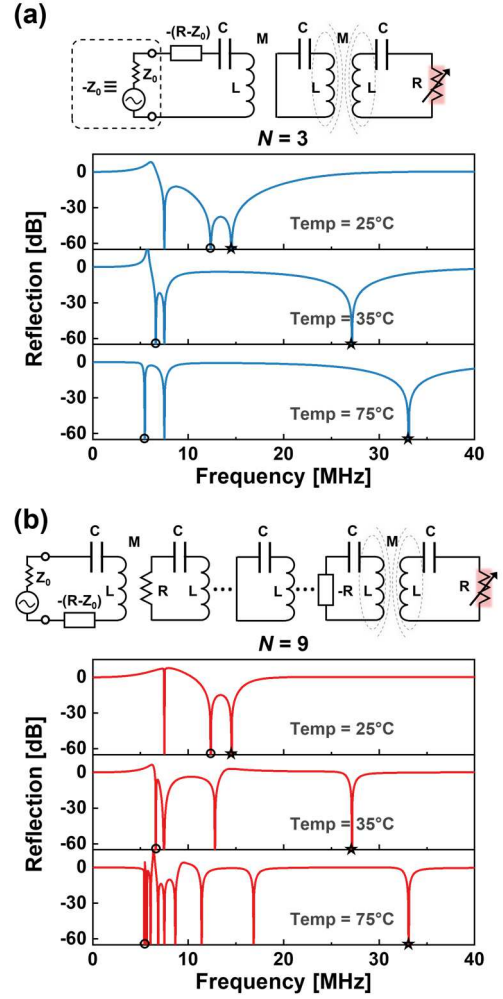


Fig. 5. Reflection spectra for the wireless temperature sensor interrogated by (a) third- and (b) ninth-order PT -symmetric telemetry methods. In both cases, one of the eigenbranches of the system is operated near the DEP. The eigenfrequency (resonance frequency) corresponding to the greatest shift is highlighted with star and another eigenfrequency on the same branch is highlighted with circle.

RLC oscillators. Eigenfrequencies of the even-order systems can be likewise derived from the zeros of recurring polynomials listed in the following:

$$\begin{aligned} U_0^{even} &= 1 \\ U_1^{even} &= x' - \kappa'^2 \\ U_{a+1}^{even} &= (x' - 2\kappa'^2)U_a^{even} - \kappa'^4 U_{a-1}^{even} \end{aligned} \quad (12a)$$

or

$$U_a^{even} = \prod_{\beta=0}^{a-1} x' - (2\kappa' A_{a\beta})^2 \quad (12b)$$

where

$$x' = \left(\omega^2 - \frac{1}{1 - 4\kappa^2 A_{a\beta}^2} \right)^2 + (\gamma\omega)^2 \quad (13)$$

$$\kappa' = \frac{\kappa}{1 - 4\kappa^2 A_{a\beta}^2} \quad (14)$$

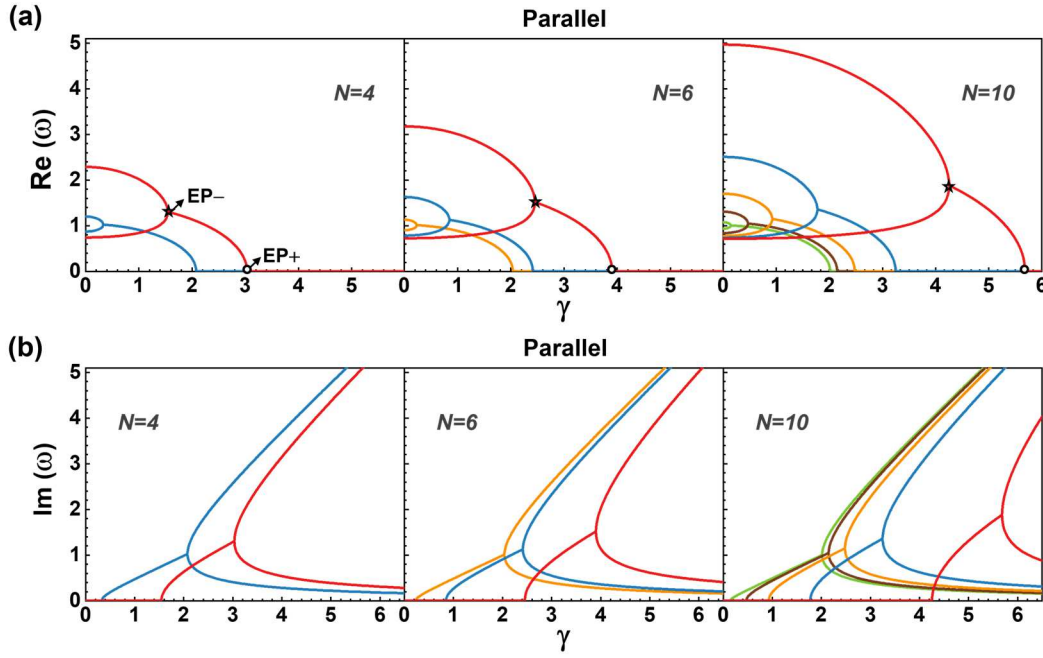


Fig. 6. (a) Real and (b) imaginary eigenspectra for the *PT*-symmetric systems with parallel-type oscillators of order $N = 4, 6$, and 10 ; here, κ is fixed to $1/2$. The eigenfrequencies with the largest variation are highlighted as stars in (a). The exact-*PT* phase is obtained when $\gamma \in [0, \gamma_{EP-}]$, whereas the system is in the broken-*PT* phase when $\gamma > \gamma_{EP-}$.

$\alpha = N/2, \beta = 0, 1, 2, \dots, \alpha - 1$, and the expression of $A_{\alpha\beta}$ can be found in (3). By solving $U_{\alpha}^{even} = 0$, the eigenfrequencies of the even-order system are given by

$$\omega_n^{even} = \pm \sqrt{\frac{2 - \gamma^2(1 - 4\kappa^2 A_{\alpha\beta}^2) \pm \sqrt{16\kappa^2 A_{\alpha\beta}^2 - 4\gamma^2(1 - 4\kappa^2 A_{\alpha\beta}^2) + \gamma^4(1 - 4\kappa^2 A_{\alpha\beta}^2)^2}}{2(1 - 4\kappa^2 A_{\alpha\beta}^2)}}. \quad (15)$$

From (15), N EPs of the system are obtained as

$$\gamma_{EP\pm}^{even} = \sqrt{\frac{2(1 \pm \sqrt{1 - 4\kappa^2 A_{\alpha\beta}^2})}{1 - 4\kappa^2 A_{\alpha\beta}^2}}. \quad (16)$$

This clearly points out the DEP condition given by

$$\kappa_D^{even} = \frac{1}{2A_{\alpha\beta}}. \quad (17)$$

Such a result is similar to what obtained in the even-numbered system with series-type oscillators (6), implying that the DEP condition for the even-order electronic *PT* multimers is determined only by the number and arrangement of gain and loss and is independent of whether each lumped oscillator is in series or parallel connection. Here, we should note that the DEP does not exist in an electronic *PT* trimer composed of parallel-type oscillators. When $N \geq 5$, the necessary DEP conditions and κ_D for the odd-order electronic *PT* multimers formed by parallel oscillators are the same as those formed by series oscillators of order $(N - 1)/2$. This makes odd-order *PT* systems with parallel oscillators less interesting for sensing applications because they demand more oscillators to achieve similar DEP-enabling divergent bifurcation effect.

Fig. 6(a) and (b) shows real and imaginary eigenspectra, respectively, for even-order *PT*-symmetric systems formed by multiple parallel *-RLC* and *RLC* oscillators with $\kappa = 1/2$. Similar to what was observed for the *PT* multimers with series oscillators, when the number of oscillatory elements increases, γ_{EP+}^{even} and γ_{EP-}^{even} will degenerate at the DEP, and κ_D^{even} will ultimately reach the lower bound, leading to a drastic eigenfrequency splitting [see Fig. 6(a)]. Again, as shown in Fig. 7, increasing the order of electronic circuitry also leads to a complex, multiresonance system with several cluttered resonant dips in the reflection spectrum. Given the modal interferences, it is sometimes problematic to resolve and track drifts in resonance frequencies.

IV. GENERALIZED NTH-ORDER *PT*-SYMMETRIC ELECTRONIC SYSTEMS WITH GAIN-NEUTRAL ARRAY-LOSS STRUCTURES

The above results can be extended to type III of electronic *PT* multimer systems in Fig. 1(a), where an active *-RLC* oscillator and an *RLC* oscillator are coupled via $N - 2$ ($N > 3$) neutral intermediators. Such a telemetry method is commonly adopted in real-world wireless sensing and power transfer systems. Through mathematical investigations, we figure out that by setting the coupling strength between each neutral intermediary to be κ and that between the first/last oscillator and its neighboring intermediary to be $(2)^{1/2}\kappa$, the system's eigenfrequencies can still be roots of the following transcendental equations that have recursive relations:

$$(x^2 - 4\kappa^2)V_{\alpha}^{even} = 0 \text{ and } \alpha = (N - 2)/2 \quad (18a)$$

for $N = 4, 6, 8, \dots$

$$(\omega^2 - 1)(x^2 - 4\kappa^2)V_{\alpha}^{odd} = 0 \text{ and } \alpha = (N - 3)/2 \quad (18b)$$

for $N = 5, 7, 9, \dots$

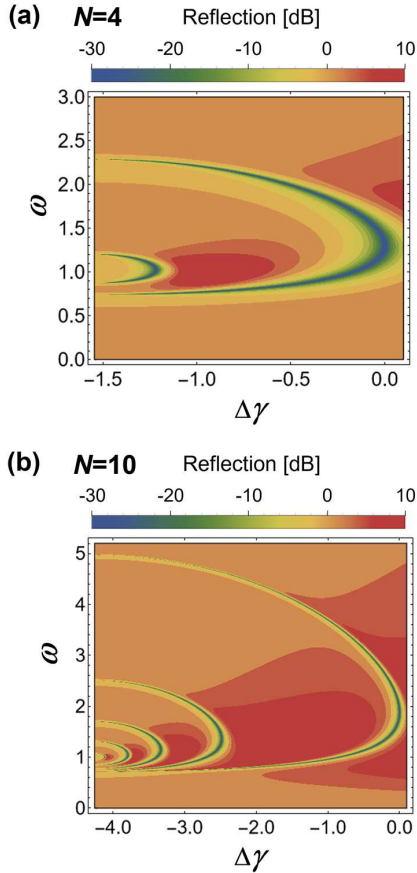


Fig. 7. Similar to Fig. 4, but for (a) fourth- and (b) tenth-order *PT*-symmetric electronic systems formed by parallel oscillators [type I in Fig. 1(a)]; here, $\Delta\gamma = \gamma - \gamma_{EP-}^{N/2}$, where $\gamma_{EP-}^{N/2}$ is the EP found with the maximum bifurcation effect, the ratio of R or $|-R|$ to the generator impedance $\eta = R/Z_0 = 1/2$, and the coupling strength $\kappa = 1/2$.

where

$$V_{\alpha}^{even} = \prod_{\beta=0}^{\alpha-1} (1 - 1/\omega^2)^2 - (2\kappa A_{\alpha\beta})^2 \quad (19a)$$

$$V_{\alpha}^{odd} = \prod_{\beta=0}^{\alpha-1} (1 - 1/\omega^2)^2 - (2\kappa B_{\alpha\beta})^2 \quad (19b)$$

$\beta = 0, 1, 2, \dots, \alpha - 1$, and the expressions for x , $A_{\alpha\beta}$, and $B_{\alpha\beta}$ can be found in (2), (3), and (8), respectively. The eigenfrequencies of these systems with series-type oscillators can be derived as

$$\omega_n^{even} = \pm \sqrt{\frac{2\gamma^2 - 1 \pm \sqrt{1 - 4\gamma^2 + 16\gamma^4\kappa^2}}{2\gamma^2(1 - 4\kappa^2)}}, \quad \pm \frac{1}{\sqrt{1 \pm 2\kappa A_{\alpha\beta}}} \quad (20a)$$

$$\omega_n^{odd} = \pm 1, \pm \sqrt{\frac{2\gamma^2 - 1 \pm \sqrt{1 - 4\gamma^2 + 16\gamma^4\kappa^2}}{2\gamma^2(1 - 4\kappa^2)}}, \quad \pm \frac{1}{\sqrt{1 \pm 2\kappa B_{\alpha\beta}}} \quad (20b)$$

As can be observed from (20), the systems exhibit γ -dependent eigenfrequencies that are identical to those of

an electronic *PT* trimer in [46] [i.e., applying $N = 3$ to (9)]. In addition, there are $N - 2$ positive γ -independent and nonbifurcated eigenfrequencies bounded in the range: $\omega_n \in [1/(1 + 2\kappa)^{1/2}, 1/(1 - 2\kappa)^{1/2}]$. These modes are of little interest for sensor applications. Thus, if one gets rid of these stationary roots, a bifurcation effect similar to that of the electronic *PT* trimer can be obtained. In this scenario, although adding more intermediate elements can increase the total wireless interrogation distance, it has no effect on tailoring the DEP and the slope of splitting for the bifurcating eigenfrequencies, while the stationary and cluttered eigenfrequencies could represent a drawback for data acquisition. Here, we should note that DEP does not exist in the type-III *PT* multimer systems with parallel $-RLC$ and RLC oscillators, analogous to what observed in the *PT* trimer composed of parallel oscillators. Finally, we should emphasize that the proposed *PT*-symmetric electronic multimers are stable in the exact-*PT* phase due to their purely real eigenfrequencies. Stability is of great significance for RF telemetry systems, especially when active components are involved [64]–[66]. The stability dynamics in the exact-*PT* phase have been discussed in [66]–[68]. In the broken-*PT* phase, the imaginary part of eigenfrequency may lead to unbounded and unstable behavior of the system [27], [66]–[69]. Nevertheless, this does not influence the robustness of the proposed *PT*-multimer sensing systems operating in the exact-*PT* phase.

V. CONCLUSION

In summary, we have presented the theoretical basis for the generalized *PT*-symmetric electronic multimers and showed that the complex eigenfrequencies can exhibit a divergent singularity near the DEP. The DEP allows an ever-booster branching effect in the eigenfrequencies or resonance frequencies of the system, which can be more drastic than any bifurcation process around a regular EP. The potential of DEP in sensor telemetry applications has been experimentally demonstrated with the *PT* trimer structure [1]. In this work, we have comprehensively studied different categories of structural topologies for *PT*-symmetric electronic multimers beyond the common dimer and trimer structures and have derived the lower bound on the DEP in terms of the inductive coupling strength. Even though adopting a very high-order multimer can help reduce the inductive coupling strength (inductive coupling coefficient) necessary for discovering the DEP, the minimally required value cannot be lower than 1/2. On the other hand, the appearance of higher order resonant modes could be a potential cause of modal interferences in data interpretation, thus affecting the assessing resolvability and accuracy for applications in wireless sensing and telemetering. Our results will provide universally applicable guidelines for the design of short-/mid-range wireless sensor and actuator systems, which are keys to the future ubiquitous sensing, the IoT, and industry 4.0 systems.

REFERENCES

- [1] R. El-Ganainy, K. G. Makris, D. N. Christodoulides, and Z. H. Musslimani, "Theory of coupled optical *PT*-symmetric structures," *Opt. Lett.*, vol. 32, no. 17, p. 2632, Sep. 2007, doi: 10.1364/OL.32.002632.

- [2] A. Guo *et al.*, "Observation of PT -symmetry breaking in complex optical potentials," *Phys. Rev. Lett.*, vol. 103, no. 9, Aug. 2009, Art. no. 093902, doi: [10.1103/PhysRevLett.103.093902](https://doi.org/10.1103/PhysRevLett.103.093902).
- [3] C. E. Rüter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev, and D. Kip, "Observation of parity-time symmetry in optics," *Nature Phys.*, vol. 6, no. 3, pp. 192–195, Mar. 2010, doi: [10.1038/nphys1515](https://doi.org/10.1038/nphys1515).
- [4] Y. D. Chong, L. Ge, and A. D. Stone, "PT-symmetry breaking and laser-absorber modes in optical scattering systems," *Phys. Rev. Lett.*, vol. 106, no. 9, Mar. 2011, Art. no. 093902, doi: [10.1103/PhysRevLett.106.093902](https://doi.org/10.1103/PhysRevLett.106.093902).
- [5] Z. Lin, H. Ramezani, T. Eichelkraut, T. Kottos, H. Cao, and D. N. Christodoulides, "Unidirectional invisibility induced by PT -symmetric periodic structures," *Phys. Rev. Lett.*, vol. 106, no. 21, May 2011, Art. no. 213901, doi: [10.1103/PhysRevLett.106.213901](https://doi.org/10.1103/PhysRevLett.106.213901).
- [6] L. Feng *et al.*, "Experimental demonstration of a unidirectional reflectionless parity-time metamaterial at optical frequencies," *Nature Mater.*, vol. 12, no. 2, pp. 108–113, Feb. 2013, doi: [10.1038/nmat3495](https://doi.org/10.1038/nmat3495).
- [7] S. Savoia, G. Castaldi, V. Galdi, A. Alù, and N. Engheta, "Tunneling of obliquely incident waves through PT -symmetric epsilon-near-zero bilayers," *Phys. Rev. B, Condens. Matter*, vol. 89, no. 8, Feb. 2014, Art. no. 085105, doi: [10.1103/PhysRevB.89.085105](https://doi.org/10.1103/PhysRevB.89.085105).
- [8] S. Savoia, G. Castaldi, and V. Galdi, "Non-Hermiticity-induced wave confinement and guiding in loss-gain-loss three-layer systems," *Phys. Rev. A, Gen. Phys.*, vol. 94, no. 4, Oct. 2016, Art. no. 043838, doi: [10.1103/PhysRevA.94.043838](https://doi.org/10.1103/PhysRevA.94.043838).
- [9] R. El-Ganainy, K. G. Makris, M. Khajavikhan, Z. H. Musslimani, S. Rotter, and D. N. Christodoulides, "Non-hermitian physics and PT symmetry," *Nature Phys.*, vol. 14, no. 1, pp. 11–19, Jan. 2018, doi: [10.1038/nphys4323](https://doi.org/10.1038/nphys4323).
- [10] M.-A. Miri and A. Alù, "Exceptional points in optics and photonics," *Science*, vol. 363, no. 6422, Jan. 2019, doi: [10.1126/science.aar7709](https://doi.org/10.1126/science.aar7709).
- [11] A. F. Abdelshafy, M. A. K. Othman, D. Oshmarin, A. T. Almutawa, and F. Capolino, "Exceptional points of degeneracy in periodic coupled waveguides and the interplay of gain and radiation loss: Theoretical and experimental demonstration," *IEEE Trans. Antennas Propag.*, vol. 67, no. 11, pp. 6909–6923, Nov. 2019, doi: [10.1109/TAP.2019.2922778](https://doi.org/10.1109/TAP.2019.2922778).
- [12] M. A. K. Othman and F. Capolino, "Theory of exceptional points of degeneracy in uniform coupled waveguides and balance of gain and loss," *IEEE Trans. Antennas Propag.*, vol. 65, no. 10, pp. 5289–5302, Oct. 2017, doi: [10.1109/TAP.2017.2738063](https://doi.org/10.1109/TAP.2017.2738063).
- [13] Y. Ra'Di *et al.*, "On-site wireless power generation," *IEEE Trans. Antennas Propag.*, vol. 66, no. 8, pp. 4260–4268, Aug. 2018, doi: [10.1109/TAP.2018.2835560](https://doi.org/10.1109/TAP.2018.2835560).
- [14] P.-Y. Chen and J. Jung, "PT symmetry and singularity-enhanced sensing based on photoexcited graphene metasurfaces," *Phys. Rev. A, Gen. Phys.*, vol. 5, no. 6, Jun. 2016, Art. no. 064018, doi: [10.1103/PhysRevApplied.5.064018](https://doi.org/10.1103/PhysRevApplied.5.064018).
- [15] M. Sakhdari, M. Farhat, and P.-Y. Chen, " PT -symmetric metasurfaces: Wave manipulation and sensing using singular points," *New J. Phys.*, vol. 19, no. 6, Jun. 2017, Art. no. 065002, doi: [10.1088/1367-2630/aa6bb9](https://doi.org/10.1088/1367-2630/aa6bb9).
- [16] X. Zhu, H. Ramezani, C. Shi, J. Zhu, and X. Zhang, " PT -symmetric acoustics," *Phys. Rev. X*, vol. 4, no. 3, Sep. 2014, Art. no. 031042, doi: [10.1103/PhysRevX.4.031042](https://doi.org/10.1103/PhysRevX.4.031042).
- [17] R. Fleury, D. Sounas, and A. Alù, "An invisible acoustic sensor based on parity-time symmetry," *Nature Commun.*, vol. 6, no. 1, pp. 1–7, May 2015, doi: [10.1038/ncomms6905](https://doi.org/10.1038/ncomms6905).
- [18] R. Fleury, D. L. Sounas, and A. Alu, "Parity-time symmetry in acoustics: Theory, devices, and potential applications," *IEEE J. Sel. Topics Quantum Electron.*, vol. 22, no. 5, pp. 121–129, Sep. 2016, doi: [10.1109/JSTQE.2016.2549512](https://doi.org/10.1109/JSTQE.2016.2549512).
- [19] M. Farhat, P. Y. Chen, S. Guenneau, and Y. Wu, "CPA-lasing in thin-elastic plates via exceptional points," 2020, *arXiv:2007.01674*.
- [20] J. Schindler, Z. Lin, J. M. Lee, H. Ramezani, F. M. Ellis, and T. Kottos, "PT-symmetric electronics," *J. Phys. A, Math. Theor.*, vol. 45, no. 44, Art. no. 44, Nov. 2012, doi: [10.1088/1751-8113/45/44/444029](https://doi.org/10.1088/1751-8113/45/44/444029).
- [21] Y. Ra'di, D. L. Sounas, A. Alù, and S. A. Tretyakov, "Parity-time-symmetric teleportation," *Phys. Rev. B, Condens. Matter*, vol. 93, no. 23, Jun. 2016, Art. no. 235427, doi: [10.1103/PhysRevB.93.235427](https://doi.org/10.1103/PhysRevB.93.235427).
- [22] P.-Y. Chen *et al.*, "Generalized parity-time symmetry condition for enhanced sensor telemetry," *Nature Electron.*, vol. 1, no. 5, pp. 297–304, May 2018, doi: [10.1038/s41928-018-0072-6](https://doi.org/10.1038/s41928-018-0072-6).
- [23] M. Sakhdari, M. Hajizadegan, Y. Li, M. M.-C. Cheng, J. C. H. Hung, and P.-Y. Chen, "Ultrasensitive, parity-time-symmetric wireless reactive and resistive sensors," *IEEE Sensors J.*, vol. 18, no. 23, pp. 9548–9555, Dec. 2018, doi: [10.1109/JSEN.2018.2870322](https://doi.org/10.1109/JSEN.2018.2870322).
- [24] M. Hajizadegan, M. Sakhdari, S. Liao, and P.-Y. Chen, "High-sensitivity wireless displacement sensing enabled by PT -symmetric telemetry," *IEEE Trans. Antennas Propag.*, vol. 67, no. 5, pp. 3445–3449, May 2019, doi: [10.1109/TAP.2019.2905892](https://doi.org/10.1109/TAP.2019.2905892).
- [25] Z. Dong, Z. Li, F. Yang, C.-W. Qiu, and J. S. Ho, "Sensitive readout of implantable microsensors using a wireless system locked to an exceptional point," *Nature Electron.*, vol. 2, no. 8, pp. 335–342, Aug. 2019, doi: [10.1038/s41928-019-0284-4](https://doi.org/10.1038/s41928-019-0284-4).
- [26] Z. Ye, M. Farhat, and P.-Y. Chen, "Tunability and switching of Fano and Lorentz resonances in PT -symmetric electronic systems," *Appl. Phys. Lett.*, vol. 117, no. 3, Jul. 2020, Art. no. 031101, doi: [10.1063/5.0014919](https://doi.org/10.1063/5.0014919).
- [27] M. Sakhdari, M. Hajizadegan, and P.-Y. Chen, "Robust extended-range wireless power transfer using a higher-order PT -symmetric platform," *Phys. Rev. Res.*, vol. 2, no. 1, Feb. 2020, Art. no. 013152, doi: [10.1103/PhysRevResearch.2.013152](https://doi.org/10.1103/PhysRevResearch.2.013152).
- [28] M. Yang, Z. Ye, M. Farhat, and P.-Y. Chen, "Ultrarobust wireless interrogation for sensors and transducers: A non-hermitian telemetry technique," *IEEE Trans. Instrum. Meas.*, vol. 70, pp. 1–9, 2021, doi: [10.1109/TIM.2021.3107057](https://doi.org/10.1109/TIM.2021.3107057).
- [29] G. W. Hanson, A. B. Yakovlev, M. A. K. Othman, and F. Capolino, "Exceptional points of degeneracy and branch points for coupled transmission lines—Linear-algebra and bifurcation theory perspectives," *IEEE Trans. Antennas Propag.*, vol. 67, no. 2, pp. 1025–1034, Feb. 2019, doi: [10.1109/TAP.2018.2879761](https://doi.org/10.1109/TAP.2018.2879761).
- [30] P.-Y. Chen and R. El-Ganainy, "Exceptional points enhance wireless readout," *Nature Electron.*, vol. 2, no. 8, pp. 323–324, Aug. 2019, doi: [10.1038/s41928-019-0293-3](https://doi.org/10.1038/s41928-019-0293-3).
- [31] H. Ramezani, T. Kottos, R. El-Ganainy, and D. N. Christodoulides, "Unidirectional nonlinear PT -symmetric optical structures," *Phys. Rev. A, Gen. Phys.*, vol. 82, no. 4, Oct. 2010, Art. no. 043803, doi: [10.1103/PhysRevA.82.043803](https://doi.org/10.1103/PhysRevA.82.043803).
- [32] R. Fleury, D. L. Sounas, and A. Alù, "Negative refraction and planar focusing based on parity-time symmetric metasurfaces," *Phys. Rev. Lett.*, vol. 113, no. 2, Jul. 2014, Art. no. 023903, doi: [10.1103/PhysRevLett.113.023903](https://doi.org/10.1103/PhysRevLett.113.023903).
- [33] S. Longhi, " PT -symmetric laser absorber," *Phys. Rev. A, Gen. Phys.*, vol. 82, no. 3, Sep. 2010, Art. no. 031801, doi: [10.1103/PhysRevA.82.031801](https://doi.org/10.1103/PhysRevA.82.031801).
- [34] M. Sakhdari, N. M. Estakhri, H. Bagci, and P.-Y. Chen, "Low-threshold lasing and coherent perfect absorption in generalized PT -symmetric optical structures," *Phys. Rev. A, Gen. Phys.*, vol. 10, no. 2, Aug. 2018, Art. no. 024030, doi: [10.1103/PhysRevApplied.10.024030](https://doi.org/10.1103/PhysRevApplied.10.024030).
- [35] Z. J. Wong *et al.*, "Lasing and anti-lasing in a single cavity," *Nature Photon.*, vol. 10, no. 12, pp. 796–801, Dec. 2016, doi: [10.1038/nphoton.2016.216](https://doi.org/10.1038/nphoton.2016.216).
- [36] M. Farhat, M. Yang, Z. Ye, and P.-Y. Chen, "PT-symmetric absorber-laser enables electromagnetic sensors with unprecedented sensitivity," *ACS Photon.*, vol. 7, no. 8, pp. 2080–2088, Aug. 2020, doi: [10.1021/acsp Photonics.0c00514](https://doi.org/10.1021/acsp Photonics.0c00514).
- [37] M. Yang, Z. Ye, M. Farhat, and P.-Y. Chen, "Enhanced radio-frequency sensors based on a self-dual emitter-absorber," *Phys. Rev. A, Gen. Phys.*, vol. 15, no. 1, Jan. 2021, Art. no. 014026, doi: [10.1103/PhysRevApplied.15.014026](https://doi.org/10.1103/PhysRevApplied.15.014026).
- [38] Z. Ye, M. Yang, L. Zhu, and P.-Y. Chen, "PTX-symmetric metasurfaces for sensing applications," *Frontiers Optoelectron.*, vol. 14, no. 2, pp. 211–220, Jun. 2021, doi: [10.1007/s12200-021-1204-6](https://doi.org/10.1007/s12200-021-1204-6).
- [39] M. Farhat, W. W. Ahmad, A. Khelif, K. N. Salama, and Y. Wu, "Enhanced acoustic pressure sensors based on coherent perfect absorber-laser effect," *J. Appl. Phys.*, vol. 129, no. 10, Mar. 2021, Art. no. 104902, doi: [10.1063/5.0041771](https://doi.org/10.1063/5.0041771).
- [40] P.-Y. Chen, "High-order parity-time-symmetric electromagnetic sensors," in *Proc. Int. Conf. Electromagn. Adv. Appl. (ICEAA)*, Sep. 2018, pp. 460–463, doi: [10.1109/ICEAA.2018.8520393](https://doi.org/10.1109/ICEAA.2018.8520393).
- [41] H. Hodaei *et al.*, "Enhanced sensitivity at higher-order exceptional points," *Nature*, vol. 548, no. 7666, pp. 187–191, Aug. 2017, doi: [10.1038/nature23280](https://doi.org/10.1038/nature23280).
- [42] Z. Xiao, H. Li, T. Kottos, and A. Alù, "Enhanced sensing and nondegraded thermal noise performance based on PT -symmetric electronic circuits with a sixth-order exceptional point," *Phys. Rev. Lett.*, vol. 123, no. 21, Nov. 2019, Art. no. 213901, doi: [10.1103/PhysRevLett.123.213901](https://doi.org/10.1103/PhysRevLett.123.213901).

- [43] M. Yang, Z. Ye, M. Farhat, and P.-Y. Chen, "Cascaded PT-symmetric artificial sheets: Multimodal manipulation of self-dual emitter-absorber singularities, and unidirectional and bidirectional reflectionless transparencies," *J. Phys. D, Appl. Phys.*, vol. 55, no. 8, Nov. 2021, Art. no. 085301, doi: [10.1088/1361-6463/ac3300](https://doi.org/10.1088/1361-6463/ac3300).
- [44] W. Langbein, "No exceptional precision of exceptional-point sensors," *Phys. Rev. A, Gen. Phys.*, vol. 98, no. 2, Aug. 2018, Art. no. 023805, doi: [10.1103/PhysRevA.98.023805](https://doi.org/10.1103/PhysRevA.98.023805).
- [45] N. A. Mortensen, P. A. D. Gonçalves, M. Khajavikhan, D. N. Christodoulides, C. Tserkezis, and C. Wolff, "Fluctuations and noise-limited sensing near the exceptional point of parity-time-symmetric resonator systems," *Optica*, vol. 5, no. 10, p. 1342, Oct. 2018, doi: [10.1364/OPTICA.5.001342](https://doi.org/10.1364/OPTICA.5.001342).
- [46] M. Sakhdari, M. Hajizadegan, Q. Zhong, D. N. Christodoulides, R. El-Ganainy, and P.-Y. Chen, "Experimental observation of PT symmetry breaking near divergent exceptional points," *Phys. Rev. Lett.*, vol. 123, no. 19, Nov. 2019, Art. no. 193901, doi: [10.1103/PhysRevLett.123.193901](https://doi.org/10.1103/PhysRevLett.123.193901).
- [47] K. G. Ong, C. A. Grimes, C. L. Robbins, and R. S. Singh, "Design and application of a wireless, passive, resonant-circuit environmental monitoring sensor," *Sens. Actuators A, Phys.*, vol. 93, no. 1, pp. 33–43, Aug. 2001, doi: [10.1016/S0924-4247\(01\)00624-0](https://doi.org/10.1016/S0924-4247(01)00624-0).
- [48] S. Novikov, N. Lebedeva, A. Satrapinski, J. Walden, V. Davydov, and A. Lebedev, "Graphene based sensor for environmental monitoring of NO₂," *Sens. Actuators B, Chem.*, vol. 236, pp. 1054–1060, Nov. 2016, doi: [10.1016/j.snb.2016.05.114](https://doi.org/10.1016/j.snb.2016.05.114).
- [49] J. M. López-Higuera, L. Rodríguez-Cobo, A. Q. Incera, and A. Cobo, "Fiber optic sensors in structural health monitoring," *J. Lightw. Technol.*, vol. 29, no. 4, pp. 587–608, Feb. 15, 2011, doi: [10.1109/JLT.2011.2106479](https://doi.org/10.1109/JLT.2011.2106479).
- [50] R. W. Ngigi, C. Pislariu, A. Ball, and F. Gu, "Modern techniques for conduction monitoring of railway vehicle dynamics," *J. Phys., Conf.*, vol. 364, May 2012, Art. no. 012016, doi: [10.1088/1742-6596/364/1/012016](https://doi.org/10.1088/1742-6596/364/1/012016).
- [51] S. Rodriguez, S. Ollmar, M. Waqar, and A. Rusu, "A batteryless sensor ASIC for implantable bio-impedance applications," *IEEE Trans. Biomed. Circuits Syst.*, vol. 10, no. 3, pp. 533–544, Jun. 2016, doi: [10.1109/TBCAS.2015.2456242](https://doi.org/10.1109/TBCAS.2015.2456242).
- [52] L. Y. Chen *et al.*, "Continuous wireless pressure monitoring and mapping with ultra-small passive sensors for health monitoring and critical care," *Nature Commun.*, vol. 5, p. 5028, Dec. 2014, doi: [10.1038/ncomms6028](https://doi.org/10.1038/ncomms6028).
- [53] P.-J. Chen, S. Saati, R. Varma, M. S. Humayun, and Y.-C. Tai, "Wireless intracocular pressure sensing using microfabricated minimally invasive flexible-coiled LC sensor implant," *J. Microelectromech. Syst.*, vol. 19, no. 4, pp. 721–734, Aug. 2010, doi: [10.1109/JMEMS.2010.2049825](https://doi.org/10.1109/JMEMS.2010.2049825).
- [54] S.-Y. Wu, C. Yang, W. Hsu, and L. Lin, "3D-printed microelectronics for integrated circuitry and passive wireless sensors," *Microsyst. Nanoeng.*, vol. 1, no. 1, pp. 1–9, Jul. 2015, doi: [10.1038/micronano.2015.13](https://doi.org/10.1038/micronano.2015.13).
- [55] A. C. Godoy and I. G. Pérez, "Integration of sensor and actuator networks and the SCADA system to promote the migration of the legacy flexible manufacturing system towards the industry 4.0 concept," *J. Sensor Actuator Netw.*, vol. 7, no. 2, p. 23, May 2018, doi: [10.3390/jsan7020023](https://doi.org/10.3390/jsan7020023).
- [56] C.-C. Lin, D.-J. Deng, Z.-Y. Chen, and K.-C. Chen, "Key design of driving industry 4.0: Joint energy-efficient deployment and scheduling in group-based industrial wireless sensor networks," *IEEE Commun. Mag.*, vol. 54, no. 10, pp. 46–52, Oct. 2016, doi: [10.1109/MCOM.2016.7588228](https://doi.org/10.1109/MCOM.2016.7588228).
- [57] C. Zhu, V. C. M. Leung, L. Shu, and E. C.-H. Ngai, "Green Internet of Things for smart world," *IEEE Access*, vol. 3, pp. 2151–2162, 2015, doi: [10.1109/ACCESS.2015.2497312](https://doi.org/10.1109/ACCESS.2015.2497312).
- [58] X. Zhang, X. Zhang, and L. Han, "An energy efficient Internet of Things network using restart artificial bee colony and wireless power transfer," *IEEE Access*, vol. 7, pp. 12686–12695, 2019, doi: [10.1109/ACCESS.2019.2892798](https://doi.org/10.1109/ACCESS.2019.2892798).
- [59] M. Swan, "Sensor mania! The Internet of Things, wearable computing, objective metrics, and the quantified self 2.0," *J. Sensor Actuator Netw.*, vol. 1, no. 3, pp. 217–253, Nov. 2012, doi: [10.3390/jsan1030217](https://doi.org/10.3390/jsan1030217).
- [60] Q.-A. Huang, L. Dong, and L.-F. Wang, "LC passive wireless sensors toward a wireless sensing platform: Status, prospects, and challenges," *J. Microelectromech. Syst.*, vol. 25, no. 5, pp. 822–841, Oct. 2016, doi: [10.1109/JMEMS.2016.2602298](https://doi.org/10.1109/JMEMS.2016.2602298).
- [61] R. Nopper, R. Niekrawietz, and L. Reindl, "Wireless readout of passive LC sensors," *IEEE Trans. Instrum. Meas.*, vol. 59, no. 9, pp. 2450–2457, Sep. 2010, doi: [10.1109/TIM.2009.2032966](https://doi.org/10.1109/TIM.2009.2032966).
- [62] J. Bai and B. Zhou, "Titanium dioxide nanomaterials for sensor applications," *Chem. Rev.*, vol. 114, pp. 10131–10176, Oct. 2014, doi: [10.1021/cr400625j](https://doi.org/10.1021/cr400625j).
- [63] M. Yang, Z. Ye, and P.-Y. Chen, "A quantum-inspired biotelemetry system for robust and ultrasensitive wireless intracranial pressure monitoring," in *Proc. IEEE Sensors*, Oct. 2021, pp. 1–4, doi: [10.1109/SENSOR547087.2021.9639684](https://doi.org/10.1109/SENSOR547087.2021.9639684).
- [64] X.-L. Zhang, S. Wang, B. Hou, and C. T. Chan, "Dynamically encircling exceptional points: *In situ* control of encircling loops and the role of the starting point," *Phys. Rev. X*, vol. 8, no. 2, Jun. 2018, Art. no. 021066, doi: [10.1103/PhysRevX.8.021066](https://doi.org/10.1103/PhysRevX.8.021066).
- [65] Y. Zhiyenbayev, Y. Kominis, C. Valagiannopoulos, V. Kovanis, and A. Bountis, "Enhanced stability, bistability, and exceptional points in saturable active photonic couplers," *Phys. Rev. A, Gen. Phys.*, vol. 100, no. 4, Oct. 2019, Art. no. 043834, doi: [10.1103/PhysRevA.100.043834](https://doi.org/10.1103/PhysRevA.100.043834).
- [66] B. Yerezhep and C. Valagiannopoulos, "Approximate stability dynamics of concentric cylindrical metasurfaces," *IEEE Trans. Antennas Propag.*, vol. 69, no. 9, pp. 5716–5724, Sep. 2021, doi: [10.1109/TAP.2021.3060124](https://doi.org/10.1109/TAP.2021.3060124).
- [67] H. Ramezani, J. Schindler, F. M. Ellis, U. Günther, and T. Kottos, "Bypassing the bandwidth theorem with PT symmetry," *Phys. Rev. A, Gen. Phys.*, vol. 85, no. 6, Jun. 2012, Art. no. 062122, doi: [10.1103/PhysRevA.85.062122](https://doi.org/10.1103/PhysRevA.85.062122).
- [68] J. Wiersig, "Review of exceptional point-based sensors," *Photon. Res.*, vol. 8, no. 9, pp. 1457–1467, Sep. 2020, doi: [10.1364/PRJ.396115](https://doi.org/10.1364/PRJ.396115).
- [69] M. I. Molina, "The fractional nonlinear PT dimer," *Sci. Rep.*, vol. 11, no. 1, pp. 1–8, May 2021, doi: [10.1038/s41598-021-89484-x](https://doi.org/10.1038/s41598-021-89484-x).



Maryam Sakhdari received the B.Sc. degree in electronics from the Iran University of Science and Technology, Tehran, Iran, in 2010, the M.Sc. degree from Tarbiat Modares University, Tehran, in 2013, and the Ph.D. degree in electrical engineering from the University of Illinois at Chicago, Chicago, IL, USA, in 2020.

She had been a Research Assistant with Wayne State University, Detroit, MI, USA, and the University of Illinois at Chicago from 2015 to 2020, while her research interests include low-noise harmonic transponder sensors; ultrasensitive PT symmetric wireless sensors; electromagnetic metamaterials and plasmonics; and their RF, THz, infrared, and optical applications.



Zhilu Ye received the B.S. degree in microelectronics from the Huazhong University of Science and Technology, Wuhan, China, in 2018, and the master's degree in electrical engineering from Arizona State University, Tempe, AZ, USA, in 2019. She is currently pursuing the Ph.D. degree with the University of Illinois at Chicago, Chicago, IL, USA.

Her research interests include RF and microwave sensors, wireless communication, and wearable electronics.



Mohamed Farhat received the master's degree in theoretical physics and the Ph.D. degree in optics and electromagnetism from Aix-Marseille University, Marseille, France. His Ph.D. dissertation was entitled Metamaterials for Harmonic and Biharmonic Cloaking and Superlensing.

He is currently a Research Scientist with the King Abdullah University of Science and Technology (KAUST), Thuwal, Saudi Arabia. He has authored over 170 publications, including one edited book, 85 journal articles, seven book chapters, and five international patents, as of August 2021. His research is in the fields of plasmonics and metamaterials with applications spanning optics and acoustics waves.

Dr. Farhat has organized several special sessions at META conferences and is an Active Reviewer of many international journals in physics, including *Physical Review Letters* and *Nature Physics*. He has also co-edited the book *Transformation Wave Physics: Electromagnetics, Elastodynamics, and Thermodynamics* (Pan Stanford Publishing).



Pai-Yen Chen (Senior Member, IEEE) received the B.S. and M.S. degrees from National Chiao Tung University, Hsinchu, Taiwan, in 2006 and 2004, respectively, and the Ph.D. degree from the University of Texas at Austin, Austin, TX, USA, in 2013.

He was a Research Scientist with the Intellectual Ventures' Metamaterials Commercialization Center, Bellevue, WA, USA, from 2013 to 2014; and a Research Staff with National Nano Device Laboratories, Hsinchu, from 2006 to 2009. He is currently an Associate Professor with the Department of Electrical

and Computer Engineering, University of Illinois at Chicago, Chicago, IL, USA. He has been involved in multidisciplinary research on applied electromagnetics, RF/microwave antennas and circuits, wireless micro/nano-sensors and integrated systems, and nanoelectromagnetism in plasmonics and nanophotonics.

Dr. Chen has received quite a few prestigious awards, including the National Science Foundation (NSF) CAREER Award, the IEEE Sensors Council Young Professional Award, the IEEE Raj Mittra Travel Grant (RMTG) Award, the SPIE Rising Researcher Award, the ACES Early Career Award, the PIERS Young Professional Award, the IOP Measurement Science and Technology

Emerging Leader, the Young Scientist Awards from URSI General Assembly and URSI Commission B: Electromagnetics, the Air Force Research Laboratory Faculty Fellowship, the College of Engineering Faculty Research Excellence Award, the Donald Harrington Fellowship, the Taiwan Ministry of Education Study Abroad Award, the United Microelectronics Corporation Scholarship, and a number of student paper awards and travel grants from major IEEE conferences, including the USNC-URSI Ernest K. Smith Student Paper Award in 2012; the Best Paper Award at the IEEE Sensors in 2016; the First Prize in the Student Paper Contest at the IEEE Wireless Power Transfer Conference in 2021; the Finalist in 2011, 2013, 2016 (twice), and 2019, and the Honorable Mention in 2010, 2017 (twice), and 2018 in the Student Paper Contest at the International Symposium on Antennas and Propagation; and the Finalist in the Student Paper Contest at the IEEE international Microwave Symposium. He is on Board of Directors of Applied Computational Electromagnetics Society (ACES) for the term 2021–2024. He also serves as an Associate Editor of IEEE SENSORS JOURNAL, IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, IEEE JOURNAL OF RADIO FREQUENCY IDENTIFICATION (IEEE JRFID), and IEEE JOURNAL OF ELECTROMAGNETICS, RF AND MICROWAVES IN MEDICINE AND BIOLOGY (IEEE-JERM); and a guest editor of several international journals. He was an Associate Editor of *Applied Electromagnetics*.