A Joint Transmit and Receive Design for Dimmable High Speed MC MU VLC Systems

Yujie Yang*, Yang Yang*, Hailun Xia*, Caili Guo†, Mahdi Chehimi‡, Walid Saad ‡
*Beijing Key Laboratory of Network System Architecture and Convergence, School of Information and Communication Engineering, Beijing University of Posts and Telecommunications, Beijing 100876, China
†Beijing Laboratory of Advanced Information Networks, School of Information and Communication Engineering, Beijing University of Posts and Telecommunications, Beijing 100876, China
‡ Wireless@VT Group, Bradley Department of Electrical and Computer Engineering, Virginia Tech, Blacksburg, VA 24061, USA
Email: yangyujie@bupt.edu.cn, yangyang01@bupt.edu.cn, xiahailun@bupt.edu.cn, guocaili@bupt.edu.cn, mahdic@vt.edu, walids@vt.edu

Abstract—**In multi-cell multi-user multiple-input multiple-output (MC MU-MIMO) visible light communication (VLC) systems, each user is equipped with multiple closely-placed photodiodes (PDs) with similar channel gains, leading to severe inter-cell interference and intra-cell interference. To address this problem, this paper proposes a hybrid dimming (HD) scheme with MIMO VLC transceiver design, which jointly optimizes transmit and receive antenna selection (TRAS), cell clustering and precoding (TRASP-HD) for MC MU-MIMO VLC systems. In this scheme, a sum-rate maximization problem under the dimming level and illumination uniformity is formulated and solved by being divided into two sub-problems. In particular, The first sub-problem is on TRAS and cell formation based on the criterion of sum-rate maximization under the illumination uniformity constraint. With the same goal, the second sub-problem is on optimizing the precoding matrices of each cell. Finally, these two sub-problems are iteratively solved to obtain a convergent solution. Simulation results verify that in a typical indoor scenario, the mean bandwidth efficiency of TRASP-HD scheme is 2.36 bit/s/Hz higher than the conventional MC MU-MIMO system.**

I. Introduction

Visible light communication (VLC) is a promising technology for high speed data transmission and accurate positioning. VLC utilizes light-emitting diodes (LEDs) as transmitters and photodiodes (PDs) as receivers. Typical indoor scenarios require multiple LEDs to provide sufficient illumination. Moreover, due to the increasing number of mobile users in the wireless network, the multi-user (MU) VLC system attracts a lot of research attention. To increase the data rates in MU VLC systems, a number of technologies are usually utilized, such as multiple-input-multiple-output (MIMO) system and orthogonal frequency division multiplexing (OFDM). In this work, MU-MIMO systems are studied for high speed VLC.

A key challenge in MU-MIMO systems is the multiple user interference (MUI) due to the overlapped emission of LEDs, which may lead to system performance degradation. Prior works on mitigating MUI in MU MIMO VLC systems adopt two main approaches, which are transmitter design and receiver design. In particular, transmitter design-based approaches include transmit antenna selection schemes (TAS) [1] and precoding schemes such as zero forcing (ZF) precoding [2], minimum mean-squared error (MMSE) precoding [3], and block diagonalization (BD) precoding [4]. Moreover, receiver design-based approaches mainly focus on receiver antenna selection (RAS) to reduce channel correlation [5] and angle diversity receiver design to improve the illumination and communication coverage [6]. Furthermore, when considering a multi-cell (MC) scenario where multiple LEDs simultaneously serve multiple users, both inter-cell interference (inter-CI) and intra-cell interference (intra-CI) become more severe than in a single cell scenario. In this regard, the joint design of transmitter and receiver, named as transceiver designs is crucial to MC MU-MIMO VLC systems. To illustrate, the authors in [7] jointly optimize the transmitter precoding matrix and the receiving equalizer on interference alignment. Moreover, in [8], the modulation-mode assignment, the power allocation and RAS are jointly managed in a dynamic manner based on the sum throughput maximization criterion.

However, prior works considering transceiver designs in MC MU-MIMO VLC systems [7], [8] consider improving the communication performance without taking into consideration controlling the dimming levels of the visible light signals. In fact, in VLC, the visible light signals must satisfy certain dimming levels due to the inherent illumination functions of LEDs. Hence, controlling the dimming levels in transceiver designs of MC MU-MIMO VLC systems by changing the number of activated LEDs has a significant impact on the communication performance of those systems [9]. Accordingly, *there is a need for studying dimmable transceiver designs in order to satisfy both the communications and illumination requirements in MC MU-MIMO VLC systems.*

Henceforth, in this work, we propose a hybrid dimming (HD) scheme with MIMO VLC transceiver design, which jointly optimizes transmit and receive antenna selection (TRAS), cell clusterings, and precoding (TRASP-HD) for dimmable MC MU-MIMO VLC systems. The main objective of the proposed scheme is to maximize the sum-rate of users under the constraints of achieving illumination uniformity and satisfying certain illumination levels. When considering the proposed scheme in scenarios with a large number of transmitters and receivers, one notices a clear tradeoff between the computational complexity and the accuracy of the obtained optimal solution. Therefore, the original problem is divided into several sub-problems that are solved iteratively. The simulation results show that, compared with existing cell formation schemes, our proposed scheme achieves a higher mean bandwidth efficiency. In particular, in a typical indoor scenario, the mean bandwidth efficiency of the proposed TRASP-HD scheme is 2.36 bit/s/Hz higher than the conventional approaches in MC MU-MIMO VLC systems.

Notations: Bold upper case letters represent matrices and black-

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board bold letters represent sets. $\boldsymbol{A}^{\mathrm{T}}$ is the transpose of matrix $\boldsymbol{A}$, $\boldsymbol{A}_{(i,j)}$ is the element at the *i*th row and *j*th column of $\boldsymbol{A}$, $\boldsymbol{A}_{(k,:)}$ is the *k*th row vector of $\boldsymbol{A}$ and $\boldsymbol{A}_{(k,:)}$ is the *k*th column of $\boldsymbol{A}$. The notation $\|\cdot\|_1$ is the L_1 norm operator, $\mathcal{R}$ is the real number sets, and the $\lfloor\cdot\rfloor$ and $|\cdot|$ are the round down and absolute value operators, respectively.

II. System Model

We consider an MC MU-MIMO VLC system consisting of K users and N_{T} LEDs to form N_c cells. The *c*th VLC cell is formed by $N_{c,\mathrm{T}}$ LEDs and K_c users, where $N_{\mathrm{T}} = \sum_{c=1}^{N_c} N_{c,\mathrm{T}}$ and $K = \sum_{c=1}^{N_c} K_c$. Each user is equipped with $N_{c,\mathrm{R},i}$ PD. The total number of PDs in this system and the number of PDs in the *c*th cell are denoted as $N_{\mathrm{R}} = \sum_{c=1}^{N_c} N_{c,\mathrm{R}}$ and $N_{c,\mathrm{R}} = \sum_{i=1}^{K_c} N_{c,\mathrm{R},i}$, respectively.

A. VLC Channel Model

We assume that each LED obeys the Lambertian beam distribution, and the channel matrix of the *i*th user in the *c*th cell is denoted as $\mathbf{H}_{c,i} \in \mathcal{R}^{N_{c,\mathrm{R},i} \times N_{c,\mathrm{T}}}$. For the *j*th LED and the *p*th PD of the *i*th user in the *c*th cell, the channel gain is expressed as $(\mathbf{H}_{c,i})_{p,j}$ [9].

At the transmitter, LEDs serve multiple users simultaneously. To eliminate intra-CI, the signals transmitted to the users within the *c*th cell are precoded by a block diagonalization precoder (BDP) given by $\mathbf{W}_c \in \mathcal{R}^{N_{c,\mathrm{T}} \times N_{c,\mathrm{R}}}$. To ensure that the amplitude of the transmitted signal is within the dynamic range of LEDs, a DC bias I_{B} is added. The transmitted signal from the *j*th LED to the users in the *c*th cell is written as

$$x_{c,j} = \sum_{i=1}^{N_{c,\mathrm{R}}} \mathbf{W}_{c,i(j,:)} \mathbf{d}_c + I_{\mathrm{B}}, \tag{1}$$

where $\mathbf{W}_{c,i(j,:)} \in \mathcal{R}^{1 \times N_{c,\mathrm{T}}}$ is the *j*th row of the precoding matrix of the *i*th user in *c*th cell. $\mathbf{d}_c = \left[\, \mathbf{d}_{c,1}^{\mathrm{T}}, \; \mathbf{d}_{c,2}^{\mathrm{T}}, \; \cdots, \; \mathbf{d}_{c,K_c}^{\mathrm{T}} \,\right]^{\mathrm{T}} \in \mathcal{R}^{N_{c,\mathrm{R}} \times 1}$ is the data vector of all users in the *c*th cell, where $\mathbf{d}_{c,i} \in \mathcal{R}^{N_{c,\mathrm{R},i} \times 1}$ represents the data vector of the *i*th user in the *c*th cell, whose elements are assumed to be normalized pulse amplitude modulation (PAM) symbols and is normalized to $[-1, 1]$. Since $x_{c,j}$ must satisfy $x_{c,j} \in [I_{\mathrm{l}}, I_{\mathrm{h}}]$ where I_{l} and I_{h} are the lower and upper bounds of the dynamic range of LEDs, respectively, we have

$$\sum_{i=1}^{N_{c,\mathrm{R}}} \left\| \mathbf{W}_{c,i(j,:)} \right\|_1 \leq \Delta I, \tag{2}$$

where $\Delta I = \min\left(I_{\mathrm{B}} - I_{\mathrm{l}}, I_{\mathrm{h}} - I_{\mathrm{B}}\right)$. In this work, BDP is applied, which satisfy the constraint $\mathbf{H}_{c,i}\mathbf{W}_{c,q} = 0, i \neq q$ implying that the intra-CI can be completely eliminated.

To calculate the precoding matrix, we first define the channel matrix $\tilde{\mathbf{H}}_{c,i} \in \mathcal{R}^{(N_{c,\mathrm{R}} - N_{c,\mathrm{R},i}) \times N_{c,\mathrm{T}}}$ as $\tilde{\mathbf{H}}_{c,i} = \left[\, \mathbf{H}_{c,1}^{\mathrm{T}}, \; \cdots, \; \mathbf{H}_{c,N_{c,\mathrm{R},i-1}}^{\mathrm{T}}, \; \mathbf{H}_{c,N_{c,\mathrm{R},i+1}}^{\mathrm{T}}, \; \cdots, \; \mathbf{H}_{c,N_{c,\mathrm{R}}}^{\mathrm{T}} \,\right]^{\mathrm{T}}$. Then we calculate the singular value decomposition (SVD) of $\tilde{\mathbf{H}}_{c,i}$ as

$$\tilde{\mathbf{H}}_{c,i} = \tilde{\mathbf{U}}_{c,i} \tilde{\mathbf{\Lambda}}_{c,i} \left[\tilde{\mathbf{V}}_{c,i}^{(1)} \middle| \tilde{\mathbf{V}}_{c,i}^{(0)} \right]^{\mathrm{H}}, \tag{3}$$

where $\tilde{\mathbf{U}}_{c,i} \in \mathcal{R}^{(N_{c,\mathrm{R}} - N_{c,\mathrm{R},i}) \times (N_{c,\mathrm{R}} - N_{c,\mathrm{R},i})}$ contains the left singular vectors of $\tilde{\mathbf{H}}_{c,i}$, while $\tilde{\mathbf{V}}_{c,i}^{(1)} \in \mathcal{R}^{N_{c,\mathrm{T}} \times (N_{c,\mathrm{R}} - N_{c,\mathrm{R},i})}$ and $\tilde{\mathbf{V}}_{c,i}^{(0)} \in \mathcal{R}^{N_{c,\mathrm{T}} \times (N_{c,\mathrm{T}} - (N_{c,\mathrm{R}} - N_{c,\mathrm{R},i}))}$. Since $\tilde{\mathbf{V}}_{c,i}^{(0)}$ forms an orthogonal basis for the null space of $\tilde{\mathbf{H}}_{c,i}$, the precoding matrix can be any linear combination of $\tilde{\mathbf{V}}_{c,i}^{(0)}$, given by

$$\mathbf{W}_{c,i} = \tilde{\mathbf{V}}_{c,i}^{(0)} \mathbf{q}_{c,i}, \tag{4}$$

where $\mathbf{q}_{c,i}$ can be any arbitrary matrix, which is optimized in the next section.

At the receiver, the signal received by the *i*th user after removing the DC components can be expressed as (5), where γ and ς are the responsivity of the PD and the electrical-to-optical conversion coefficient, respectively. Moreover, $\mathbf{x}_c = \left[\, x_{c,1}, \; x_{c,2}, \; \cdots \; x_{c,N_{c,\mathrm{T}}} \,\right]^{\mathrm{T}} \in R^{N_{c,\mathrm{T}} \times 1}$, $c \in \{1, \cdots, N_c\}$ is the transmitted signal vector. $\mathbf{H}_{c,i}\mathbf{W}_{c,i}\mathbf{d}_{c,i}$ is the desired signal part, and $\mathbf{H}_{c,i} \sum_{k \in u_c, k \neq i}^{K_c} \mathbf{W}_{c,k}\mathbf{d}_{c,k}$ and $\sum_{c' \neq c}^{N_c} \sum_{q \in u_{c'}}^{K_{c'}} \mathbf{H}_{c',q}\mathbf{W}_{c',q}\mathbf{d}_{c',q}$ are the intra-CI and inter-CI, respectively. Besides, $n_{c,i}$ denotes the receiver noise, which is assumed to be additive white Gaussian noise (AWGN) with zero mean and variance $\sigma_{c,j}^2$ defined by $\sigma_{c,i}^2 = 2\gamma e \overline{P_r^{c,i}} B + 4\pi e_{\mathrm{ch}} A_r \gamma \chi_{amp} \left(1 - \cos\left(\Psi\right)\right) B + i_{amp}^2 B$, where e_{ch} is the elementary charge, B denotes the system bandwidth, and i_{amp} is the pre-amplifier noise current density. Moreover, $\overline{P_r^{c,i}} = E[P_r^{c,i}] = \zeta\left(\boldsymbol{H}_{c,i,:}\boldsymbol{I}_{\mathrm{B}}^{c}\right)$ is the average received optical power of user i in the *c*th cell, and χ_{amp} is the ambient light photocurrent.

B. Hybrid Dimming

The dimming level in HD scheme is defined as: [9]

$$\eta = \frac{\|\boldsymbol{A}\|_1 (I_{\mathrm{B}} - I_{\mathrm{l}})}{N_{\mathrm{T}}(I_0 - I_{\mathrm{l}})} \times 100\%, \tag{6}$$

where $\boldsymbol{A} = \mathrm{diag}\left\{\boldsymbol{A}_1, \boldsymbol{A}_2, \cdots, \boldsymbol{A}_{N_c}\right\} \in \mathcal{R}^{N_{\mathrm{T}} \times N_{\mathrm{T}}}$ is the combination form of the LED selection matrices of all N_c cells. In addition, $\|\boldsymbol{A}\|_1 = n_t$ is the number of activated LEDs and $I_0 = (I_{\mathrm{l}} + I_{\mathrm{h}})/2$. Similar to our former work [10], we adopt a two-step method to satisfy the dimming constraint. In the first step, the coarse dimming control is achieved by adjusting the number of activated LEDs $n_t = \lfloor \eta N_{\mathrm{T}} \rfloor$. Then the DC bias can be obtained as $I_B = \frac{\eta N_T I_0}{n_t}$.

Furthermore, the coefficient of variation of root mean square error (CV(RMSE)) is used to quantify the illumination uniformity [11], which is defined as:

$$\mathrm{CV(RMSE)} = \frac{\upsilon}{\overline{E}}, \tag{7}$$

where υ is the root mean square error of illumination given by $\upsilon = \sqrt{\frac{1}{K} \sum_{\mu=1}^{K} \left(\|\boldsymbol{E}_\mu \boldsymbol{A}\|_1 - \overline{E}\right)^2}$, and $\overline{E}$ is the average illumination given by $\overline{E}\left(\mathbf{E}_\mu, \boldsymbol{A}\right) = \underset{\mu \in \{1,2,\cdots,K\}}{\mathrm{avg}} \left\{\|\mathbf{E}_\mu \boldsymbol{A}\|_1\right\}$, where K is the total number of the sample points on the receiver plane, $\boldsymbol{E}_\mu = \left[\, E_{\mu,1} \;\; E_{\mu,2} \;\; \cdots \;\; E_{\mu,N_{\mathrm{T}}} \,\right] \in \mathcal{R}^{1 \times N_{\mathrm{T}}}$ is the illuminance vector of the μth sample point, whose element $E_{\mu,j}, \forall j \in \{1, 2, \cdots, N_{\mathrm{T}}\}$ is the horizontal illuminance in lux of the jth LED received at the μth sample point [12].

C. Channel Capacity Bound

To introduce the sum-rate maximization problem for the TRASP-HD scheme in MC MU-MIMO system, we first

$$\mathbf{y}_{c,i} = \gamma\varsigma \begin{bmatrix} \mathbf{H}_{1,i} & \mathbf{H}_{2,i} & \cdots & \mathbf{H}_{N_c,i} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_{N_c} \end{bmatrix} + n_{c,i}$$

$$= \gamma\varsigma \left(\mathbf{H}_{c,i}\mathbf{W}_{c,i}d_{c,i} + \mathbf{H}_{c,i}\sum_{k\in u_c, k\neq i}^{K_c} \mathbf{W}_{c,k}d_{c,k} + \sum_{c'\neq c}^{N_c}\sum_{q\in u_{c'}}^{K_{c'}} \mathbf{H}_{c',q}\mathbf{W}_{c',q}d_{c',q} + \mathbf{H}_c\mathbf{I}_{\mathrm{B}}^{c} + \mathbf{H}_{c'}\mathbf{I}_{\mathrm{B}}^{c'} \right) + n_{c,i}$$

$$= \gamma\varsigma \left(\mathbf{H}_{c,i}\mathbf{W}_{c,i}d_{c,i} + \sum_{c'\neq c}^{N_c}\sum_{q\in u_{c'}}^{K_{c'}} \mathbf{H}_{c',q}\mathbf{W}_{c',q}d_{c',q} \right) + n_{c,i}. \quad (5)$$

define $\mathbf{M}_{c,i} = diag\{\mathbf{m}_{c,i}\} \in \mathcal{R}^{N_{c,\mathrm{R},i}\times N_{c,\mathrm{R},i}}$ as the receive antenna selection (RAS) matrix of the *i*th user in the *c*th cell, where $\mathbf{m}_{c,i} = \begin{bmatrix} m_{c,i,1}, & m_{c,i,2}, & \cdots, & m_{c,i,N_{c,\mathrm{R},i}} \end{bmatrix}^{\mathrm{T}} \in \mathcal{R}^{N_{c,\mathrm{R},i}\times 1}$ is a binary vector. If the *p*th PD is selected, $m_{c,i,p} = 1$; otherwise, $m_{c,i,p} = 0$. With the RAS matrix $\mathbf{M}_{c,i}$ and the TAS matrix $\mathbf{A}_c$, the SINR $\xi_{c,i}$ is rewritten as

$$\xi_{c,i} = \frac{2(\gamma\varsigma)^2 |(\mathbf{M}_{c,i}\mathbf{H}_{c,i}\mathbf{A}_c)\,\mathbf{W}_{c,i}|^2}{\pi e\left(\frac{(\gamma\varsigma)^2}{3}\sum_{c'\neq c}^{N_c}\sum_{q\in u_{c'}}^{K_{c'}}\left[(\mathbf{M}_{c,i}\mathbf{H}_{c',i}\mathbf{A}_{c'})\,\mathbf{W}_{c',q}\right]^2 + \sigma_{c,i}^2\right)}. \quad (8)$$

Therefore, the sum-rate of the system in terms of the number of cells, receive and transmit antenna selection and precoding matrix design is written as

$$R\left(N_c, \mathbf{M}_{c,i}, \mathbf{A}_c, \mathbf{W}_{c,i}\right) = \frac{1}{2}\sum_{c=1}^{N_c}\sum_{i=1}^{K_c}\log\left(1+\xi_{c,i}\right). \quad (9)$$

III. Joint Design of TRAS, Cell Formation and Precoding Matrix

In this section, we propose the joint optimization of the proposed TRASP-HD scheme in MC MU-MIMO VLC systems as a sum-rate maximization problem. This problem is a non-convex mixed-integer problem and its direct solution is computationally intractable. To provide a tradeoff between the achieved performance and computational complexity, a sub-optimal algorithm is proposed by splitting the joint problem into three separate sub-problems.

A. Problem Formulation

The proposed optimization problem for maximizing the sum rate of all users in the MC MU-MIMO VLC system under the constraints of given dimming level and uniform illumination distribution is formulated as follows:

$$\max_{N_c,\mathbf{M}_{c,i},\mathbf{A}_c,\mathbf{W}_{c,i}} R\left(N_c, \mathbf{M}_{c,i}, \mathbf{A}_c, \mathbf{W}_{c,i}\right), \quad (10)$$

$$\text{s.t.} \quad \mathbf{W}_{c,i} = \tilde{\mathbf{V}}_{c,i}^{(0)}\mathbf{q}_{c,i}, \quad (10a)$$

$$\sum_{i=1}^{N_{c,\mathrm{R}}} \left\|\mathbf{W}_{c,i(j,:)}\right\|_1 \leq \Delta I, \quad (10b)$$

$$\mathrm{CV(RMSE)} = \frac{v}{\overline{E}}, \quad (10c)$$

$$\eta = \frac{\sum_{c=1}^{N_c}\left\|\mathbf{A}_c\right\|_1\left(I_{\mathrm{B}} - I_{\mathrm{l}}\right)}{N_{\mathrm{T}}\left(I_0 - I_{\mathrm{l}}\right)} \times 100\%, \quad (10d)$$

$$\sum_{c=1}^{N_c}\sum_{i=1}^{K_c}\left\|\mathbf{M}_{c,i}\right\|_1 = n_{\mathrm{R}}, \quad (10e)$$

$$a_{c,j} \in \{0,1\} \quad \forall j \in \{1,2,\cdots,N_{c,\mathrm{T}}\}, \quad (10f)$$

$$m_{c,i,p} \in \{0,1\} \quad \begin{array}{l}\forall i \in \{1,2,\cdots,K_c\},\\ \forall p \in \{1,2,\cdots,N_{c,\mathrm{R},i}\},\end{array} \quad (10g)$$

where $a_{c,j}$ and $m_{c,i,p}$ are binary elements of the TRAS matrices to indicate whether an antenna is activated. The objective function (10) is the achievable sum rate of the users according to (8) and (9). Also, equation (10a) is the BDP constraint explained in Section II. Moreover, equation (10b) implies that the amplitude of the precoding matrix must be in the range of ΔI to satisfy $x_{c,j} \in [I_{\mathrm{l}}, I_{\mathrm{h}}]$. In addition, equations (10c) and (10d) represent the illumination uniformity constraint and illumination level constraint, respectively. Further, equation (10e) is the constraint of the activated number of PDs. Finally, equations (10f) and (10g) indicates that the diagonal elements of $\mathbf{A}_c$ and $\mathbf{M}_{c,i}$ are binary.

B. Proposed Algorithm

Since the problem (10) is a non-convex mixed integer problem and is nondeterministic polynomial time hard (NP-hard), a sub-optimal algorithm is proposed by splitting the problem into three parts: TRAS, UC cell formation and precoding design.

1. TRAS: In MC MU-MIMO VLC systems, each user is equipped with multiple PDs, and thus the channel correlation between the different users is increased which yields a higher MUI. The receive and transmit antenna deployment is crucial to alleviate this problem. Unfortunately, the antenna selection (AS) problem is an integer programming problem, whose exact solution is NP-hard. Furthermore, the inter-CI and intra-CI change dynamically and cannot be determined since the AS must be performed before cell formation. Therefore, the proposed TRAS sub-problem considers the scenario where all the users and LEDs are in the same cell, which can be treated as a single cell MU-MIMO system, written as

$$\max_{\mathbf{M}_{c,i},\mathbf{A}_c} R\left(\mathbf{M}_{c,i}, \mathbf{A}_c\right) = \frac{1}{2}\sum_{c=1}^{N_c}\sum_{i=1}^{K_c}\log\left(1 + m_{c,i}|(\mathbf{M}_{c,i}\mathbf{H}_{c,i}\mathbf{A}_c)\,\mathbf{W}_{c,i}|^2\right), \quad (11)$$

$$\text{s.t.} \quad (10\text{c}), (10\text{d}), (10\text{e}), (10\text{f}), (10\text{g}),$$

where $m_{c,i} = \frac{2(\gamma\varsigma)^2}{\pi e\sigma_{c,i}^2}$. As shown in [5], the signal to noise ratio (SNR) of a spatial multiplexing system with linear receivers is lower bounded by the monotonically increasing function of the minimum singular value (MSV) of the equivalent channel. To provide a tradeoff between the performance and computational complexity, a simplified receive antenna selection algorithm based on the maximum MSV (MMSV) criteria is adopted. The algorithm is summarized as follows:

Denote Ω_p as the set of all the possible deployments of users, and denote $\mathcal{M}_p \in \Omega_p$ as the *p*th candidate subset, respectively. Then, we have

$$\mathbf{H}_{i,\mathcal{M}_p}\tilde{\mathbf{V}}_{i,\mathcal{M}_p}^{(0)} = \mathbf{U}_{i,\mathcal{M}_p}\mathbf{\Lambda}_{i,\mathcal{M}_i}\mathbf{V}_{i,\mathcal{M}_p}^{(1)\mathrm{H}}, \quad (12)$$

where $\mathbf{H}_{i,\mathcal{M}_p}$ and $\tilde{\mathbf{V}}_{i,\mathcal{M}_p}^{(0)}$ represent the effective channel of *i*th user in $\mathcal{M}_p$, and the corresponding right singular vector given by (3), respectively. We can obtain the MSV of the *i*th user in $\mathcal{M}_p$ as $\lambda_{i,M_p}^{\min} = \min\left\{\operatorname{diag}\left(\mathbf{\Lambda}_{i,M_p}\right)\right\}$, where $\operatorname{diag}\left(\mathbf{\Lambda}_{i,M_p}\right)$ is the operation that forms a set containing all elements on the diagonal of $\mathbf{\Lambda}_{i,M_p}$. Finally, the user set $\hat{M}_p$ that satisfies $\hat{M}_p = \arg\max_h \left\{\lambda_{i,M_p}^{\min}\right\}$ is selected as the RAS.

Next, the TAS is solved using the penalty method. In particular, denote the diagonal entries of $\mathbf{A}_c$ as $\boldsymbol{a}_c = \{0,1\}^{1\times N_{c,\mathrm{T}}}$, the exterior penalty function is constructed as follows:

$$\max_{a_{c,j}} R\left(a_{c,j}\right) - \lambda \sum_{c=1}^{N_c}\sum_{j=1}^{N_{c,\mathrm{T}}}\left(a_{c,j} - a_{c,j}^2\right), \tag{13}$$
$$\text{s.t.} \qquad (10\text{c}), (10\text{d}), (10\text{f}),$$

where $\lambda \gg 1$ is a large constant acting as a penalty factor, and $-\lambda \sum_{c=1}^{N_c}\sum_{j=1}^{N_{c,\mathrm{T}}}\left(a_{c,j} - a_{c,j}^2\right)$ is a penalty term which penalizes the objective function for any value of $a_{c,j}$ other than 0 and 1. Therefore, the optimal value of $a_{c,j}$ must be infinitely close to 0 or 1 when maximizing the objective function. This way, the first sub-problem with integer variables is transformed into a nonlinear programming (NLP) problem with continuous variables, which can be solved by known optimization algorithms such as interior point method. In this work, we adopt the *fmincon()* in MATLAB optimization toolbox to implement the interior point method.

2. Cell Formation Based on Hierarchical Clustering Algorithm: With the selected TRAS, the cells are formed by a low-complexity hierarchical clustering algorithm. First, each user forms an independent cell to construct an initial cell formation. Then, the cells are merged iteratively according to a suitability function, which is used for deciding whether to merge two cells or not.

Before explaining the procedures of the cell formation process, we first define a suitability function $S\left(C_x, C_y\right) = R_{C_{x,y}} - \left(R_{C_x} + R_{C_y}\right)$ [13], where C_x, C_y and $C_{x,y}$ are two cells to be combined and their combination, respectively. When $S\left(C_x, C_y\right) > 0$, this means there exists a sum-rate gain to combine C_x and C_y, and thus $C_{x,y}$ is applied instead of C_x and C_y.

In particular, the steps of cell formation are as follows: *Step 1-Initialization:* Denote the cluster of users in the *c*th cell as $\mathcal{U}_c$, and the set of LEDs in the *c*th cell as $\mathcal{T}_c$, $c \in \{1, 2, \cdots, N_{c,\mathrm{R}}\}$. In this step, each cluster is simply constructed by a single user.

Step 2-PD-LED association: Denote the association matrix of the *i*th user in the *c*th cell as $\mathbf{M}_{c,i} = \mathbf{H}_{c,i} \in R^{N_{c,\mathrm{R},i}\times N_{c,\mathrm{T}}}$. For each PD, find the best PD-LED association [p, $j*$] with the strongest LOS channel amongst the *p*th rows of $\mathbf{M}_{c,i}$. Then set the $j*$ column of $\mathbf{M}_{c,i}$ to $\mathbf{0}$. Then, for the remaining non-zero $N_{c,\mathrm{T}} - N_{c,\mathrm{R},i}$ columns of $\mathbf{M}_{c,i}$, find the strongest channel gain amongst each column, and then set the column to $\mathbf{0}$. For example, if the *n*th entry of the *m*th column is the strongest LOS channel, then the *m*th LED is allocated to the cluster where the *n*th PD is in, written as PD-LED pair [n, m]. Finally $\mathbf{M}_{c,i} = \mathbf{0}$ and all the $N_{c,\mathrm{T}}$ LEDs are allocated to $N_{c,\mathrm{R}}$ PDs.

Step 3-Cells merging: Next, merge the cells iteratively by the hierarchical clustering method. In the first iteration, start with N_{R} cells, and then in each iteration the two cells with the largest value of suitability function are merged, such that the number of cells in the *t*th iteration is $c_t = N_{\mathrm{R}} - (t-1)$. Here, define the suitability matrix $\mathbf{S}_t \in R^{c_t\times c_t}$ for the *t*th iteration, which can be calculated using the suitability function, defined as:

$$\mathbf{S}_{t(x,y)} = \begin{cases} S\left(C_x, C_y\right) & x < y, \\ -\mathrm{Inf} & otherwise \end{cases}. \tag{14}$$

Moreover, the indexes of the cells that need to be merged, denoted as $x*, y*$, are determined by

$$x*, y* = \arg\max\left(\mathbf{S}_{t(x,y)}\right) \tag{15}$$

Finally each the iteration stops when $\max\left(\mathbf{S}_{t(x,y)}\right) \le 0$.

3. Precoding Matrix Design: Given the transmit and receive antenna deployment and cell formation, the third sub-problem is a sum-rate maximization problem in terms of the precoding matrix design. Based on (10a) this sub-problem can be reformulated as

$$\max_{\mathbf{q}_{c,i}} R\left(\mathbf{q}_{c,i}\right), \tag{16}$$
$$\text{s.t.} \sum_{i=1}^{N_{c,R}} \left\|\left(\tilde{\mathbf{V}}_{c,i}^{(0)}\mathbf{q}_{c,i}\right)_{(j,:)}\right\|_1 \le \Delta I. \tag{16a}$$

The objective function (16) is non-convex with respect to $\mathbf{q}_{c,i}$. To transform (16) into a convex problem, define $\mathbf{E}_{c.i} = \mathbf{q}_{c,i}\mathbf{q}_{c,i}^{\mathrm{T}}$. Then, according to the mean inequality [14], square the constraint (16a) as

$$\sum_{i=1}^{N_{c,R}} \left\|\left(\tilde{\mathbf{V}}_{c,i}^{(0)}\mathbf{E}_{c.i}\tilde{\mathbf{V}}_{c,i}^{(0)\mathrm{T}}\right)_{(j,:)}\right\|_1 \le \frac{\Delta I^2}{N_{c,R}} \quad \forall j \in \{1, 2, \cdots, N_{c,T}\}. \tag{17}$$

Thus, the third sub-problem can be rewritten as

$$\max_{\mathbf{E}_{c,i}} R\left(\mathbf{E}_{c,i}\right) = \frac{1}{2}\sum_{c=1}^{N_c}\sum_{i=1}^{K_c}\log\left(1 + \xi_{c,i}\left(\mathbf{E}_{c,i}\right)\right), \tag{18}$$
$$\text{s.t.} \qquad (17),$$

where $\xi_{c,i}\left(\mathbf{E}_{c,i}\right)$ is the SINR in terms of $\mathbf{E}_{c,i}$, defined as

$$\xi_{c,i}\left(\mathbf{E}_{c,i}\right) = \frac{2(\gamma\varsigma)^2\left|\hat{\mathbf{H}}_{c,i}\tilde{\mathbf{V}}_{c,i}^{(0)}\mathbf{E}_{c.i}\tilde{\mathbf{V}}_{c,i}^{(0)\mathrm{T}}\hat{\mathbf{H}}_{c,i}^{\mathrm{T}}\right|}{\pi e\left(\frac{(\gamma\varsigma)^2}{3}\sum_{c'\neq c}^{N_c}\sum_{q\in u_{c'}}^{K_{c'}}\left(\hat{\mathbf{H}}_{c',i}\tilde{\mathbf{V}}_{c',q}^{(0)}\mathbf{E}_{c.i}\tilde{\mathbf{V}}_{c',q}^{(0)\mathrm{T}}\hat{\mathbf{H}}_{c',i}^{\mathrm{T}}\right)+\sigma_{c,i}^2\right)}. \tag{19}$$

It can be easily verified that (18) is a convex problem since the objective function is concave and the constraint (17) is linear. When the number of cells is large, the amount of exchanged channel state information between cells is overwhelmingly increased, and thus leads to high computational complexity. In this work, we adopted a distributed method via ADMM, where the optimization process in each iteration is divided into a local and a global optimization problem. In the local optimization problem, only the precoding matrices of each cell are optimized by maximizing its own capacity. Then, the precoding matrices of all the cells are updated in the global optimization. This method can be realized without any backhaul information exchange and can achieve a fast convergence [15]. Particularly, in this method, each cell optimizes its own precoding matrices and treates inter-CI as a constant. Therefore, the optimal problem and constraint are

TABLE I Simulation Parameters.

Parameter	Value
Semiangle of half power, $\Phi_{1/2}$	$80°$
Detect area of PDs, A_r	1 cm^2
FOV of the PD, Ψ	$60°$
Optical filter gain $T_s(\psi_{c,j})$	1
Refractive index of optical concentrator, κ	1
Height from TXs to RX, h	1.75 m
System bandwidth, B	100 MHz
PD responsivity, γ	0.54 A/W
LED conversion factor, ζ	0.44 W/A
Ambient light photocurrent, χ_{amb}	10.93 A/m^2·Sr
Preamplifier noise current density, i_{amb}	5 pA/Hz$^{-1/2}$

separable in terms of $\mathbf{E}_{c.i}$ [16].

Therefore, to solve the third sub-problem by ADMM, (18) can be rewritten with local variable $\mathbf{E}_{c.i}$ and a common global variable $\mathbf{z}_c$. We first define the feasible region of the constraint (17) as

$$\mathcal{B}_{c,i} = \left\{ \mathbf{E}_{c,i} \middle| \begin{array}{l} \sum\limits_{i=1}^{N_{c,R}} \left\| \left(\tilde{\mathbf{V}}_{c,i}^{(0)} \mathbf{E}_{c.i} \tilde{\mathbf{V}}_{c,i}^{(0)\mathrm{T}} \right)_{(j,:)} \right\|_1 \leq \frac{\Delta I^2}{N_{c,R}}, \\ \forall j \in \{1, 2, \cdots, N_{c,T}\} \end{array} \right\}. \quad (20)$$

Define $\mathbb{I}_{\mathcal{B}_{c,i}}(\mathbf{E}_{c,i})$ as the indicator function of the feasible region $\mathcal{B}_{c,i}$, *i.e.*, $\mathbb{I}_{\mathcal{B}_{c,i}}(\mathbf{E}_{c,i}) = 0$ for $\mathbf{E}_{c,i} \in \mathcal{B}_{c,i}$ and $\mathbb{I}_{\mathcal{B}_{c,i}}(\mathbf{E}_{c,i}) = -\infty$ otherwise. Thus, we obtain the equivalent ADMM reformulation of problem as follows

$$\max_{\mathbf{E}_{c,i} \in \mathcal{B}_{c,i}} \quad R(\mathbf{E}_{c,i}) + \mathbb{I}_{\mathcal{B}_{c,i}}(\mathbf{E}_{c,i}), \quad (21)$$

$$\text{s.t.} \quad \mathbf{E}_{\mathrm{c.i}} - \mathbf{z}_{\mathrm{c}} = 0. \quad (21a)$$

The augmented Lagrangian of problem (21) can be formulated as

$$L(\mathbf{E}_{c,i}, \mathbf{z}_c, \lambda_{c,i}) = \frac{1}{2} \sum_{c=1}^{N_c} (l_c(\mathbf{E}_{c,i}, \mathbf{z}_c, \lambda_{c,i})), \quad (22)$$

where $l_c(\mathbf{E}_{c,i}, \mathbf{z}_c, \lambda_{c,i})$ is the objective function to be solved in each cell, given by

$$\begin{aligned} l_c(\mathbf{E}_{c,i}, \mathbf{z}_c, \lambda_{c,i}) = I_{B_{c,i}}(\mathbf{E}_{c,i}) + \sum_{i=1}^{K_c} \log(1 + \xi_{c,i}(\mathbf{E}_{c,i})) \\ + \sum_{i=1}^{K_c} \lambda_{c,i}(\mathbf{E}_{c,i} - \mathbf{z}_c) + \frac{\rho}{2} \|\mathbf{E}_{c,i} - \mathbf{z}_c\|_2^2, \end{aligned} \quad (23)$$

where $\lambda_{c,i} > 0$ and ρ are the penalty parameter and the dual variable parameter, respectively. In the $(t+1)$th iteration, $\mathbf{E}_{c,i}, \mathbf{z}_c, \lambda_{c,i}$ are updated as follows

$$\mathbf{E}_{c,i}^{(t+1)} = \arg\min_{\mathbf{E}_{c,i}} l_c\left(\mathbf{E}_{c,i}, \mathbf{z}_c^{(t)}, \lambda_{c,i}^{(t)}\right), \quad (24)$$

$$\mathbf{z}_c^{(t+1)} = \frac{1}{N_c} \sum_{c=1}^{N_c} \left(\mathbf{E}_{c,i}^{(t+1)} + (1/\rho)\lambda_{c,i}^{(t)}\right), \quad (25)$$

and

$$\lambda_{c,i}^{(t+1)} = \lambda_{c,i}^{(t)} + \rho\left(\mathbf{E}_{c,i}^{(t+1)} - \mathbf{z}_c^{(t+1)}\right). \quad (26)$$

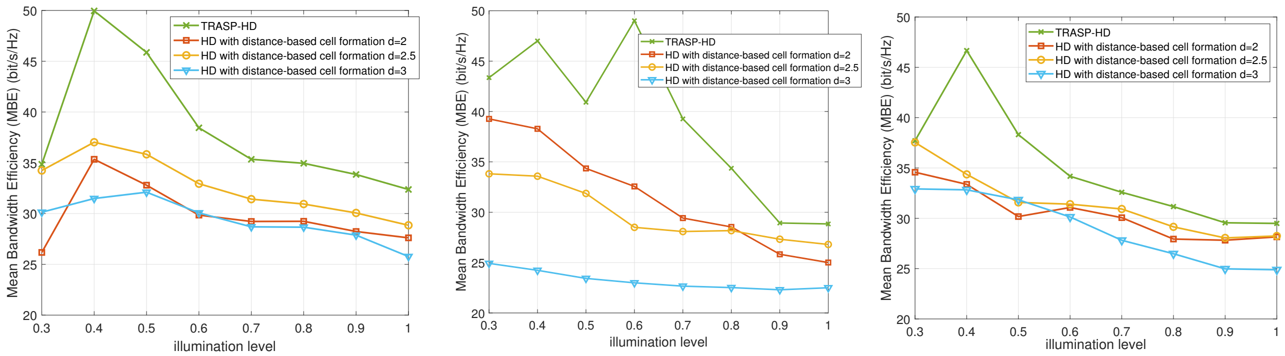
The optimal value of $\mathbf{E}_{c,i}^{(t+1)}$ in iteration can be efficiently solved by using some off-the-shelf convex optimization tools such as CVX.

IV. Performance Evaluation

In this section, we consider an MC MU-MIMO VLC system employing TRASP-HD scheme in a 8m × 8m × 3m square room, where $K_c = 20$ users that are randomly located at the height of 0.75 m above the floor and each user is equipped with $N_{c,\mathrm{R},i}$ PDs. Moreover, $N_\mathrm{T} = 64, 81$ and 100 LEDs are uniformly installed at 2.5m height. The detailed simulation parameters are listed in Table I.

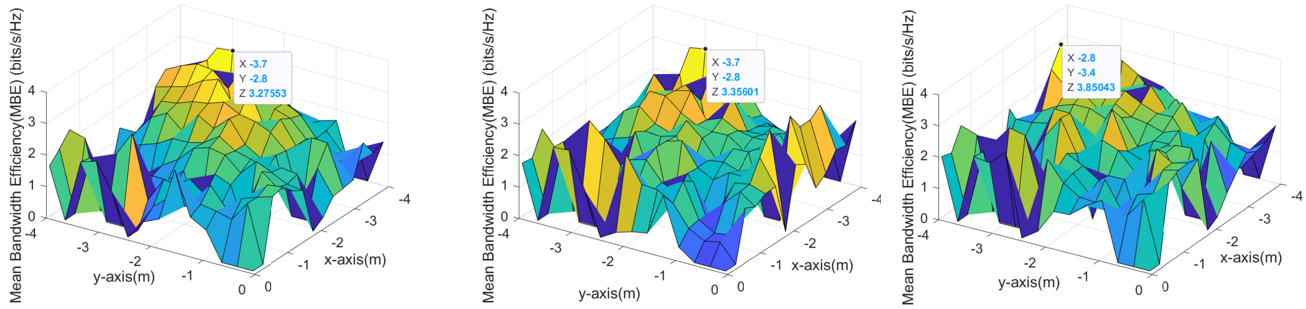
Fig. 1 presents the mean bandwidth efficiency (MBE) performance of the proposed TRASP-HD scheme and adopts the HD schemes with distance-based cell formation as the baseline. Fig. 1(a) presents the MBE of TRASP-HD scheme with $N_\mathrm{T} = 64$ and $N_\mathrm{R} = 20$. The distance thresholds of the baseline schemes are set to 2m, 2.5m and 3m. From Fig. 1(a), we can observe that TRASP-HD scheme always achieves the best MBE compared to all other considered schemes. For instance, the MBE of TRASP-HD scheme are 6.73, 2.77, 2.36 bit/s/Hz higher than that of the HD schemes with distance-based cell formation when the dimming levels are 30%, 50% and 70%, respectively. This is because the cells are formed as long as there exists capacity gains, while the distance-based cell formation is performed by a given distance threshold, which cannot achieve the best sum-rate performance in most cases. It can also be observed that the MBE performances of all the dimming schemes do not increase with dimming levels. This is because the number of activated LEDs is increased with dimming levels, and thus, the inter-CI becomes worse and leads to lowering the data-rate of users. We further present the MBE of TRASP-HD scheme under different values of N_T to prove that our proposed TRASP-HD scheme has a superior performance compared with the state-of-the-art distance-based cell formation schemes shown in Fig. 1(b) and Fig. 1(c). For instance, in Fig. 1(b), where we set $N_\mathrm{T} = 81$, it can be seen that the MBE of our proposed TRASP-HD scheme are 4.09, 6.57, 9.84 bits/s/Hz higher than that of the HD scheme with distance-based cell formation with distance thereshold $d = 2$ when the dimming levels are 30%, 50% and 70%, respectively. In addition, in Fig. 1(c) with $N_\mathrm{T} = 100$, the MBE of our proposed TRASP-HD scheme are 8.14, 2.52 and 1.73 bit/s/Hz higher than that of the HD scheme with distance-based cell formation with distance thereshold $d = 2$ when the dimming levels are 50%, 70% and 90%, respectively. This results show the superiority of our proposed TRASP-HD scheme compared to state-of-the-art approaches.

To evaluate the communication performance with different user locations in the indoor space, the MBE performance of one mobile user with the proposed TRASP-HD scheme and the HD scheme with distance-based cell formation are shown in Fig. 2. The user moves randomly within the range of -4 m to 0 m on both the horizontal and vertical axis while the positions of the rest of users are fixed. As shown in Fig. 2, the TRASP-HD scheme significantly improves the MBE for different user locations. In particular, for the HD scheme with the distance-based cell formation with distance thereshold $d = 2$ in Fig. 2(a), the MBEs in the locations of (0,-4 m) and (-2 m, -2 m) are 1.58 bits/s/Hz and 2.21 bits/s/Hz, respectively. For the HD scheme with the distance-based cell formation with distance thereshold $d = 3$ in Fig. 2(b), the MBEs in the same locations are 1.68 bits/s/Hz and 1.23 bits/s/Hz, respectively, while the MBEs in the same locations for the hierarchical cell formation in Fig. 2(c) are 1.94 bits/s/Hz and 3.06 bits/s/Hz, respectively. This results further prove that our proposed TRASP-HD scheme achieves a superior MBE performance with a scattered distribution of users.



(a) $N_{\mathrm{T}} = 64$, $N_{\mathrm{R}} = 20$ (b) $N_{\mathrm{T}} = 81$, $N_{\mathrm{R}} = 20$ (c) $N_{\mathrm{T}} = 100$, $N_{\mathrm{R}} = 20$

Fig. 1. MBE of TRASP-HD scheme under different cell formations.



(a) HD with distance-based cell formation, $d = 2$ m (b) HD with distance-based cell formation, $d = 3$ m (c) TRASP-HD

Fig. 2. MBE distribution of different cell formations under η= 70%, $N_{\mathrm{T}} = 64$, $N_{\mathrm{R}} = 20$.

V. Conclusion

This paper proposed an HD scheme based on the joint design of TRAS, cell formation, and precoding design in MC MU-MIMO VLC systems. In the proposed TRASP-HD scheme, both the number of activated LEDs and DC bias are adjusted to achieve dimming control. With the goal of alleviating the inter-CI, we formulated a sum-rate maximization problem under varied dimming levels and illumination constraints. Then, to solve this non-convex mixed integer problem, we divided the original problem into three sub-problems, and solved them iteratively. Numerical and simulation results showed that the proposed TRASP-HD scheme improves the sum-rate of users compared to state-of-the-art approaches. Finally, the achieved MBE of the proposed TRASP-HD scheme is 2.36 bits/s/Hz higher than the HD scheme with distance-based cell formation under a fixed distance threshold with a dimming level of 70%.

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