

QuickSHiFT: Rapid High-Frequency Transformer Simulation and Optimization

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Abstract—Transformer characteristics such as current distribution, loss, and leakage inductance are often predicted through hand calculations or FEA analysis, which are slow and labor-intensive. A recently-developed analysis technique can predict transformer characteristics six orders of magnitude faster than FEA and is promising for use in analysis and especially optimization. The work presented here drastically improves the usability of this rapid prediction method by creating a software tool that can automatically generate winding configurations and analyze them at speeds of thousands-per-second. This paper covers the implementation of such a general software tool, called the Quick Simulator of High-Frequency Transformers (QuickSHiFT), demonstrates its user interface and utility for human analysis, and further provides examples of rapid brute-force optimization of 4-layer planar transformers.

Index Terms—High Frequency Transformer, Core Loss, Copper Loss, Simulation, Optimization, Finite Element Analysis

I. INTRODUCTION

As power converters increasingly operate at switching frequencies in the MHz regime, efficiency and size become more limited by the performance of magnetic components. As such, it is important to be able to predict winding loss, core loss, and leakage inductance, and optimize based on those predictions. This task is especially difficult in both high-power and high-turns-ratio transformers where large copper cross-sections and/or parallel turns are often used, as the current distribution within and between turns becomes more complicated.

General heuristics for design exist, such as interleaving of primary and secondary windings to minimize inter-winding H-fields and current crowding; for some simple winding arrangements, analytical solutions may be available [1], [2]. However, interleaving can be done in many ways and the analysis of loss is further complicated when parallel turns are used. Finite element analysis (FEA) software tools are the most commonly used tool for verifying the performance of such structures [3]–[5]. This verification process is accurate but relatively slow, limited by the computational intensity of FEA approaches. Ultimately, these software tools can only validate or optimize over a narrow design space, chosen by human designers.

Alternative approaches for analyzing current distribution within such structures have been developed and verified based

on building SPICE models from physical parameters [3], [6]–[9]. These methods are particularly useful exercise in connecting the language of fields and current densities to the language of circuits. Other methods simply utilize Maxwell’s equations to create systems of coupled differential equations [10], [11]. These approaches are faster than FEA but not always fast enough to use in optimization algorithms, especially when scaled to complex designs.

An alternative approach to solving the problem of current distribution is presented in [12]. By leveraging an assumption that the structure is operating at the high-frequency limit, this approach drastically reduces the complexity of calculating the current distribution, ac resistance, and magnetic fields in a transformer. While FEA software tools spend computational resources meshing a geometry and solving Maxwell’s equations across the identified nodes, the approach of [12] reduces the problem to a lightweight system of linear algebraic equations readily solvable. This reduces the computational time needed to identify current distributions in a given structure by over six orders of magnitude compared to traditional FEA [12]. This increase in speed holds great promise for accelerating human-in-the-loop design (where results are available almost instantaneously) and especially for automated optimization.

The prior work of [12] used bespoke MATLAB scripts, created by hand for each test case, to implement this accelerated method. Further maximizing the impact of this approach requires a method to capture physical magnetic structures in a computational data structure, automatically generate such structures, evaluate and cull identical designs, visualize the results and, optionally, an optimization engine. This paper presents methods for achieving these goals.

In this work, we greatly improve the usability of this approach by creating an open-source software application that implements it for use by human designers and by optimization engines. The software tool is called the Quick Simulator of High Frequency Transformers, stylized as QuickSHiFT. It includes (a) a tool to solve for and visualize the current distribution for a given transformer geometry and winding configuration (directly comparable to the usage of FEA software); and (b) a tool that identifies the optimal winding configuration(s) for a particular set of physical constraints and turns ratio. This optimizer tool is directly enabled by the rapidity of the high-frequency current analysis approach. In addition to assuming high-frequency operation (and thus,

skin depth limitation), the tool further assumes that structures will have a one dimensional distribution of conductors and that structures will not store significant energy in the core, so that magnetizing current is minimal. As such, QuickSHiFT is most applicable to planar designs for PWM converters, gate-drive transformers, dual active bridge transformers, current transformers, series-resonant transformers, etc.

To demonstrate the speed of this tool, we implement a brute force search over the set of possible transformer stackups, analyzing thousands of configurations per second. We describe the implementation of this tool, then utilize it to build several planar transformers, whose performance is compared to that of several un-optimized but perhaps intuitive designs. QuickSHiFT is publically available at github.com/Power-Electronics-and-Magnetics-Group/quickshift.

II. SIMULATOR IMPLEMENTATION

QuickSHiFT's first tool is a simulator. For a given transformer winding configuration and geometric parameters, it can solve for the current distribution within and between conductors, much like a traditional FEA solver (except much faster). For inputs, the simulator takes a set of geometric parameters and the transformer's winding configuration, specified using a custom tree format described in Section II-A. The optimizer then uses a mathematical method, presented in [12] and reviewed in Section II-B to solve for current distribution and ac resistance in the structure.

A. Winding Representation

A data structure is necessary to uniquely represent a variety of winding configurations, both for human input and for automated optimization. It must be possible to represent any possible winding, whether those layers be connected in series, parallel, or more complex combinations of the two (for example, a winding consisting of two series-connected layers with two turns each, in parallel with a layer with four turns, with that combination in series with another layer). We accomplish this through the use of a binary tree structure.

The tree is formed from three types of nodes: 'Parallel', 'Series', and 'Layer'. Parallel and Series nodes represent that their two children (the two nodes connected directly beneath them in the tree) are connected in parallel or series, respectively. Note that their child nodes need not be individual layers, but can also be Parallel or Series nodes themselves. As such, these nodes can be chained together to specify multiple nodes connected in parallel or series, parallel connections of series-connected nodes, series connections of parallel-connected nodes, or even more complex configurations.

Layer nodes form the leaves of the tree (the bottom nodes, with no sub-trees), and must specify two parameters: the layer number (which layer is it, numbered from top to bottom of the stack), and the number of turns on the layer.

For a given transformer with two windings and N layers, the final encoded representation consists of two binary trees with a cumulative N leaf nodes. The two trees (representing the primary and secondary windings) must be mutually exclusive

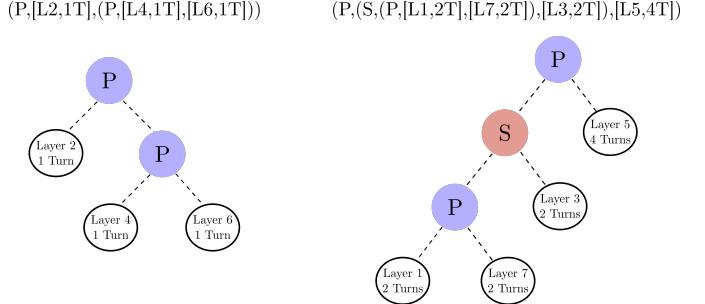


Fig. 1. Tree and text representation of the primary and secondary windings for a 7 layer, 1:4 transformer. P nodes represent parallel connections, S nodes represent series connections.

and totally complete. In a valid winding configuration, a layer can only be a member of one tree, and every layer must be represented on one of the two trees. Note that it is possible to analyze structures with more than two windings, or with a layer acting as a shield, using the method presented in [12] - this completeness restriction is one introduced by the software implementation, not the method itself.

An example pair of trees for a 7 layer, 1:4 transformer is represented in Fig. 1. In this case, the left tree represents a primary winding consisting of three layers (layers 2, 4, and 6) with one turn each, placed in parallel. The right tree specifies a secondary consisting of the remaining layers being placed in a complex configuration: Layers 1 and 7 have two turns and are connected in parallel; their combination is placed in series with layer 3, which has 2 turns; that combination is then placed in parallel with layer 5 with 4 turns. As demonstrated, this tree structure allows for both the specification of arbitrarily complex winding configurations by a user and the automatic generation of those configurations by the optimizer itself.

The two trees that represent the two windings are encoded in a Stackup object. If the Stackup object is combined with a set of geometric parameters representing the window breadth, window height, layer thickness and spacing, and so on, then a numeric solution can be found. Otherwise, a general, symbolic solution can be found and dimensions can be substituted in later with no loss of generality. Note that due to the realities of existing linear algebra solvers, the numeric solver is orders of magnitude faster than the symbolic one (which in turn, is orders of magnitude faster than FEA simulation). For applications where dimensions are known early in the design process, the numeric solution may be most suitable. The QuickSHiFT application and GUI utilizes the numeric solver to maximize speed, but the open-source code library includes a symbolic solver as well.

B. Solver

The solver runs the method presented in [12], summarized here. The method relies on several simplifying assumptions, the most important being that the transformer is operating at the high frequency limit. As such, current distributions are treated as surface current densities, with units of A/m - this

drastically simplifies the application of Maxwell's equations, and enables the rapid nature of the method.

For an N -layer structure, $3N$ unknowns are identified: N layer currents, and $2N$ surface current densities (one for the top and bottom of each layer). $3N$ equations are generated which represent the winding configuration of the transformer and Maxwell's equations in various simplified forms: 1 definition of primary current; N surface current identities (i.e. the two surface currents in a conductor sum to the total current); $(N + 1)$ amperian loops around adjacent conductor surfaces; and $(N - 2)$ equations that are either current definitions for series-connected turns or faraday loops around paralleled conductors.

The majority of these equations are very simple and independent of the winding configuration of the transformer. For example, no matter what interconnections between layers exist, total layer current will always be equal to the sums of the surface current densities within that layer.

However, some equations do require knowledge about the structure of the transformer winding, namely the applications of Faraday's law to parallel connected layers and the definitions of series connected layers. Constructing these equations is made simple due to the binary tree representation of the winding structure. QuickSHiFT traverses each winding tree recursively from the top-down to generate the structure-specific equations. At each parallel node, it generates an equation representing an application of Faraday's law between the child nodes. If the child node contains multiple layers, then one layer is selected from the children. At every series node, it generates an equation equating the current in one child node to the other child node.

Consider the winding tree shown on the right in Fig. 1. At the top node, an equation representing a Faraday loop between layers 1 and 5 is made. This process proceeds down the tree: The left child of the top node is a series node - as such an equation equating the current between its children is made ($I_1 + I_7 = I_3$). At the bottom parallel node, a final equation representing a Faraday loop between layers 1 and 7 is generated. Because of its recursive nature, this approach extends straightforwardly to any possible tree.

The equations are all linear, algebraic expressions and so can be collected into a single matrix equation, $\mathbf{Mx} = \mathbf{b}$ where the entries of x consist of unknown layer currents and surface currents, and the entries of b are empty except for the definition of the known winding current. Once the (symbolic or numeric) matrix is generated, an existing linear algebra solver from a computational math library is called, and the current distribution solution is generated. The resulting vector \mathbf{x} contains both the layer level currents and the surface current densities for each layer. Loss, ac resistance, and leakage/magnetizing inductance can be readily calculated from this result. The accuracy of this result has been extensively validated, by confirmation with both finite element analysis simulation and measurements of experimental transformers.

[12] considered structures with both 1-D and 2-D distribution of conductors. In order to make the automated represen-

tation and generation of winding configurations possible, this tool limits its implementation to simply structures with 1-D distribution of conductors. This makes it suitable for commonly used, easily manufacturable, PCB planar transformers.

III. OPTIMIZER IMPLEMENTATION

QuickSHiFT's second tool is an optimizer. Given a set of constraints (turns ratio, number of layers, minimum number of turns, etc.), it generates a set of candidate designs. Relying on the simulator described in Section II, the optimizer then calculates the expected performance of the candidates, then outputs the best performing designs.

A. Winding Configuration Generation

For the purpose of 'brute force' optimization of a given set of design parameters, the search space of possible winding configurations must be generated. This is nontrivial because: a) the space of configurations searched must be complete; b) duplication and thus excess computational burden must be avoided.

To begin, the user must constrain the problem by inputting the available number of layers, the desired turns ratio, and the maximum turns per layer. If it is desired, (i.e., if a target magnetizing inductance must be reached) minimum turns on the primary winding can be specified. Any turns ratio can be specified (although QuickSHiFT always treats the primary as the high-current winding, potentially inverting the user-specified turns ratio). Each individual layer can contain turns from either the primary or secondary, but not both. This is a constraint of the tool, in its current implementation, and not of the simulation method, which can handle two dimensional distribution of currents. A set of combinations of primary & secondary turn counts that meet the specifications are generated - e.g. for a 6 layer, 1:3 transformer with up to 3 turns per layer, primary/secondary turn counts could be 1/3, 2/6, 3/9, or 4/12. These turns could be placed on many different combinations of layers.

For each pair of turn counts (representing a combination of primary and secondary turns), a set of valid layer assignments is generated. The program identifies the possible acceptable number of layers that can be used for a given winding. For example, if a winding consists of 6 turns, with a maximum of 2 turns per layer, that winding can be put on no fewer than 3 layers. This is done for both windings to identify mutually acceptable layer quantities for each winding. All possible combinations of layers for the given requirements are generated (using an existing Python mathematical library, `itertools`). Layer combinations that would be duplicated if the transformer is flipped upside-down are culled by checking if the 'mirrored' assignment is also in the generated set of assignments.

Then, for each layer assignment, a set of valid interconnections are generated using recursive generation of the winding tree. The algorithm places a series node at the top of the tree, then identifies acceptable layer assignments (ones that can achieve the user's specifications) to each of the children

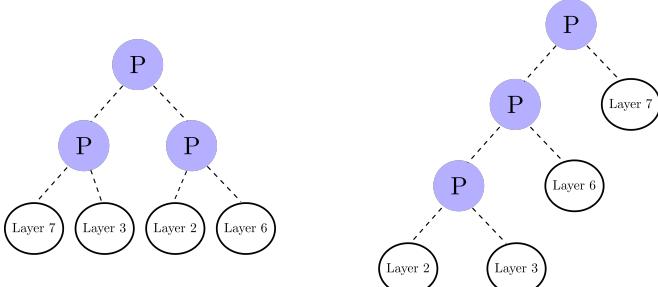


Fig. 2. Two electrically identical winding configurations, showing an arbitrarily-organized tree (left) and the standard form tree (right).

nodes. The algorithm is duplicated for a parallel node at the top of the tree, then ran recursively on the child nodes & layer assignments. This is repeated until all available layers are used. As a simple example, if 3 turns must be placed on 2 layers, this can be accomplished by putting 1 turn in series with 2 turns on the other layer, or putting 3 turns on both layers, then connecting the layers in parallel. Due to its recursive nature and the flexibility of the tree format though, this process extends straightforwardly to cases with more turns or available layers. Then, all of the possible sub-trees are collected and combined with their parent nodes to construct a set of valid interconnections.

Then, each winding tree is sorted into a standardized form: Any block of series- or parallel- connected nodes is sorted based on the smallest layer number contained by each node, then assembled back into a left-branching tree. This proceeds recursively down the tree, until it is entirely sorted. An example of an unsorted tree and its sorted equivalent is shown in Fig. 2. Duplicate trees can easily and quickly be culled (because set membership can be checked in constant time) during the connection generation process. This process proceeds for both the primary and secondary windings. Once complete, all valid combinations of primary and secondary interconnections are made and the full list of candidate designs is produced.

B. Brute-Force Optimization

The user can take advantage of the solver and candidate generation tools combined with any optimization engine. We have implemented a brute-force optimizer that combines the list of candidates with geometric parameters and solves them all in across multiple parallel threads. The optimizer can process well over one thousand candidate transformer designs per second (depending on the power of the hardware it is run on). Figure 3 shows design quantity and speed data from the optimizer across a range of turns ratios for a 6 layer structure with a maximum of 5 layers per turn, run on an AMD Ryzen 3800X with 16 threads. Note that the variability in calculation time relative to number of configurations comes primarily from the candidate generation process (which is not parallel processed), rather than from variability in the time needed to analyze individual designs. This represents a drastic increase

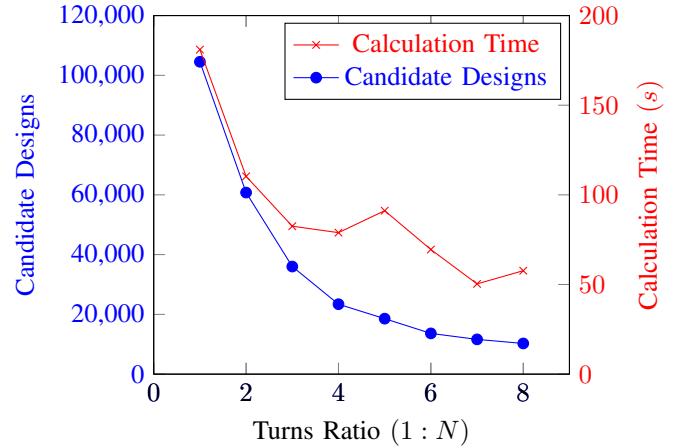


Fig. 3. Statistics from the implemented optimizer, showing number of candidate configurations and total calculation time for a 6 layer transformer with varying turns ratio and a maximum of 5 turns per layer.

in speed from FEA simulation, which takes on the order of 10 to 100 seconds per design, not including the substantial time it takes to build the individual transformer models. Compare this to the roughly 1-3 milliseconds per candidate using the tool presented here.

Upon completion of processing candidate designs, QuickSHiFT reports a list of the best performing winding configurations (the length of that list is a user-set parameter), as shown in Fig. 4.

While every candidate design is guaranteed to meet the specifications set by the user (turns ratio, turn count, etc.), the tool does not attempt to evaluate the feasibility of implementing each design. For example, a planar transformer winding consisting of a set of parallel-connected layers with multiple turns may have one terminal on the outside of the windings, and another on the inside near the transformer center post. To get current out of the transformer, it may be necessary to use an additional PCB layer (in which case the user-specified number of layers should be reduced relative to the total layers in the PCB) or an external fly-wire to complete the winding. There may be other practical considerations involved in constructing each winding, such as via placement and quantity. The tool (and the underlying method) also does not consider the losses associated with interconnects and terminations, which may often be significant in high-frequency transformers [13]. As such, a designer might use QuickSHiFT's optimizer to rapidly identify a set of high-performing winding configurations, then select a final design based on further 3-D FEA simulations and manufacturability concerns.

IV. EXPERIMENTAL VALIDATION

The analysis method has been thoroughly validated experimentally in [12]. As long as its assumptions are met, the tool's predictions match ANSYS Maxwell FEA and experimental results very well. Nevertheless, to validate the utility of the tool, we built pairs of planar transformers, each consisting of one designed with the optimizer tool, and an 'intuitive' design

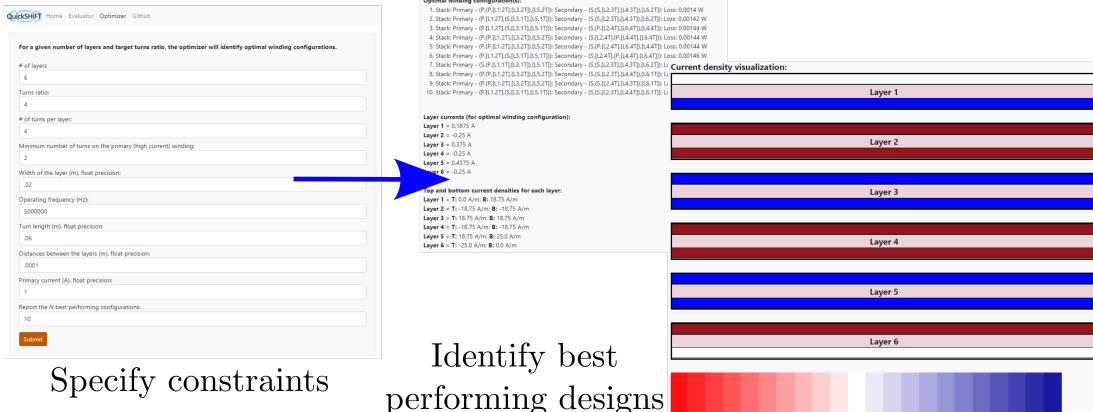


Fig. 4. Screenshots from the QuickSHiFT application showing the optimizer inputs and outputs.

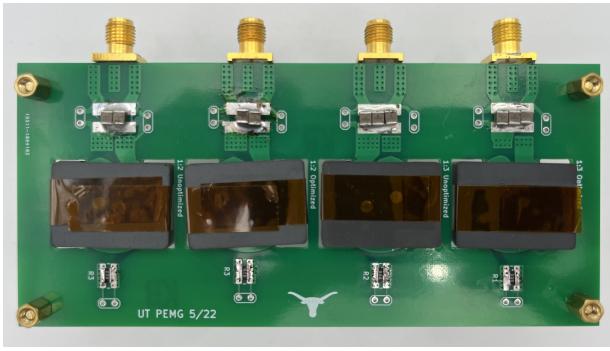


Fig. 5. Test board consisting of two pairs of planar transformers, each with one tool-optimized design and one ‘intuitive’ design.

built using standard interleaving. The final transformers are shown in Fig. 5.

All test transformers were built using four layer PCBs with 2 ounce copper on both inner and outer layers. All measurements were conducted at 10 MHz to ensure that our assumptions of operating while skin depth limited were fulfilled. We used Fair-Rite 67 planar cores due to their low-loss performance in the chosen frequency range [14].

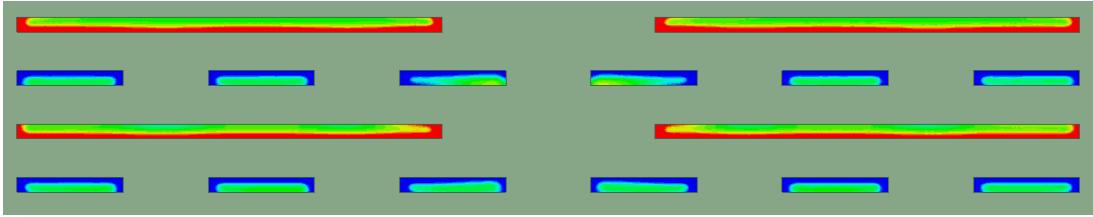
The first pair consisted of transformers with a 1:3 turns ratio, with a minimum of 4 turns on the primary (corresponding to a target magnetizing inductance of $3.87 \mu\text{H}$) and a maximum of 6 turns placed on any layer. Both the optimized and unoptimized designs interleaved the primary and secondary turns across the transformer layers, with the primary on layers 1 and 3, and the secondary on layers 2 and 4. Both designs had identical secondaries, with 6 turns on layer 2 placed in series with 6 turns on layer 4. Their primaries differed: the un-optimized design ‘intuitively’ places 2 turns on layer 1 in series with 2 turns on layer 3. The optimized design places 1 turn on layer 1 and puts it in series with 3 turns on layer 3.

This, while not necessarily intuitive, does make sense: If there is not significant magnetizing current in the core (as required by our assumptions) - the outer layers have half as

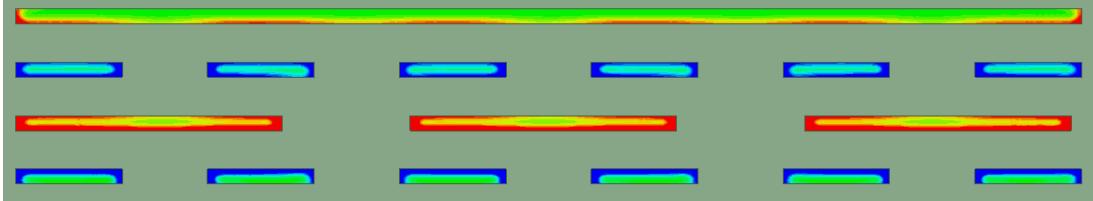
much effective conduction area than the inner layers. Because there is no H-field in the core, current does not crowd on the side of the conductors directly adjacent to the core. Fig. 6 shows FEA simulations of the two designs in question. Notice how in both cases, the primary turns (with positive current density in red) on the top layer do not have a ‘skin’ of current traveling on their topmost edge. Likewise, the conductors on the inner layers in the un-optimized design seem only to have current traveling on one of their long edges. In the optimized design, the tool has identified that adjusting the distribution of turns in the primary winding can change the MMF distribution between windings such that current distributes evenly across all edges of each conductor, akin to a ‘double sided conduction’ effect [15]. This more even distribution of current results in lower loss and ac resistance. As such, QuickSHiFT often identifies that designs that put more turns on inner layers end up performing better. In this case, the optimized design is predicted to have 25% lower ac resistance than its un-optimized counterpart.

The second pair consists of transformers with a 1:2 turns ratio with a minimum of 5 turns on the primary (corresponding to a magnetizing inductance of $6.05 \mu\text{H}$) and a maximum of 8 turns placed on any layer. In this case, both designs had identical primaries: 2 turns on layer 1 placed in series with 3 turns on layer 3. The optimized secondary placed 7 turns on layer 2, and 3 turns on layer 4. The un-optimized secondary split turns evenly between layers 2 and 4, with 5 on each. The rationale for why the optimized design performed better is identical to the first test case: with more available conduction area on inner layers, putting more turns on inner layers results in reduced loss. In this case, the optimized design is predicted to have 19% lower AC resistance than its un-optimized counterpart.

To evaluate the relative performance of these structures, their ac resistance was evaluated using ANSYS Maxwell and experimentally using a series resonant method at 10 MHz, as described in [16]. Two versions of the structures were simulated in ANSYS Maxwell 2-D: One where virtually all



(a) Simulation of the un-optimized 1:3 transformer



(b) Simulation of the optimized 1:3 transformer

Fig. 6. FEA simulations (performed in ANSYS Maxwell 2-D) of the intuitively and QuickSHiFT designed 1:3 test transformers, showing how the optimized design achieves greater conduction on the surfaces of the turns placed on inner layers.

Turns Ratio	Design	QuickSHiFT Predicted R	Small Clearance FEA R	Manufactured Dimensions FEA R	Experimental R
1:3	Optimized	98 mΩ	101 mΩ	146 mΩ	196 mΩ
1:3	Intuitive	130 mΩ	127 mΩ	170 mΩ	225 mΩ
1:2	Optimized	138 mΩ	148 mΩ	240 mΩ	337 mΩ
1:2	Intuitive	171 mΩ	189 mΩ	266 mΩ	327 mΩ

TABLE I: Predicted (both from QuickSHiFT and ANSYS Maxwell) and experimentally measured ac resistances of the test transformers operating at 10 MHz.

the available area of a given layer is used (with small clearances between windings) and one with realistic, manufacturer-constrained clearances between turns. A comparison of the predicted, simulated, and measured ac resistances are shown in Fig. I.

In all cases, experimentally measured resistance is higher in magnitude than the QuickSHiFT and both versions of FEA predicted resistance. A major contributing factor to this may be via and interconnect losses, which can be significant in high frequency transformers [13]. These effects are modeled neither in Maxwell 2-D nor in QuickSHiFT. In all four cases, the small clearance FEA and QuickSHiFT predicted resistances match with extremely high accuracy. Considering these simulate structures with almost exactly the same conductor dimensions, this further validates the accuracy of the method used by QuickSHiFT.

In terms of relative magnitudes, the 1:3 test case matches our predictions well. In all three cases, the optimized design has approximately 30 milliohms less absolute resistance than the intuitive design (in relative terms, our method, FEA with manufactured dimensions, and experimental results show 25%, 14.1%, and 12.9% lower loss in the optimized design respectively). This indicates that QuickSHiFT was able to successfully optimize the transformer design.

For the 1:2 test case, experimental results suggest no meaningful difference between the two designs (in fact, the optimized design performs about 3% worse than its intuitive

counterpart). We note that QuickSHiFT does show agreement with FEA simulation - both show higher performance for the optimized design. This suggests that three dimensional effects, not modeled by either simulation method, are a substantial contributing factor for losses in these structures. Another source of additional loss, which can be modeled by FEA but not currently by QuickSHiFT, is the reduced winding cross-sectional area caused by clearance requirements between turns on the same layer. The differences between the idealized and manufactured dimensions FEA predicted resistances are significant, on the order of 30 to 50% of the idealized prediction. For structures with many turns on a single layer, clearance requirements can significantly reduce the available cross-sectional area and balloon loss. QuickSHiFT assumes full utilization of cross-sectional area, even when many turns are placed on the same layer. Future versions of the software will allow for control over clearance requirements, resulting in more accurate loss predictions.

Nevertheless, strong and established agreement between the proposed method, traditional FEA simulation, and experimental results [12], indicate the utility of the proposed method. QuickSHiFT enables designers to A) identify top performing candidate designs regardless of requirements (arbitrary turns ratios, layer counts, etc.), B) quickly verify whether their proposed designs are top performing. This tool is entirely enabled by the extremely rapid computational method at its core, allowing the processing of thousands of winding config-

urations per second. This tool could be potentially extremely useful across a variety of magnetics design problems.

V. CONCLUSION

This work has presented a software tool, QuickSHiFT, which rapidly solves for current distribution within and between conductors in high-frequency planar transformers. The tool can be used as a direct replacement for traditional FEA as a way of validating individual candidate designs. Alternatively, its great speed enables the rapid, automated generation and evaluation of all possible designs. The optimizer can evaluate thousands of designs a second, assisting designers in the creation of novel transformer winding configurations. The tool was then used to design several planar transformers, whose performance was compared favorably against un-optimized, intuitive designs. The authors hope QuickSHiFT will prove useful to magnetics designers looking to do rapid simulation and optimization.

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