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On the Local Structure of Stochastic Parker Spirals in the Solar Wind

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Abstract

We present a Langevin model describing the local structure of the interplanetary magnetic field lines. It is established on the basis of the analysis of the Lagrangian properties of strong Alfvénic turbulence, which provides a new perspective on the critical balance condition. The model is consistent with the $k_{\parallel}^{-2}$ spectrum of magnetic fluctuations derived from in situ measurements. We show that the magnetic field line diffusivity at the spacecraft position can be inferred from the wavelet analysis of one-point measurements of the fluctuating magnetic fields in the solar wind independently of the three-dimensional nature of the anisotropy.

Unified Astronomy Thesaurus concepts: Magnetohydrodynamics (1964); Interplanetary turbulence (830)

1. Introduction

The large-scale structure of the interplanetary magnetic field is on average given by Parker spirals threading the entire heliosphere (Parker 1958). To a first approximation, they represent the trajectories of suprathermal particles accelerated at the Sun (Parker & Tidman 1958). Soon after the first diagnosis of turbulence in the solar wind, Jokipii & Parker (1968) argued that these spirals are stochastic. Their primary motivation was to provide an explanation for the angular spread of solar energetic particles inferred from the measurements by the Pioneer missions (Fan et al. 1968). Stochastic Parker spirals can be well represented by realizations of a random walk on a sphere of varying radii superimposed on the angular drift due to the solar rotation (Bian & Li 2021, 2022). Exploring the mechanisms that transport the charged particles accelerated to suprathermal energies during solar eruptions remains the fundamental objective of the Parker Solar Probe and the Solar Orbiter missions. The velocity and the magnetic field fluctuations measured by spacecraft in the solar wind are anisotropic (Matthaeus et al. 1990). They display power spectra proportional to $k_{\parallel}^{-\alpha}$ with $\alpha \simeq 2$ over a wide range of $k_{\parallel}$, the component of the fluctuation wavevector in the direction parallel to the local magnetic field (Horbury et al. 2008; Podesta 2009; Wicks et al. 2010). The extraction of a parallel power index close to -2 from the time series of the field fluctuations involves a complex wavelet analysis of the data set. It also requires that the Taylor hypothesis, which connects one-point temporal statistics and two-point spatial statistics, is valid. A parallel spectral index equal to -2 was originally predicted by Goldreich & Sridhar (1995) in their theory enlightening the role of the critical balance condition in strong Alfvénic turbulence. Their prediction was also confirmed by numerical simulations of magnetohydrodynamics (MHD) turbulence at large Reynolds numbers (Cho & Vishniac 2000; Maron & Goldreich 2001; Beresnyak 2015). The effects of nonlinear interactions between Alfvén waves can effectively be singled out within the framework of reduced MHD. The reduced MHD equations, which decouple Alfvén and compressive modes, can be obtained either from an asymptotic expansion of the compressible MHD equations or from the gyrokinetic equations for the guiding centers (Schekochihin et al. 2009).

2. The Lagrangian Properties of Strong Alfvénic Turbulence and the Local Structure of Stochastic Parker Spirals

From Alfvén wave polarized magnetic field fluctuations $\delta \boldsymbol{B}_{\perp}(\boldsymbol{r}, t)$ perpendicular to the guiding magnetic field $B_0 z$, one can define the magnetic field lines. The magnetic field lines are the curves $\boldsymbol{r}_{\perp}(z)$ everywhere tangent to the magnetic field $\boldsymbol{B}(\boldsymbol{r}, t) = \boldsymbol{B}_0 z + \delta \boldsymbol{B}_{\perp}(\boldsymbol{r}, t)$ at a given instant in time. They are the solutions of the ordinary differential equations

$$\frac{d\boldsymbol{r}_{\perp}(z)}{dz} = \frac{\delta \boldsymbol{B}_{\perp}(\boldsymbol{r}_{\perp}, z)}{B_0}, \quad (1)$$

which is valid to first order in the fluctuation amplitude. The velocity and the magnetic field fluctuations can be evaluated along the magnetic field lines in order to investigate their turbulent structure. The set of magnetic field lines provides a map between planes orthogonal to the guide field direction. Because Alfvén-polarized fluctuations are transverse, the magnetic field lines cannot cross the same perpendicular plane twice. In addition to being area preserving, this map is therefore invertible. Thus, the velocity and the magnetic field fluctuations evaluated along the magnetic field lines only depend on the coordinate z along the guide field: $\delta \boldsymbol{V}_{\perp}(z) = \delta \boldsymbol{V}_{\perp}(\boldsymbol{r}_{\perp}(z), z)$, $\delta \boldsymbol{B}_{\perp}(z) = \delta \boldsymbol{B}_{\perp}(\boldsymbol{r}_{\perp}(z), z)$. We work in a frame where there is no fluid particle flux along the guide field but where there remains a continuous Poynting flux of electromagnetic Alfvén waves in this direction.

The magnetic field line motions are correlated to the fluid particle motions in highly conducting fluids. A given magnetic field line always connects the same fluid particles drifting with the velocity $\delta \boldsymbol{V}_{\perp}(\boldsymbol{r}, t) = \delta \boldsymbol{E}_{\perp}(\boldsymbol{r}, t) \times \boldsymbol{B}_0/c$, where $\delta \boldsymbol{E}_{\perp}(\boldsymbol{r}, t)$ is the perpendicular component of the electric field fluctuations and c is the speed of light. Therefore, the motions of fluid particles are two dimensional in planes perpendicular to the guide field. Each individual fluid particle moves with the Lagrangian velocity $\delta \boldsymbol{V}_{\perp}(t) = \delta \boldsymbol{V}_{\perp}(\boldsymbol{r}_{\perp}(t), t)$: the Eulerian velocity field evaluated along the fluid particle trajectories, which depends on time only. The perpendicular fluid velocity derives from the common electric drift velocity of the ions and the electrons in the plasma, which is independent of their charge and their mass. The perpendicular electric field fluctuations measured in the solar wind display a frequency spectrum following a power law $\omega^{-\beta}$, with $\beta \simeq -5/3$ over a wide range of frequencies (Bale et al. 2005). While the spatial anisotropy of the electric field fluctuations in the solar wind, including its magnetic field aligned component (Bian & Kontar 2010), remains an open subject for investigation, the measurements by Bale et al. (2005) show that the flow of electromagnetic energy is consistent with the Alfvén speed in this frequency regime. The Lagrangian velocity $\delta \boldsymbol{V}_\perp(t)$ has a different dependence on time t than the Eulerian velocity field evaluated at a fixed position in space, inasmuch as the fluid particle moves a distance of the order of $\Delta r_\perp$ in a time Δt. It is also true that the field fluctuations $\delta \boldsymbol{V}_\perp(z)$ and $\delta \boldsymbol{B}_\perp(z)$ have a different dependence on z than the Eulerian velocity and magnetic field fluctuations evaluated at a given time along the fixed direction of the guide field, inasmuch as the turbulent magnetic field lines meander by an amount $\Delta r_\perp \sim (\delta B_\perp/B_0)\Delta z$ over a distance Δz due to the random fluctuations in the magnetic field direction $\theta \sim \Delta r_\perp/\Delta z \sim \delta B_\perp/B_0 \sim \delta V_\perp/V_{\rm A}$. The Alfvén speed $V_{\rm A} = B_0/\sqrt{4\pi\rho}$, where ρ is the mass density, represents the guide field intensity in velocity units. In the Iroshnikov–Kraichnan phenomenology of strong MHD turbulence (Iroshnikov 1963; Kraichnan 1965), the cascade efficiency is depleted with respect to the nominal Kolmogorov value by a factor of the order of the magnetic inclination angle θ: $\tau_{\rm NL}^{\rm IK} \sim \tau_{\rm NL}^{K}/\theta$, where the nonlinear Kolmogorov cascade time is $\tau_{\rm NL}^{K} \sim l/\Delta_l V_\perp$ and $\Delta_l V_\perp$ is the velocity increment. Combined with the Kolmogorov hypothesis of a constant rate ϵ of kinetic energy transfer through scales l in the inertial range $\epsilon = (\Delta_l V_\perp)^2/\tau_{\rm NL}$, this yields the Iroshnikov–Kraichnan scaling $\Delta_l V_\perp \sim (\epsilon V_{\rm A} l)^{1/4}$ instead of the Kolmogorov scaling $\Delta_l V_\perp \sim (\epsilon l)^{1/3}$. Alfvénic turbulence is anisotropic. The recent theory by Boldyrev (2006), the first to introduce a scale-dependent dynamic alignment, also predicts the Iroshnikov–Kraichnan scaling while capturing the strong nature of the turbulence as well as its anisotropy. We shall now develop a phenomenological description of strong Alfvénic turbulence based on the analysis of its Lagrangian properties. A closely related hypothesis will be adopted in order to justify the scaling with time of the second-order structure function of the Lagrangian velocity $\delta \boldsymbol{V}_\perp(t)$ and the scaling with distance of the second-order structure functions of both $\delta \boldsymbol{V}_\perp(z)$ and $\delta \boldsymbol{B}_\perp(z)$.

We evaluate the velocity increment $\delta \boldsymbol{V}_\perp(t+\Delta t, \boldsymbol{r}_\perp(t+\Delta t)) - \delta \boldsymbol{V}_\perp(t, \boldsymbol{r}_\perp(t))$ along a fluid particle trajectory $\boldsymbol{r}_\perp(t)$ and assume the existence of an inertial range of time increments Δt, where the mean square only depends on the average amount of kinetic energy ϵ dissipated per unit mass and per unit time and on the time increment Δt. Dimensional analysis yields the scaling of the Lagrangian second-order structure function in the form given by Landau & Lifshitz (1959) and Tennekes & Lumley (1972):

$$<(\delta \boldsymbol{V}_\perp(t+\tau) - \delta \boldsymbol{V}_\perp(t))^2> = C_0 \epsilon \Delta t, \tag{2}$$

where C_0 is a constant. Equation (2) corresponds to the Kolmogorov spectrum,

$$E_V(\omega) \propto \epsilon \omega^{-2}. \tag{3}$$

The Lagrangian frequency spectrum given by Equation (3) has been observed in numerical simulations of three-dimensional MHD turbulence (Busse et al. 2010). Such a ω^2 spectrum is well established from laboratory experiments and numerical simulations of hydrodynamic turbulence (Pope 1994; Mordant et al. 2001), which is isotropic at small scales. Note that the linear dependence on Δt of the Kolmogorov scaling law (2) is suggestive of a velocity space diffusion with $C_0\epsilon$ representing the average rate of the stochastic acceleration of the fluid particles. We now evaluate the velocity increment $\delta \boldsymbol{V}_\perp(z+\Delta z, \boldsymbol{r}_\perp(z+\Delta z)) - \delta \boldsymbol{V}_\perp(z, \boldsymbol{r}_\perp(z))$ along a magnetic field line $\boldsymbol{r}_\perp(z)$ and assume the existence of a range of spatial increments Δz, where the mean square only depends on the average rate of kinetic energy dissipated per unit mass and per unit length in the parallel direction, denoted here by $\epsilon_\parallel^V$, and on Δz. Dimensional analysis yields a linear dependence on Δz of the second-order structure function: $<(\delta \boldsymbol{V}_\perp(z+\Delta z) - \delta \boldsymbol{V}_\perp(z))^2> = C_\parallel^V \epsilon_\parallel^V \Delta z$, where $C_\parallel^V$ is a constant. Similarly, assuming that $<(\delta \boldsymbol{B}_\perp(z+\Delta z) - \delta \boldsymbol{B}_\perp(z))^2>$ only depends on the average rate of magnetic energy dissipated per unit length in the parallel direction, denoted by $\epsilon_\parallel^B$, and on Δz, then

$$<(\delta \boldsymbol{B}_\perp(z+\Delta z) - \delta \boldsymbol{B}_\perp(z))^2> = C_\parallel^B \epsilon_\parallel^B \Delta z. \tag{4}$$

Therefore, the velocity fluctuations evaluated along fluid particle trajectories and the velocity and magnetic field fluctuations evaluated along the magnetic field lines all share the same regularity properties as those of realizations of a Wiener process, corresponding to the Holder continuous functions with an H-exponent equal to $1/2$. The scaling law (4) is tantamount to the inertial range energy spectrum

$$E_B(k_\parallel) \propto \epsilon_\parallel^B k_\parallel^{-2}. \tag{5}$$

A two-way bridge between the Lagrangian frequency spectrum $E_V(\omega)$ and the perpendicular wavenumber spectrum $E_V(k_\perp)$, where $k_\perp$ is the component of the wavevector transverse to the local magnetic field direction, can be built upon the relation $\omega \sim \tau_{\rm NL}^{-1}(k_\perp)$ together with $E_V(\omega)d\omega \sim E_V(k_\perp)dk_\perp$. Taking $\tau_{\rm NL}(k_\perp) \sim \tau_{\rm NL}^{K}(k_\perp)$ leads from the Kolmogorov form (3) of the Lagrangian frequency spectrum to the Eulerian spectrum $E_V^K(k_\perp) \propto \epsilon^{2/3}k_\perp^{-5/3}$, as it was originally predicted by the Goldreich–Sridhar phenomenological description of strong Alfvénic turbulence. However, from the estimate of the nonlinear energy transfer timescale in the form given by $\tau_{\rm NL}^{\rm IK}(k_\perp) \sim \tau_{\rm NL}^{K}(k_\perp)/\theta$, the very same bridge recovers the Iroshnikov–Kraichnan spectrum $E_V^{\rm IK}(k_\perp) \propto (\epsilon V_{\rm A})^{1/2}k_\perp^{-3/2}$ but is restricted to $k_\perp$, a result consistent with the analysis by Boldyrev (2006; see also Schekochihin 2020). Note that choosing the length scale Λ, which represents the onset of the inertial range scale-dependent dynamic alignment, as $\Lambda \sim V_{\rm A}^3/\epsilon$ in $E_V(k_\perp) \propto \Lambda^{1/6}\epsilon^{2/3}k_\perp^{-3/2}$ taken from Perez et al. (2012), also yields $E_V^{\rm IK}(k_\perp)$. A Lagrangian–Eulerian bridge based on the relation $\omega \sim \tau_{\rm NL}^{-1}(k)$ is discussed by Tennekes & Lumley (1972) in the context of three-dimensional isotropic hydrodynamic turbulence. Now following Beresnyak (2015), a two-way bridge relation between $E_V(\omega)$ and $E_V(k_\parallel)$ can be built upon the Alfvén wave dispersion $\omega \sim V_{\rm A}k_\parallel$ leading from the Kolmogorov spectrum (3) to $E_V(k_\parallel) \propto (\epsilon/V_{\rm A})k_\parallel^{-2}$ and vice versa.

Therefore, the average amount of kinetic energy dissipated per unit length in the parallel direction can be related to ϵ by $\epsilon_{\parallel}^{V} \sim (\epsilon / V_{\rm A})$. And similarly, the average amount of magnetic energy dissipated per unit length in the parallel direction can be related to ϵ by $\epsilon_{\parallel}^{B} \sim B_0^2 \epsilon / V_{\rm A}^3$. The rates of kinetic and magnetic energy dissipation are expected to be spatially fragmented due to processes associated with turbulent magnetic reconnections (Lazarian & Vishniac 1999; Eyink et al. 2011). Finally, eliminating the frequency between these two Lagrangian–Eulerian bridges results in the critical balance condition $\tau_{\rm NL}^{-1}(k_\perp) \sim k_\parallel V_{\rm A}$ that connects the kinetic energy spectra $E_V(k_\perp)$ and $E_V(k_\parallel)$, as well as the magnetic energy spectra $E_B(k_\perp)$ to $E_B(k_\parallel)$. The situation can be summarized as follows:

$$\tau_{\rm NL}^{-1}(k_\perp) \sim \omega \sim k_\parallel V_{\rm A}. \tag{6}$$

Overall, the Lagrangian perspective clarifies the reasons why conflicting phenomenological descriptions of strong Alfvénic turbulence predict a similar form of the $k_\parallel$ spectra differing only in the form of the predicted scale-dependent anisotropy. Our aim is now to establish a Langevin model describing the local structure of the turbulent magnetic field lines in the solar wind in a way consistent with measurements of $E_B(k_\parallel) \propto k_\parallel^{-2}$ and remaining valid for any form of the scale-dependent anisotropy. We shall motivate it further by first considering few fundamental properties of stochastic magnetic field line dispersion.

The running magnetic field line diffusivity $D_{\rm m}(z)$ is, by definition, the rate of variation of the field line displacement variance per unit distance along the parallel direction: $D_{\rm m}(z) = d < r_\perp^2(z) > /2dz$ (Jokipii & Parker 1969). Using the magnetic field line Equation (1), it can be shown that $D_{\rm m}(z) = (1/B_0^2) \int_0^z < \delta \boldsymbol{B}_\perp(0) \cdot \delta \boldsymbol{B}_\perp(z') > dz'$. Therefore, $D_{\rm m}(z)$ can equivalently be expressed in terms of the parallel wavenumber spectrum $E_B(k_\parallel)$ via the relation

$$D_{\rm m}(z) = \int \frac{E_B(k_\parallel)}{B_0^2} \frac{\sin k_\parallel \Delta z}{k_\parallel} dk_\parallel. \tag{7}$$

Taylor's relation (7) explicitly shows how each individual parallel wavenumber component of the spectrum $E_B(k_\parallel)$ contributes to the perpendicular displacement variance of the magnetic field lines as a function of z. When $z \to 0$, the wavenumber dependent weighting function $\sin(k_\parallel z)/k_\parallel \to z$, and thus all the $k_\parallel$ components of the spectrum $E_B(k_\parallel)$ contribute equally to the integral on the right-hand side of Equation (7), resulting in $< r_\perp^2(z) > = (< \delta B_\perp^2 > / B_0^2) z^2$, where $< \delta B_\perp^2 > = \int E_B(k_\parallel) dk_\parallel$ is the turbulent magnetic energy density. In the opposite far field limit, the weighting function converges to the Dirac distribution $\sin(k_\parallel z)/k_\parallel \to \pi \delta(k_\parallel)$. The large $k_\parallel$ components of the spectrum are progressively filtered out, resulting in only the $k_\parallel = 0$ that contributes to the field line perpendicular displacement variance when $z \gg \lambda_L$. Here, $\lambda_L = \pi E_B(k_\parallel = 0) / < \delta B_\perp^2 >$ stands for the integral correlation length scale of the magnetic field fluctuations evaluated along magnetic field lines. The diffusive regime is thus attained when the running magnetic field line diffusivity becomes sufficiently close to the constant given by

$$D_{\rm m} = \lambda_L \frac{< \delta B_\perp^2 >}{B_0^2}, \tag{8}$$

which is, by definition, the magnetic field line diffusivity, and hence, $< r_\perp^2(z) > = 2 D_{\rm m} z$ in this regime. The inertial range of the turbulence is involved in the intermediate behavior that connects these two "universal" asymptotic regimes of magnetic field line dispersion. The quasilinear approximation is tantamount to taking $k_\parallel = k_z$ in Equation (7). The quasilinear magnetic field line diffusivity is thus given in this case by $D_{\rm m}^{QL} = \lambda_z < \delta B_\perp^2 > / B_0^2$ (Jokipii 1966), which is an approximation of the definition (Equation (8)), valid for a sufficiently small magnetic Kubo number. In the pathological cases when the turbulent magnetic field fluctuations lack any power at zero parallel wavenumber, i.e., $E_B(k_\parallel = 0) = 0$, or when it is divergent, i.e., $E_B(k_\parallel = 0) = +\infty$, the far field diffusive behavior cannot be guaranteed anymore, and a refined analysis should be carried out, if necessary.

A finite variance stochastic process describing the magnetic field fluctuations in a way consistent with the inertial range energy spectrum (Equation (5)) is the Ornstein–Uhlenbeck process (Chandrasekhar 1943). Its spectral energy density is the Lorentzian distribution

$$E_B(k_\parallel) = \frac{1}{\pi} \frac{\lambda_L < \delta B_\perp^2 >}{1 + (\lambda_L k_\parallel)^2}, \tag{9}$$

which smoothly rolls-over the uniform distribution when $\lambda_L k_\parallel \to 0$. As a consequence, the turbulent magnetic field lines $\boldsymbol{r}_\perp(z)$ can be modeled by the solutions of the following differential equations

$$\frac{d r_\perp(z)}{dz} \frac{\delta B_\perp(z)}{B_0}, \tag{10}$$

$$\frac{d \boldsymbol{\delta B}_\perp(z)}{dz} = -\frac{\delta \boldsymbol{B}_\perp(z)}{\lambda_L} + \sqrt{\frac{< \delta B_\perp^2 >}{\lambda_L}} \boldsymbol{\zeta}(z), \tag{11}$$

where $\boldsymbol{\zeta}(z)$ is the two-dimensional unit Gaussian white noise. From Equation (11), one can deduce that $< (\delta \boldsymbol{B}_\perp(z + \Delta z) - \delta \boldsymbol{B}_\perp(z))^2 > = (< \delta B_\perp^2 > / \lambda_L) \Delta z$ when $\Delta z \ll \lambda_L$. Therefore, it is required that $< \delta B_\perp^2 > / \lambda_L = C_\parallel^B \epsilon_\parallel^B$ in order for the Langevin model to yield the multiplicative constant of Δz in the scaling law (4). Using the Duhamel principle in Equation (11), the magnetic field fluctuations are expressed in terms of a convolution of two-dimensional Wiener processes:

$$\delta \boldsymbol{B}_\perp(z) = \sqrt{\frac{< \delta B_\perp^2 >}{\lambda_L}} \int_0^z e^{-\frac{(z - z')}{\lambda_L}} d\boldsymbol{W}_{z'}, \tag{12}$$

where we have used the notation $d\boldsymbol{W}_z = \boldsymbol{\zeta}(z) dz$. The magnetic field line equation can in turn be integrated to give

$$\boldsymbol{r}_\perp(z) = \sqrt{D_{\rm m}} \int_0^z (1 - e^{-\frac{(z - z')}{\lambda_L}}) d\boldsymbol{W}_{z'}. \tag{13}$$

The magnetic field line distribution function $P(\boldsymbol{r}_\perp, z)$ is therefore the two-dimensional Gaussian distribution, and its variance, which is given by

$$<r_\perp^2(z)> = 2D_{\rm m}[z + \lambda_L(e^{-\frac{z}{\lambda_L}} - 1)], \qquad (14)$$

satisfies ipso facto the two asymptotic constraints imposed by the exact relation (Equation (7)). In the singular Markov limit where the integral length scale $\lambda_L \to 0$ while keeping the magnetic field line diffusivity $D_{\rm m}$ finite, the magnetic field fluctuations evaluated along the magnetic field lines tend to the Gaussian white noise $\delta \boldsymbol{B}_\perp(z) = \sqrt{\lambda_L < \delta B_\perp^2 >}\boldsymbol{\zeta}(z)$. Their spectral energy density becomes independent of $k_{\|}$, and the magnetic field lines become realizations of a Wiener process $\boldsymbol{r}_\perp(z) = \sqrt{D_{\rm m}}\boldsymbol{W}(z)$. The Brownian description of turbulent magnetic field lines in the solar wind was introduced in the seminal works of Jokipii & Parker (1968, 1969). Since then, it has remained the main framework for modeling the turbulent cross-field transport of solar energetic particles and their angular spread in the heliosphere, which is observed to only weakly depend on the charge-to-mass ratio (Cohen et al. 2017).

3. Discussion and Conclusions

Our work provides a resolution to the long-standing conundrum that magnetic field lines undergoing a heliospheric Brownian diffusion and thus, locally described by the equation $\boldsymbol{r}_\perp(z) = \sqrt{D_{\rm m}}\boldsymbol{W}(z)$ (Jokipii & Parker 1968, 1969), are nowhere differentiable and therefore have infinite path lengths. Instead, the sample paths of the integrated Ornstein–Uhlenbeck process $\boldsymbol{r}_\perp(z) = \sqrt{D_{\rm m}}\int_0^z (1 - e^{-\frac{(z-z')}{\lambda_L}})d\boldsymbol{W}_{z'}$, which is not Markov, are smooth differentiable functions with finite path lengths. The variance $<r_\perp^2(z)>$ has the properties that it satisfies: in this case the two asymptotic constraints imposed by Taylor's relation (Equation (7)): $<r_\perp^2(z)> = (<\delta B_\perp^2>/B_0^2)z^2$ when $z \to 0$ and $<r_\perp^2(z)> = 2D_{\rm m}z$ when $z \gg \lambda_L$. Moreover, the Langevin equation (Equation (11)), describing here the local structure of stochastic Parker spirals, is consistent with the inertial range spectrum $E_B(k_{\|}) \propto \epsilon_{\|}^B k_{\|}^{-2}$ of the magnetic field fluctuations measured in the solar wind. This suggests that the Lagrangian frequency spectrum of the perpendicular velocity fluctuations obeys the Kolmogorov law $E_V(\omega) \sim \epsilon\omega^{-2}$ in the inertial range of solar wind turbulence. The running magnetic field line diffusivity can be obtained from in situ observations by using Taylor's relation (Equation (7)) together with the spectrum $E_B(k_{\|})$ that is derived from the wavelet analysis of one-point measurements of the fluctuating magnetic fields. The field line diffusivity of stochastic Parker spirals, which is related to $E_B(k_{\|})$ by $D_{\rm m} = \pi E_B(k_{\|} = 0)/B_0^2$, can therefore be inferred from measurements by extrapolating the spectrum $E_B(k_{\|})$ toward $k_{\|} \to 0$. The diffusivity $D_{\rm m}$ does not depend on the scale-dependent anisotropy of the turbulence. The Langevin model was motivated by the analysis of the Lagrangian properties of Alfvénic turbulence. The Lagrangian perspective tends to unify theories, which are based on the critical balance condition (Equation (6)). They predict different scale-dependent anisotropy but a similar form of the parallel wavenumber spectrum $E_B(k_{\|})$ in the inertial range. In passing, we note that dimensional arguments in anisotropic turbulence can lead to a family of spectral indices for $E(k_\perp)$ while $E(k_{\|}) \sim k_{\|}^{-2}$. The form of the parallel wavenumber spectrum $E_B(k_{\|})$ is also fundamental to the description of the resonant scattering of charged particles by turbulent magnetic fields in the solar wind and other astrophysical environments.

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References

Bale, S. D., Kellogg, P. J., Mozer, F. S., Horbury, T. S., & Reme, H. 2005, PhRvL, 94, 215002
Beresnyak, A. 2015, ApJL, 801, L9
Bian, N. H., & Kontar, E. P. 2010, PhPl, 17, 062308
Bian, N. H., & Li, G. 2021, ApJ, 908, 45
Bian, N. H., & Li, G. 2022, ApJ, 924, 120
Boldyrev, S. 2006, PhRvL, 96, 115002
Busse, A., Müller, W. C., & Gogoberidze, G. 2010, PhRvL, 105, 235005
Chandrasekhar, S. 1943, RvMP, 15, 1
Cho, J., & Vishniac, E. T. 2000, ApJ, 539, 273
Cohen, C. M. S., Mason, G. M., & Mewaldt, R. A. 2017, ApJ, 843, 132
Eyink, G. L., Lazarian, A., & Vishniac, E. T. 2011, ApJ, 743, 51
Fan, C. Y., Pick, M., Pyle, R., Simpson, J. A., & Smith, D. R. 1968, JGR, 73, 1555
Goldreich, P., & Sridhar, S. 1995, ApJ, 438, 763
Horbury, T. S., Forman, M., & Oughton, S. 2008, PhRvL, 101, 175005
Iroshnikov, P. S. 1963, AZh, 40, 742
Jokipii, J. R. 1966, ApJ, 146, 480
Jokipii, J. R., & Parker, E. N. 1968, PhRvL, 21, 44
Jokipii, J. R., & Parker, E. N. 1969, ApJ, 155, 777
Kraichnan, R. H. 1965, PhFl, 8, 1385
Landau, L. D., & Lifshitz, E. M. 1959, Fluid Mechanics (Amsterdam: Elsevier),
Lazarian, A., & Vishniac, E. T. 1999, ApJ, 517, 700
Maron, J., & Goldreich, P. 2001, ApJ, 554, 1175
Matthaeus, W. H., Goldstein, M. L., & Roberts, D. A. 1990, JGR, 95, 20673
Mordant, N., Metz, P., Michel, O., & Pinton, J.-F. 2001, PhRvL, 87, 214501
Parker, E. N. 1958, ApJ, 128, 664
Parker, E. N., & Tidman, D. A. 1958, PhRv, 111, 1206
Perez, J. C., Mason, J., Boldyrev, S., & Cattaneo, F. 2012, PhRvX, 2, 041005
Podesta, J. J. 2009, ApJ, 698, 986
Pope, S. B. 1994, AnRFM, 26, 23
Schekochihin, A. A. 2020, arXiv:2010.00699
Schekochihin, A. A., Cowley, S. C., Dorland, W., et al. 2009, ApJS, 182, 310
Tennekes, H., & Lumley, J. L. 1972, First Course in Turbulence (Cambridge, MA: MIT Press)
Wicks, R. T., Horbury, T. S., Chen, C. H. K., & Schekochihin, A. A. 2010, MNRAS, 407, L31