

## Conformation of Network Strands in Polymer Gels

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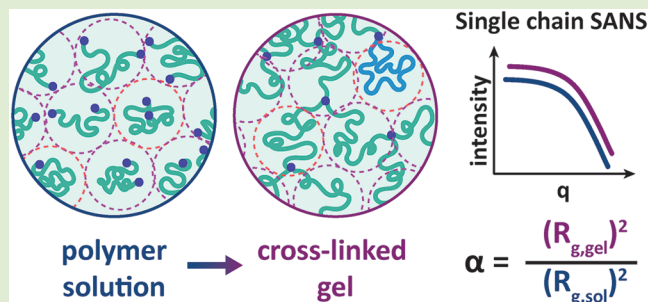
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**ABSTRACT:** Small angle neutron scattering was used to measure single chain radii of gyration of end-linked polymer gels before and after cross-linking to calculate the prestrain, which is the ratio of the average chain size in a cross-linked network to that of a free chain in solution. The prestrain increased from  $1.06 \pm 0.01$  to  $1.16 \pm 0.02$  as gel synthesis concentration decreased near the overlap concentration, indicating that the chains are slightly more stretched in the network than in solution. Dilute gels with higher loop fractions were found to be spatially homogeneous. Form factor and volumetric scaling analyses independently confirmed that elastic strands stretch by 2–23% from Gaussian conformations to create a space-spanning network, with increased stretching as network synthesis concentration decreases. Prestrain measurements reported here serve as a point of reference for network theories that rely on this parameter for the calculation of mechanical properties.



Polymer networks are important materials with an array of applications ranging from common tires<sup>1–3</sup> to biomedical materials such as tissue scaffolds and drug delivery systems.<sup>4–7</sup> Quantitatively predicting the mechanical properties of networks, even the relatively simple linear elasticity, is a long-standing challenge with significant technological importance. This challenge arises because real networks are difficult to accurately characterize due to the formation of defects such as dangling ends and loops which are not fully elastically effective and therefore do not contribute to the mechanical properties of the material to the same extent.<sup>8–12</sup> While recent efforts to incorporate loops into elasticity,<sup>11,13–17</sup> swelling,<sup>18,19</sup> and fracture<sup>20,21</sup> theories have shown improved agreement with experimental data, one uncertainty underlying these updates is the role chain conformation plays in property prediction.

Prestrain represents the change in chain conformations during the cross-linking process, described by the following ratio:

$$\alpha = \frac{\langle R^2 \rangle_0}{Nb^2} \quad (1)$$

where  $\langle R^2 \rangle_0$  is the mean square end-to-end distance of strands in the network,  $N$  is the number of Kuhn segments, and  $b$  is the Kuhn length, assuming a Gaussian conformation in solution. Until recently, most network theories assumed that the average end-to-end distance of a chain was equal before and after gelation.<sup>1,11,22–24</sup> However, loop defects have more compact conformations than free chains in solution; in the case of a primary loop, the end-to-end distance decreases from its Gaussian conformation in solution to zero after the looping reaction. Loops have been shown to become more prevalent as

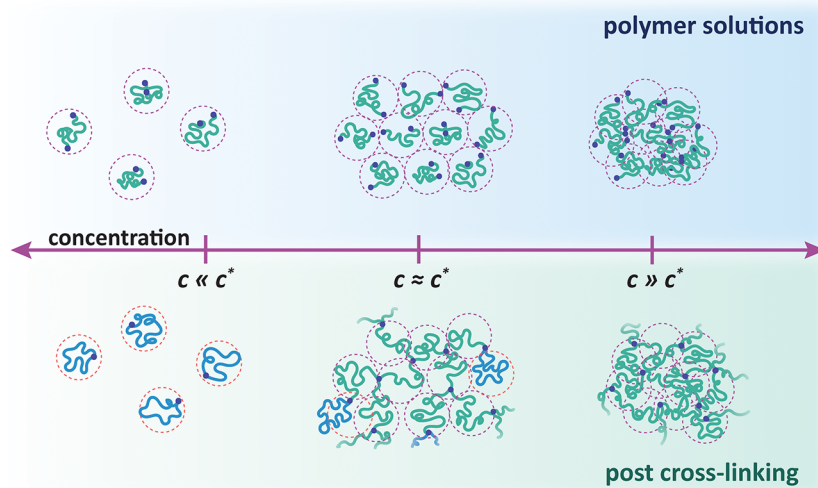
the synthesis concentration decreases, impacting network properties in the semidilute regime.<sup>9–11,25</sup>

Theoretical prestrain differs for strands in loops of different sizes, and these effects contribute significantly to the elasticity of the network.<sup>14,26,27</sup> Lang initially pointed out that elastic modulus predictions vary by a factor of 4, depending on if and how prestrain is incorporated into elasticity theories.<sup>14,15</sup> Lin et al. updated the phantom network-based real elastic network theory to show that a quantitative match between theory and experimental elasticity measurements was achieved only when prestrain of all loops was included.<sup>11,13</sup> Gusev similarly argued that reliable property predictions require a representative 3D network, which can be used to extract key topological factors, including prestrain, to correct a traditional affine or phantom network model.<sup>16</sup> However, neither loop populations larger than second order nor prestrain have been measured experimentally in gels, making various theoretical assumptions difficult to confirm.

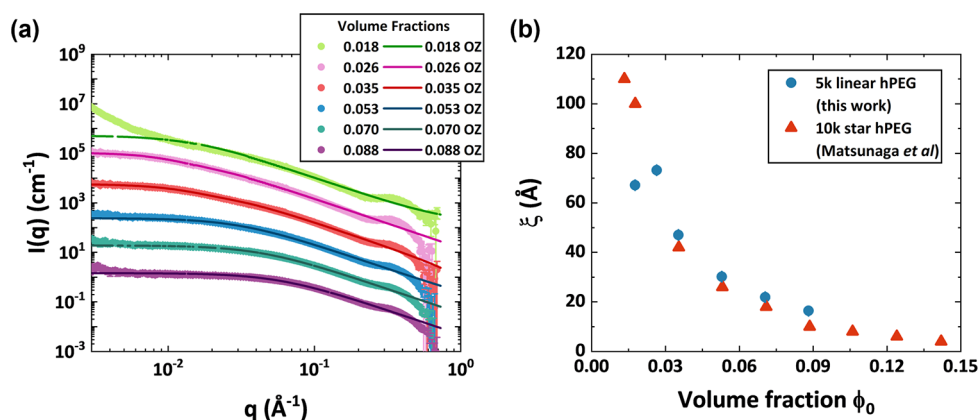
Small angle neutron scattering (SANS) has previously been used to measure single chain conformations of both ring polymers and neat networks. SANS is ideally suited for precise substructure measurements using isotope labeling.<sup>28–31</sup> Beltzung et al. measured the radii of gyration of polymer melts and the corresponding end-linked networks in the concentrated regime where loop fractions are expected to be

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**Figure 1.** Polymers in the dilute (left), semidilute (center), and concentrated (right) regimes before (top) and after (bottom) cross-linking take on different conformations depending on the concentration, resulting in a wide array of prestrain-dependent properties.



**Figure 2.** (a) SANS intensity data of hydrogenous PEG gels fit with the Ornstein–Zernike equation. Data were vertically shifted for clarity by multiplicative 10-fold increments for  $\phi_0 = 0.070$ – $0.018$  data. (b) Correlations lengths for the  $A_2 + B_4$  hPEG networks compared to previously reported correlation lengths of a 10 kDa tetra-PEG network as a function of volume fraction.<sup>42</sup>

negligible.<sup>32</sup> Their results showed no change in the radius of gyration upon cross-linking, providing strong evidence that  $\alpha = 1$  when networks are prepared well above the overlap concentration. Ring polymers have radii of gyration that have been widely confirmed to be approximately half that of the free chain in solution, depending on solvent quality and concentration.<sup>33–37</sup> As shown in Figure 1, these two cases fall on opposite ends of the concentration spectrum. Here, SANS measurements are presented to provide prestrain data in the regime around overlap concentration (center of Figure 1).

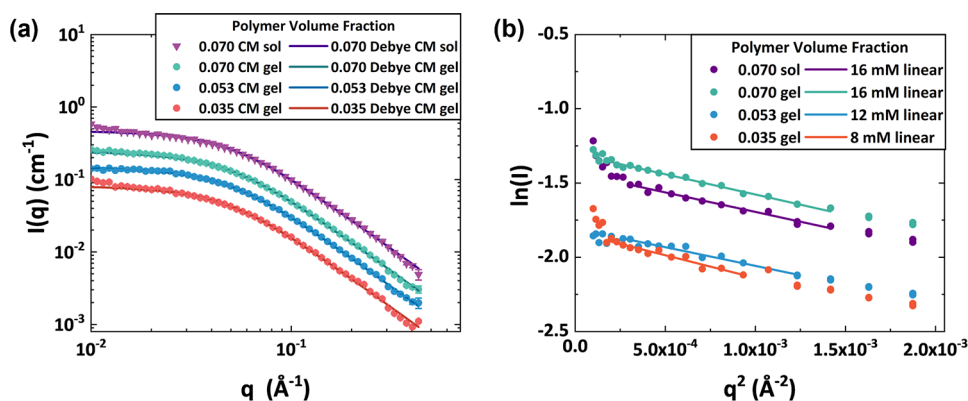
To this end, end-linked poly(ethylene glycol) (PEG) gels with primary loop fractions previously quantified via network disassembly spectrometry (NDS) were synthesized and analyzed with SANS.<sup>11</sup> Gels with a mixture of hydrogenous and deuterated chains in deuterated solvent were synthesized at the zero average contrast match (CM) condition to provide single chain information,<sup>30</sup> while pure hydrogenous (hPEG) networks yield information about the overall network. Scattering intensity data for the hPEG networks are plotted as a function of the scattering vector  $q$  in Figure 2a. The plateau regions at low  $q$  in the hPEG gels show that the gels are relatively homogeneous, even below the overlap polymer

volume fraction of  $\phi_0^* \approx 0.057$ .<sup>38,39</sup> The transition to inhomogeneous scattering at  $\phi_0 = 0.018$  corresponds to the gel–sol concentration threshold; lower concentration solutions did not form a gel after cross-linking. The bump in the high- $q$  regime corresponds to a length scale on the order of 2–3 Å, a range where features are typically attributed to the amorphous halo generated by short-range order in amorphous polymers.<sup>40</sup> The bump does not influence fitting results, as discussed in the SI.

To quantify these observations, the data were fit to a correlation length model. The Ornstein–Zernike equation captures both inter- and intramolecular thermal concentration fluctuations in networks:<sup>41,42</sup>

$$I(q) = I(0) \left( \frac{1}{1 + q^2 \xi^2} \right) + b \quad (2)$$

where  $I(0)$  is the low- $q$  intensity plateau,  $q$  is the scattering vector,  $\xi$  is the correlation length of the network, and  $b$  is the incoherent scattering contribution at high- $q$ . Correlation lengths from the  $A_2 + B_4$  hPEG are plotted in Figure 2b and compared to correlation lengths of 10 kDa tetra-arm PEG  $A_4 +$



**Figure 3.** (a) SANS curves for zero average contrast matched PEG gel and solution samples, fit with Debye scattering functions. Data were shifted vertically for clarity by a multiplicative factor of 0.6 for  $\phi_0 = 0.035$  and 2 for  $\phi_0 = 0.070$  sol. (b) Guinier plots of  $\ln(I)$  vs  $q^2$  for the same contrast matched samples, with the slope (drawn as a solid line) equal to  $R_g^2/3$ .

$B_4$  networks reported by Matsunaga et al., which have a nearly identical length between cross-links of 5 kDa.<sup>42–44</sup> Although the tetra-PEG  $A_4 + B_4$  chemistry prevents primary loop formation while the gels in this work have primary loop fractions ranging from 0.09 to 0.33, the correlation lengths are in good agreement between the two materials.<sup>43,45–47</sup> This suggests that both of these systems have a similar level of spatial homogeneity, despite known differences in topology. Scaling of correlation length with volume fraction demonstrated a scaling exponent consistent with near- $\theta$  solvent conditions, as shown in the [Correlation Length Scaling section of the SI](#).

Single chain measurements, necessary to calculate the prestrain of the network, were generated via Debye fitting and Guinier analysis of the CM SANS data and compared to a prestrain estimate based on primary loop defects. Because SANS yields a measure of chain radius of gyration as opposed to end-to-end distance, prestrain was calculated from the extracted  $R_g$  values with a modified prestrain equation:

$$\alpha_{R_g} = \frac{R_{g,\text{network}}^2}{R_{g,\text{solution}}^2} \quad (3)$$

In this work  $R_{g,\text{solution}}$  was measured at  $\phi_0 = 0.070$ , which is shown below to be in close agreement with the melt  $R_g$ . As the samples were all close to the overlap concentration and near  $\theta$  conditions, the 16 mM solution sample was taken to be invariant to concentration for the purpose of calculating prestrain.<sup>29,42,48</sup>

Contrast matched SANS data were fit with the Debye function for a Gaussian coil to extract the average radius of gyration,  $R_g$ :

$$I(q) = I_0 \left( \frac{2 \exp(-Z) + Z - 1}{Z^2} \right) + b \quad (4)$$

where  $Z = (qR_g)^2$ . These fits are shown in [Figure 3a](#), with the  $R_g$  values and corresponding prestrain reported in [Table 1](#). The Debye equation assumes  $\theta$  conditions and a single chain signal, which is supported based on the [Correlation Length Scaling and Contrast Matching Analysis](#) detailed in the SI ([Figures S3 and S4](#)). An error estimate calculated with the bootstrapping method<sup>49</sup> resulted in similar uncertainties, as reported from the confidence interval of the fit ([Figure S5](#)). An alternative

**Table 1. Radius of Gyration from Debye and Guinier Analyses with the Corresponding Prestrain Reported at Different Synthesis Concentrations**

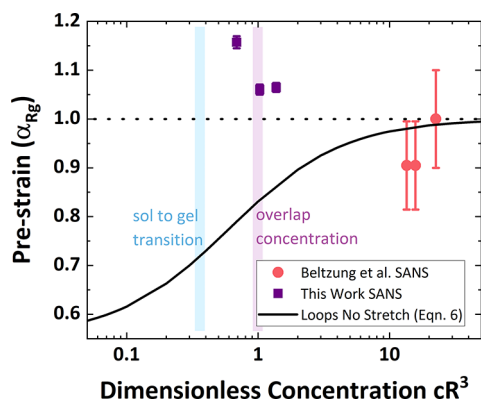
Synthesis volume fraction	Debye radius of gyration (Å)	Debye prestrain	Guinier radius of gyration (Å)	Guinier prestrain
0.035 gel	30.9 ± 0.2	1.16 ± 0.02	29.9 ± 1.1	1.14 ± 0.06
0.053 gel	29.6 ± 0.1	1.06 ± 0.01	27.5 ± 0.4	0.96 ± 0.04
0.070 gel	29.6 ± 0.1	1.06 ± 0.01	28.6 ± 0.3	1.04 ± 0.03
0.070 solution	28.7 ± 0.1		28.0 ± 0.6	

method used to extract the radius of gyration is the Guinier approximation:

$$\ln(I) \sim -\frac{q^2 R_g^2}{3} \quad \text{where } qR_g < 1 \quad (5)$$

Guinier plots of  $\ln(I)$  versus  $q^2$  can thus be used to determine the radius of gyration without any assumptions related to the model or solvent quality; plots for the contrast matched gels are shown in [Figure 3b](#), with corresponding  $R_g$  values and prestrain values reported in [Table 1](#). The slope of the fit line is sensitive to the exact region used for the fitting, which is reflected in the relatively higher error associated with Guinier versus Debye analysis. All of the Debye  $R_g$  values were 1–2 Å greater than the Guinier results, but this bias was consistent across samples such that it resulted in comparable prestrain values. The theoretical radius of gyration for a PEG chain with  $b = 1.1$  nm and  $m_0 = 137$  g/mol<sup>22</sup> is 27.5 Å (see the calculation in [SI](#)), in good agreement with the measured solution radius of gyration of 28.7 ± 0.1 Å (Debye) and 28.0 ± 0.6 Å (Guinier) at  $\phi_0 = 0.070$ . Debye prestrain values are greater than one for all concentrations measured and show a slight increase as concentration decreases. The measured values of prestrain are relatively close to Lang's average estimation of  $\alpha = 1.03$  at  $\phi_0 = 0.070$  and 0.053, deviating as loops become more prevalent at the lower concentration.<sup>14</sup>

The measured prestrain data were compared to estimated prestrain values based on previously reported primary loop fractions and the complementary fraction of bridging strands in [Figure 4](#).<sup>11,12</sup> To compare directly with prestrain from SANS data, an estimated  $R_g$  was calculated by weighing the form factors for a Gaussian chain and Gaussian ring by the mole fraction of bridging chains and primary loops, respectively, and



**Figure 4.** Prestrain as a function of  $c_{\text{dim}}$  for the gels measured in this work (squares), concentrated PDMS networks measured by Beltzung et al.<sup>32</sup> (circles), and estimated by accounting for primary loops and nonstretching bridging strands (solid line).

setting the sum equal to the form factor used to fit the data as follows:

$$P(R_{g,\text{avg}}) = (1 - x_{\text{loop}})P(R_{g,\text{bridge}}) + (x_{\text{loop}})P(R_{g,\text{ring}}) \quad (6)$$

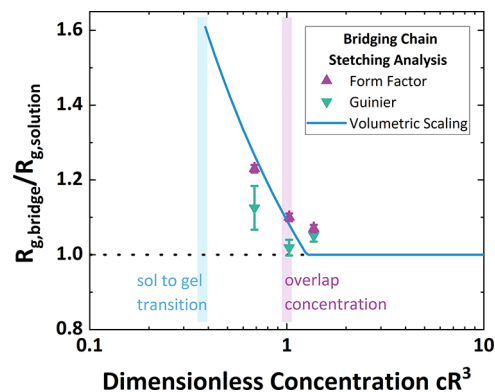
where  $x_{\text{loop}}$  is the primary loop fraction,  $P(R_{g,\text{bridge}})$  is the form factor for a linear chain, and  $P(R_{g,\text{ring}})$  is the form factor for a ring. Theoretical prestrain was calculated using a simplifying assumption previously invoked by Lin et al. whereby primary loops are Gaussian rings and all other strands in the network are bridging chains which do not change conformation upon gelation; this curve is therefore labeled “loops no stretch”. Equation 6 was solved for the average predicted  $R_g$  (details in the [Weighted Form Factor Analysis section in the SI](#)), and the resulting prestrain estimate is plotted as a function of dimensionless concentration in [Figure 4](#). Dimensionless concentration is used here because it enables a direct comparison across different chemical systems and molecular weights, which is challenging when working in traditional concentration or volume fraction units. Dimensionless concentration is defined as<sup>50</sup>

$$c_{\text{dim}} = cR^3 = c\langle R^2 \rangle^{3/2} = cb^3 \left( \frac{M}{m_0} \right)^{3/2} \quad (7)$$

where  $c$  is the molar polymer synthesis concentration and  $R$  denotes the root-mean-square end-to-end distance for the linear precursor, equivalent to  $b \left( \frac{M}{m_0} \right)^{1/2}$ , where  $M$  is the polymer molar mass and  $m_0$  is the Kuhn segment molar mass. [Figure 4](#) shows that the loops no stretch estimate agrees with high concentration poly(dimethylsiloxane) (PDMS) network prestrain measurements reported by Beltzung et al.,<sup>32</sup> but does not capture the prestrain data collected around the overlap concentration.

The measured prestrain clearly deviates from the loops no stretch estimate around the overlap concentration, suggesting that the elastically effective bridging strands must also deviate from their pregel conformations to offset the contraction of loops. Stretching of the bridging strands was estimated independently with both a form factor and volumetric scaling analysis. In the form factor analysis, the average  $R_g$  of a bridging strand was calculated with [eq 6](#) using the average  $R_g$  extracted from the SANS data fitting to calculate  $R_{g,\text{bridge}}$ .

Values of  $R_{g,\text{bridge}}$  were calculated with both Debye and Guinier analyses, as detailed in the [SI](#), and were then compared to the  $R_g$  of free chains in solution. The ratio  $R_{g,\text{bridge}}/R_{g,\text{solution}}$  captures the effective stretching experienced by the linear bridging strands, which is plotted in [Figure 5](#) and reported in [Table S4](#). The extent of stretching ranged from 2–23% near the overlap concentration and increased as the synthesis concentration decreased.



**Figure 5.** Ratio of the radius of gyration of bridging strands in a gel over the size in solution is plotted as a function of dimensionless concentration. The ratio was calculated with Debye form factor and Guinier analysis based on the SANS data and compared to a volumetric scaling analysis.

To independently estimate anticipated chain stretching, a scaling relationship was employed that relates the number of effective chains to the increase in radius the average chain would experience to fill a constant volume. The total volume,  $V_{\text{tot}}$  scales as

$$V_{\text{tot}} \sim nV_{\text{chain}} \sim nr^3 \quad (8)$$

where  $n$  is the number of polymer molecules and  $V_{\text{chain}}$  is the volume each individual chain occupies, which scales as  $r^3$ . Since  $n$  is directly related to the concentration, changes in radius as the concentration falls below the overlap concentration should scale as

$$\left( \frac{n_{\text{overlap}}}{n_{\text{bridge}}} \right)^{1/3} = \left( \frac{r_{\text{bridge}}}{r_{\text{overlap}}} \right) \quad (9)$$

To account for topological defects in the gels, an effective elastic chain concentration of bridging molecules was calculated by subtracting the fraction of strands that are primary loops from the synthesis concentration; these adjusted chain concentrations are reported in the [Chain Stretching Analysis section of the SI](#) and are the basis for the calculation of  $r_{\text{bridge}}$ . Using [eq 9](#) and  $\phi_0^* \approx 0.057$  as the overlap concentration,<sup>11,39</sup> the percent change in  $r$  was calculated based on the concentration of bridging strands, reported as volumetric scaling in [Table S2](#) and plotted as a function of dimensionless concentration in [Figure 5](#).

Based on [Figure 5](#) it is evident that the bridging polymer chains stretch by up to 25% when necessary to allow the cross-linking reaction to proceed to complete conversion. This stretching is well within the entropic regime for Gaussian chains, and there is precedent for stretching of much greater magnitude during gel swelling. In the case of prestrain,

however, it is the kinetic process of network formation that drives chain extension. It should be stressed that while the prestrain measured here is an accurate reflection of the ensemble average of strands in the network, it does not differentiate between different populations of strand topologies.

In summary, prestrain was experimentally measured in dilute gels and found to increase as the concentration decreased, despite the increased prevalence of loop defects. Radii of gyration extracted from contrast matched SANS curves showed that single chains in the gel exhibit slightly larger conformations, on average, than single chains in solution. The extent of elastic strand stretching was quantitatively estimated to show that bridging strands will stretch up to 25% to create a space-spanning network around the overlap condition, emphasizing the need to consider individual strand behavior within a material. The measured prestrain provides a useful benchmark for theoretical calculations and future model refinement.

## EXPERIMENTAL DETAILS

Hydrogenous and partially deuterated  $A_2 + B_4$  PEG gels were synthesized over a range of concentrations (4–20 mM) following a previously published protocol.<sup>20,25</sup> Briefly, bifunctional linear PEG–OH (MW = 4600 g/mol,  $D = 1.04$ , Sigma-Aldrich) and dPEG–OH (MW = 4800 g/mol,  $D = 1.26$ , Creative PEGworks) were modified with azide-containing functional groups, dried, and brought into the glovebox to react with a tetra-functional alkyne cross-linker via copper-catalyzed alkyne azide click chemistry. Conversion reached >97% and >98% for the deuterated and hydrogenous coupling reactions, respectively (Figures S15–S17). Contrast matched gels were synthesized with 10% hydrogenous PEG and 90% deuterated PEG by molar ratio to match the scattering length density of the solvent, deuterated  $N,N$ -dimethylformamide. Synthesis and sample preparation details are included in the SI. Scattering length densities were calculated with the NIST web calculator;<sup>51</sup> densities and the resultant coherent and incoherent SLD's are found in Table S1. Small-angle neutron scattering was conducted at Oak Ridge National Laboratory Spallation Neutron Source and High Flux Isotope Reactor; SANS measurement details are included in the SI.

## ASSOCIATED CONTENT

### Supporting Information

The Supporting Information is available free of charge at <https://pubs.acs.org/doi/10.1021/acsmacrolett.3c00006>.

Contrast matching calculation; SANS sample preparation and experimental details; calculation of radius of gyration; uncertainty analysis; correlation length scaling; weighted form factor and chain stretching analysis; synthesis protocols, polymer GPC, and small molecule  $^1\text{H}$  and  $^2\text{H}$  NMR spectra (PDF)

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## Author Contributions

CRedit: Haley K. Beech conceptualization (supporting), data curation (lead), formal analysis (lead), investigation (lead), methodology (lead), writing-original draft (lead), writing-review & editing (equal); Jeremiah A. Johnson conceptualization (supporting), funding acquisition (supporting), project administration (supporting), resources (supporting), writing-review & editing (equal); Bradley D. Olsen conceptualization (lead), data curation (supporting), formal analysis (supporting), funding acquisition (lead), investigation (supporting), methodology (supporting), project administration (lead), resources (lead), writing-review & editing (equal).

## Notes

The authors declare no competing financial interest.

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