Reply to comment by D. R. Lester et al. on "Plume spreading in groundwater by stretching and folding"

David C. Mays[1] and Roseanna M. Neupauer[2]

Received 12 October 2012; accepted 24 October 2012.

Citation: Mays, D. C., and R. M. Neupauer (2013), Reply to comment by D. R. Lester et al. on "Plume spreading in groundwater by stretching and folding," *Water Resour. Res.*, *49*, doi:10.1002/wrcr.20081.

[1] We thank *Lester et al.* [2013] for their thought-provoking comment on our recent work on plume spreading by chaotic advection [*Mays and Neupauer*, 2012]. We agree completely with their opinion that chaotic advection provides a framework for deepened understanding of fluid mixing and transport processes in porous media, and that stretching and folding via engineered injection and extraction shows promise for improved groundwater remediation. In this commentary, we address two questions posed by their comment.

[2] The first question is the following. Does relaxing the assumption of reinjection at a deterministic angle remove chaos from periodic injection and extraction schemes, such as the pulsed dipole [e.g., *Jones and Aref*, 1988] and the periodically reoriented dipole [e.g., *Lester et al.*, 2009]? Here we agree with the comment that the answer is no: relaxing the assumption of reinjection at a deterministic angle does *not* remove chaos from the system but instead produces stochastic chaos rather than deterministic chaos in the region of enhanced mixing. As a technical point, the analysis we presented in *Mays and Neupauer* [2012] to confirm the presence of deterministic chaos (finding periodic points, classifying them as elliptic or hyperbolic, plotting stable and unstable manifolds, and then identifying heteroclinic points) would not apply to the case of stochastic chaos, in which periodic points are absent. But we agree that adopting realistic assumptions about the nature of reinjection, specifically with regard to reinjection angle, would not change the chaotic nature of the underlying dynamics of the pulsed dipole and related approaches. This point has been made clearly by the comment and previous works cited therein.

[3] The second question is the following. Are injection and extraction schemes requiring reinjection practical and feasible groundwater intervention and engineering tools? Here we disagree with the comment because in our opinion reinjection presents a number of practical and theoretical concerns in the context of groundwater remediation. Turning first to the practical concerns, reinjection demands attention from a regulatory perspective, at least in cases where the reinjection fluid has the potential to endanger drinking water, because injection of the contaminated water into aquifers is regulated for the purposes of environmental protection. In the United States, for example, the Safe Drinking Water Act establishes the state-administered underground injection control programs that regulate injection of contaminants into aquifers [*42 U.S.C. 300f–300j–9*, 2002, §1421 et seq], and the Resource Conservation and Recovery Act (RCRA) prohibits the injection of hazardous waste by underground injection into or within 0.4 km (1/4 mile) of an underground source of drinking water or above such a formation. The RCRA provides an exception for reinjection during groundwater remediation, but only when the remediation is part of a response action under the RCRA [*42 U.S.C. 6901–6992k*, 2002, §3020] or the Comprehensive Environmental Response, Compensation, and Liability Act [*42 U.S.C. 9601–9675*, 2002, §104 or §106], and only when a treatment solution is added to the groundwater prior to reinjection. In the context of in situ remediation, the requirement to add treatment solution presents a hydraulic concern, because the treatment solution will trigger the in situ remediation reactions that may result in chemical or biological byproducts that clog the aquifer in the vicinity of the injection wells [*Bagtzoglou and Oates*, 2007; *Li et al.*, 2009, 2010; *MacDonald et al.*, 1999]. Clogging would also be a concern near extraction wells in cases where the reinjection scheme extracts the interface between the treatment solution and the contaminated groundwater because that interface is where the in situ remediation reactions take place and their byproducts accumulate. These regulatory and hydraulic concerns were raised in our paper [*Mays and Neupauer*, 2012].

[4] A more theoretical concern is that the stochastic chaotic flows may be equivalent, in a statistical sense, to stochastic noise superposed on a regular flow, i.e., the opposite of a chaotic flow. To illustrate this point, consider the four examples of dipole flow in Figure 1. Each panel shows the dipole flow from an injection well at $(-1, 0)$ to an extraction well at $(1, 0)$, assuming passive scalar transport without dispersion in a confined, homogeneous, isotropic aquifer of infinite extent, such that the resulting flow is symmetric about the x axis. Starting with a cross formation, comprising 16 red particles vertically from $(0, 1/16)$ to $(0, 1)$ and 16 blue particles horizontally from $(-1/2, 1/2)$ to $(1/2, 1/2)$, each panel shows the particle positions after injection of the first pore volume, the second pore volume, and so on for the first 1000 pore volumes, where a pore

[1]Department of Civil Engineering, University of Colorado Denver, Denver, Colorado, USA.
[2]Department of Civil, Environmental, and Architectural Engineering, University of Colorado Boulder, Boulder, Colorado, USA.

Corresponding author: D. C. Mays, Department of Civil Engineering, University of Colorado Denver, Campus Box 113, PO Box 173364, Denver, CO 80217-3364, USA. (david.mays@ucdenver.edu)
This article is a reply to *Lester et al.* [2013] doi:10.1002/wrcr.20082.

0043-1397/13/10.1002/wrcr.20081

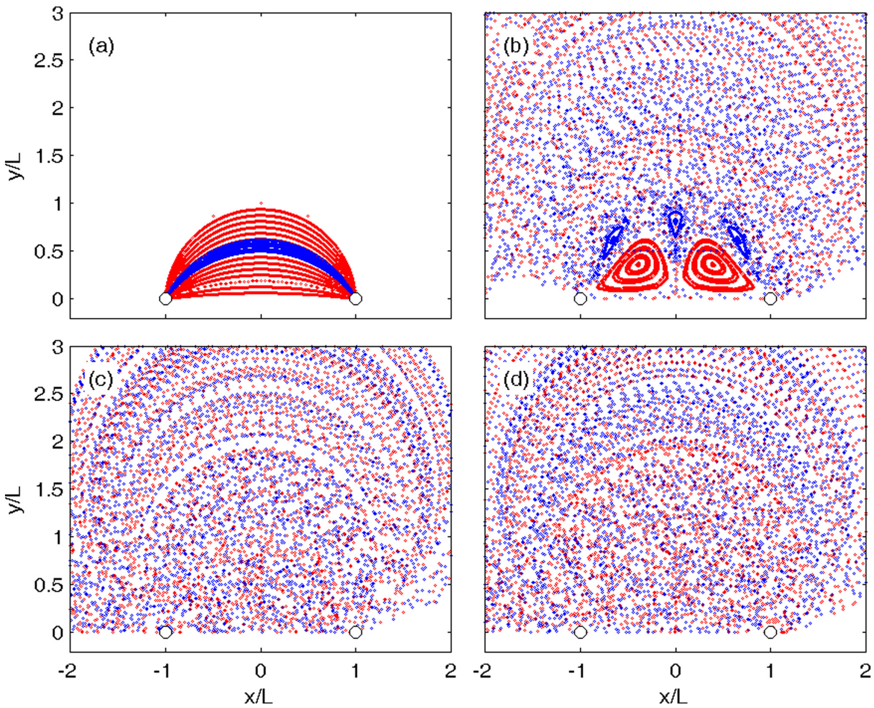

Figure 1. Poincaré maps showing particle positions after injection of each of 1000 pore volumes for the four variations of dipole flow: (a) the standard dipole, (b) the pulsed dipole, (c) the standard dipole with the random reinjection angle, and (d) the pulsed dipole with the random reinjection angle.

volume is the volume of the fluid contained within a vertical cylinder centered at the origin and passing through the wells. To use the terminology of dynamical systems, therefore, each panel is a Poincaré map. Figure 1 shows the results for four different flows: (a) the standard dipole, (b) the pulsed dipole, (c) the standard dipole with random reinjection, and (d) the pulsed dipole with random reinjection. All flows assume the first-out-first-in particle ordering; flows in Figures 1a and 1b assume $\theta_i = \pi - \theta_e$, where θ_i is the injection angle, and θ_e is the extraction angle, both measured at their respective well counterclockwise from the positive x axis; flows in Figures 1c and 1d assume random θ_i uniformly distributed on $0 < \theta_i < \pi$. The pulsed dipole (Figure 1b) replicates the coherent structures in *Jones and Aref* [1988, Figure 3(g)] for the case of $\Lambda^2 = 1$, where Λ^2 is defined in *Mays and Neupauer* [2012, equation (5)] and can be interpreted as the number of pore volumes injected during each iteration of the pulsed dipole. Figure 2 shows the average interparticle distance (AIPD) between the red particles and the blue particles for each flow, calculated with *Bagtzoglou and Oates* [2007, equation (1)]. Since flows (c) and (d) include a random component, Figure 2 shows the averages and standard deviations of the AIPD calculated from a Monte Carlo simulation repeated 30 times. For the standard dipole, the AIPD is essentially constant for all the iterations, while the AIPD increases with iteration for all the other flows. Physically, the increase in the AIPD means that particles are being spread through a larger region of the aquifer with each iteration. Although the AIPD is much larger for the pulsed dipole than for the standard dipole when the reinjection angle is deterministic, the AIPD is similar for the standard and pulsed dipoles when the reinjection angle is random. Thus, the added benefit of the pulsed dipole may be minimal.

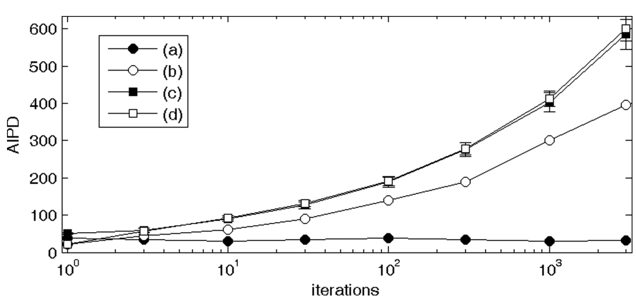

Figure 2. AIPD between the red particles and the blue particles for the four variations of dipole flow shown in Figure 1: (a) the standard dipole, (b) the pulsed dipole, (c) the standard dipole with random reinjection, and (d) the pulsed dipole with random reinjection.

[5] Returning to the practical concerns, all four flows in Figure 1 would introduce regulatory and clogging concerns because they require reinjection. Avoiding reinjection was therefore a primary motivation for our approach to generate a chaotic flow by stretching and folding via engineered injection and extraction [*Mays and Neupauer*, 2012].

[6] **Acknowledgments.** The authors thank Felix Flechas for helpful discussion and James Meiss for reviewing a draft of this commentary. This work was supported by the National Science Foundation (EAR-1113996 and EAR-1114060).

References

42 U.S.C. 300f–300j–9 (2002), Safe Drinking Water Act, *Public Law 107-377*, as amended, U.S. Gov. Print. Off., Washington, D. C., 31 Dec.

42 U.S.C. 6901–6992k (2002), Resource Conservation and Recovery Act, *Public Law 107-377*, as amended, U.S. Gov. Print. Off., Washington, D. C., 31 Dec.

42 U.S.C. 9601–9675 (2002), Comprehensive Environmental Response, Compensation, and Liability Act, *Public Law 107-377*, as amended, U.S. Gov. Print. Off., Washington, D. C., 31 Dec.

Bagtzoglou, A. C., and P. M. Oates (2007), Chaotic advection and enhanced groundwater remediation, *J. Mater. Civ. Eng.*, *19*(1), 75–83.

Jones, S. W., and H. Aref (1988), Chaotic advection in pulsed source sink systems, *Phys. Fluids*, *31*(3), 469–485.

Lester, D., G. Metcalfe, M. Telfry, A. Ord, B. Hobbs, and M. Rudman (2009), Lagrangian topology of a periodically reoriented potential flows: Symmetry, optimization, and mixing, *Phys. Rev. E*, *80*, 036298, doi:10.1103/PhysRevE.80.036208.

Lester, D., M. Trefry, G. Metcalfe, A. Ord, and K. Regenauer-Lieb (2013), Comment on ''Plume spreading in groundwater by stretching and folding,'' *Water Resour. Res.*, *49*, doi:10.1002/wrcr.20082.

Li, L., C. I. Steefel, K. H. Williams, M. J. Wilkins, and S. S. Hubbard (2009), Mineral transformation and biomass accumulation associated with uranium bioremediation at Rifle, Colorado, *Environ. Sci. Technol.*, *43*(14), 5429–5435.

Li, L., C. I. Steefel, M. B. Kowalsky, A. Englert, and S. S. Hubbard (2010), Effects of physical and geochemical heterogeneities on mineral transformation and biomass accumulation during biostimulation experiments at Rifle, Colorado, *J. Contam. Hydrol.*, *112*(1–4), 45–63.

MacDonald, T. R., P. K. Kitanidis, P. L. McCarty, and P. V. Roberts (1999), Mass-transfer limitations for macroscale bioremediation modeling and implications on aquifer clogging, *Ground Water*, *37*(4), 523–531.

Mays, D. C., and R. M. Neupauer (2012), Plume spreading in groundwater by stretching and folding, *Water Resour. Res.*, *48*, W07501, doi:10.1029/2011WR011567.