Fundamental Thickness Bounds for Wide-Field-of-View Metalenses

Shiyu Li and Chia Wei Hsu
Ming Hsieh Department of Electrical and Computer Engineering, University of Southern California, Los Angeles, California 90089, USA
cwhsu@usc.edu

Abstract: We show that any aberration-free wide-field-of-view lens system must have a minimal thickness—depending on the field of view, lens diameter, and numerical aperture—that originates from the Fourier transform relation between space and angle.

1. Introduction

Metalenses [1]—compact lenses made with metasurfaces—have the potential to enable thinner, lighter, cheaper, and better imaging systems for applications where miniaturization is critical (e.g. for mobile devices, medical imaging, and augmented reality). Achieving a sufficient field of view (FOV) within a compact size is crucial for these applications, but conventional hyperbolic metalenses suffer from coma, astigmatism, and field-curvature aberrations at oblique incidence [2]. To expand the FOV, one can use the phase profile of an equivalent spherical lens [3] or a quadratic phase profile [4], which reduce off-axis aberrations at the expense of having a low Strehl ratio. To achieve wide FOV with diffraction-limited focusing, one can use metasurface doublets [5,6] or triplets [7] analogous to conventional multi-lens systems, add an aperture stop [8] so incident light from different angles reach different regions of the metasurface, or use inverse-designed multi-layer structures [9]; these approaches are schematically illustrated in Fig. 1(a,b). Notably, all of these approaches involve a much thicker system where the overall thickness (e.g., separation between the aperture stop and the metasurface) plays a critical role. Meanwhile, miniaturization is an important consideration and motivation for metalenses. This points to the scientifically and technologically important questions: is there a fundamental trade-off between the FOV and the thickness of a metalens system, or lenses in general? If so, what is the minimal thickness allowed by physical laws?

2. Results

We look for universal bounds applicable to all designs, including those illustrated in Fig. 1(b) and still-unknown ones. To do so, we adopt the transmission matrix formalism, which captures all spatial and angular dependencies of wave transport through arbitrarily complex linear systems. For any linear system, the input vector and the output vector are related through the transmission matrix t by $E_t(k_k; z = h) = \int t(k_k; k^0_k) E_{in}(k^0_k; z = 0) dk^0_k$, where $k_k = (k_x; k_y)$ is the transverse momentum and h the thickness of the lens system. We can use discrete Fourier transform F to convert the standard transmission matrix in momentum basis to a transmission matrix in spatial basis, $t(\mathbf{r}; \mathbf{r}^0) = F^{-1} t(k_k; k^0_k) F$, where $\mathbf{r} = (x; y)$ is the transverse coordinate.

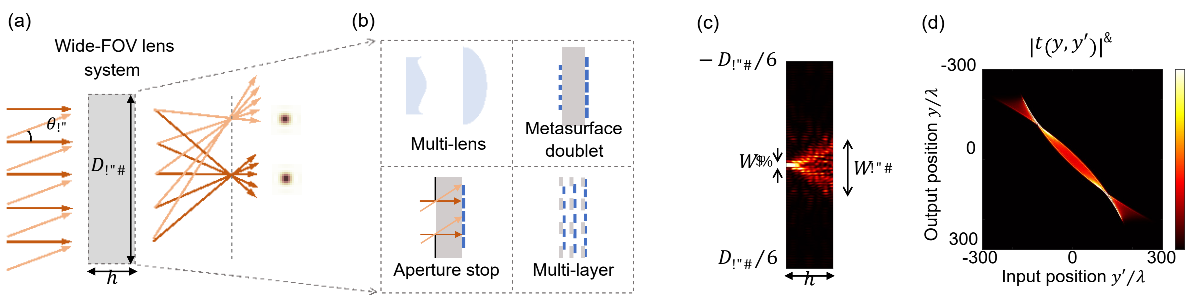

Fig. 1: (a) Schematic of a diffraction-limited lens system with a wide FOV. (b) Systems that realize wide-FOV diffraction-limited focusing. (c) Intensity profile corresponding to the $y^0 = 0$ column of the spatial transmission matrix. (d) Ideal spatial transmission matrix $jt(y; y^0)j^2$ when $D_{out} = 400l$, NA = 0.45, FOV = 80.

The spatial transmission matrix provides a link to the device thickness h of interest. Intuitively, we can expect that given a thicker device, incident light at $z = 0$ can potentially spread more laterally when it reaches the other end at $z = h$. The extent of such a lateral spreading ΔW is the difference between the widths of the output and the input, $\Delta W(y') = W_{out}(y') - W_{in}$, as indicated in Fig. 1(c). For a 2D system, the output width W_{out} is also the vertical width of the near-diagonal elements of the spatial transmission matrix, which we quantify using the inverse participation ratio (IPR): $W_{out}(y') = \left[\int |t(y;y')|^2 dy\right]^2 / \int |t(y;y')|^4 dy$. Similarly, we quantify the width of the incident sinc profile through its IPR, $W_{in} = 3\lambda/[4\sin(FOV/2)]$. With comprehensive numerical simulations, we quantitatively establish the relation

$$h \propto \Delta W, \tag{1}$$

as intuitively expected. This relation connects the transport properties of an arbitrary system to its thickness, which we use to establish a universal thickness bound for wide-FOV lenses.

To focus incident waves from different angles θ_{in}, the ideal phase profile [1, 8] must be angle-dependent, as $\phi_{ideal}(y;\theta_{in}) = \psi(\theta_{in}) - \frac{2\pi}{\lambda}\left(\sqrt{f^2 + [y - y_f(\theta_{in})]^2} + y\sin\theta_{in}\right)$, where f and λ are the focal length and wavelength. The transmission matrix of such an ideal lens system is $t(y;\theta_{in}) = w(y)\exp[i\phi_{ideal}(y;\theta_{in})]$ where $w(y)$ is a rectangular window function; we then do a Fourier transform on the input side to obtain $t(y;y')$. Figure 1(d) shows the transmission matrix $|t(y;y')|^2$ for an ideal lens with $D_{out} = 400\lambda$, NA = 0.45, FOV = 80. We optimize the angle-dependent global phase $\psi(\theta_{in})$ and the focal spot position $y_f(\theta_{in})$ to minimize the maximal lateral spreading $\Delta W_{max} \equiv \max_{y'} \Delta W(y')$, which limits the overall device thickness. The resulting ΔW_{max} is shown in Fig. 2, which grows linearly with D_{out} and with the numerical aperture $NA = \sin(\arctan(D_{out}/(2f)))$, and grows as $\sin\frac{\pi}{2}\sin\frac{FOV}{2}$ with the FOV. Similar dependencies are observed for other lens parameters. These results, together with Eq. (1), provide a fundamental thickness bound on aberration-free wide-FOV lenses.

This bound appears tight, as some inverse-designed multi-layer structures [9] have thicknesses that approach this bound. Our transmission-matrix approach can also be used to establish thickness bounds for other nonlocal metasurfaces beyond lenses, such as retroreflectors and photovoltaic concentrators where a wide FOV is desirable. This work provides guidance for the design of future wide-FOV metasurfaces while establishing an intrinsic relation between angular diversity and spatial footprint in multi-mode systems.

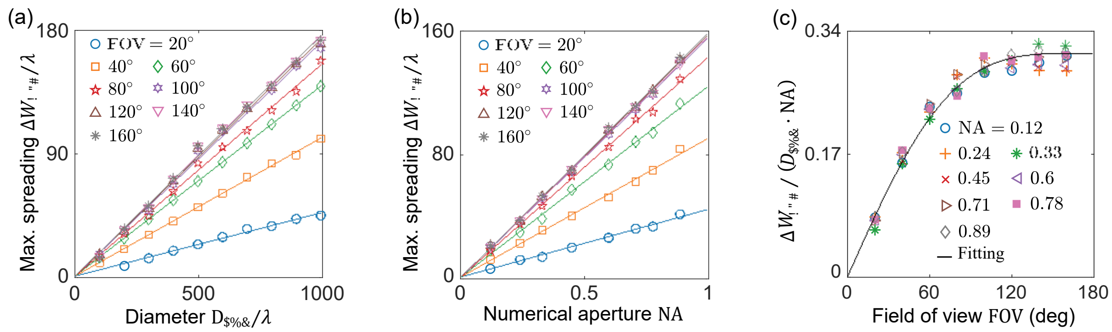

Fig. 2: Maximal lateral spreading ΔW_{max} of aberration-free wide-FOV lenses, as a function of (a) lens diameter D_{out} when NA = 0.6, (b) numerical aperture NA when $D_{out} = 500\lambda$, and (c) FOV. Solid lines are fitting curves.

References

1. P. Lalanne and P. Chavel, Laser Photonics Rev. 11, 1600295 (2017).
2. F. Aieta, P. Genevet, M. Kats and F. Capasso, Opt. Express 21, 31530 (2013).
3. H. Liang, A. Martins, B.-H. V. Borges, J. Zhou, E. R. Martins, J. Li and T. F. Krauss, Optica 6, 1461–1470 (2019).
4. M. Pu, X. Li, Y. Guo, X. Ma and X. Luo, Opt. Express 25, 31471–31477 (2017).
5. A. Arbabi, E. Arbabi, S. M. Kamali, Yu Horie, S. Han and A. Faraon, Nat. Commun. 7, 13682 (2016).
6. B. Groever, W. T. Chen and F. Capasso, Nano Lett. 17, 4902–4907 (2017).
7. S. Shrestha, A. Overvig and N. Yu, in 2019 Conference on Lasers and Electro-Optics (CLEO) (Optical Society of America, 2019), paper FF2B.
8. M. Y. Shalaginov, S. An, F. Yang, P. Su, D. Lyzwa, A. M. Agarwal, H. Zhang, J. Hu and T. Gu, Nano Lett. 20, 7429–7437 (2020).
9. Z. Lin, C. Roques-Carmes, R. E. Christiansen, M. Soljačić and S. G. Johnson, Appl. Phys. Lett. 118, 041104 (2021).