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Bounds on Substructure Dielectric Resonator Antennas Using Characteristic Modes

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***Abstract*—This paper presents a recent development on the fundamental limits of substructure dielectric resonator antennas (DRAs). Specifically, the substructure DRAs are found to have higher resonance frequencies than the superstructure DRA, and a bounding relation on Q factor is also obtained. Numerical examples based on a ring DRA and a cylindrical DRA is examined here to demonstrate the bounding relations.**

I. Introduction

Due to the high radiation efficiency, compact size and large design freedom in both material options and 3D geometries, dielectric resonator antennas (DRAs) have drawn significant attention from researchers and engineers. DRA designs reported in literature cover various applications, such as broadband designs, multi-port applications, beamforming arrays, etc. However, DRAs with complex structures are generally less investigated than their metallic counterparts. While metallic antennas often take complex geometries based on heuristic modification of canonical antennas or pixel-based optimization [1], [2], the majority of reported DRA designs are based on canonical shapes, such as rectangular, cylindrical and spherical resonators. In particular, miniaturization of metallic antennas by adding inductive energy with meandering lines, helix or spiral structures [3], has been thoroughly investigated. However, few studies investigate DRAs with complex shapes and their general resonance behaviors.

In this paper, leveraging the theory of characteristic modes for analysis of DRAs, we study the performance of DRAs with arbitrary geometry, and establish bounds on the resonance frequencies and quality factors of DRAs before and after partial material removal, providing general insight and guiding principles on DRA design and synthesis.

II. Bounds on Substructure DRAs

It has recently been observed that DRAs display only capacitive characteristic modes [4], i.e. modes with negative eigenvalues at the low frequency limit. More recently, a mathematical proof the capacitive nature of DRAs at the low frequency limit was established in [5]. Based on [5], the characteristic eigenvalues of all DRAs are negative at the low frequency limit, namely,

$$\lambda_n < 0 \text{ as } \text{freq} \to 0. \tag{1}$$

In the following analysis, the capacitive nature of DRAs characteristic modes will be leveraged to study the performance bounds of DRAs of arbitrary geometries. Consider an arbitrary 3D dielectric body as the superstructure DRA (Ω), we consider any DRA that is obtained through partial material removal from the superstructure DRA as its substructure ($\bar{\Omega}$). Following similar basis transformation analysis as that in [6], it can be shown that the characteristic eigenvalues of the complete and substructure antennas are bounded through eigenvalue interlacing as:

$$\lambda_k \geqslant \bar{\lambda}_k \geqslant \lambda_{k+K-\bar{K}}, 1 \geqslant k \geqslant \bar{K}. \tag{2}$$

for all frequencies, where K is the rank of the super-structure MoM Z matrix, and $\bar{K}$ is the rank of the substructure matrices and the λ_k and $\bar{\lambda}_k$ are the k-th characteristic eigenvalues of the superstructure and the substructure DRAs respectively. The mode numbering are based on the algebraic descending order of the eigenvalues.

The above characteristic eigenvalue bound has immediate implication on the antenna resonance frequencies. In characteristic modal analysis (CMA), the antenna's modal resonance frequency is the frequency when the corresponding eigenvalue reaches zero. Due to the capacitive nature of DRAs' characteristic modes at low frequencies, the eigenvalue bound in (2) suggests that the when the k-th characteristic eigenvalue of the superstructure reaches zero, the k-th characteristic eigenvalue of the substructures are generally below zero. In other words, the k-th characteristic eigenvalue of the substructures crosses zero at a higher frequency than the superstructure. Therefore, the bounding relation in (2) suggests a bound on the modal resonance frequencies:

$$f_k \leqslant \bar{f}_k \leqslant f_{k+K-\bar{K}}, 1 \geqslant k \geqslant \bar{K}. \tag{3}$$

where f_k and $\bar{f}_k$ are the resonant frequencies of the k-th characteristic mode for the complete and the substructure DRAs respectively. In other words, all substructure DRAs will resonate at a frequency higher (or no less) than the superstructure DRA, regardless of the DRA shapes.

A bounding relation on modal Q factors of DRAs are also established in [5], which states that at frequencies below the fundamental resonance of the superstructure DRA, the modal Q factors of all substructure DRAs are bounded below by those of the superstructure DRA, namely,

$$Q_k \leqslant \bar{Q}_k \leqslant Q_{k+K-\bar{K}}, 1 \geqslant k \geqslant \bar{K}, \tag{4}$$

for $f \leq f_1$, with f_1 being the first modal resonance of the superstructure DRA.

TABLE I: Comparison of the Characteristic Modal Resonant Frequencies of the Cylindrical and the Ring DRA.

DRAs\f (GHz)	f_1^{res}	f_2^{res}	f_3^{res}	f_4^{res}	f_5^{res}	f_6^{res}
Cylindrical DRA	0.939	0.985	0.985	1.303	1.303	1.333
Ring DRA	1.389	1.711	1.711	1.945	1.982	1.982

III. Numerical Examples

To demonstrate the bounding relations in (2)- (4), we here consider a cylindrical DRA (radius 30 mm, height 50 mm) as the superstructure DRA and a ring DRA (outer radius 30 mm, inner radius 25 mm, height 50 mm) as the substructure DRA, both with the permittivity of 23.

Fig. 1 (a) and (b) shows the two structures with the current distribution of the fourth characteristic mode (TE_{01p} mode). The characteristic eigenvalues spectrum of both structures are calculated and compared in Fig. 1 (c), where the dashed lines are the results of the cylindrical DRA, and the marked lines are for the ring DRA. The eigenvalues of the ring DRA are found bounded above by the cylindrical DRA, confirming the result in (2). The eigenvalue bound directly translates to the bound on resonance frequencies in (3). Table I compares the first 6 modal resonances of the two DRA structures, and it's clearly shown that all the modal resonances of the substructure DRA (ring) is higher than that of the superstructure (cylindrical). Fig. 1 (d) shows the comparison of the modal Q factors of the two structures calculated from the source formulation of the stored energies [7]. The bounding relation on Q factors in (4) implies that the Q factor of the ring DRA should be bounded below by the cylindrical DRA for frequencies below the fundamental resonance of the cylindrical DRA (0.939 GHz), and Fig. 1 (d) confirms that. Note that for the cylindrical DRA, modes 1 and 2 are degenerate, and modes 5 and 6 are degenerate. For the ring DRA, modes 2 and 3 are degenerate and modes 5 and 6 are degenerate. In Table I, the resonance frequency is arranged in algebraic ascending order, and may not correspond to the same ordering of the eigenvalues.

It is worth noting that the Q factor bound is only valid for the DRA antenna operating below the fundamental resonance of the superstructure. In this case, the ring DRA resonates at a higher frequency (1.389 GHz) than the cylindrical DRA (0.939 GHz), therefore its Q at resonance is not necessarily bounded by the cylindrical DRAs. In fact, the Q factor comparison in Fig. 1 also shows that the Q factor of the ring DRA are lower than the cylindrical DRA at higher frequency ranges. However, if we compare the Q factor at and below the fundamental resonance (0.939 GHz), it's found out that the substructure DRA always possesses a higher Q factor, therefore a narrower bandwidth. These bounding relations on antenna resonance frequency and Q factors will provide insightful guidance for DRA design and synthesis.

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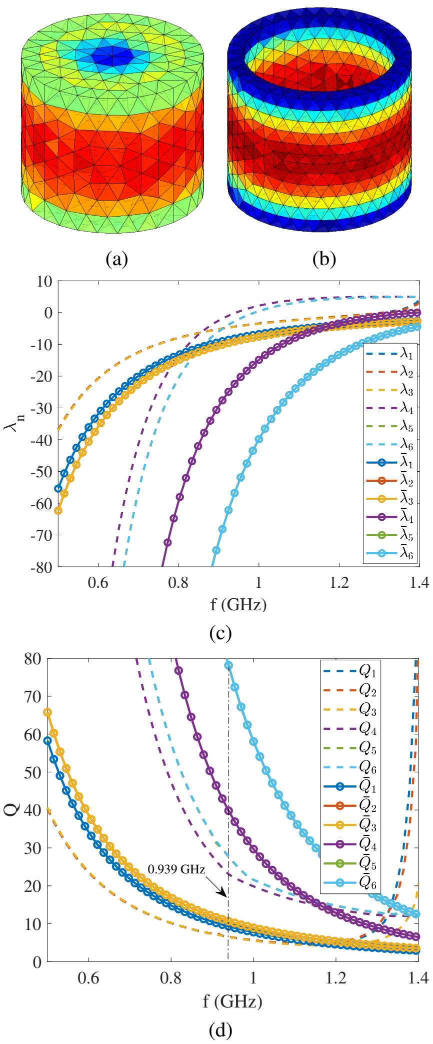

Fig. 1: (a) Cylindrical DRA, (b) ring DRA, (c) comparison of the characteristic eigenvalues of the cylindrical DRA (dashed lines) and the eigenvalues of the ring DRA, (d) comparison of the modal Q factors between the two structures.

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