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Disruption and Rerouting in Supply Chain Networks

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Abstract. We study systemic risk in a supply chain network where firms are connected through purchase orders. Firms can be hit by cost or demand shocks, which can cause defaults. These shocks propagate through the supply chain network via input-output linkages between buyers and suppliers. Firms endogenously take contingency plans to mitigate the impact generated from disruptions. We show that, as long as firms have large initial equity buffers, network fragility is low if both buyer diversification and supplier diversification are low. We find that a single-sourcing strategy is beneficial for a firm only if the default probability of the firm's supplier is low. Otherwise, a multiple-sourcing strategy is ex post more cost effective for a firm.

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Keywords: supply chain networks • disruption risk • contingent rerouting • secondary markets • systemic risk

1. Introduction

Over the past decade, supply disruptions caused by various types of shocks, including bankruptcies of suppliers, natural and man-made disasters, labor strikes, and public health crisis, have presented serious concerns to firms. Major examples of supply chain disruptions include bankruptcy filings of 30% of preexisting North American automotive suppliers by 2008; the 2011 Great East Japan Earthquake that caused wide damages to supply chains in automobile and electronics industries; the fire at a Philips plant in New Mexico in 2000 that stopped the supply of semiconductor chips to major cell phone manufacturers for several months; the labor disputes, wage strikes, and walk-outs that occurred in China, Bangladesh, Cambodia, and Vietnam in 2010 and exposed firms in developing markets to high supply disruption risk¹; and the most recent coronavirus disease 2019 (COVID-19) pandemic that triggered both supply and demand shocks across a wide spectrum of industries. We refer to Brinca et al. (2020, 2021) for studies on supply and demand shocks that occurred during the COVID-19 pandemic and to Guerrieri et al. (2022) for the causal relationship between supply shocks and demand shortages experienced during the pandemic.

If a firm defaults after being hit by a shock, downstream firms that purchased goods from it will be negatively affected. These firms would experience a decline in their

outputs if they are not able to replace their suppliers and may themselves default because of a drop in revenues. Moreover, the shock hitting the defaulting firm also propagates upstream to its suppliers, which will not deliver orders to a firm defaulting on its payments. If a supplier cannot reroute the undelivered orders and the drop in revenues caused by the defaulting firm is significant, the supplier may also default and create further distress through the network. Hence, the default of a firm has implications on the overall system. We refer to the risk of contagious failures as *systemic risk*. The negative externalities created by defaults are material, as empirical evidence suggests. For instance, the study of Kolay et al. (2016) considers trading partners of firms that file for Chapter 11. They find that suppliers and buyers of a bankrupt firm experience losses when they lose the bankrupt partner, and the suppliers suffer larger losses than buyers. Suppliers and buyers may, in turn, default if the negative externalities imposed by the bankrupt firm are too large, as in the bankruptcy cases described next.

Pipeline Foods, a Minnesota-headquartered organic food company, filed for bankruptcy in July 2021 because of the reduced customer orders during the COVID-19 pandemic. Many farms were hit hard by this bankruptcy; in addition to their claims on delivered grain, they could not sell undelivered grain that was under contract to

Pipeline Foods to other buyers. Assisted by Oliver Larson, an assistant attorney general who petitioned the court on behalf of Minnesota farmers for permissions to sell the undelivered grain, Chris and Andrea Koller's farm was granted permission to sell the grain to other buyers. Proceeds from this sale provided the farm with enough funds to survive (see Pates 2021 for additional details). Another noticeable case is that of Hanjin Shipping, a world top 10 container carrier that filed for bankruptcy in September 2016 because of sluggish freight rates caused by weak demand and soaring global capacity. The bankruptcy affected global supply chains because half of Hanjin's container ships were denied access to ports. Major U.S. retailers, such as J.C. Penney and Walmart, began to divert and switch demand of carriers for their containers to other suppliers (e.g., Hyundai Merchant Marine) right after assessing the impact. We refer to *AP News* (2016), *DW News* (2016), and Dong-chan (2017) for additional details.

Pipeline Foods and Hanjin are two examples of firms that went bankrupt because of demand shocks. The Pipeline Foods' supplier managed to reroute its grain, and Hanjin's buyers were able to switch suppliers to prevent further distress. This should be contrasted with the many suppliers and buyers of firms damaged by the Great East Japan Earthquake in 2011, which were unable to withstand the shock induced by the earthquake after trying to find alternative buyers and switch suppliers, respectively.²

Although systemic risk has been the subject of considerable investigation in the finance literature,³ there are only a handful of studies on systemic risk in supply chain networks. Nevertheless, supply chain systemic risk is economically significant considering that the gross output in the manufacturing industry is more than twice that in the finance and insurance industry (\$5,712 billion against \$2,408 billion in the United States in 2016 according to the U.S. Department of Commerce).

The goal of our study is to analyze the systemic implications of shocks to the supply chain network when firms are reactive and ex post take contingency plans to mitigate the adverse effects from disruptions. We develop a network model for an industry whose firms are interconnected through purchase orders of different goods. Each firm in the network incurs a net production cost in the time from order placement until order delivery. The net production cost includes the cost of fulfilling orders minus the revenue generated from transactions outside the network (e.g., income generated from consumer sales of retail firms). Upon the occurrence of a shock, if the net production cost of a firm plus its safety stock cost exceeds its initial equity plus the revenue generated from the sale of goods, then the firm will fundamentally default. The firm's default will have consequences on the entire network via the input-output linkages between buyers and suppliers.

Each firm optimally mitigates the potential losses triggered by a default of its buyers or suppliers by rerouting undelivered orders to alternative buyers and switching excess demand to alternative suppliers. If the firm's net worth is still negative after these mitigating actions, then the firm will default and trigger another wave of contagious defaults in the network. A unique equilibrium is reached when the default cascade stops and simultaneously, buyers and suppliers of any defaulting firm agree on an efficient profile of switched demands and rerouted supplies. We develop an algorithm to compute this equilibrium. Under such equilibrium, all firms have the largest ex post net worth, and the number of defaults in the network is minimized.

We analyze the impact of buyer and supplier diversification on systemic risk in tiered supply chain networks. A tiered network is more diversified than another if each firm in this network distributes its orders to a larger number of suppliers and buyers than the other. We measure the ex ante performance of a supply chain network using two metrics: *resilience* and *fragility*.

The resilience of a network is measured by its reduction of out-of-stock risk quantified by the fraction of losses caused by switching suppliers, which can be reduced by holding safety stocks. The fragility of a network is measured by the expected total loss conditioned on the event that at least one contagious default occurs (i.e., that the net production costs are sufficiently high to trigger contagion).

We show that higher diversification always results in a more resilient network. With respect to fragility, instead the result depends on the capitalization of firms in the network. If firms' initial capital is sufficiently low, less diversification amplifies losses from defaults and leads to a more fragile network. However, if firms' initial capital is high, higher diversification results in a more fragile network. Despite that higher diversification yields larger loss-sharing benefits to firms in the supply chain network, it also presents a cost for two main reasons: (i) a larger number of contagious defaults and as a result, (ii) a larger amount of rerouted supply and switched demand. We show that the loss-sharing benefits are outweighed by the losses resulting from higher contagion. These results stand in contrast with the findings of Acemoglu et al. (2015) in the context of financial networks. They show that if banks have a sufficiently large cash buffer to absorb a shock, denser interconnections among banks result in a less fragile financial system.

On a firm level, we compare the two types of sourcing most commonly used in supply chain systems, namely single and multiple sourcing. We begin by considering the event (i) in which a common supplier to both the single-sourcing and multiple-sourcing firms defaults. Then, the single-sourcing firm needs to switch the entire demand, whereas the multiple-sourcing firm only needs to switch part of it because the remaining demand is served by other solvent suppliers. As a result,

the single-sourcing firm ends up with a lower net worth than the multiple-sourcing firm. The recent study of Crognani et al. (2020) lends support to this model implication. They find that if the buyer of victims of cyberattacks has fewer alternative suppliers, then it will experience a larger revenue reduction than those firms with more suppliers. Next, we consider event (ii) in which the supplier of the single-sourcing firm remains solvent while at least one of the multiple-sourcing firm's suppliers defaults. In this circumstance, the entire demand of the single-sourcing firm is served, whereas the multiple-sourcing firm needs to switch the demand that is unserved by its defaulted suppliers. As a result, the net worth of the single-sourcing firm would be higher. If the probability of event (i) is higher than the probability of event (ii), the expected cost under the single-sourcing strategy would be higher. Otherwise, the multiple-sourcing strategy would result in higher expected costs.

Our findings provide theoretical support to empirically observed patterns. According to Chopra and Sodhi (2004), Nokia swiftly adopted a multiple-sourcing strategy after the fire at Philips' Albuquerque plant in the United States. This strategy was based on the belief that the supply disruption would have had long-lasting consequences. By contrast, the communication technology company Ericsson opted for a single source of supply, believing that the disruption effect would have only been temporary. Mukherjee (2008) reported that Nokia's purchasing manager had worked in the semiconductor industry before and was able to estimate a very high probability that the fire would cause supply disruption. Under these conditions, our theoretical results imply that the supply disruption resulting from the fire would cause a smaller loss to Nokia than Ericsson. This is consistent with realized facts as the disruption had severe consequences, and Ericsson suffered much larger losses than Nokia.

The rest of the paper is organized as follows. In Section 2, we develop the model of the supply chain network. In Section 3, we formalize the notion of partial and general equilibrium in a supply chain network. In Section 4, we characterize the unique general equilibrium and develop an algorithm that recovers the equilibrium. In Section 5, we study the impact of buyer and supplier diversification on network performance. We also compare single-sourcing with multiple-sourcing strategies. We conclude in Section 6. All technical proofs and auxiliary results are relegated to the e-companion. A supplementary study on the impact of firms' mergers on network performance is also provided in the e-companion.

1.1. Related Literature

Our paper is related to a branch of literature on supplier selection and risk in supply chains. Anupindi and Akella (1993), Tomlin (2006), and Babich et al. (2007, 2012) characterize the optimal multisourcing strategies of a firm that is exposed to supply disruption risk. Deng

and Elmaghraby (2005) evaluate the performance of the tournament selection among suppliers in the face of unknown supplier quality and unverifiable investment. Chod et al. (2019) focus on buyer default risk. They study how the buyer's own riskiness affects its supplier selection strategy. Deo and Corbett (2009) and Tang and Kouvelis (2011) model supply chain disruptions using a Cournot competition model. Behzadi et al. (2020) use the net present value of the loss of profit, time to recovery, and level of recovery to measure supply chain resilience. The authors evaluate the effectiveness of a port backup strategy in reducing the port closure disruption risks in a supply chain with fresh perishable goods. All these papers deal with suppliers' selection strategies for mitigating the adverse consequences of disruptions from the perspective of an individual firm. Our study instead focuses on the implications of supplier diversification on systemic risk in the supply chain network.

A separate branch of literature has focused on operational tools that can mitigate the impact of supply disruption. Babich (2010) and Wadecki et al. (2012) analyze the optimal subsidy (on suppliers) decisions of manufacturers under different network topologies in the presence of supply disruption risk. Serel et al. (2001), Kouvelis and Milner (2002), and Babich (2006) investigate the impact of supplier default risk, capacity reservation requirements, and demand (supply) uncertainty on manufacturer procurement and production decisions. Tomlin and Wang (2005) consider the resource investment problem in the presence of demand and supply uncertainty. Shan et al. (2022) study the problem of a retailer that orders from two competing strategic suppliers, subject to disruption risk, and responds by setting the retail price upon delivery. Schmitt et al. (2015) compare a centralized with a decentralized inventory strategy in a two-echelon system that is subject to supply or demand disruptions. They find that the decentralized strategy is preferred to the centralized one if there is demand or supply uncertainty. Lim et al. (2011) study a bipartite network consisting of products and plants owned by a single firm. They investigate the design of the optimal network architecture, in which each plant is ex ante assigned to a certain product so to minimize the expected costs account for supply disruptions. Further, they solve for optimal allocation of a plant's capacity to its products after demand is realized. Hopp et al. (2008) use a two-stage model to identify the optimal strategies that minimize losses caused by supply disruptions. In the predisruption stage, two firms determine the investment in precautionary measures that facilitate quick detection of a supply disruption. Then, in the postdisruption stage, firms compete for alternative suppliers in a Stackelberg game. In most of these studies, firms determine ex ante the optimal strategies to prevent losses that may be caused by supply and demand disruptions, whereas in our model, firms ex post take

contingent actions, such as order rerouting and supplier switching, to mitigate the negative impact from disruptions.

Most of the literature surveyed focuses on supply chain models with a single product and a two-tier setting; the retailer at the downstream tier sources from the suppliers at the upstream tier. In contrast, we analyze a multitier supply chain network consisting of multiple firms that supply and purchase several goods. In particular, the multitier network allows us to analyze how the default of a firm affects the firms that are more than one tier apart. A noticeable exception is Bimpikis et al. (2019), who also consider multitier supply chain networks accounting for supply disruption risk. They develop an equilibrium model to determine how firms optimally source their inputs from suppliers and set prices of intermediate goods (i.e., those produced by firms in intermediate tiers). Although their focus is on the formation of such a supply chain network, in our paper the supply chain network is exogenously specified. However, firms are strategic ex post and optimally choose contingency plans to mitigate the adverse effects of default contagion.

Battiston et al. (2007) develop a multiperiod framework to simulate the propagation of bankruptcy shocks, accounting for the cost of supply disruption. In their model, the propagation of contagious defaults in the network is caused by delayed trade credit payments rather than supply and demand disruptions. Kim et al. (2015) consider a supply network consisting of a collection of facility nodes and transportation arcs that connect different facilities. They define a disruption as the event that, after local disruptions occur in the nodes or arcs, there no longer exists a path between the source and sink nodes. Different from our paper, they do not consider the adverse effect caused by a failed node or arc on its adjacent nodes or arcs. In their model, the local disruptions are exogenous and do not cause contagion. In contrast, in our model, fundamental defaults may lead to default contagion via input-output linkages.

On the empirical side, Carvalho et al. (2021) provide evidence that input-output linkages contributed to propagation and amplification of economic shocks triggered by the Great East Japan Earthquake of 2011. Ellis et al. (2010) provide a survey on purchasing managers and buyers that identifies how firms make decisions in the face of supply disruptions. We refer to Yano and Lee (1995), Snyder et al. (2006), and Tang (2006) for excellent surveys on supply chain risk.

2. Model

We develop a model to quantify the linkages between firms triggered by cost and demand shocks in the supply chain network.

In Section 2.1, we describe the firms' response to cost and demand shocks at the time when orders are due.

Hence, firms have zero pipeline inventory, and one does not need to specify the replenishment lead time between order placement and order delivery. In Section 2.2, we introduce the downstream markets where firms reroute their supply and upstream markets where firms switch their excess demand. We quantify the firms' ex post net worth after all contingency actions are taken in Section 2.3.

We consider an industry consisting of a set of firms $\mathcal{N} := \{1, \dots, N\}$. Each firm supplies, consumes, or purchases at least one good, and we denote the set of available goods by $\mathcal{M} := \{1, \dots, M\}$, where goods 1 and M represent raw materials and finished products, respectively. For example, in the steel industry, miners supply iron ore, steel producers purchase and consume iron ore and supply steel, and steel consumers purchase steel. The order o_{ij}^m specifies the quantity of good m that firm i commits to deliver to firm j . We use p^m to denote the exogenously specified price paid by buyers for each unit of the good m .⁴ Each firm i is assumed to be initially solvent with equity w_i and endowed with safety stock θ_i^m of good m whose unit holding cost is λ_i^m per unit time. The safety stock acts as a buffer stock to protect firm i from a supply disruption. We use a supply chain network to describe the interconnected market between firms in the same industry, defined as a five-tuple $(\mathbf{O}, \mathbf{p}, \mathbf{w}, \Theta, \Lambda)$, where $\mathbf{O} := \{o_{ij}^m\}_{m \in \mathcal{M}, i, j \in \mathcal{N}}$, $\mathbf{p} := \{p^m\}_{m \in \mathcal{M}}$, $\mathbf{w} := \{w_i\}_{i \in \mathcal{N}}$, $\Theta := \{\theta_i^m\}_{m \in \mathcal{M}, i \in \mathcal{N}}$, and $\Lambda := \{\lambda_i^m\}_{m \in \mathcal{M}, i \in \mathcal{N}}$.

2.1. Cost and Demand Shocks and Firms' Response

Each firm i incurs a random net production cost c_i , which captures the expenses required to produce the good minus revenues from transactions outside the supply chain network (e.g., sale of goods to consumers). We define the *cost shock* and *demand shock* to be sudden and unexpected change, respectively, in the expenses to produce a good and in the demand for a good. Uncertainty in the net production cost comes from the occurrence of cost and demand shocks caused by unforeseeable events occurring in the time interval from order placements until final delivery. Cost shocks include labor strikes, fire at plants, and changes in wages or energy prices. For example, the 2019 General Motors strike was estimated to cost General Motors up to \$100 million a day (Wayland 2019). In 2000, the fire at a semiconductor chip plant in Albuquerque, New Mexico caused the loss of millions of cell phone chips, damaged and contaminated cleanrooms, and led to a loss of approximately six weeks of production for Philips (Sheffi 2005). Another example is the bankruptcy of about 30 UK energy suppliers in 2021 because of the rising wholesale prices of natural gas for power generation (Cyrus 2022). Demand shocks, instead, are induced by natural or human-made disasters, such as earthquakes and terrorist attacks. The most recent COVID-19

pandemic has caused large demand shocks in many sectors. We refer to Miron and Zeldes (1988) for a discussion of cost shocks and to Brinca et al. (2020) for an analysis of demand shocks caused by the COVID-19 pandemic.

When orders are due, the net production costs of all firms are realized. The financial state of each firm i is determined by its net worth e_i (i.e., assets minus liabilities), which is observable by all firms. We use a structural default model (i.e., firm i defaults if $e_i < 0$ and stays solvent if $e_i \geq 0$). A default event is common knowledge among all firms in the supply chain network. This is consistent with current practices, where a defaulting firm files for either Chapter 7 or Chapter 11 bankruptcy. If a firm's net production cost is too high to be absorbed by the firm's initial equity plus revenues from its buyers' payments, then the firm defaults. The suppliers do not deliver any good to a defaulting firm because they are not compensated for such a delivery. Hence, the amount of good m delivered by firm i to firm j , denoted by q_{ij}^m , is given by

$$q_{ij}^m := \begin{cases} o_{ij}^m & \text{if } e_j \geq 0, \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

and it is referred as the *delivery amount*. The amount of *undelivered orders* is then given by the difference between the ordered and delivered amount: that is,

$$\gamma_{ij}^m := o_{ij}^m - q_{ij}^m \quad \text{for } m \in \mathcal{M}, i, j \in \mathcal{N}. \quad (2)$$

It follows that firm i 's total amount of undelivered orders for good m is given by

$$\bar{r}_i^m := \sum_{j \in \mathcal{N}} \gamma_{ij}^m. \quad (3)$$

These undelivered orders reduce firm i 's revenue by an amount equal to its claims on the defaulting buyers. To mitigate this loss, firm i will reroute those undelivered orders to alternative buyers that are either part of the supply chain network or outside it. We refer to those orders as the *rerouted supply* and use r_i^m to denote firm i 's rerouted supply of good m . Notice that $r_i^m \in [0, \bar{r}_i^m]$, and we will quantify r_i^m precisely in the next section. If, after rerouting, the initial equity plus revenue is still lower than the costs for these suppliers, they will also default and in turn, negatively affect their own suppliers and buyers.

A defaulting firm delivers the orders to its buyers but ceases its operations afterward.⁵ This, in turn, forces the firm's buyers that do not possess sufficient safety stock to switch to new suppliers to fill their future orders. Such an operation is costly because of the transaction costs in switching between identical brands; market research expenditures to learn about new brands of the same product; loss of benefits resulting from long-term relationships with the old suppliers; and adaptation costs incurred by new suppliers, which may need to

scale up their production processes to satisfy additional demand. These costs are inversely proportional to product substitutability (i.e., they are high when product substitutability is low and low when product substitutability is high). We will refer to the total costs as the switching costs incurred by the buyers. See also Klemperer (1987), Burnham et al. (2003), and Swinney and Netessine (2009) for additional details. If the buyers cannot find new suppliers, they incur the back-order (penalty) costs of not filling the orders on time as they had committed (see Tomlin 2006).

Example 2.1. A prominent example of the mechanism described is the fire at the Philips semiconductor chip plant in Albuquerque, New Mexico, which occurred in 2000. The fire severely affected the cell phone production of Ericsson. After the fire, Ericsson started looking into new suppliers of microchips but failed to obtain the chips needed for a new generation of cell phone products. In the end, Ericsson reported before-tax losses ranging from U.S. \$430 million to U.S. \$570 million because of a lack of semiconductor chips (see chapter 1 of Sheffi 2005 for more details).

Suppose firm j defaults, and let firm i be a buyer of goods from firm j . We use the amount of orders placed by firm i on firm j to approximate the amount of input lost by firm i if it does not switch to a new supplier. This is the residual loss of inputs after accounting for available inventory of firm i but not for safety stock. Our approximation is accurate if firms are under supply contracts with quantity commitments. This type of contract is widely used when a buyer commits to purchase a fixed quantity of good from the same supplier per period over a certain time horizon (see, for instance, Anupindi and Bassok 1999, Bassok and Anupindi 2008). The firm i 's *unserved demand* of good m because of firm j is then given by

$$\delta_{ji}^m := \begin{cases} 0 & \text{if } e_j \geq 0, \\ o_{ji}^m & \text{otherwise.} \end{cases} \quad (4)$$

In practice, a safety stock carried by a firm is used to reduce the risk of stockouts only during the lead time between order placement and delivery. Its amount is typically smaller than the order quantity if a reorder point/reorder quantity policy is used.

Example 2.2. For a firm with a single supplier, let τ be the replenishment lead time in days. Suppose this firm's daily customer demand is random and follows a normal distribution with mean μ and standard deviation ϑ . The order quantity of the firm will be the maximum between its average demand $\tau \times \mu$ during the lead time and economic order quantity $\sqrt{2k\mu/\lambda}$, where k denotes a fixed cost incurred every time an order is placed and λ denotes a per-unit per-time inventory holding cost.

The safety stock of the firm will be $z_\alpha \times \vartheta \times \sqrt{\tau}$, where z_α denotes the service factor chosen to guarantee that the probability of stockouts during the lead time is $1 - \alpha$. In practice, we expect $\mu > \vartheta$ and $\tau \geq (z_{99.9\%})^2 = 3.08^2 \approx 9$ days. This directly leads to the inequality

$$\tau\mu = \sqrt{\tau} \times \sqrt{\tau} \times \mu > \sqrt{\tau} \times z_{99.9\%} \times \vartheta \geq \sqrt{\tau} \times z_\alpha \times \vartheta$$

for any $0 < \alpha \leq 99.9\%$.

Hence,

$$\begin{aligned} \text{Order quantity} &= \max\left\{\tau\mu, \sqrt{2k\mu/\lambda}\right\} > z_\alpha \times \vartheta \times \sqrt{\tau} \\ &= \text{Safety stock with } \alpha \text{ service level.} \end{aligned}$$

This example supports the following assumption on θ_i^m .

Assumption 2.1. The safety stock of good m carried by firm i is smaller than the minimum order of good m made by firm i : that is, $\theta_i^m \leq \min_{j \in \mathcal{N}} \{o_{ji}^m\}$ for $i \in \mathcal{N}$.

Then, the firm i 's total unserved demand of good m after accounting for safety stock is given by

$$\begin{aligned} \bar{\sigma}_i^m &:= \left(\sum_{j \in \mathcal{N}} \delta_{ji}^m - \theta_i^m \right)^+ \\ &= \begin{cases} 0 & \text{if } \sum_{j \in \mathcal{N}} \delta_{ji}^m = 0 \\ \sum_{j \in \mathcal{N}} \delta_{ji}^m - \theta_i^m & \text{otherwise,} \end{cases} \end{aligned} \quad (5)$$

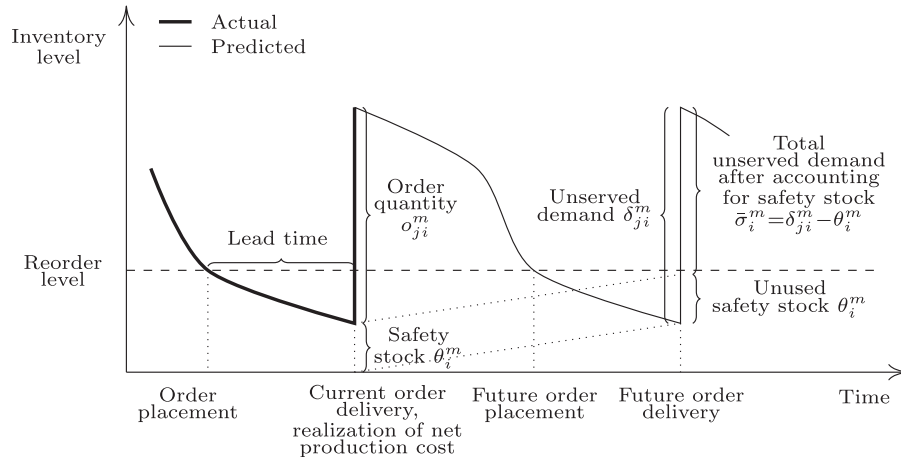
where we use the notation $x^+ := \max\{0, x\}$, for $x \in \mathbb{R}$. See Figure 1 for an illustration of the total unserved demand on an inventory-level curve. Each buyer aims at filling up the unserved demand, if any, by switching to new suppliers so to reduce the back-order costs

caused by not filling up the orders placed by its own buyers. We refer to the portion of unserved demand, which is filled after switching suppliers as the *switched demand*, and use σ_i^m to denote the switched demand of firm i on good m . It clearly holds that $\sigma_i^m \in [0, \bar{\sigma}_i^m]$. The switching cost incurred by firm i depends on the amount of firm i 's switched demand. If switching demand cannot effectively reduce the back-order costs, the buyers of the defaulting firm may go bankrupt and adversely affect their own buyers and suppliers.

Remark 2.1. It is worth mentioning that traditional literature on disruptions considers supply shocks in the form of reduced (or zero) replenishment capacity. In our model, although the cost and demand shocks are exogenous and may cause fundamental defaults, the supply shocks are endogenous and triggered by fundamental defaults. Whenever a firm defaults, it creates a supply shock to its buyer. If the buyer is not able to replace the defaulted supplier, it will experience a decline in its output and then, incur back-order costs. Our model does not capture how exogenous supply shocks (not triggered by fundamental defaults) affect a supply chain network, unless they are large enough to cause fundamental defaults. If a firm hit by an exogenous supply shock remains solvent, it will not create a supply shock to its buyer in our model.

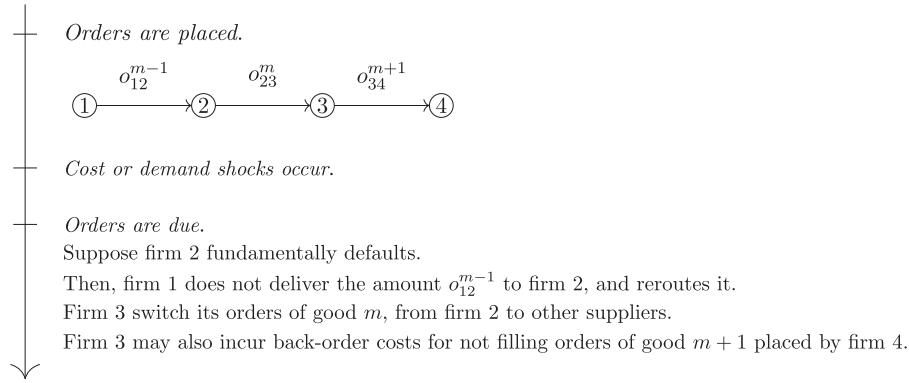
Figure 2 describes the time line of events following a default. The default of a firm is said to be *fundamental* if the firm is not able to honor its promises, even though all its buyers and suppliers are solvent (i.e., even if all the firm's orders are delivered and the entire demand is served). We call the default of a firm *contagious* if it is only caused by defaults of other firms but would otherwise be avoided if all the firm's orders are delivered and the demand is served.

Figure 1. The Inventory of Good m Held by Firm i Over Time Under a Reorder Point/Reorder Quantity Policy



Notes. Suppose firm j is the only supplier to firm i under a supply contract with a quantity commitment. If firm j fundamentally defaults at the order delivery time, the current order will be delivered, but future orders will be stopped. This results in an input loss for firm i . Although firm i can still fill existing customer orders using its cycle stock (i.e., o_{ji}^m), its pipeline inventory is zero. Hence, firm i cannot use future orders delivered from j to fill new/recurring customer orders. Such input loss can be reduced by firm i 's safety stock.

Figure 2. The Sequence of Events Caused by Firm 2's Fundamental Default



2.2. Sourcing and Secondary Markets

Secondary markets are those where firms sell the surplus of stock caused by random shifts in demand, the bullwhip effect, and inaccurate forecasting models. See Lee and Whang (2002) and Angelus (2011) for a more in-depth analysis. Examples of such markets include Virtual Chip Exchange and tradinghubs.com for electronic components. Firms reroute their supply of good m ; let $\mathcal{R}_m := \{i \in \mathcal{N} | \bar{r}_i^m > 0\}$ be the set of firms with undelivered orders and $d_m(\pi)$ be the demand function. If $\pi \in \mathbb{R}$ is higher than the exogenously specified reservation price p^m , potential buyers gain zero utility from buying good m in the rerouted market; hence, the total demanded quantity is zero.

Assumption 2.2. The demand function $d_m(\pi)$ of each rerouted good m is twice-continuously differentiable, concave, and strictly decreasing for all π such that $d_m(\pi) > 0$. Moreover, $d_m(\pi) = 0$ for all $\pi \geq p^m$ and $d_m(0) \geq \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} o_{ij}^m$.

Given a profile of prices, we next describe the allocation mechanism for the aggregate demand of a rerouted good among the $|\mathcal{R}_m|$ firms. We assume that the buyers choose the firm offering the lowest price (of the rerouted good) if such a firm has any remaining undelivered orders (it is worth remarking that a similar notion has been used by Acemoglu et al. (2009) to define the flow equilibrium). If demand exceeds this firm's capacity, buyers are served in decreasing order of their valuations (i.e., starting with high-valuation buyers). Further, if two or more firms charge the same price, buyers are served by each of these firms with a probability proportional to the relative size of undelivered orders (hence, a firm with a higher amount of undelivered orders has a larger probability of being chosen relative to a firm with smaller amount of those orders). Formally, we capture this mechanism through the concept of *efficient rerouted supply*.

Definition 2.1 (Efficient Rerouted Supply). For a given profile of total undelivered orders $\{\bar{r}_i^m\}_{i \in \mathcal{R}_m}$ and prices

$\{\pi_i^m\}_{i \in \mathcal{R}_m}$, define

$$\mathcal{X} := \left\{ \{x_i\}_{i \in \mathcal{R}_m} \mid \frac{x_i}{x_j} = \frac{\bar{r}_i^m}{\bar{r}_j^m} \text{ for any } i, j \in \mathcal{R}_m \right. \\ \left. \text{such that } \pi_i^m = \pi_j^m \right\}. \quad (6)$$

$\{r_i^m\}_{i \in \mathcal{R}_m}$ is an efficient rerouted supply profile if

$$\{r_i^m\}_{i \in \mathcal{R}_m} \in \arg \max_{x_i \in [0, \bar{r}_i^m] \forall i \in \mathcal{R}_m} \left(\int_0^{\sum_{i \in \mathcal{R}_m} x_i} d_m^{-1}(x) dx - \sum_{i \in \mathcal{R}_m} \pi_i^m x_i \right) \cap \mathcal{X}. \quad (7)$$

The first and second terms in the arg max expression are the consumers' willingness to pay and the amount actually paid when the total quantity purchased is $\sum_{i \in \mathcal{R}_m} x_i$, respectively. The difference between those two quantities is the consumer surplus. Maximizing the consumer surplus guarantees that the buyers with high valuation are served first by the set of firms offering low prices. Taking the intersection of the arg max set and \mathcal{X} ensures that, at the same time, buyers allocate their demands to the firms that charge the same price in the amount proportional to the relative size of those firms' undelivered orders.

The sourcing markets (e.g., Mouser Electronics for electronic components), where firms search for alternative suppliers, behave symmetrically to the secondary market where supply is rerouted. Given good m , let $\mathcal{S}_m := \{i \in \mathcal{N} | \bar{\sigma}_i^m > 0\}$ be the set of firms with strictly positive unserved demand, and let $s_m(\kappa)$ be the aggregate quantity supplied if the cost of switching suppliers is $\kappa \in \mathbb{R}$.

Assumption 2.3. The supply function of any switched good m , $s_m(\kappa)$, is twice-continuously differentiable, concave, strictly increasing, and $s_m(\kappa) > 0$ for all $\kappa > 0$.

Next, we define the concept of efficient switched demand profile.

Definition 2.2 (Efficient Switched Demand). For a given profile of total unserved demands $\{\bar{\sigma}_i^m\}_{i \in \mathcal{S}_m}$ and costs $\{\kappa_i^m\}_{i \in \mathcal{S}_m}$, let

$$\mathcal{X} := \left\{ \{x_i\}_{i \in \mathcal{S}_m} \left| \frac{x_i}{x_j} = \frac{\bar{\sigma}_i^m}{\bar{\sigma}_j^m} \text{ for any } i, j \in \mathcal{S}_m \right. \right. \\ \left. \left. \text{such that } \kappa_i^m = \kappa_j^m \right\}. \quad (8)$$

A switched demand profile $\{\sigma_i^m\}_{i \in \mathcal{S}_m}$ is efficient if

$$\{\sigma_i^m\}_{i \in \mathcal{S}_m} \in \arg \max_{x_i \in [0, \bar{\sigma}_i^m] \forall i \in \mathcal{S}_m} \left(\sum_{i \in \mathcal{S}_m} \kappa_i^m x_i - \int_0^{\sum_{i \in \mathcal{S}_m} x_i} s_m^{-1}(x) dx \right) \cap \mathcal{X}. \quad (9)$$

The difference between the two terms in the arg max expression is the switching cost incurred by firms with unserved demand net of the firms' expenses for finding new suppliers. Fixing a profile of switching costs, the maximization criterion ensures that new suppliers try to sell to the firm incurring high switching costs first. Whenever the market supply exceeds this firm's capacity, new suppliers exhaust their supply in increasing order of switching costs (i.e., starting with suppliers whose cost is lower). Furthermore, if two or more firms incur the same switching cost, we break the tie by letting the suppliers serve each of these firms with a probability proportional to the firm's total unserved demand. This is implemented by taking the intersection of the arg max set and \mathcal{X} in Equation (9).

We remark that the secondary and sourcing markets of a specific good are two separate markets. The reason is that sellers in the secondary market aim to sell and deliver the good immediately to cash out payments. The buyers in the sourcing market, instead, submit an order of the good and only make payments at the time the good is delivered.

2.3. Firm's Ex Post Net Worth

The ex post net worth of a firm includes its initial equity plus the profits earned from filling the orders and selling the rerouted supply minus the safety stock holding, back-order, and supplier switching costs. The profits earned by firm i from rerouting good m are given by $(\pi_i^m - l_i^m)r_i^m$, where π_i^m represents the unit price of rerouted supply, l_i^m is the (average) unit cost of rerouting good m , and we recall that r_i^m is the amount of rerouted supply. The total costs of firm i , including its back-order and switching costs, are given by $\kappa_i^m \sigma_i^m + b_i^m(\bar{\sigma}_i^m - \sigma_i^m)$, where κ_i^m is the unit cost of switching suppliers and b_i^m is the back-order cost per unit of remaining unserved demand (i.e., which cannot be switched) of good m . We recall that $\bar{\sigma}_i^m$ denotes the unserved demand after disruption and before mitigation, whereas

σ_i^m denotes the switched demand. The term $b_i^m \sigma_i^m - \kappa_i^m \sigma_i^m$ may also be interpreted as the net reduction in back-order costs. Altogether, firm i 's ex post net worth is

$$e_i = w_i + \underbrace{\sum_{m \in \mathcal{M}} p^m \left(\sum_{j \in \mathcal{N}} o_{ij}^m - \bar{r}_i^m \right)}_{\text{profits from filling the orders}} - c_i - \underbrace{\sum_{m \in \mathcal{M}} \theta_i^m \lambda_i^m}_{\text{safety stock holding cost}} \\ + \underbrace{\sum_{m \in \mathcal{M}} (\pi_i^m r_i^m - l_i^m r_i^m)}_{\text{profits from rerouted supply}} - \underbrace{\sum_{m \in \mathcal{M}} b_i^m \bar{\sigma}_i^m}_{\text{back-order costs in the absence of switched demand}} \\ + \underbrace{\sum_{m \in \mathcal{M}} (b_i^m \sigma_i^m - \kappa_i^m \sigma_i^m)}_{\text{net reduction in back-order costs}}, \quad (10)$$

where we recall that $\sum_{j \in \mathcal{N}} o_{ij}^m - \bar{r}_i^m = \sum_{j \in \mathcal{N}} q_{ij}^m$ is the quantity of good m delivered by firm i .

Our aim is to understand how fragility of a tiered supply chain network is affected by buyer and supplier diversification. Figure 3 illustrates the main economic forces at play in our model. For the sake of illustration, we assume the losses caused by rerouted supply and switched demand to be exogenous and set the amount of each firm's safety stock to zero.

3. Supply Chain Network Equilibrium

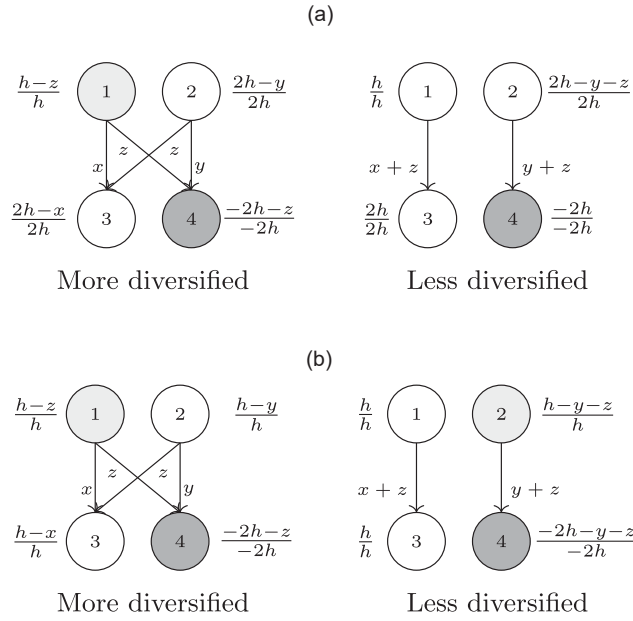
In this section, we introduce the equilibrium concept. In Section 3.1, we begin by examining the decision problem of a firm in a single secondary and sourcing market. We then provide the definition of a partial equilibrium for a given market, where the prices of rerouted goods and switching costs in other markets are taken as given. In Section 3.2, we introduce the notion of a market stable state (i.e., the state reached when the default cascade in the supply chain network stops). We then use it to define the general supply chain network equilibrium, where all secondary and sourcing markets in the network are simultaneously taken into account.

3.1. Partial Equilibrium

We model the secondary and sourcing markets as Bertrand oligopoly and oligopsony markets with capacity constraints, respectively. We first analyze the secondary market. Taking the profile of undelivered orders of good m as fixed, firms with strictly positive undelivered orders simultaneously choose prices of rerouted supply. Each firm chooses the price of good m that maximizes its own ex post net worth given the prices of the same good chosen by any other firms. That is, in the secondary market of good m , firm $i \in \mathcal{R}_m$ chooses the price π_i^m , which solves the following maximization problem:

$$\max_{\pi_i^m} e_i(\pi_i^m; \pi_{-i}^m, \{\bar{r}_i^m\}_{i \in \mathcal{R}_m}),$$

Figure 3. The Impact of Buyer and Supplier Diversification on the Fragility of a Supply Chain Network



Notes. The denominator in the ratio denotes the firm's ex post net worth, after shocks are realized and before they propagate through the system. The numerator in the ratio denotes the firm's ex post net worth after the propagation of shocks. The edge labels are the amount of orders. They are chosen to satisfy $z > h > y > x > 0$ and $2h > \max\{x + z, y + z\}$. Both unit losses of rerouted supply and switched demand are exogenous and equal to one. The fundamentally defaulting firms are shaded in dark grey, and those that default because of contagion are shaded in light grey. (a) Highly capitalized networks. Each firm in panel (a) has initial equity higher than or equal to the corresponding firm in panel (b). The more diversified network is more fragile than the less diversified network. This is the case because in a highly capitalized network, the loss-sharing benefits are outweighed by higher contagion losses. In the less diversified network, contagion effects are lower, and fewer firms default. Moreover, firms whose loss-sharing benefits are smaller than in the more diversified network remain solvent (e.g., firm 2). (b) Lowly capitalized networks. The more diversified network is less fragile than the less diversified network. Firms that default in the less diversified network (e.g., firm 2) no longer default in the more diversified network where loss-sharing benefits are higher.

where π_{-i}^m denotes the price profile of good m (i.e., the set of prices chosen by all firms in $\mathcal{R}_m \setminus \{i\}$).

Next, we turn to the sourcing market. Taking the profile of unserved demands of good m as fixed, firms with strictly positive unserved demand all incur switching costs. Each of these firms controls the switching cost of good m to maximize its own ex post net worth given the cost chosen by any other firm for the same good. Firm $i \in \mathcal{S}_m$ solves the maximization problem:

$$\max_{\kappa_i^m} e_i(\kappa_i^m, \kappa_{-i}^m, \{\bar{\sigma}_i^m\}_{i \in \mathcal{S}_m}),$$

where κ_{-i}^m denotes the cost profile of good m (i.e., the set of costs incurred by firms $j \in \mathcal{S}_m \setminus \{i\}$). Because the secondary and sourcing markets are two separate markets in our model,⁶ the choice of π_i^m does not depend on

κ_i^m , κ_{-i}^m and $\{\bar{\sigma}_i^m\}_{i \in \mathcal{S}_m}$, and vice versa, the choice of κ_i^m is not influenced by π_i^m , π_{-i}^m and $\{\bar{\tau}_i^m\}_{i \in \mathcal{R}_m}$. Hence, π_i^m and κ_i^m can be determined separately.

Definition 3.1 (Partial Rerouted Supply Equilibrium). Fix a profile of undelivered orders $\{\bar{\tau}_i^m\}_{i \in \mathcal{R}_m}$ for good m . A profile of prices $\{\pi_i^m\}_{i \in \mathcal{R}_m}$ is a partial equilibrium in the oligopoly market if the corresponding rerouted supply $\{\tau_i^m\}_{i \in \mathcal{R}_m}$ is efficient, and for all $i \in \mathcal{R}_m$,

$$e_i(\pi_i^m, \pi_{-i}^m, \{\bar{\tau}_i^m\}_{i \in \mathcal{R}_m}) \geq e_i(\pi; \pi_{-i}^m, \{\bar{\tau}_i^m\}_{i \in \mathcal{R}_m}) \quad \text{for any } \pi \geq 0. \quad (11)$$

Definition 3.2 (Partial Switched Demand Equilibrium). Fix a profile of unserved demands $\{\bar{\sigma}_i^m\}_{i \in \mathcal{S}_m}$ for good m . Then, $\{\kappa_i^m\}_{i \in \mathcal{S}_m}$ is a partial equilibrium in the oligopoly market if the corresponding switched demand $\{\sigma_i^m\}_{i \in \mathcal{S}_m}$ is efficient, and for all $i \in \mathcal{S}_m$,

$$e_i(\kappa_i^m, \kappa_{-i}^m, \{\bar{\sigma}_i^m\}_{i \in \mathcal{S}_m}) \geq e_i(\kappa; \kappa_{-i}^m, \{\bar{\sigma}_i^m\}_{i \in \mathcal{S}_m}) \quad \text{for any } \kappa \geq 0. \quad (12)$$

3.2. Cascading Defaults and General Equilibrium

In our model, the suppliers of fundamentally defaulting firms reroute the excess supply to mitigate the costs from demand disruption. If these costs are large, those suppliers may themselves default and negatively impact their own suppliers and buyers. The buyers of fundamentally defaulting firms also suffer from supply disruption. They switch to new suppliers to reduce the back-order costs. If, after this mitigation plan, the back-order costs are still high, those buyers may also default. This will in turn further force their own buyers to switch suppliers and their own suppliers to stop delivery. These effects propagate upstream and downstream through the entire supply chain network, starting from the fundamentally defaulting firms and continuing until a stable state is reached (i.e., a state where no additional firm defaults). Reaching this state means that no new firm wants to enter the secondary and sourcing markets, and thus, aggregate undelivered orders and unserved demands no longer increase. We refer to such a state as a *market stable state* and formally define it next. Set $\Gamma := \{\gamma_{ij}^m\}_{m \in \mathcal{M}, i, j \in \mathcal{N}}, \Delta := \{\delta_{ij}^m\}_{m \in \mathcal{M}, i, j \in \mathcal{N}}$.

Definition 3.3 (Market Stable State). Define a sequence of undelivered order and unserved demand profiles $(\Gamma^{(n)}, \Delta^{(n)}), n = 0, 1, \dots$ recursively by $\Gamma^{(0)} := \mathbf{0}^{N \times N \times M}, \Delta^{(0)} := \mathbf{0}^{N \times N \times M}, \Gamma^{(n+1)} := \Phi(\Gamma^{(n)}, \Delta^{(n)})$, and $\Delta^{(n+1)} := \Psi(\Gamma^{(n)}, \Delta^{(n)})$, where the functions Φ and Ψ specify the relation between the input and the output of each term in the sequence. A market stable state in the supply chain network $(\mathbf{O}, \mathbf{p}, \mathbf{w}, \Theta, \Lambda)$ is a profile of undelivered orders and unserved demands, (Γ, Δ) , such that $(\Gamma, \Delta) = \lim_{n \rightarrow \infty} (\Gamma^{(n)}, \Delta^{(n)})$ and $(\Gamma, \Delta) = (\Phi(\Gamma, \Delta), \Psi(\Gamma, \Delta))$.

In this definition, the condition $(\Gamma, \Delta) = (\Phi(\Gamma, \Delta), \Psi(\Gamma, \Delta))$ guarantees that undelivered order and unserved demand profiles no longer change, and thus, no additional firm defaults when the market stable state is reached. This condition is needed because $(\Gamma, \Delta) = \lim_{n \rightarrow \infty} (\Gamma^{(n)}, \Delta^{(n)})$ does not necessarily guarantee that (Γ, Δ) is a fixed point. Because Φ and Ψ are not continuous everywhere in (Γ, Δ) , then $(\Gamma, \Delta) = (\Phi(\Gamma, \Delta), \Psi(\Gamma, \Delta))$ is not guaranteed to hold.

A general supply chain network equilibrium is reached when (i) no new firm defaults, (ii) excessively supplied goods are optimally rerouted, and (iii) all supplier switching decisions have been made. Hence, in the general supply chain network equilibrium, undelivered orders, unserved demands, and prices are determined simultaneously accounting for interactions between secondary and sourcing markets of different goods.

Definition 3.4 (General Supply Chain Network Equilibrium). A general supply chain network equilibrium consists of a profile of rerouted supply prices $\Pi := \{\pi_i^m\}_{m \in \mathcal{M}, i \in \mathcal{N}}$, switching demand costs $\mathbf{K} := \{\kappa_i^m\}_{m \in \mathcal{M}, i \in \mathcal{N}}$, undelivered orders $\Gamma := \{\gamma_{ij}^m\}_{m \in \mathcal{M}, i, j \in \mathcal{N}}$, and unserved demands $\Delta := \{\delta_{ij}^m\}_{m \in \mathcal{M}, i, j \in \mathcal{N}}$, such that (i) a partial equilibrium in each secondary and sourcing market and (ii) a market stable state in the supply chain network are simultaneously reached.

4. Finding a General Equilibrium

In this section, we first characterize the partial equilibria described in Section 4.1. Next, we show existence and uniqueness of a general equilibrium in Section 4.2.

4.1. Characterization of Partial Equilibria

We begin by imposing the following assumption on the amount of undelivered orders.

Assumption 4.1. The marginal revenue from a differential increase in r at $\sum_{i \in \mathcal{R}_m} \bar{r}_i^m$ is strictly greater than the maximum unit rerouting cost of each firm $i \in \mathcal{R}_m$: that is,

$$\left. \frac{d(rd_m^{-1}(r))}{dr} \right|_{r=\sum_{i \in \mathcal{R}_m} \bar{r}_i^m} > \max_{i \in \mathcal{R}_m} \{\iota_i^m\}, \quad (13)$$

where $d_m^{-1}(r)$ is the inverse demand function of the rerouted good m .

Assumption 4.1 imposes that the aggregate amount of undelivered orders is sufficiently small to make it profitable to reroute every unit of it. Despite this assumption being stated in terms of the endogenous variable \bar{r}_i^m , we can provide a sufficient condition for it to hold depending only on the model primitives $\{\sigma_{ij}^m\}$:

$$\left. \frac{d(rd_m^{-1}(r))}{dr} \right|_{r=\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \sigma_{ij}^m} > \max_{i \in \mathcal{N}} \{\iota_i^m\}. \quad (14)$$

This condition is sufficient because $\max_{i \in \mathcal{N}} \{\iota_i^m\} \geq \max_{i \in \mathcal{R}_m} \{\iota_i^m\}$, $\sum_{i \in \mathcal{R}_m} \bar{r}_i^m \leq \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \sigma_{ij}^m$, and the marginal revenue is decreasing on r .

Under the assumption, we can show that the partial equilibrium for each rerouted good m exists and is unique.

Proposition 4.1. For each good m , fix the total amount of undelivered orders $\{\bar{r}_i^m\}_{i \in \mathcal{R}_m}$. Then, there exists a unique partial equilibrium given by $\pi_i^m = d_m^{-1}(\sum_{j \in \mathcal{R}_m} \bar{r}_j^m)$ for $i \in \mathcal{R}_m$, and the corresponding efficient rerouted supply is given by $r_i^m = \bar{r}_i^m$ for $i \in \mathcal{R}_m$.

Next, we characterize the partial equilibrium of switched demand. We make the following assumption on the total unserved demand.

Assumption 4.2. The marginal cost from a differential increase in σ at $\sum_{i \in \mathcal{S}_m} \bar{\sigma}_i^m$ is strictly smaller than the minimum unit back-order cost of each firm $i \in \mathcal{S}_m$: that is,

$$\left. \frac{d(\sigma s_m^{-1}(\sigma))}{d\sigma} \right|_{\sigma=\sum_{i \in \mathcal{S}_m} \bar{\sigma}_i^m} < \min_{i \in \mathcal{S}_m} \{b_i^m\}, \quad (15)$$

where $s_m^{-1}(\sigma)$ is the inverse supply function of good m .

This assumption guarantees that the aggregate unserved demand is small enough that the cost of sourcing alternative material suppliers is always lower than the back-order cost of not filling an order on time. A sufficient condition for Assumption 4.2 to hold, given in terms of model primitives, is

$$\left. \frac{d(\sigma s_m^{-1}(\sigma))}{d\sigma} \right|_{\sigma=\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \sigma_{ij}^m} < \min_{i \in \mathcal{N}} \{b_i^m\}. \quad (16)$$

To see it, observe that $\min_{i \in \mathcal{N}} \{b_i^m\} \leq \min_{i \in \mathcal{S}_m} \{b_i^m\}$, $\sum_{i \in \mathcal{S}_m} \bar{\sigma}_i^m \leq \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \sigma_{ij}^m$, and the marginal cost is increasing in σ .

Proposition 4.2. For each good m , fix the total amount of unserved demands $\{\bar{\sigma}_i^m\}_{i \in \mathcal{S}_m}$. Then, there exists a unique partial equilibrium given by $\kappa_i^m = s_m^{-1}(\sum_{j \in \mathcal{S}_m} \bar{\sigma}_j^m)$ for $i \in \mathcal{S}_m$, and the corresponding efficient switched demand is given by $\sigma_i^m = \bar{\sigma}_i^m$ for $i \in \mathcal{S}_m$.

In the rest of the paper, we use e_i^* to denote firm i 's expected net worth under the partial equilibrium of rerouted supply and switched demand. The quantity e_i^* is monotonically decreasing with respect to undelivered orders (Γ) and unserved demands (Δ). We refer to Lemma A.7 in the e-companion for the technical proof.

4.2. Existence and Uniqueness of a General Equilibrium

Before we proceed to show existence and uniqueness of a general equilibrium, we observe that under the partial equilibrium, each firm $i \in \mathcal{R}_m$ charges the same price

for its rerouted supply, and each firm $i \in \mathcal{S}_m$ incurs the same cost for its switched demand. Without loss of generality, we set $\pi_i^m = d_m^{-1}(\sum_{j \in \mathcal{R}_m} \bar{r}_j^m)$ for $i \in \mathcal{N} \setminus \mathcal{R}_m$ (i.e., for the firms without undelivered orders) and $\kappa_i^m = s_m^{-1}(\sum_{j \in \mathcal{S}_m} \bar{o}_j^m)$ for $i \in \mathcal{N} \setminus \mathcal{S}_m$ (i.e., for the firms without excess demand). Such a specification will not affect the ex post equities of those firms and will not alter the analytical results in the paper.

Let Φ^* and Ψ^* be the undelivered orders and unserved demands, respectively, given the ex post net worth under the partial equilibrium. They are defined by

$$\begin{aligned}\phi_{ij}^{m,*}(\Gamma, \Delta) &:= \begin{cases} 0 & \text{if } e_i^*(\Gamma, \Delta) \geq 0, \\ o_{ij}^m & \text{otherwise,} \end{cases} \quad \text{and} \\ \psi_{ij}^{m,*}(\Gamma, \Delta) &:= \begin{cases} 0 & \text{if } e_i^*(\Gamma, \Delta) \geq 0, \\ o_{ij}^m & \text{otherwise} \end{cases} \end{aligned} \quad (17)$$

for all $m \in \mathcal{M}$, $i, j \in \mathcal{N}$. Observe that the difference between Φ^* and Ψ^* here and Φ and Ψ in Definition 3.3 is that the Φ^* and Ψ^* are evaluated using the ex post net worth under the partial equilibrium, whereas Φ and Ψ are not. By definition, both Φ^* and Ψ^* are bounded. Moreover, they are monotonically increasing in Γ and Δ because $\phi_{ij}^{m,*}$ and $\psi_{ji}^{m,*}$ are decreasing in e_j^* , and e_j^* is decreasing in Γ and Δ . We refer to Lemma A.8 in the e-companion for the precise statement.

Next, we show the existence and uniqueness of a general equilibrium. We first construct a sequence of sets $\{\Gamma^{(n)}, \Delta^{(n)}, \Pi^{(n)}, \mathbf{K}^{(n)}\}$, $n = 0, 1, \dots$, defined recursively by

$$\begin{aligned}\Gamma^{(0)} &:= \mathbf{0}^{M \times N \times N}, \Delta^{(0)} := \mathbf{0}^{M \times N \times N}, \Gamma^{(n+1)} := \Phi^*(\Gamma^{(n)}, \Delta^{(n)}), \\ \Delta^{(n+1)} &:= \Psi^*(\Gamma^{(n)}, \Delta^{(n)}), \pi_i^{m,(n)} := d_m^{-1} \left(\sum_{j \in \mathcal{R}_m} \bar{r}_j^{m,(n)} \right), \\ \kappa_i^{m,(n)} &:= s_m^{-1} \left(\sum_{j \in \mathcal{S}_m} \bar{o}_j^{m,(n)} \right) \quad \text{for all } m \in \mathcal{M}, i \in \mathcal{N},\end{aligned} \quad (18)$$

where we recall that $\bar{r}_j^{m,(n)} = \sum_{g \in \mathcal{N}} \gamma_{jg}^{m,(n)}$ and $\bar{o}_j^{m,(n)} = \left(\sum_{g \in \mathcal{N}} \delta_{gj}^{m,(n)} - \theta_j^m \right)^+$ (see Equations (3) and (5)). Fix a profile of undelivered orders $\Gamma^{(n)}$ and unserved demands $\Delta^{(n)}$. Then, the partial equilibrium rerouted supply price and switching demand cost are given by $d_m^{-1} \left(\sum_{j \in \mathcal{R}_m} \bar{r}_j^{m,(n)} \right)$ and $s_m^{-1} \left(\sum_{j \in \mathcal{S}_m} \bar{o}_j^{m,(n)} \right)$, respectively (see Propositions 4.1 and 4.2). Moreover, Φ^* and Ψ^* are defined using the ex post net worth under the partial equilibrium, $e_i^*(\Gamma^{(n)}, \Delta^{(n)})$ for $i \in \mathcal{N}$, which depends on $\Pi^{(n)}$ and $\mathbf{K}^{(n)}$. Because of the boundedness and monotonicity of Φ^* and Ψ^* , we can show that the sequence defined by Equation (18) converges monotonically to a limit. This limit is the unique general equilibrium.

Proposition 4.3. *In a supply chain network $(\mathbf{O}, \mathbf{p}, \mathbf{w}, \Theta, \Lambda)$, there exists a unique supply chain network general equilibrium.*

We can construct the general equilibrium using an algorithm, reported in Section A.5 of the e-companion, that traces the sequence of contagious defaults propagating in the supply chain network. This sequence allows us to measure each firm's resilience to systemic risk. More precisely, the algorithm mimics how contagious defaults propagate in the supply chain network. The firms defaulting in the n th iteration have a lower resilience than those defaulting in the $(n+1)$ th iteration because the latter remain solvent in the n th iteration, while the former cannot absorb the losses incurred by the firms defaulting in the $(n-1)$ th iteration.

5. Performance Analysis of Supply Chain Networks

In this section, we define two metrics, resilience and fragility, to quantify the performance of a supply chain network. We then analyze how the performance of the network depends on buyer and supplier diversification. We also compare the cost effectiveness of single- versus multiple-sourcing strategies from the viewpoint of an individual firm.

Remark 5.1. We study resilience of the entire supply chain network. This is different from the resilience of a single firm in the network, which we have defined to be the number of iterations necessary to induce a given firm into default at the end of previous section. Both single-firm and network resilience measures depend on how orders connect firms in the supply chain network. Low resilience of the supply chain network means that firms are connected through orders in such a way that default contagion is likely to occur in the network. As a result, it takes fewer iterations for each firm to default, resulting in low idiosyncratic firm's resilience. Vice versa, a high resilience of the supply chain network implies a high resilience of the individual firms.

5.1. Performance Measures

The ex ante performance of the supply chain network (i.e., before net production costs are realized) is evaluated using two metrics: resilience and fragility. In the remainder of the paper, we use $\sigma_i^m(\mathcal{X}, y)$ to stress the dependence of firm i 's efficient switched demand of good m on a set of defaulted firms \mathcal{X} and firm i 's safety stock y . Specifically, $\sigma_i^m(\mathcal{X}, y) = (\sum_{k \in \mathcal{X}} o_{ki}^m - y)^+$ (which follows from the result $\sigma_i^m = \bar{\sigma}_i^m$ in Proposition 4.2).

The *resilience* is measured by the reduction in out-of-stock risk, defined as the fraction of losses from switching demand, triggered by fundamental defaults, that can be reduced by the safety stock. Specifically, we use $\mathcal{D}_0(\theta)$ to denote the set of fundamentally defaulting

firms if θ is the safety stock of each good held by any firm: that is,

$$\mathcal{D}_0(\theta) := \left\{ i \in \mathcal{N} \left| w_i + \sum_{m \in \mathcal{M}} \left(p^m \sum_{j \in \mathcal{N}} o_{ij}^m - \theta \lambda_i^m \right) - c_i < 0 \right. \right\}.$$

For each $i \in \mathcal{N}$,

$$\begin{aligned} \zeta_i(\theta) := & \sum_{m \in \mathcal{M}} \left[\underbrace{\theta \lambda_i^m}_{\text{safety stock inventory cost}} + \underbrace{\sigma_i^m(\mathcal{D}_0(\theta), \theta)}_{\text{amount of switched demand}} \right. \\ & \left. \times \underbrace{s_m^{-1} \left(\sum_{j \in \mathcal{N}} \sigma_j^m(\mathcal{D}_0(\theta), \theta) \right)}_{\text{unit cost of switching suppliers}} \right] \end{aligned}$$

is the inventory cost plus the costs from switching demand because of fundamental defaults after accounting for the safety stock θ . Holding safety stock reduces firm i 's out-of-stock risk by $\zeta_i(0) - \zeta_i(\theta)$. The total percentage reduction of all firms in the network is then given by

$$\zeta(\theta) := \frac{\sum_{i \in \mathcal{N}} \zeta_i(0) - \zeta_i(\theta)}{\sum_{i \in \mathcal{N}} \zeta_i(0)}.$$

The higher $\zeta(\theta)$, the more resilient the supply chain network, as formalized in the next definition.

Definition 5.1. Consider two supply chain networks $A := (\mathbf{O}^A, \mathbf{p}, \mathbf{w}, \mathbf{\Theta}, \mathbf{\Lambda})$ and $B := (\mathbf{O}^B, \mathbf{p}, \mathbf{w}, \mathbf{\Theta}, \mathbf{\Lambda})$ subject to the same ex post realization of net production costs. Let $\zeta^A(\theta)$ and $\zeta^B(\theta)$ be their corresponding total percentage reduction in out-of-stock risk. Network A is more resilient than B if $\mathbb{P}[\zeta^A(\theta) \geq \zeta^B(\theta)] = 1$ for any θ satisfying Assumption 2.1.

The *fragility* of a network depends on its systemic loss. The latter consists of three components: (i) the total loss from undelivered orders net of the income from rerouted supply, (ii) the back-order costs after the implementation of contingency plans, and (iii) the costs of switching the unserved demand. The systemic loss is the difference between the aggregate net worth of all firms in the absence of fundamental defaults and the corresponding quantity under the general equilibrium (where defaults occur). Specifically, we use \underline{e}_i to denote the net worth of firm i under the general equilibrium. The precise expression of the systemic loss, denoted by ℓ , is given by

$$\begin{aligned} \ell := & \sum_{i \in \mathcal{N}} \mathbb{E} \left[\underline{e}_i \left| w_i + \sum_{m \in \mathcal{M}} \left(p^m \sum_{j \in \mathcal{N}} o_{ij}^m - \theta_i^m \lambda_i^m \right) - c_i \geq 0 \text{ for all } i \in \mathcal{N} \right. \right] \\ & \underbrace{\hspace{10em}}_{\text{event that no firm fundamentally defaults}} \\ & - \sum_{i \in \mathcal{N}} \mathbb{E}[\underline{e}_i]. \end{aligned} \quad (19)$$

Define the *spread of contagion* to be the event that at least one contagious default occurs.

Definition 5.2. Consider two supply chain networks $A := (\mathbf{O}^A, \mathbf{p}, \mathbf{w}, \mathbf{\Theta}, \mathbf{\Lambda})$ and $B := (\mathbf{O}^B, \mathbf{p}, \mathbf{w}, \mathbf{\Theta}, \mathbf{\Lambda})$ subject to the same ex post realization of net production costs $\{c_i\}_{i \in \mathcal{N}}$. Let ℓ^A and ℓ^B be the corresponding systemic losses. Network A is more fragile than B if, conditioning on the spread of contagion, $\ell^A \geq \ell^B$.

5.2. Tiered Supply Chain Network

We analyze the performance of *tiered* supply chain networks (i.e., those that satisfy the following criteria); its underlying directed graphs are weakly connected (replacing all directed edges with undirected edges produces a connected graph). Each firm must (i) supply good m and buy good $m-1$ if $m > 1$, (ii) supply good 1 but buy nothing, or (iii) supply nothing but buy good M . Each firm also holds safety stock for the good it buys. Firms supplying good $m \in \mathcal{M}$ are said to be at tier m of the supply chain network, and those purchasing good M are said to be at tier $M+1$. The set of firms in tier m is denoted by \mathcal{F}_m . The firms at the most downstream tier (tier $M+1$) are retailers of finished products, and those at the most upstream tier (tier 1) are suppliers of raw material. The net production cost of firm i in tier $m \in \mathcal{M}$ is the cost of supplying good m , whereas the net production cost of each firm in tier $M+1$ includes both the cost of selling good M to consumers and the revenue generated from those sales. Each net production cost c_i is a continuous random variable taking values in the interval $[\underline{c}_i, \bar{c}_i]$. The average unit cost of rerouting supply is set to be the same for all firms in the same tier (i.e., $\iota_i^m = \iota^m$ for $i \in \mathcal{F}_m, m \in \mathcal{M}$).

5.3. Buyer and Supplier Diversification

For each firm $i \in \mathcal{N}$ in a tiered supply chain network, we use m_i to denote firm i 's tier. We denote the set of firm i 's buyers and suppliers by \mathcal{L}_i and \mathcal{U}_i , respectively, and next use them to compare buyer and supplier diversification between two networks. We use y^X to specify the variable y associated to network X whenever the value of y is not the same in different networks.

Definition 5.3. Consider two tiered supply chain networks: $A := (\mathbf{O}^A, \mathbf{p}, \mathbf{w}, \mathbf{\Theta}, \mathbf{\Lambda})$ and $B := (\mathbf{O}^B, \mathbf{p}, \mathbf{w}, \mathbf{\Theta}, \mathbf{\Lambda})$. Network B is more diversified than network A if it has both higher buyer and supplier diversification. That is, for $i \in \mathcal{N}$,

- $\sum_{j \in \mathcal{L}_i^A} o_{ji}^{A, m_i} = \sum_{j \in \mathcal{L}_i^B} o_{ji}^{B, m_i}$ and $\sum_{j \in \mathcal{L}_i^A} o_{ij}^{A, m_i} = \sum_{j \in \mathcal{L}_i^B} o_{ij}^{B, m_i}$,
- $\mathcal{U}_i^A \subseteq \mathcal{U}_i^B$ and $\mathcal{L}_i^A \subseteq \mathcal{L}_i^B$, and

$$\text{iii. } \begin{cases} \min_{j \in \mathcal{U}_i^A} \{o_{ji}^{A, m_i}\} \geq \max_{j \in \mathcal{U}_i^B} \{o_{ji}^{B, m_i}\} & \text{if } \mathcal{U}_i^A \subset \mathcal{U}_i^B \text{ and} \\ o_{ji}^{A, m_i} = o_{ji}^{B, m_i} \text{ for } j \in \mathcal{U}_i^A & \text{if } \mathcal{U}_i^A = \mathcal{U}_i^B \end{cases}$$

$$\begin{cases} \min_{j \in \mathcal{L}_i^A} \{o_{ij}^{A, m_i}\} \geq \max_{j \in \mathcal{L}_i^B} \{o_{ij}^{B, m_i}\} & \text{if } \mathcal{L}_i^A \subset \mathcal{L}_i^B \\ o_{ij}^{A, m_i} = o_{ij}^{B, m_i} \text{ for } j \in \mathcal{L}_i^A & \text{if } \mathcal{L}_i^A = \mathcal{L}_i^B \end{cases}.$$

In the definition, the first condition fixes the total orders placed and received by each firm to be the same in both networks. The second and third conditions require that such total orders are distributed to more suppliers and buyers in the network with higher diversification. See Figure 4 for an illustration.

Theorem 5.1. Let $A := (\mathbf{O}^A, \mathbf{p}, \mathbf{w}, \mathbf{\Theta}, \mathbf{\Lambda})$ and $B := (\mathbf{O}^B, \mathbf{p}, \mathbf{w}, \mathbf{\Theta}, \mathbf{\Lambda})$ be two tiered supply chain networks. If B is more diversified than A , then B is more resilient than A .

The statement in the theorem is intuitive. Suppose firm i fundamentally defaults. The total unserved demand of all firm i 's buyers, before accounting for safety stocks, is equal to the total quantity firm i commits to deliver. This is the same in both networks (i.e., it is not affected by the degree of diversification). Instead, it is the accumulated safety stock buffer over firm i 's buyers that matters. The degree of diversification determines the total amount of safety stocks that can be used to reduce the total unserved demand. Less diversification yields a smaller accumulated amount of safety stocks. This, in turn, generates a larger amount of unserved demand for firm i 's buyers after accounting for safety stocks and hence, imposes larger out-of-stock risk on the supply chain network.

Suppose one firm defaults fundamentally in a tiered supply chain network. Then, for each $i \in \mathcal{N}$,

$$\rho_{i,1} := \underline{c}_i - p^{m_i} \sum_{k \in \mathcal{L}_i} o_{ik}^{m_i} + \theta_i^{m_i-1} \lambda_i^{m_i-1}$$

$$+ \underbrace{\min_{k \in \mathcal{L}_i} \left\{ o_{ik}^{m_i} \times \left(p^{m_i} + l^{m_i} - d_{m_i}^{-1} \left(\sum_{j \in \mathcal{F}_{m_i}} o_{jk}^{m_i} \right) \right) \right\}}_{\text{lower bound for the loss generated from rerouting supply when one of firm } i \text{'s buyers defaults fundamentally}} \text{ and}$$

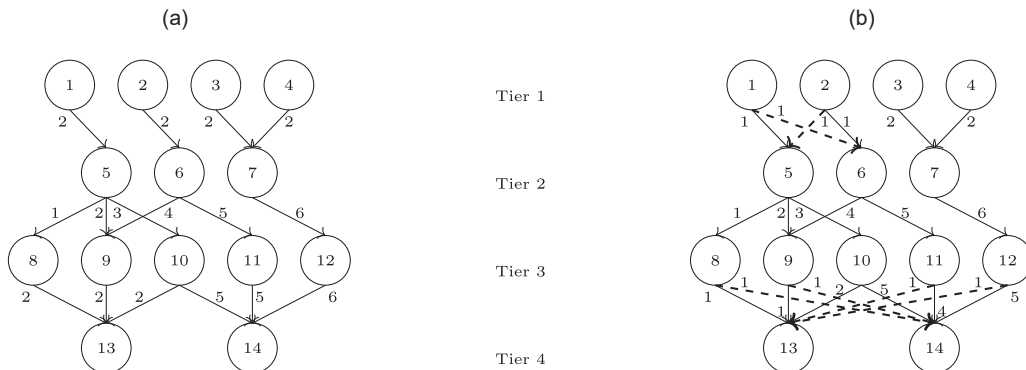
$$\rho_{i,2} := \underline{c}_i - p^{m_i} \sum_{k \in \mathcal{L}_i} o_{ik}^{m_i} + \theta_i^{m_i-1} \lambda_i^{m_i-1}$$

$$+ \underbrace{\min_{k \in \mathcal{L}_i} \left\{ o_{ik}^{m_i-1}(\{k\}, \theta_i^{m_i-1}) \times s_{m_i-1}^{-1} \left(\sum_{j \in \mathcal{F}_{m_i}} o_{jk}^{m_i-1}(\{k\}, \theta_j^{m_i-1}) \right) \right\}}_{\text{lower bound for the loss generated from switching demand when one of firm } i \text{'s suppliers defaults fundamentally}}$$

are the levels of firm i 's initial equity below which firm i defaults with probability one because of contagion when, respectively, any of its buyers and suppliers defaults fundamentally. If $\rho_{i,1} \geq \rho_{i,2}$, then the fundamental default of a downstream firm will cause higher losses to firm i than the fundamental default of an upstream firm. If instead $\rho_{i,1} < \rho_{i,2}$, firm i will suffer a larger loss from the fundamental default of an upstream, rather than a downstream, firm. Next, we analyze the impact of buyer and supplier diversification on the fragility of the network. Before stating the main results, we introduce terminology and notation. First, we define two collectively exhaustive and mutually exclusive sets of firms $\mathcal{N}_1 := \{i \in \mathcal{N} | \rho_{i,1} \geq \rho_{i,2}\}$ and $\mathcal{N}_2 := \{i \in \mathcal{N} | \rho_{i,1} < \rho_{i,2}\}$. Then, let $\mathcal{C} := \{i \in \mathcal{N} | w_i < \max\{\rho_{i,1}, \rho_{i,2}\}\}$. If firm $i \in \mathcal{C} \cap \mathcal{N}_1$, then it defaults because of contagion if any of its buyers defaults; otherwise, if $i \in \mathcal{C} \cap \mathcal{N}_2$, then it defaults because of contagion if at least one of its suppliers defaults.

Given a tiered supply chain network $(\mathbf{O}, \mathbf{p}, \mathbf{w}, \mathbf{\Theta}, \mathbf{\Lambda})$, we construct a directed graph $G := (\mathcal{V}, E)$, which will be used to identify vulnerabilities in the network. The set of nodes $\mathcal{V} = \mathcal{N}$ and set of links $E = \{(j, i) \in \mathcal{N}^2 | i \in \mathcal{U}_j \cap \mathcal{N}_1 \cap \mathcal{C}\} \cup \{(j, i) \in \mathcal{N}^2 | i \in \mathcal{L}_j \cap \mathcal{N}_2 \cap \mathcal{C}\}$. If $(j, i) \in E$, then firm i must be either firm j 's supplier with $w_i < \rho_{i,1}$ or firm j 's buyer with $w_i < \rho_{i,2}$. This means that the default of firm j imposes a large-enough loss on firm i to induce its default. For a subset $\mathcal{X} \subseteq \mathcal{V}$, we denote by $\mathcal{V}_{\mathcal{X}}$ the set of nodes that can be reached via directed paths from any node $j \in \mathcal{X}$ (by convention, $\mathcal{X} \subseteq \mathcal{V}_{\mathcal{X}}$). If \mathcal{X} is the set of fundamentally defaulted firms, we refer to $\mathcal{V}_{\mathcal{X}}$ as the *most vulnerable set* induced by \mathcal{X} because each firm in

Figure 4. Network A in Panel (a) Is Less Diversified Than Network B in Panel (b)



Notes. We set $N = 14, M = 3$. The dashed arrows in network B represent the diversified orders. The number beside each arrow denotes the size of orders that one firm receives from the other (e.g., $o_{15}^A = 2$ and $o_{15}^B = 1$).

this set would default because of contagion. Each directed path started at $j \in \mathcal{X}$ is a chain of sequential defaults triggered by firm j 's fundamental default. Hence, the graph captures the domino effect of defaults in the supply chain network and will be referred to as the *domino graph* throughout the rest of the paper. See Figure 5 for an illustration.

Remark 5.2. The most vulnerable set, $\mathcal{V}_{\mathcal{X}}$, may not include all firms that default in the general equilibrium. In the presence of multiple directed paths or of a single directed path heading first upstream and then, downstream (or vice versa), it is possible that multiple firms from the same tier can belong to the most vulnerable set. As a result, those firms can lead to the default of others that are not in $\mathcal{V}_{\mathcal{X}}$.

For $i \in \mathcal{N}$, we define

$$\begin{aligned} v_i := & c_i - p^{m_i} \sum_{k \in \mathcal{L}_i} o_{ik}^{m_i} + \theta_i^{m_i-1} \lambda_i^{m_i-1} \\ & + \underbrace{\sum_{k \in \mathcal{V}_{D_0}} o_{ik}^{m_i} \times \left(p^{m_i} + l^{m_i} - d_{m_i}^{-1} \left(\sum_{j \in \mathcal{F}_{m_i}} \sum_{k \in \mathcal{V}_{D_0}} o_{jk}^{m_i} \right) \right)}_{\text{loss generated from rerouting supply because of the defaults of all firms in the most vulnerable set}} \\ & + \underbrace{\sigma_i^{m_i-1}(\mathcal{V}_{D_0}, \theta_i^{m_i-1}) \times s_{m_i-1}^{-1} \left(\sum_{j \in \mathcal{F}_{m_i}} \sigma_j^{m_i-1}(\mathcal{V}_{D_0}, \theta_j^{m_i-1}) \right)}_{\text{loss generated from switching demand because of the defaults of all firms in the most vulnerable set}} \end{aligned}$$

to be the minimum initial equity needed by firm $i \notin \mathcal{V}_{D_0}$ to stay solvent whenever the set of fundamentally defaulted firms is \mathcal{D}_0 . Depending on the value of v_i , the extent of buyer and supplier diversification may impact the fragility of the network differently.

Theorem 5.2. Let $A := (\mathbf{O}^A, \mathbf{p}, \mathbf{w}, \mathbf{\Theta}, \mathbf{\Lambda})$ and $B := (\mathbf{O}^B, \mathbf{p}, \mathbf{w}, \mathbf{\Theta}, \mathbf{\Lambda})$ be two tiered supply chain networks such that

$\mathcal{N}_1^A = \mathcal{N}_1^B$, $\mathcal{N}_2^A = \mathcal{N}_2^B$, and $\mathcal{C}^A = \mathcal{C}^B$. Suppose the amount of each firm's safety stock is zero and the same set of firms defaults fundamentally in both networks. Let $\mathcal{Q}_i := \mathcal{L}_i \cup \mathcal{U}_i \cap \mathcal{V}_{D_0}$ denote firm i 's suppliers and buyers, which are part of the most vulnerable set (i.e., firm i is a supplier or buyer of at least one firm in the most vulnerable set if $\mathcal{Q}_i \neq \emptyset$).

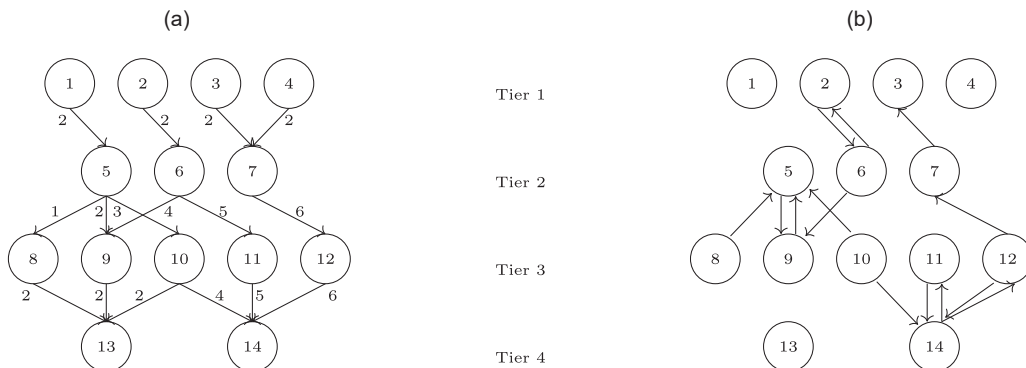
Suppose network B is more diversified than network A . Then, we have the following implications.

- Network A is more fragile than network B if $\mathbb{P}[w_i < v_i^A | i \notin \mathcal{V}_{D_0}^A \text{ and } \mathcal{Q}_i^A \neq \emptyset] = 1$ holds for any $i \in \mathcal{N}$.
- Network A is less fragile than network B if $\mathbb{P}[w_i \geq v_i^A | i \notin \mathcal{V}_{D_0}^A \text{ and } \mathcal{Q}_i^A \neq \emptyset] = 1$ holds for any $i \in \mathcal{N}$.

We next discuss the result stated in the theorem. We condition on the event that firm i is not in the set of those firms that will surely default because of contagion, but some of its buyers or suppliers are. If firms in both networks are lowly capitalized (i.e., have small initial equity), then a lower buyer and supplier diversification will lead to a more fragile network. This is because contagious defaults propagate not only locally in the neighborhood of fundamentally defaulting firms but also, globally to all firms in the network that is less diversified; this may not be the case in the more diversified network, where the amount of switched demand (rerouted supply) is lower and may not drive the buyers (suppliers) of defaulting firms to default.

If firms in both networks are highly capitalized (i.e., have high initial equity), then a lower diversification would result in a less fragile network. In both networks, contagious defaults would spread only locally around defaulting firms. Higher diversification makes each firm more likely to default because of its higher connectivity in the network. Moreover, it makes the number of defaulting firms in each tier larger, which increases the aggregate amount of rerouted supply and switched

Figure 5. (a) Supply Chain Network and (b) the Domino Graph of the Supply Chain Network



Notes. Suppose in the network in panel (a) we have $\mathcal{N}_1 = \{1, 2, 3, 4, 5, 7, 10, 11, 12\}$, $\mathcal{N}_2 = \{6, 8, 9, 13, 14\}$, and $\mathcal{C} = \{2, 3, 5, 6, 7, 9, 11, 12, 14\}$. Then, we have $\mathcal{N}_1 \cap \mathcal{C} = \{2, 3, 5, 7, 11, 12\}$ and $\mathcal{N}_2 \cap \mathcal{C} = \{6, 9, 14\}$, based on which we derive the corresponding domino graph in panel (b). In the graph, we can identify the most vulnerable set induced by a set of fundamentally defaulted firms (e.g., if $\mathcal{D}_0 = \{5, 10, 14\}$, then $\mathcal{V}_{D_0} = \{3, 5, 6, 7, 9, 10, 11, 12, 14\}$). Although firm 8 is not in \mathcal{V}_{D_0} , it may still default in the general equilibrium because of the costs of switching demand due to the defaults of firms 5 and 7.

demand, leading to higher losses from rerouted supply and switched demand.

The result that more diversified networks are more fragile if highly capitalized stands in contrast with results from the literature on financial networks. Acemoglu et al. (2015) show that, in the small shock regime, a completely diversified financial network is the least fragile. (There is an analogy between small/large shock regimes and highly/lowly capitalized networks. If the shock is small, the cash held by each bank remains high. Hence, the small shock regime corresponds to a highly capitalized network.) Although increasing diversification distributes the potential losses among a larger set of firms, similarly to the effect of diversifying interbank liabilities in financial networks, in our model it also increases the individual default probability of each firm (because of the higher connectivity of each firm). This, in turn, generates a higher amount of rerouted supply and switched demand, leading to larger losses that offset the benefits of loss sharing and make the network more fragile.

5.4. Single Vs. Multiple Sourcing

We next compare the net worth of the firm that chooses single sourcing against that of a firm that uses a multiple-sourcing strategy. This comparison serves to quantify how supplier diversification impacts an individual firm.

Theorem 5.3. *In a tiered network $(\mathbf{O}, \mathbf{p}, \mathbf{w}, \Theta, \Lambda)$, suppose i and j are two otherwise identical firms in the same tier except that i has firm k as its single supplier, whereas firm j has multiple suppliers, including k (i.e., $\mathcal{U}_i = \{k\} \subset \mathcal{U}_j$, $\mathcal{L}_i = \mathcal{L}_j$, $o_{ki}^{m_i-1} = \sum_{\eta \in \mathcal{U}_j} o_{\eta j}^{m_j-1}$, $o_{ij}^{m_i} = o_{j\eta}^{m_j}$ for any $\eta \in \mathcal{L}_i$, $w_i = w_j$, $\theta_i^{m_i-1} = \theta_j^{m_i-1}$, $\lambda_i^{m_i-1} = \lambda_j^{m_i-1}$, and c_i and c_j follow the same probability distribution). Denote by A the event that firm k*

defaults in the general equilibrium and by B the event that firm k remains solvent and at least another supplier of firm j defaults in the general equilibrium. There exists $\beta > 0$ such that

- i. $\mathbb{E}[\underline{e}_j] \geq \mathbb{E}[\underline{e}_i]$ if $\mathbb{P}[A] \geq \beta \times \mathbb{P}[A^c \cap B]$ and
- ii. $\mathbb{E}[\underline{e}_j] < \mathbb{E}[\underline{e}_i]$ if $\mathbb{P}[A] < \beta \times \mathbb{P}[A^c \cap B]$.

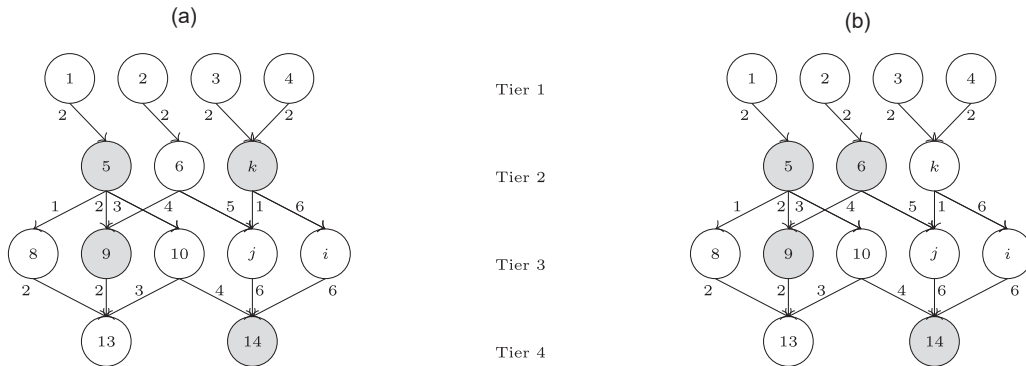
In this theorem, we consider two firms that are identical, except that one uses a single-sourcing strategy and the other uses a multiple-sourcing strategy. We then study under which conditions a firm prefers a single-sourcing strategy over a multiple-sourcing strategy. We consider two default events (also see Figure 6 for an illustration).

- Event A . Firm i 's single supplier k defaults. Despite k being also one of the suppliers of firm j , firm i always incurs a larger loss because it needs to switch the entire demand compared with firm j , whose demand is in part served by other solvent suppliers. This makes firm i 's net worth smaller than firm j 's. These findings are consistent with empirical patterns reported in Crognani et al. (2020). They analyze the impact of cyberattacks on the supply chains using the Petya malware in 2017. Based on the amount of losses reported by buyers of the attacked suppliers, the authors find that buyers with fewer alternative suppliers experienced larger losses than buyers with more suppliers.

- Event $A^c \cap B$. Firm k remains solvent, but at least another supplier of firm j defaults. Then, the entire demand of firm i is served, and no switch occurs, unlike firm j , which needs to switch the demand unserved by its defaulted suppliers. Because of the demand switching costs incurred by j , firm i has a higher net worth than firm j .

If the probability of event A is sufficiently higher than that of $A^c \cap B$, firm i will have on average a lower net worth than firm j . Hence, single sourcing is a less cost-effective strategy than multiple sourcing. Otherwise,

Figure 6. Single vs. Multiple Sourcing



Notes. \mathcal{D} denotes the set of defaulted firms in the general equilibrium. In panel (a), $\mathcal{D} = \{5, 9, 14, k\}$, $k \in \mathcal{D}$, and $6 \notin \mathcal{D}$. In panel (b), $\mathcal{D} = \{5, 6, 9, 14\}$, $k \notin \mathcal{D}$, and $6 \in \mathcal{D}$. (a) In equilibrium, both firm i and j reroute six units of the supplied good 3. Although firm i switches the demanded six units of good 2, firm j only switches one demanded unit of good 2. Hence, firm i ends up with lower net worth than firm j . (b) As in panel (a), both firms i and j reroute the same supplied amount of good 3. However, because firm k remains solvent, firm i has no demand to switch. Firm j instead needs to switch the demand of good 2 in the amount of five. Therefore, firm i is left with a higher net worth than firm j .

firm i will have a higher expected net worth than firm j (i.e., single sourcing is on average more profitable than multiple sourcing).

Theorem 5.3 implies that a firm that strongly believes that one of its suppliers will fail to deliver the ordered goods should adopt a multiple-sourcing strategy. Such a strategy would reduce potential losses from supply disruption. Vice versa, if such a firm believes that one of its suppliers will default with low probability, then it would be better off with a single-sourcing strategy. This model implication is consistent with empirical evidence from past accidents. In 2000, after being informed about the fire at the Philips microchip plant in Albuquerque, New Mexico, Nokia and Ericsson took completely different actions. Nokia estimated the supply disruption to last longer than the one week promised by Philips, so it reacted swiftly to secure spare capacity at other Philips and non-Philips plants. Ericsson, instead, estimated the disruption to be only temporary and opted for the same single source of supply. Nokia's chief component-purchasing manager Tapio Markki, who had past working experience in the semiconductor industry, estimated that the postfire cleanup would take more than one week. Hence, Nokia estimated the probability of Philips failing to deliver on time to be quite high. Under these circumstances, Theorem 5.3 states that Nokia would suffer a smaller loss because of its swift switch from single to multiple sourcing, which turned out to be the most successful strategy. As it turned out, the supply disruption from Philips lasted for six weeks and imposed much larger losses on Ericsson than on Nokia. We refer to Chopra and Sodhi (2004) and Mukherjee (2008) for more details.

6. Concluding Remarks

This paper provides a tractable framework to quantify the relationship between the structure of the supply chain network and systemic risk. Specifically, we use out-of-stock risk reduction and systemic loss as performance measures to quantify network resilience and fragility. We then analytically quantify the implications of buyer and supplier diversification on the performance of tiered networks.

Our main result is that in a highly capitalized network, higher diversification leads to a more fragile network. In addition, we find that a single-sourcing strategy is less cost effective than a multiple-sourcing strategy if the probability that the single supplier defaults is high. This result implies that during economic downturns when the probability of supplier defaults is high, maintaining fewer suppliers makes firms more vulnerable to shocks, and conversely, expanding the set of suppliers reduces firms' vulnerability. Such an action was taken during the 2008 financial

crisis by Innocent Ltd., a UK-based premium drink producer. In early 2008, Innocent was using a single supplier for copacking smoothies into polyethylene terephthalate bottles. Unlike most firms that focused on cutting costs and consolidating suppliers during the global financial crisis, Innocent noticed that its sole supplier was at a high financial risk and started bringing in a new supplier. This early move protected Innocent from the bankruptcy of its original supplier in January 2009. See Hoberg and Aliche (2014) and Purvis et al. (2016) for a more detailed discussion.

Our model is based on fully rational firms, which maximize their ex post net worth. In practice, firms may face constraints that prevent them from freely optimizing to achieve the highest possible net worth. For example, there may be limitations in the amount that can be rerouted. These frictions introduce discontinuities in the system of equations characterizing undelivered orders and unserved demands and may lead to the existence of multiple equilibria. These frictions are especially acute in times of crisis, where firms would also face bounded rationality constraints. We expect our results would still hold in the presence of extreme frictions, such as when no supply can be rerouted or no demand can be switched. Reduced mitigation strategies would make default contagion stronger, which in turn, leads to a smaller ex post net worth for all firms and thus, strengthens the main conclusions of our study. We leave the design of a framework, which incorporates bounded rationality constraints and allows for a large set of frictions, for future research.

Our model produces testable implications, which set the ground for follow-up empirical research. Our analytical results imply that higher diversification leads to higher fragility during economic expansion when firms have strong balance sheets, and the opposite holds during economic contractions when firms' balance sheets are weaker. These implications can be tested using firm-level supply chain data.

Acknowledgments

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Endnotes

¹ See the report "Building the supply chain of the future" released by McKinsey & Company in 2011 (<https://www.mckinsey.com/capabilities/operations/our-insights/building-the-supply-chain-of-the-future>).

² According to *Business Today* (2011) and *The Japan Times* (2016), 8.3% of the bankruptcies were linked to direct causes with cost implications, such as damage to offices or plants because of the earthquake. However, about 90% of these failures were attributed to indirect factors, such as the loss of buyers or suppliers directly damaged by the earthquake. We refer to World Bank (2020) for a more detailed treatment.

³ See, for instance, the pioneering works of Allen and Gale (2000) and Eisenberg and Noe (2001) on counterparty risk networks and further developments in recent years by Elliott et al. (2014), Acemoglu et al. (2015), Glasserman and Young (2015), and Capponi et al. (2016).

⁴ Firm i may use trade credit to delay payments. This would reduce the cash position of firm j and increase the account receivable of firm j , but it would not affect firm j 's total assets and net worth. Hence, different payment methods will not affect the analytical results in the paper, which all depend just on a firm's net worth.

⁵ In 2006, General Motors had made a deal with the defaulting company Clark-Cutler-McDermott, its longtime supplier of auto parts. Such a deal allowed the automaker General Motors to buy any product completed by its supplier during the bankruptcy procedure (see Korosec 2016).

⁶ This separation is empirically supported; in practice, manufacturers tend to build long-term relationships with their suppliers, as also pointed out by Kalwani and Narayandas (1995). Searching suppliers is time consuming, and it is unlikely to find them in the secondary market, where transactions are not made on the premise of long-term relationships.

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