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Brief paper

Adaptive optimal output regulation of linear discrete-time systems based on event-triggered output-feedback☆

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ABSTRACT

This paper presents novel event-triggered control approaches to solve the adaptive optimal output regulation problem for a class of linear discrete-time systems. Different from most existing research on output regulation problems, the developed adaptive optimal control approaches are based on (1) output-feedback instead of full-state or partial-state feedback, (2) adaptive dynamic programming (ADP) which provides approximate solutions of the optimal control problem without requiring the precise knowledge of the plant dynamics, and (3) an event-triggering mechanism that reduces the communication between the controller and the plant. It is shown that the system in closed-loop with the developed controllers is asymptotically stable at an equilibrium of interest, and the tracking errors asymptotically converge to zero. Moreover, the suboptimality of the closed-loop system is directly determined by the relative threshold, which is a ratio between the triggering threshold and the actual state. A numerical simulation example is employed to verify the effectiveness of the proposed methodologies.

1. Introduction

The output regulation problem addresses the design of a feedback controller to achieve the disturbance rejection and asymptotic tracking for dynamical systems with disturbance and reference signal generated by certain exosystems. Due to its wide application, the problem of output regulation has received tremendous attention (see Bonivento et al. (2001), Huang (2004), Isidori et al. (2003), Krener (1992) and Saberi et al. (2003)). The optimal output regulation problem is studied in Krener (1992) and Saberi et al. (2003), where the transient performance can be optimized by minimizing a predefined cost function. However, a general limitation in these solutions is that the system dynamics is assumed to be perfectly known.

Adaptive dynamic programming (ADP) is a non-model based control approach inspired by biological systems (see Gao et al. (2016), Jiang and Jiang (2012), Kiumarsi et al. (2015), Lewis and Liu (2012), Li et al. (2017), Werbos (1974) and Song et al. (2016)). It can be used to approximate the optimal control solutions for uncertain systems whose uncertain system model is missing. Therefore, it is a good candidate to handle optimal output regulation problems with unknown system dynamics. In our previous work (Gao & Jiang, 2016; Gao et al., 2018), we have, for the fist time, achieved adaptive optimal output regulation for linear systems via ADP.

Recently, event-triggered strategies (Åström & Bernhardsson, 2002; Donkers & Heemels, 2012; Heemels et al., 2013; Lemmon, 2011; Liu & Jiang, 2015; Tabuada, 2007; Xing et al., 2019) have been employed in ADP-based control to save control system resources. An adaptive optimal controller is developed to solve the event-triggered Hamilton–Jacobi-Bellman (HJB) equation for nonlinear systems in Vamvoudakis (2014). Vamvoudakis and Ferraz (2018) further proposes a class of event-triggered controllers for uncertain systems by using Q-learning techniques. In Sahoo et al. (2016), a suboptimal event-triggered condition is provided for a class of discrete-time nonlinear systems in the affine form. Moreover, input constraints and robustness with respect to uncertain terms and unmatched dynamics have also been studied within the scope of event-triggered ADP (Dong et al., 2017; Wang & Liu, 2018; Zhang et al., 2018; Zhu et al., 2017). In Xue et al. (2020), Zhang et al. (2017) and Zhao et al. (2019), the methods of event-triggered H_∞ optimal control via

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ADP are studied. Besides stabilization, some event-triggered optimal methods have been proposed to solve the tracking problems. An event-triggered optimal tracking control algorithm for nonlinear systems is presented in Batmani et al. (2017), but no external disturbance is taken into account and the dynamics of the system are assumed to be known. In Li and Yang (2018), a simultaneous design of a neural network based adaptive control law and an event-triggered condition for a class of strict feedback nonlinear discrete-time systems is proposed, and the uniform ultimate boundedness property is achieved for the closed-loop system.

It should be noticed that most previously proposed ADP-based event-triggered control designs are based on state-feedback. In many practical tracking control problems, the state of a system is not measurable, and only the measured output can be utilized for feedback. The observer-based control design is usually utilized to develop measurement feedback controllers. However, observer design is a challenging problem for the linear optimal output regulation of uncertain systems. Besides this technical challenge, due to sampling errors, unknown system dynamics and unknown disturbances, the main difficulty lies in the design of adaptive optimal controller with event-triggered output-feedback for unknown systems to achieve the disturbance rejection and asymptotic tracking. Moreover, it is a non-trivial task to quantify the relationship between the suboptimality and triggering threshold ratio.

In this paper, we consider a class of unknown discrete-time linear systems with unknown exosystem. The unmeasured states and exostates are reconstructed by using the measured input/output data. A dynamic compensator is designed to compensate the effect of the states of the exosystem. Then, we design a novel event-triggering mechanism which only depends on the tracking errors. The event-triggered ADP method is implemented based on the reconstructed state. It is proved that the discrete-time algebraic Riccati equation and the regulator equation can be iteratively solved by utilizing the event-triggered ADP based on both the policy iteration (PI) and value iteration (VI) methods. With the proposed ADP designs, the event-triggered adaptive optimal output regulation problem is achievable for uncertain linear systems by only using the input/output data. Moreover, the communication between the plant and the controller can be reduced.

The major contributions of this paper are threefold. As the first contribution, we, for the first time, solve the model-free event-triggered output regulation problem via output-feedback. Compared with Batmani et al. (2017), Li and Yang (2018) and Vamvoudakis et al. (2017), the solution in this paper can deal with the optimal output regulation problem for unknown discrete-time linear systems with unmeasurable states, exostates, and disturbances. The second contribution is that we propose two successive approximation algorithms, PI and VI, to approximate the optimal control policy. The closed-loop stability and convergence of proposed algorithms are rigorously guaranteed in this paper. Last but not the least, different from existing studies on optimal control and event-triggering mechanisms, we have analyzed the sensitivity of triggering threshold. To be more specific, we quantify the relationship between the suboptimality of the closed-loop system and the triggering threshold ratio.

The remainder of this paper is organized as follows. In Section 2, we review the linear optimal output regulation problem, and provide a brief review on the even-triggering mechanism. In Section 3, PI and VI based output-feedback event-triggered adaptive optimal controllers are designed, respectively. The main results of the stability and suboptimality analysis are given in Section 4. Simulation for an LCL coupled inverter-based distributed generation system is given in Section 5. Finally, conclusions are given in Section 6.

Notations: Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}_+$ the set of nonnegative real numbers, $\mathbb{Z}_+$ the set of non-negative integers, and $\|\cdot\|$ the Euclidean norm of vectors or matrices. $A\otimes B$ stands for the Kronecker product of matrices A and B. For a symmetric matrix $A\in\mathbb{R}^{n\times n}$, $\bar{\lambda}(A)$ and $\underline{\lambda}(A)$ are the maximum eigenvalue and the minimum eigenvalue of A, respectively. A continuous function $g(s):\mathbb{R}_+\to\mathbb{R}_+$ is a $\mathcal{K}$-function if it is strictly increasing and $g(0)=0$; it is a $\mathcal{K}_\infty$-function if it is a $\mathcal{K}$-function and $g(s)\to\infty$ as $s\to\infty$. For a matrix $L\in\mathbb{R}^{n\times m}$, $\mathrm{vec}(L)=[l_1^T,l_2^T,\ldots,l_m^T]^T\in\mathbb{R}^{mn}$. For a symmetric matix $F\in\mathbb{R}^{m\times m}$, $\mathrm{vecs}(F)=[f_{11},2f_{12},\ldots,2f_{1m},f_{22},2f_{23},\ldots,2f_{m-1},f_{mm}]^T\in\mathbb{R}^{\frac{1}{2}m(m+1)}$. For an arbitrary column vector $a\in\mathbb{R}^n$, $\mathrm{vecv}(a)=[a_1^2,a_1a_2,\ldots,a_1a_n,a_2^2,a_2a_3,\ldots,a_{n-1}a_n,a_n^2]^T\in\mathbb{R}^{\frac{1}{2}n(n+1)}$.

2. Problem formulation and preliminaries

2.1. Basics of optimal output regulation

Consider the following linear system:

$$x_{k+1}=Ax_k+Bu_k+Cz_k, \tag{1}$$

$$z_{k+1}=Dz_k, \tag{2}$$

$$\omega_k=Ex_k+Fz_k \tag{3}$$

where $x_k\in\mathbb{R}^n$ is the state vector, $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times m}$, $C\in\mathbb{R}^{n\times d}$, $D\in\mathbb{R}^{d\times d}$, $E\in\mathbb{R}^{l\times n}$, and $F\in\mathbb{R}^{l\times d}$ are constant matrices. The exosystem described by (2) generates both the disturbances $\zeta_k=Cz_k$ and the reference signal $y_{r_k}=-Fz_k$. $\omega_k\in\mathbb{R}^m$ denotes the tracking error, $u_k\in\mathbb{R}^m$ stands for the control input of the system. The following assumptions are made in this paper.

Assumption 1. The pair (A,B) is controllable.

Assumption 2. $\mathrm{rank}\begin{bmatrix}A-\lambda I & B\\ E & 0\end{bmatrix}=n+l,\ \forall\lambda\in\sigma(D)$.

Assumption 3. All the eigenvalues of D are semisimple with modulus equal to 1, and the minimal polynomial of D is available, which is

$$h_m(s)=\prod_{i=1}^{M}(s-\lambda_i)^{a_i}\prod_{j=1}^{N}(s^2-2p_js+\mu_j^2+q_j^2)^{b_j} \tag{4}$$

with its degree $r\le d$, where a_i and b_j are positive integers and $\lambda_i,p_j,q_j\in R$, for $i=1,2,\cdots,M$, $j=1,2,\cdots N$.

Under the condition of Assumption 3, we can always find a vector $\gamma_k\in R^r$ and a matrix $\hat{D}\in R^{r\times r}$, such that

$$\gamma_{k+1}=\hat{D}\gamma_k,$$
$$z_k=H\gamma_k$$

where $H\in R^{d\times r}$ is an unknown constant matrix.

Therefore, the system (1)–(3) turns into

$$\begin{aligned}x_{k+1}&=Ax_k+Bu_k+\hat{C}\gamma_k,\\ \gamma_{k+1}&=\hat{D}\gamma_k,\\ \omega_k&=Ex_k+\hat{F}\gamma_k\end{aligned} \tag{5}$$

where $\hat{C}=CH$ and $\hat{F}=FH$.

Assumption 4. The pair $(\bar{A},\bar{E})$ is observable, where $\bar{E}=\begin{bmatrix}E & \hat{F}\end{bmatrix}$, $\bar{A}=\begin{bmatrix}A & \hat{C}\\ 0 & \hat{D}\end{bmatrix}$.

The solvability of linear output regulation problem is discussed in the following theorem.

Theorem 1 (Huang, 2004). *Under the conditions of Assumptions 1–2, choose a K such that $A - BK$ is a Schur matrix. The linear output regulation problem is solvable if the controller is designed as*

$$u_k = -K(x_k - \mathbf{X}\gamma_k) + \mathbf{U}\gamma_k \tag{6}$$

where $\mathbf{X} \in \mathbb{R}^{n\times r}$ and $\mathbf{U} \in \mathbb{R}^{m\times r}$ solve the following regulator equation:

$$\begin{aligned} \mathbf{X}\hat{D} &= A\mathbf{X} + B\mathbf{U} + \hat{C}, \\ 0 &= E\mathbf{X} + \hat{F}. \end{aligned} \tag{7}$$

Remark 1. As introduced in Huang (2004), Assumption 3 is a standard assumption for solving a linear output regulation problem, which is usually necessary to develop an internal model. Assumption 2 ensures the solvability of regulator Eq. (7) for any matrices $\hat{C}$ and $\hat{F}$.

If the pair $(\mathbf{X}, \mathbf{U})$ is a solution to the regulator equation, then we can rewrite the error system of (5) as

$$\begin{aligned} \varepsilon_{k+1} &= A\varepsilon_k + Bu_k - B\mathbf{U}\gamma_k, \\ \omega_k &= E\varepsilon_k \end{aligned} \tag{8}$$

where $\varepsilon_k = x_k - \mathbf{X}\gamma_k$.

As shown in Krener (1992), the linear optimal output regulation problem (LOORP) is solvable if (1) the linear output regulation problem is solvable, and (2) the following optimization Problem 1 is solvable.

Problem 1. Along the solutions of (8), find $\bar{u}_k = u_k - \mathbf{U}\gamma_k$ that minimizes the following cost

$$J(\varepsilon_0) = \sum_{k=0}^{\infty} \omega_k^T Q \omega_k + \bar{u}_k^T R \bar{u}_k \tag{9}$$

where $Q = Q^T \geq 0$, $R = R^T > 0$.

By optimal control theory (Lewis et al., 2012), the control policy that minimizes the cost (9) is

$$\bar{u}_k^* = -(R + B^T P^* B)^{-1} B^T P^* A \varepsilon_k := -K^* \varepsilon_k \tag{10}$$

where $P^* = (P^*)^T > 0$ uniquely solves the following discrete-time algebraic Riccati equation under the conditions that the pair (A, B) is controllable and the pair $(A, Q^{\frac{1}{2}}E)$ is observable.

$$A^T PA - P - A^T PB(R + B^T PB)^{-1} B^T PA + E^T QE = 0. \tag{11}$$

Since (11) is nonlinear in P, solving P directly from (12) is difficult. A PI algorithm is proposed in Hewer (1971) to approximate the solution P. To be more specific, given K_0 such that $A - BK_0$ is a Schur matrix, sequences $\{P_j\}_{j=0}^{\infty}$ and $\{K_j\}_{j=0}^{\infty}$ are uniquely determined by

$$(A - BK_j)^T P_j (A - BK_j) - P_j + E^T QE + K_j^T RK_j = 0, \tag{12}$$

$$K_{j+1} = (R + B^T P_j B)^{-1} B^T P_j A. \tag{13}$$

If an initial stabilizing control gain K_0 is not available, one can rely on the VI algorithm (Lancaster & Rodman, 1995). Given any $P_0 = P_0^T > 0$, sequences $\{P_j\}_{j=0}^{\infty}$ and $\{K_{j+1}^v\}_{j=0}^{\infty}$ are uniquely determined by

$$P_{j+1} = A^T P_j A - A^T P_j B(R + B^T P_j B)^{-1} B^T P_j A + E^T QE \tag{14}$$

and

$$K_{j+1}^v = (R + B^T P_{j+1} B)^{-1} B^T P_{j+1} A. \tag{15}$$

2.2. *Problem formulation of event-triggered adaptive optimal output regulation*

In this paper, it is expected that the amount of the control updates can be reduced through an event-triggered design. For convenience of discussions, we use $\hat{\varepsilon}_k$ to represent the sampled value of ε_k, that is

$$\hat{\varepsilon}_k = \varepsilon_{k_j},\ k \in [k_j, k_{j+1}) \tag{16}$$

where $\{k_j\}_{j=0}^{\infty}$ is a monotonically increasing sequence of sampling instants, and the state is only updated at instants: $k_0, k_1, k_2, \cdots$. The sampling error of the state is defined as

$$e_k = \hat{\varepsilon}_k - \varepsilon_k. \tag{17}$$

Then, the error system (8) is converted into

$$\begin{aligned} \varepsilon_{k+1} &= A\varepsilon_k + B\hat{u}_k - B\mathbf{U}\gamma_k, \\ \hat{u}_k &= -K\hat{\varepsilon}_k + \mathbf{U}\gamma_k, \end{aligned} \tag{18}$$

and the event-triggered adaptive optimal output-feedback controller is

$$\hat{u}_k^* = \hat{\bar{u}}_k^* + \mathbf{U}\gamma_k \tag{19}$$

where $\hat{\bar{u}}_k^*$ is the optimal control law to be designed later.

The event-triggered adaptive optimal output regulation problem for the linear discrete-time system is formulated as follows.

Problem 2. Consider the system (1)–(3) with unknown constant matrixes A, B, C, D, E and F, unmeasurable state vector x_k, unmeasurable exostate vector z_k, unknown disturbances ζ_k, and unknown reference signal s_k, design a controller as in (19), such that the following properties hold.

- The linear output regulation problem is solved;
- The communication between the controller and the plant is reduced;
- The designed controller is suboptimal with respect to the cost (9). More specifically, the following suboptimality criterion (Berglind et al., 2012) needs satisfied:

$$\sum_{k=0}^{\infty} \omega_k^T Q \omega_k + (\hat{\bar{u}}_k^*)^T R \hat{\bar{u}}_k^* \leq \rho J^*(\varepsilon_0) \tag{20}$$

where $\rho \geq 1$ is a performance degradation index, $J^*(\varepsilon_0)$ is the optimal cost.

We will give the relationship between ε_k and the sequences of the outputs and inputs in Section 3. Then, in the event-triggered design, we utilize the sampled inputs and outputs to reconstruct the state ε_k. Moreover, due to the unknown system dynamics, we cannot get the value of $\mathbf{U}$ directly, but it can be learned through the online data.

3. Event-triggered output-feedback adaptive optimal controller design

This section employs the ADP techniques to solve the event-triggered output-feedback adaptive optimal control problem for the linear system (5).

3.1. *Event-triggered output-feedback ADP controller: PI-based design*

This subsection introduces a new class of event-triggered output-feedback adaptive optimal controller for suboptimality of the closed-loop system.

Under the conditions of Assumption 4, motivated by Aangenent et al. (2005) and Lewis and Vamvoudakis (2011), there exist two matrices N_w and N_u such that

$$\varepsilon_k = N_\omega \tilde{\omega}_k + N_u \tilde{u}_k = N v_k \tag{21}$$

where

$$\begin{aligned}
N_\omega &= \begin{bmatrix} I_n & -\mathbf{X} \end{bmatrix} \bar{A}^{n+r}(O_1^T O_1)^{-1} O_1^T \in \mathbb{R}^{n\times l(n+r)},\\
N_u &= \begin{bmatrix} I_n & -\mathbf{X} \end{bmatrix} (C_o - \bar{A}^{n+r}(O_1^T O_1)^{-1} O_1^T \Gamma)\\
&\quad \in \mathbb{R}^{n\times m(n+r)},\\
\tilde{u}_k &= [\hat{u}_{k-1}^T, \hat{u}_{k-2}^T, \cdots, \hat{u}_{k-n-r}^T]^T \in \mathbb{R}^{m(n+r)},\\
\tilde{\omega}_k &= [\omega_{k-1}^T, \omega_{k-2}^T, \cdots, \omega_{k-n-r}^T]^T \in \mathbb{R}^{l(n+r)},\\
N &= \begin{bmatrix} N_\omega & N_u \end{bmatrix} \in \mathbb{R}^{n\times(l+m)(n+r)},\\
v_k &= \begin{bmatrix} \tilde{\omega}_k^T & \tilde{u}_k^T \end{bmatrix}^T \in \mathbb{R}^{(l+m)(n+r)},\\
\bar{B} &= [B^T, 0_{m\times r}]^T \in \mathbb{R}^{(n+r)\times m},\\
C_o &= [\bar{B}, \bar{A}\bar{B}, \cdots, \bar{A}^{n+r-1}\bar{B}] \in \mathbb{R}^{(n+r)\times m(n+r)},\\
O_1 &= [(\bar{E}\bar{A}^{n+r-1})^T, \cdots, (\bar{E}\bar{A})^T, \bar{E}^T]^T \in \mathbb{R}^{l(n+r)\times(n+r)},\\
\Gamma &= \begin{bmatrix} 0 & \bar{E}\bar{B} & \bar{E}\bar{A}\bar{B} & \cdots & \bar{E}\bar{A}^{n+r-2}\bar{B}\\ 0 & 0 & \bar{E}\bar{B} & \cdots & \bar{E}\bar{A}^{n+r-3}\bar{B}\\ \vdots & \vdots & \ddots & \ddots & \vdots\\ 0 & 0 & 0 & 0 & \bar{E}\bar{B}\\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\\
&\quad \in \mathbb{R}^{l(n+r)\times m(n+r)}.
\end{aligned}$$

Let $\bar{K}_0 = K_0 N$ be a stable control gain for the error system (18), and define

$$u_k^v = -\bar{K}_0 v_k \tag{22}$$

as the control input. According to (13), we have

$$K_0 = (R + B^T P B)^{-1} B^T P A \tag{23}$$

where P is a solution of the Lyapunov equation (12). Also, define

$$\hat{v}_k = v_{k_j},\ k \in [k_j, k_{j+1}) \tag{24}$$

as the sampled value of v_k. Then, we have the sampling error as follows

$$e_k^v = \hat{v}_k - v_k. \tag{25}$$

In the PI-based learning process, the error system (18) can be rewritten as

$$\begin{aligned}
\varepsilon_{k+1} &= A\varepsilon_k + B\hat{u}_k^v - B\mathbf{U}\gamma_k,\\
\hat{u}_k^v &= -\bar{K}_0 \hat{v}_k.
\end{aligned} \tag{26}$$

The following lemma gives the stability of the transformed system (26). The proof is given in Appendix A.1.

Lemma 1. *Given any stabilizing $\bar{K}_0$, and assume $E^T Q E > \mu_1 I_m$, where $\mu_1 > 1$. With the event-triggered control law given in (26), the closed-loop system is input-to-state stable (ISS) with $\mathbf{U}\gamma_k$ as the input, if*

$$\|\bar{K}_0 e_k^v\|^2 \le \frac{\frac{(1-\alpha)(\mu_1-1)}{\bar{E}^2}\|\omega_k\|^2 + \underline{\lambda}(R)\|\hat{u}_k^v\|^2}{\eta} \tag{27}$$

where $\alpha \in (0,1)$, $\|E\| \le \bar{E}$, and $\eta \ge \bar{\lambda}(R + 2B^T P B)$.

As can be directly checked, the condition in (27) is guaranteed by the following event-triggering mechanism:

$$k_{j+1} = \inf\left\{k \in \mathbb{Z}_+ \mid k > k_j \wedge \|\bar{K}_0 e_k^v\|^2 \ge \bar{e}_k^{v_0}\right\} \tag{28}$$

with $k_0 = 0$ and $\bar{e}_k^{v_0} = \frac{\frac{(1-\alpha)(\mu_1-1)}{\bar{E}^2}\|\omega_k\|^2 + \underline{\lambda}(R)\|\hat{u}_k^v\|^2}{\eta}$.

To solve the ARE (11) by PI method, we rewrite the system (26) as

$$\varepsilon_{k+1} = A_j\varepsilon_k + B(\hat{u}_k^v + K_j\varepsilon_k) - B\mathbf{U}\gamma_k \tag{29}$$

where $A_j = A - BK_j$. Define $\bar{K}_j = K_j N$ and $\bar{P}_j = N^T P_j N$. Based on (29) and the PI algorithm (12)–(13), we have

$$\begin{aligned}
&v_{k+1}^T \bar{P}_j v_{k+1} - v_k^T \bar{P}_j v_k\\
=&\varepsilon_{k+1}^T P_j \varepsilon_{k+1} - \varepsilon_k^T P_j \varepsilon_k\\
=&-2v_k^T N^T A_j^T P_j B\mathbf{U}\gamma_k + 2(\hat{u}_k^v + \bar{K}_j v_k)^T B^T P_j A N v_k\\
&+ (\hat{u}_k^v + \bar{K}_j v_k)^T B^T P_j B(\hat{u}_k^v - \bar{K}_j v_k) - \omega_k^T Q\omega_k - v_k^T \bar{K}_j^T R\bar{K}_j v_k\\
&- 2(\hat{u}_k^v + \bar{K}_j v_k)^T B^T P_j B\mathbf{U}\gamma_k + \gamma_k^T \mathbf{U}^T B^T P_j B\mathbf{U}\gamma_k.
\end{aligned} \tag{30}$$

Define $G_{j_1} = N^T A_j^T P_j B\mathbf{U}$, $G_{j_2} = B^T P_j A N$, $G_{j_3} = B^T P_j B$, $G_{j_4} = B^T P_j B\mathbf{U}$, and $G_{j_5} = \mathbf{U}^T B^T P_j B\mathbf{U}$. Given a sufficiently large $q \in \mathbb{Z}_+$ and let $\xi_k = \hat{u}_k^v + \bar{K}_j v_k$. For two sequences $\{\mathcal{H}_k\}_{\bar{k}_0}^{\bar{k}_q}$ and $\{\mathcal{S}_k\}_{\bar{k}_0}^{\bar{k}_q}$, define

$$\begin{aligned}
\Theta(\mathcal{H}_k, \mathcal{S}_k) &= [\mathcal{H}_{\bar{k}_0} \otimes \mathcal{S}_{\bar{k}_0}, \mathcal{H}_{\bar{k}_1} \otimes \mathcal{S}_{\bar{k}_1}, \ldots, \mathcal{H}_{\bar{k}_q} \otimes \mathcal{S}_{\bar{k}_q}]^T,\\
\tilde{\Theta}(\mathcal{H}_k) &= [\mathrm{vecv}(\mathcal{H}_{\bar{k}_0}), \mathrm{vecv}(\mathcal{H}_{\bar{k}_1}), \ldots, \mathrm{vecv}(\mathcal{H}_{\bar{k}_q})]^T,\\
\Pi_j &= [(\omega_{\bar{k}_0})^T Q\omega_{\bar{k}_0} + (v_{\bar{k}_0})^T \bar{K}_j^T R\bar{K}_j v_{\bar{k}_0}, \ldots,\\
&\quad (\omega_{\bar{k}_q}) T Q\omega_{\bar{k}_q} + (v_{\bar{k}_q})^T \bar{K}_j^T R\bar{K}_j v_{\bar{k}_q}]^T.
\end{aligned}$$

For any stabilizing gain matrix $\bar{K}_j$, (30) indicates the following equation

$$\Xi_j G_j = \Pi_j \tag{31}$$

where

$$\begin{aligned}
\Xi_j &= [-2\Theta_j(\gamma_k, v_k), 2\Theta_j(v_k, \xi_k), \tilde{\Theta}_j(\hat{u}_k^v) - \tilde{\Theta}_j(\bar{K}_j v_k),\\
&\quad -2\Theta_j(\gamma_k, \xi_k), \tilde{\Theta}_j(\gamma_k), \tilde{\Theta}_j(v_k) - \tilde{\Theta}_j(v_{k+1})],\\
G_j &= [\mathrm{vec}(G_{j_1})^T, \mathrm{vec}(G_{j_2})^T, \mathrm{vecs}(G_{j_3})^T, \mathrm{vec}(G_{j_4})^T,\\
&\quad \mathrm{vecs}(G_{j_5})^T, \mathrm{vecs}(\bar{P}_j)^T]^T.
\end{aligned}$$

Eq. (31) can be uniquely solved when matrix Ξ_j is full column rank, i.e.,

$$G_j = (\Xi_j^T \Xi_j)^{-1} \Xi_j^T \Pi_j. \tag{32}$$

Assumption 2 implies that B is of full column rank, so G_{j_3} is a nonsingular matrix. Then, $\bar{K}_{j+1}$ and $\mathbf{U}$ can be obtained as follows:

$$\begin{aligned}
\bar{K}_{j+1} &= (R + G_{j_3})^{-1} G_{j_2},\\
\mathbf{U} &= G_{j_3}^{-1} G_{j_4}.
\end{aligned} \tag{33}$$

Let $\varphi_k = [v_k^T, \hat{u}_k^T, \gamma_k^T]^T$. The uniqueness of the solution to (32) is ensured by the following lemma.

Lemma 2 (Gao et al., 2018)**.** *Let $\theta = (l+m)(n+r)$, if there exists a $q^* \in \mathbb{Z}_+$ such that*

$$\mathrm{rank}[\Theta_j(\varphi_k, \varphi_k)] = \frac{m(m+1) + lr(lr+1+2m) + \theta(\theta+1+2lr+2m)}{2} \tag{34}$$

holds for all $q > q^*$, then (31) can be uniquely solved.

Then, we present our event-triggered output-feedback adaptive optimal control algorithm.

Algorithm 1. PI-based event-triggered ADP

(1) Utilize $\hat{u}_k^v = -\bar{K}_0\hat{v}_k + \eta_k$ as the control input on $[\tilde{k}_0, \tilde{k}_q]$, where η_k is an exploration noise. Set $j = 0$.
(2) Solve $\bar{P}_j$, $\bar{K}_{j+1}$, and $\mathbf{U}$ from (32) and (33).
(3) Set $j \leftarrow j+1$, for $j \geq 1$, if $\|\bar{P}_j - \bar{P}_{j-1}\| > \tau$, repeat Step 2; else set $j^* \leftarrow j$ and go to Step 4,where constant $\tau > 0$ is a stop criterion for the convergence of $\bar{P}_j$.
(4) The approximated optimal control gain is
$\bar{\mathbf{K}} := [-\bar{K}_{j^*}\ (G_{j_3^*})^{-1}G_{j_4^*}]$.

By using the event-triggered ADP controller, the error system (26) can be rewritten as

$$\begin{aligned} \varepsilon_{k+1} &= A\varepsilon_k + B\hat{u}_k^* - B\mathbf{U}\gamma_k, \\ \hat{u}_k^* &= \bar{\mathbf{K}}_{j^*}[\hat{v}_k\ \gamma_k]^T. \end{aligned} \tag{35}$$

The following lemma gives the stability of the transformed system (35). See Appendix A.2 for the proof.

Lemma 3. *Assume the weighting matrix in (9) satisfying $Q > \mu I_n$, where μ is a positive real number. With the event-triggered control law $\hat{u}_k^*$ given in (35), the system (35) is globally asymptotically stable at the origin, if*

$$\|\bar{K}_{j^*}e_k^v\|^2 \leq \frac{\epsilon\mu\|\omega_k\|^2 + \underline{\lambda}(R)\|\hat{u}_k^* - (G_{j_3^*})^{-1}G_{j_4^*}\gamma_k\|^2}{\bar{\lambda}(R + G_{j_3^*})} \tag{36}$$

where $\epsilon \in (0, 1)$.

Based on Lemma 3, to guarantee the satisfaction of the condition in (36), the following event-triggering mechanism is designed:

$$k_{j+1} = \inf\{k \in \mathbb{Z}_+ \mid k > k_j \wedge \|\bar{K}_{j^*}e_k^v\|^2 \geq \bar{e}_k^{v_p}\} \tag{37}$$

with $\bar{e}_k^{v_p} = \frac{\epsilon\mu\|\omega_k\|^2 + \underline{\lambda}(R)\|\hat{u}_k^* - (G_{j_3}^*)^{-1}G_{j_4}^*\gamma_k\|^2}{\bar{\lambda}(R + G_{j_3}^*)}$ and $k_0 = 0$. Then, the PI-based event-triggered ADP controller is designed as

$$\hat{u}_k^* = \bar{\mathbf{K}}_{j^*}[\hat{v}_k\ \gamma_k]^T \tag{38}$$

with $\|\bar{K}_{j^*}e_k^v\|^2 \leq \frac{\epsilon\mu\|\omega_k\|^2 + \underline{\lambda}(R)\|\hat{u}_k^* - (G_{j_3}^*)^{-1}G_{j_4}^*\gamma_k\|^2}{\bar{\lambda}(R + G_{j_3}^*)}$.

Theorem 2. *If (34) holds, then sequences $\{\bar{P}_j\}_{j=0}^{\infty}$ and $\{\bar{K}_j\}_{j=1}^{\infty}$ obtained from solving Algorithm 1 converge to $\bar{P}^*$ and $\bar{K}^*$, respectively, where $\bar{P}^* = N^T P^* N$ and $\bar{K}^* = K^* N$.*

Proof. See Appendix A.3. □

3.2. Event-triggered output-feedback ADP controller: VI-based design

As an alternative to the policy iteration approach, the value iteration approach does not need a known stabilizing control law to initialize.

In the VI-based learning process, the error system (18) is rewritten as

$$\begin{aligned} \varepsilon_{k+1} &= A\varepsilon_k + B\hat{u}_k - B\mathbf{U}\gamma_k \\ \hat{u}_k &= -\bar{\mathbf{K}}^v\phi(k) \end{aligned} \tag{39}$$

where $\bar{\mathbf{K}}^v = [\bar{K}^v\quad -U] \in \mathbb{R}^{m\times[r+(l+m)(n+r)]}$ is an arbitrary control gain, $\phi(k) = [\hat{v}_k^T\ \gamma_k^T]^T$, and e_k^v satisfies that

$$\|e_k^v\| \leq \alpha \tag{40}$$

where $\alpha > 0$ stands for a predefined threshold.

Define

$$\bar{\mathbf{G}}_j = \begin{bmatrix} \bar{\mathbf{G}}_j^{11} & \bar{\mathbf{G}}_j^{12} & \bar{\mathbf{G}}_j^{13} \\ \bar{\mathbf{G}}_j^{21} & \bar{\mathbf{G}}_j^{22} & \bar{\mathbf{G}}_j^{23} \\ \bar{\mathbf{G}}_j^{31} & \bar{\mathbf{G}}_j^{32} & \bar{\mathbf{G}}_j^{33} \end{bmatrix} = \begin{bmatrix} N^TA^TP_jAN & N^TA^TP_jB & -N^TA^TP_jB\mathbf{U} \\ B^TP_jAN & B^TP_jB & -B^TP_jB\mathbf{U} \\ -\mathbf{U}^TB^TP_jAN & -\mathbf{U}^TB^TP_jB & \mathbf{U}^TB^TP_jB\mathbf{U} \end{bmatrix}.$$

Besides, let

$$f(P_j) = A^TP_jA - A^TP_jB(R + B^TP_jB)^{-1}B^TP_jA.$$

From (14) and (39), we obtain

$$\begin{aligned} &\omega_{k+1}^T Q\omega_{k+1} \\ =&\varepsilon_{k+1}^T P_{j+1}\varepsilon_{k+1} - \varepsilon_{k+1}^T f(P_j)\varepsilon_{k+1} \\ =&\left[\text{vecv}([v_k\ \hat{u}_k\ \gamma_k]^T)\right]^T \text{vecs}(\bar{\mathbf{G}}_{j+1}) \\ &- v_{k+1}^T[\bar{\mathbf{G}}_j^{11} - \bar{\mathbf{G}}_j^{12}(R + \bar{\mathbf{G}}_j^{22})^{-1}(\bar{\mathbf{G}}_j^{12})^T]v_{k+1} \\ =&\varpi_k^T\text{vecs}(\bar{\mathbf{G}}_{j+1}) - \psi_{k+1}^j \end{aligned} \tag{41}$$

where

$$\begin{aligned} \psi_{k+1}^j &= v_{k+1}^T[\bar{\mathbf{G}}_j^{11} - \bar{\mathbf{G}}_j^{12}(R + \bar{\mathbf{G}}_j^{22})^{-1}(\bar{\mathbf{G}}_j^{12})^T]v_{k+1}, \\ \varpi_k &= \text{vecv}([v_k^T, \hat{u}_k^T, \gamma_k^T]^T). \end{aligned}$$

For $j = 0, 1, 2, \ldots$, define

$$\begin{aligned} \Psi_j &= [\omega_{\tilde{k}_1}^T Q\omega_{\tilde{k}_1} + \psi_{\tilde{k}_1}^j, \cdots, \omega_{\tilde{k}_q+1}^T Q\omega_{\tilde{k}_q+1} + \psi_{\tilde{k}_q+1}^j], \\ \Phi &= [\varpi_{\tilde{k}_0}, \varpi_{\tilde{k}_1}, \cdots, \varpi_{\tilde{k}_q}]. \end{aligned}$$

According to Eq. (41), we have

$$\Phi\text{vecs}(\bar{\mathbf{G}}_{j+1}) = \Psi_j. \tag{42}$$

Similarly, The uniqueness of the solution is ensured by Lemma 2. Then, we present our VI-based output-feedback event-triggered adaptive optimal control algorithm.

Algorithm 2. VI-based event-triggered ADP

(1) Select thresholds $\tau > 0$ and $\alpha > 0$. Set $j = 0$, $\bar{\mathbf{G}}_0 = 0$ and $\bar{K}_0 = 0$.
(2) Apply $\hat{u}_k = -\bar{K}_0 v_k + \eta_k$ on $[\tilde{k}_0, \tilde{k}_q]$. Solve $\bar{\mathbf{G}}_{j+1}$ from (42).
(3) Set $j + 1 \rightarrow j$, if $\|\bar{\mathbf{G}}_j - \bar{\mathbf{G}}_{j-1}\| > \tau$, repeat Step 2; else set $j^* \leftarrow j$ and go to Step 4.
(4) Use $\bar{\mathbf{K}}_{j^*}^v = [(R + \bar{\mathbf{G}}_{j^*}^{22})^{-1}(\bar{\mathbf{G}}_{j^*}^{12})^T, (\bar{\mathbf{G}}_{j^*}^{22})^{-1}(\bar{\mathbf{G}}_{j^*}^{23})]$ as the approximated optimal control gain.

Then, we can get the control law as follows

$$\hat{u}_k^* = \bar{\mathbf{K}}_{j^*}[\hat{v}_k\ \gamma_k]^T. \tag{43}$$

Let $\bar{K}_{j^*}^v = (R + \bar{\mathbf{G}}_{j^*}^{22})^{-1}(\bar{\mathbf{G}}_{j^*}^{12})^T$, according to Lemma 3, e_k^v satisfies that

$$\|\bar{K}_{j^*}^v e_k^v\|^2 \leq \frac{\epsilon\mu\|\omega_k\|^2 + \underline{\lambda}(R)\|\hat{u}_k^* - (\bar{\mathbf{G}}_{j^*}^{22})^{-1}(\bar{\mathbf{G}}_{j^*}^{23})\gamma_k\|^2}{\bar{\lambda}(R + \bar{G}_{j^*}^{22})}. \tag{44}$$

To guarantee the satisfaction of the condition in (44), we give the following event-triggering mechanism:

$$k_{j+1} = \inf\{k \in \mathbb{Z}_+ \mid k > k_j \wedge \|\bar{K}_{j^*}^v e_k^v\|^2 \geq \bar{e}_k^{v_v}\} \tag{45}$$

with $k_0 = 0$ and $\bar{e}_k^{v_v} = \frac{\epsilon\mu\|\omega_k\|^2 + \underline{\lambda}(R)\|\hat{u}_k^* - (\bar{\mathbf{G}}_{j^*}^{22})^{-1}(\bar{\mathbf{G}}_{j^*}^{23})\gamma_k\|^2}{\bar{\lambda}(R + \bar{G}_{j^*}^{22})}$.

Similar to the analysis of Theorem 2, the convergence of Algorithm 2 is shown as follows.

Theorem 3. *If (34) holds, then $\{\bar{\mathbf{G}}_j\}_{j=0}^{\infty}$ and $\{\bar{\mathbf{K}}_j^{v}\}_{j=1}^{\infty}$ obtained from solving Algorithm 2 converge to $\bar{\mathbf{G}}^*$ and $\bar{\mathbf{K}}^*$, where $\bar{\mathbf{K}}^* = [K^*N, -U]$ and*

$$\bar{\mathbf{G}}^* = \begin{bmatrix} N^TA^TP^*AN & N^TA^TP^*B & -N^TA^TP^*B\mathbf{U} \\ B^TP^*AN & B^TP^*B & -B^TP^*B\mathbf{U} \\ -\mathbf{U}^TB^TP^*AN & -\mathbf{U}^TB^TP^*B & \mathbf{U}^TB^TP^*B\mathbf{U} \end{bmatrix}.$$

Remark 2. Exploration noise (Al-Tamimi et al., 2007; Jiang & Jiang, 2012; Vamvoudakis & Lewis, 2011; Xu et al., 2012) is introduced in Algorithms 1 and 2 for persistent excitation, which is needed for convergence of the ADP algorithms. For simulation purpose, we will use the sum of sinusoidal signals with different frequencies, see e.g., Jiang and Jiang (2012).

Remark 3. Algorithm 1 assumes a known stabilizing control law, which may limit its applications. However, compared with Algorithm 2, the convergence rate of Algorithm 1 is quadratic in a neighborhood of the steady state (Hewer, 1971), which is faster. Algorithm 2 does not assume a known stabilizing control law, but the price paid for this is the possibly slower convergence rate.

4. Main results

From Theorem 2, there always exists a small enough threshold $\tau > 0$ in Algorithms 1, such that $A - BK_{j^*}$ is a Schur matrix. With the proof of Lemma 3, it obviously indicates that the closed-loop system is globally asymptotically stable at the origin. Besides, we can rewrite the error system of the closed-loop system as follows

$$\begin{aligned} \varepsilon_{k+1} &= A\varepsilon_k + B\hat{u}_k - B\mathbf{U}\gamma_k, \\ \omega_k &= E\varepsilon_k + (E\mathbf{X} + \hat{F})\gamma_k. \end{aligned} \tag{46}$$

Because the closed-loop system is globally asymptotically stable, we have $\lim_{k\to\infty} \varepsilon_k = 0$. Also, $\mathbf{X}$ solves the regulator Eq. (8), we have $\lim_{k\to\infty} \omega_k = 0$.

Based on the above analysis, the following Theorem 4 is given.

Theorem 4. *Considering the linear system (1)–(3), by using the learning based control policy (38), we have*

- *the system (1)–(3) is globally asymptotically stable at the origin.*
- *the tracking error ω_k converges to 0 as t goes to infinity.*

According to the proof of Lemma 3, we have

$$\|K^*e_k\|^2 \le \frac{\epsilon\mu\|\omega_k\|^2 + \underline{\lambda}(R)\|\hat{u}_k^* - \mathbf{U}\gamma_k\|^2}{\bar{\lambda}(R + B^TP^*B)} := \bar{e}_k^2. \tag{47}$$

For $\varepsilon_k \neq 0$, let the triggering threshold ratio be $\delta_e = \max\{\frac{\bar{e}_k}{\|\varepsilon_k\|}, k = 1, 2, \cdots\}$. The following theorem is given to characterize the suboptimality property of the closed-loop system composed of (1)–(3) and (38). See Appendix A.4 for the proof.

Theorem 5. *Under the conditions of Lemmas 2 and 3, the control strategy (38) is suboptimal for system (1)–(3) with the cost J_e^* in (9) satisfying*

$$J^*(\varepsilon_0) \le J_e^* \le J^*(\varepsilon_0) + \frac{\bar{\lambda}(R)\delta_e(\delta_e + 2\|K^*\|)}{\mu(1-\epsilon)\|\bar{E}\|^2}J^*(\varepsilon_0). \tag{48}$$

Remark 4. Theorem 5 shows the numerical relationship between the triggering ratio δ_e and the cost of the system. As we can see from (48), a smaller δ_e results in more frequent samplings but better suboptimality. A large δ_e causes larger inter-sampling periods but an unsatisfactory system performance.

In practice, there may be slight perturbations Δ_D on D and additional output noise Δ_{y_k}, satisfying $\underline{\Delta}_{y_k} \le \|\Delta_{y_k}\| \le \bar{\Delta}_{y_k}$. In this case, the system (1)–(3) can be formed as

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Cz_k, \\ z_{k+1} &= (D + \Delta_D)z_k, \\ \omega_k &= Ex_k + \hat{F}\gamma_k + \Delta_{y_k}. \end{aligned} \tag{49}$$

Under the conditions of Assumption 3 and Theorem 1, we get the following error system

$$\begin{aligned} \varepsilon_{k+1} &= A\varepsilon_k + Bu_k - B\mathbf{U}\gamma_k + \Delta_{z_k}, \\ \omega_k &= E\varepsilon_k + \Delta_{y_k} \end{aligned} \tag{50}$$

where $\Delta_{z_k} = Cz_k - \hat{C}\gamma_k$.

Motivated by Aangenent et al. (2005) and Lewis and Vamvoudakis (2011), we have $\varepsilon_k = Nv_k + \Delta_{\varepsilon_k}$, where Δ_{ε_k} is a bounded function related to Δ_{z_k} and Δ_{y_k}.

Based on Bian and Jiang (2019) and Pang et al. (2021), in the presence of Δ_D and Δ_{y_k}, by using Algorithms 1 and 2, we can obtain a suboptimal control gain $\bar{K}_d^*$ and a feedforward gain $\mathbf{U}_d$, such that the controller is designed as

$$\hat{u}_k^* = -\bar{K}_d^*\hat{v}_k + \mathbf{U}_d\gamma_k. \tag{51}$$

To explain the effect of the slight perturbations Δ_D and additional output noise Δ_{y_k} on the system (49), the following Corollary 1 is given.

Corollary 1. *With the designed event-triggered adaptive optimal control law (51), if the following event-triggered condition*

$$\|\bar{K}_d^*e_k^v\| \le \frac{\frac{(1-\alpha)(\mu_1-1)}{\bar{E}^2}\min\{\|\omega_k - \underline{\Delta}_{y_k}\|^2, \|\omega_k - \bar{\Delta}_{y_k}\|^2\}}{2\bar{\lambda}(R + B^TPB + I_n)} \tag{52}$$

is satisfied, then the closed-loop system is ISS with Δ_D and Δ_{y_k} as the inputs.

Proof. Define $A_d = A - BK_d^*$ with $\bar{K}_d^* = K_d^*N$. Then, we have

$$\varepsilon_{k+1} = A_d\varepsilon_k - BK_d^*e_k + \Delta_k \tag{53}$$

where $\Delta_k = \Delta_{z_k} + B(\mathbf{U}_d - \mathbf{U})\gamma_k$.

Along the trajectory of (53), we have

$$\begin{aligned} &\varepsilon_{k+1}^TP\varepsilon_{k+1} - \varepsilon_k^TP\varepsilon_k \\ =&\varepsilon_k^TA_dPA_d\varepsilon_k + (K_d^*e_k)^TB^TPBK_d^*e_k + \Delta_k^TP\Delta_k \\ &- 2(K_d^*e_k)^TB^TPA_d\varepsilon_k + 2\Delta_k^TPA_d\varepsilon_k - 2\Delta_k^TPBK_d^*e_k \\ &- \varepsilon_k^TP\varepsilon_k \\ \le& -\alpha(\mu_1 - 1)\|\varepsilon_k\|^2 - \frac{(1-\alpha)(\mu_1-1)}{\bar{E}^2}\|\omega_k - \Delta_{y_k}\|^2 \\ &+ 2(\bar{k}_d^*e_k^v)^T(R + B^TPB + I_n)(\bar{k}_d^*e_k^v) + 2\tilde{\Delta}_k^T(R + B^TPB + I_n)\tilde{\Delta}_k \\ &+ \Delta_k^T(P + PA_dA_d^TP + PBB^TP)\Delta_k \end{aligned}$$

where $\tilde{\Delta}_k = \bar{K}_d^*(\Delta_{\varepsilon_{k_j}} - \Delta_{\varepsilon_k})$ and $\{k_j\}_{j=0}^{\infty}$ are triggering instants.

When the condition (52) is satisfied, we have

$$\varepsilon_{k+1}^TP\varepsilon_{k+1} - \varepsilon_k^TP\varepsilon_k \le -\tilde{\alpha}_3(\|\varepsilon_k\|) + \tilde{\alpha}_4(\|\Delta_k\|, \|\tilde{\Delta}_k\|)$$

where $\tilde{\alpha}_3(\cdot)$ is a $\mathcal{K}_\infty$-function and $\tilde{\alpha}_4(\cdot)$ is a $\mathcal{K}$-function.

According to Jiang and Wang (2001), the closed-loop system is ISS with Δ_D and Δ_{y_k} as the inputs. □

5. Simulation results

We show the efficiency of the proposed method by means of a practical of LCL coupled inverter-based distributed generation

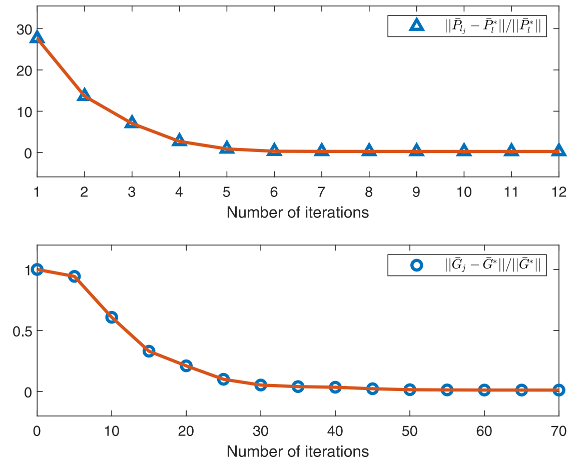

Fig. 1. Convergence of Algorithms 1 and 2.

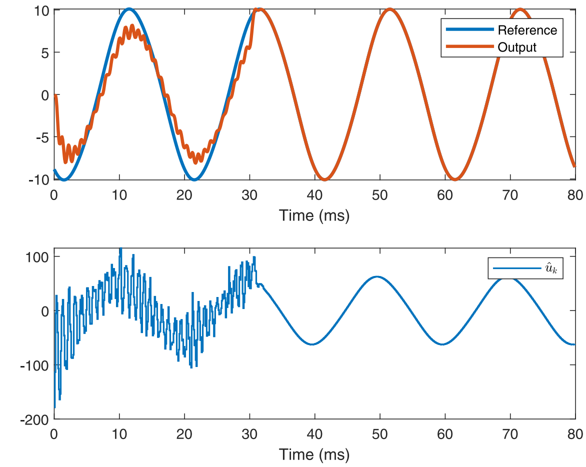

Fig. 2. Plot of the output, reference signal and control input.

system (Ahmed et al., 2011). In practice, the discrete-time system is preferred for computer implementation (Lin & Narendra, 1980). Based on the Euler discretization method, by using a sampling period $h = 0.0001$, the continuous-time system in Ahmed et al. (2011) is discretized as follows:

$$x_{k+1} = \begin{bmatrix} 1 - \frac{hR_1}{L_1} & -\frac{h}{L_1} & 0 \\ \frac{h}{c} & 1 & -\frac{h}{c} \\ 0 & \frac{h}{L_2} & 1 - \frac{hR_2}{L_2} \end{bmatrix} x_k + \begin{bmatrix} \frac{h}{L_1} \\ 0 \\ 0 \end{bmatrix} u_k + dh$$

where $x_k = [I_L, V_C, I_O]^T$, $\omega_k = I_O$, and $u_k = V_I$. The physical meanings and values of the parameters refer to Ahmed et al. (2011). The exosystem can generate both the disturbance (grid voltage) and the reference signal. The minimal polynomial of D is set as

$$h_m(s) = s^4 - 3.9901s^3 + 5.9803s^2 - 3.9901s + 1.$$

Then, we can generate the exosystem as

$$\gamma_{k+1} = \begin{bmatrix} 0.9995 & -0.0314 & 0 & 0 \\ 0.0314 & 0.9995 & 0 & 0 \\ 0 & 0 & 0.9956 & -0.0941 \\ 0 & 0 & 0.0941 & 0.9956 \end{bmatrix} \gamma_k \tag{54}$$

with initial condition $\gamma_0 = [1 \ \ 0 \ \ 1 \ \ 0]^T$.

Set the reference signal $y_{r_k} = -[5\sqrt{3} \ \ 5 \ \ 0.2 \ \ 0.1]\gamma_k$. Then, we have the tracking error $\omega_k = [0 \ \ 0 \ \ 1]x_k + y_{r_k}$.

As stated in Algorithm 1, we apply a stabilizing control strategy $\hat{u}_k^v = -\bar{K}_0\hat{v}_k + \eta_k$ as the control input on $[0, 30]$ ms for the stage of data collection. Then, $\bar{P}_j$, $\bar{K}_{j+1}$, and $\mathbf{U}$ are iteratively solved from (32) and (33) by using the collected data. In Algorithm 2, we use $\hat{u}_k = -\eta_k$ on $[0, 30]$ ms for data collection. Then, $\bar{\mathbf{G}}_{j+1}$ is iteratively solved according to (42). The convergence of Algorithms 1 and 2 is shown in Fig. 1 by choosing $\tau = 10^{-6}$.

The trajectories of the output, the reference signal and the control input of the discrete-time system are shown in Fig. 2.

To validate the reasonability of Theorem 4, by using a logarithmic coordinate for the y coordinates, Fig. 3 shows the comparisons of J^* and J_e^* at different δ_e.

Fig. 4 shows the inter-sampling steps of the event-triggered sampling and the comparison of the total sampling times under event-triggered output feedback ADP method and the ADP without event-triggered sampling. It can be seen that, the output of the plant converges to the reference signal, and the communication between the controller and the plant is reduced.

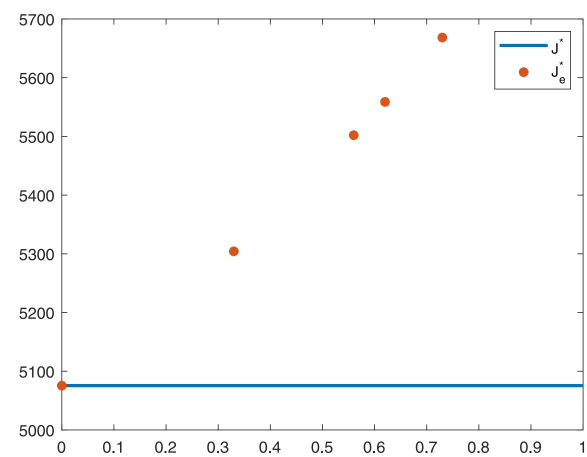

Fig. 3. Comparisons of J^* and J_e^* at different δ_e.

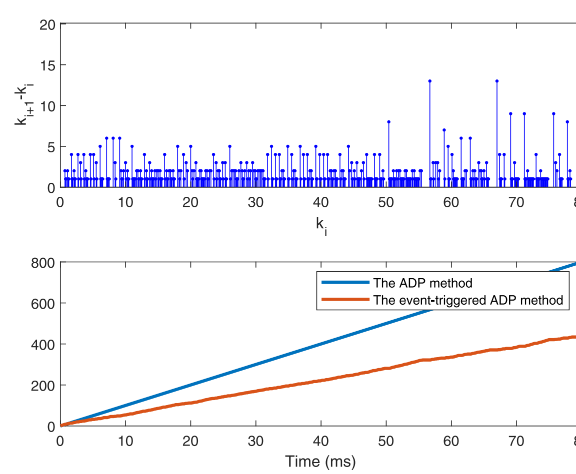

Fig. 4. The sequence of steps of event-triggered sampling and the comparison of the total sampling numbers.

6. Conclusions

This paper presents two event-triggered output-feedback approaches to address the adaptive optimal output regulation for linear systems with unknown system dynamics, unmeasurable states and disturbance. A nonmodel-based ADP scheme is proposed for the design of event-triggered adaptive optimal trackers with disturbance rejection. Simulation results have validated the effectiveness of the proposed approach. Our future work will be directed at generalizing the proposed methods to a class of nonlinear systems with output-feedback by means of recent developments in nonlinear PI and VI schemes (Bian & Jiang, 2021; Jiang et al., 2020).

Acknowledgments

The authors would like to thank the anonymous AE and reviewers for their invaluable comments that have led to the improvement of the paper.

Appendix. Proofs

A.1. Proof of Lemma 1

Define $A_g = A - BK_0$. Then, we have

$$\varepsilon_{k+1} = A_g\varepsilon_k - BK_0e_k - BU\gamma_k. \tag{A.1}$$

Define

$$V(\varepsilon_k) = \varepsilon_k^T P\varepsilon_k. \tag{A.2}$$

Obviously, there exist $\mathcal{K}_\infty$-functions $\alpha_1(\cdot)$ and $\alpha_2(\cdot)$ such that

$$\alpha_1(\|\varepsilon_k\|) \le V(\varepsilon_k) \le \alpha_2(\|\varepsilon_k\|), \forall\varepsilon_k \in \mathbb{R}^n. \tag{A.3}$$

Along the solutions of (26), we have

$$\begin{aligned}
&V(\varepsilon_{k+1}) - V(\varepsilon_k)\\
=&\varepsilon_{k+1}^T P\varepsilon_{k+1} - \varepsilon_k^T P\varepsilon_k\\
=&\varepsilon_k^T A_g^T PA_g\varepsilon_k + (\bar{K}_0e_k^v)^T B^T PB\bar{K}_0e_k^v + (U\gamma_k)^T B^T PB(U\gamma_k)\\
&-2(\bar{K}_0e_k^v)^T B^T PA_g\varepsilon_k - 2(U\gamma_k)^T B^T PA_g\varepsilon_k - \varepsilon_k^T P\varepsilon_k\\
&+2(U\gamma_k)^T B^T PB\bar{K}_0e_k^v.
\end{aligned}$$

Using (11), we also have

$$\begin{aligned}
&V(\varepsilon_{k+1}) - V(\varepsilon_k)\\
\le& -\omega_k^T Q\omega_k - \underline{\lambda}(R)(K_0\varepsilon_k + \bar{K}_0e_k^v)^2 + (\bar{K}_0e_k^v)^T(R + B^T PB)\bar{K}_0e_k^v\\
&+(U\gamma_k)^T B^T PB(U\gamma_k) - ((U\gamma_k)^T RK_0 + \varepsilon_k^T)^2\\
&+(U\gamma_k)^T RK_0K_0^T R(U\gamma_k) + (U\gamma_k)^T B^T PB(U\gamma_k)\\
&+(\bar{K}_0e_k^v)^T B^T PB\bar{K}_0e_k^v + \varepsilon_k^T\varepsilon_k.
\end{aligned}$$

Then, it follows that

$$\begin{aligned}
&V(\varepsilon_{k+1}) - V(\varepsilon_k)\\
\le& -\alpha(\mu_1 - 1)\|\varepsilon_k\|^2 - \frac{(1-\alpha)(\mu_1 - 1)}{\bar{E}^2}\|\omega_k\|^2 - \underline{\lambda}(R)\|\hat{u}_k^v\|^2\\
&+\bar{\lambda}(RK_0^T K_0R + 2B^T PB)\|U\gamma_k\|^2 + \bar{\lambda}(R + 2B^T PB)\|\bar{K}_0\|^2\|e_k^v\|^2.
\end{aligned}$$

When the condition (27) is satisfied, we have

$$\begin{aligned}
&V(\varepsilon_{k+1}) - V(\varepsilon_k)\\
\le& -\alpha(\mu_1 - 1)\|\varepsilon_k\|^2 + \bar{\lambda}(RK_0^T K_0R^T + 2B^T PB)\|U\gamma_k\|^2\\
\le& -\alpha_3(\|\varepsilon_k\|) + \alpha_4(\|U\gamma_k\|)
\end{aligned} \tag{A.4}$$

where α_3 is a $\mathcal{K}_\infty$-function and α_4 is a $\mathcal{K}$-function.

According to (A.3) and (A.4), we can know that (A.2) is an ISS-Lyapunov function (Jiang & Wang, 2001). Therefore, the closed-loop system is ISS to $\mathbf{U}\gamma_k$.

A.2. Proof of Lemma 3

Define $A_l = A - BK_{j^*}$, where $K_{j^*}N = \bar{K}_{j^*}$, according to (35), we have

$$\varepsilon_{k+1} = A_l\varepsilon_k - BK_{j^*}e_k. \tag{A.5}$$

Besides, define $V(\varepsilon_k) = \varepsilon_k^T P_{j^*}\varepsilon_k$, where P_{j^*} is the approximated solution to the ARE (11). Along the solutions of (35), the following equality holds

$$\begin{aligned}
&V(\varepsilon_{k+1}) - V(\varepsilon_k)\\
=&\varepsilon_{k+1}^T P_{j^*}\varepsilon_{k+1} - \varepsilon_k^T P_{j^*}\varepsilon_k\\
=&\varepsilon_k A_l^T P_{j^*}A_l\varepsilon_k + e_k^T K_{j^*}^T B^T P_{j^*}BK_{j^*}e_k - 2e_k^T K_{j^*}^T B^T P_{j^*}A_l\varepsilon_k\\
&-\varepsilon_k^T P_{j^*}\varepsilon_k.
\end{aligned}$$

According to (11) and (13), we have

$$\begin{aligned}
&V(\varepsilon_{k+1}) - V(\varepsilon_k)\\
=& -\omega_k^T Q\omega_k - (K_{j^*}\varepsilon_k)^T RK_{j^*}\varepsilon_k - 2e_k^T K_{j^*}^T RK_{j^*}\varepsilon_k\\
&+e_k^T K_{j^*}^T B^T P_{j^*}BK_{j^*}e_k\\
\le& -\epsilon\mu\|\omega_k\|^2 - (1-\epsilon)\mu\|\omega_k\|^2 - \underline{\lambda}(R)(\hat{u}_k - U\gamma_k)^2\\
&+\bar{\lambda}(R + B^T P_{j^*}B)\|\bar{K}_{j^*}\|^2\|e_k^v\|^2.
\end{aligned}$$

If (36) holds, then we have

$$V(\varepsilon_{k+1}) - V(\varepsilon_k) < -(1-\epsilon)\mu\|\omega_k\|^2. \tag{A.6}$$

Based on the observability of the discretized system (1)–(3), a direct application of LaSalle's Invariance Principle (Khalil, 2002) yields the GAS property of the trivial solution of the system (35).

This ends the proof of Lemma 3.

A.3. Proof of Theorem 2

Given a stabilizing K_j, if P_j is the solution to (12), then K_{j+1} can be uniquely determined by (13). Similarly, given a stabilizing $\bar{K}_j$, by solving (31) and (32), we have $\bar{P}_j$ and $\bar{K}_{j+1}$. Due to the full rank condition of Ξ_j, $\bar{P}_j$ and $\bar{K}_{j+1}$ are uniquely determined. Consider the property in the PI algorithm (Hewer, 1971), we have $\lim_{j\to\infty}\bar{P}_j = \bar{P}^*$ and $\lim_{j\to\infty}\bar{K}_j = \bar{K}^*$.

A.4. Proof of Theorem 5

Based on (21) and (37), for $\varepsilon_k \neq 0$, we have $\frac{\|K^*(\hat{\varepsilon}_k - \varepsilon_k)\|}{\|\varepsilon_k\|} \le \frac{\bar{e}_k}{\|\varepsilon_k\|}$. Thus, if $k \in [k_j, k_{j+1})$, then $K^*\hat{\varepsilon}_k = K^*\varepsilon_k + \|\varepsilon_k\|\Lambda$, where $\Lambda = \frac{K^*(\hat{\varepsilon}_k - \varepsilon_k)}{\|\varepsilon_k\|}$, and $\|\Lambda\| \le \frac{\bar{e}_k}{\|\varepsilon_k\|} \le \max\{\frac{\bar{e}_k}{\|\varepsilon_k\|}, k = 1, 2, \cdots\} := \delta_e$.

By using the designed control policy (38), we have

$$\begin{aligned}
&\omega_k^T Q\omega_k + (\hat{u}_k^*)^T R\hat{u}_k^*\\
=&\omega_k^T Q\omega_k + (K^*\varepsilon_k + \|\varepsilon_k\|\Lambda)^T R(K^*\varepsilon_k + \|\varepsilon_k\|\Lambda)\\
=&\omega_k^T Q\omega_k + (K^*\varepsilon_k)^T R(K^*\varepsilon_k) + 2(\|\varepsilon_k\|\Lambda)^T RK^*\varepsilon_k\\
&+(\|\varepsilon_k\|\Lambda)^T R\|\varepsilon_k\|\Lambda\\
\le&\omega_k^T Q\omega_k + (K^*\varepsilon_k)^T R(K^*\varepsilon_k) + \bar{\lambda}(R)\|\varepsilon_k\|^2\delta_e^2\\
&+2\bar{\lambda}(R)\|K^*\|\|\varepsilon_k\|^2\delta_e.
\end{aligned} \tag{A.7}$$

Due to (A.6), it follows that

$$\Delta V = V(\varepsilon_k) - V(\varepsilon_{k+1}) > \mu(1-\epsilon)\|\bar{E}\|^2\|\varepsilon_k\|^2. \tag{A.8}$$

Let $H_k = \bar{\lambda}(R)\delta_e(\delta_e + 2\|K^*\|)\|\varepsilon_k\|^2$. Combine (A.7) and (A.8), we have

$$H_k \le \frac{\bar{\lambda}(R)\delta_e(\delta_e + 2\|K^*\|)}{\mu(1-\epsilon)\|\bar{E}\|^2}\Delta V. \tag{A.9}$$

Let $\rho_1 = \frac{\bar{\lambda}(R)\delta_e(\delta_e+2\|K^*\|)}{\mu(1-\epsilon)\|\bar{E}\|^2}$, for $k = 0, 1, 2 \cdots$, we have

$$H_0 \leq \rho_1(V(\varepsilon_0) - V(\varepsilon_1)) \tag{A.10}$$

and

$$\begin{aligned} &\sum_{k=0}^{1} H_k \leq \rho_1(V(\varepsilon_0) - V(\varepsilon_2)) \\ &\sum_{k=0}^{2} H_k \leq \rho_1(V(\varepsilon_0) - V(\varepsilon_3)) \\ &\sum_{k=0}^{3} H_k \leq \rho_1(V(\varepsilon_0) - V(\varepsilon_4)) \\ &\cdots . \end{aligned} \tag{A.11}$$

Based on the event-triggering condition (36), there exist $\lim_{k\to\infty} \varepsilon_k = 0$, that is $\lim_{k\to\infty} V(\varepsilon_k) = 0$. Then, we have

$$\sum_{k=0}^{\infty} H_k \leq \rho_1 V(\varepsilon_0). \tag{A.12}$$

According to Melzer and Kuo (1971), we have $J^*(\varepsilon_0) = \varepsilon_0^T P^* \varepsilon_0$. Based on Algorithms 1 and 2, the following inequation is satisfied

$$\begin{aligned} J_e^* &= \sum_{k=0}^{\infty} \omega_k^T Q \omega_k + (\hat{u}_k^*)^T R \hat{u}_k^* \\ &\leq J^*(\varepsilon_0) + \frac{\bar{\lambda}(R)\delta_e(\delta_e + 2\|K^*\|)}{\mu(1-\epsilon)\|\bar{E}\|^2} J^*(\varepsilon_0). \end{aligned} \tag{A.13}$$

That is

$$J^*(\varepsilon_0) \leq J_e^* \leq J^*(\varepsilon_0) + \frac{\bar{\lambda}(R)\delta_e(\delta_e + 2\|K^*\|)}{\mu(1-\epsilon)\|\bar{E}\|^2} J^*(\varepsilon_0). \tag{A.14}$$

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