Highlights

An Improved Closed-Circuit RO (CCRO) System: Design and Cyclic Simulation

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- An improved CCRO design is proposed to achieve the energetics of BRO.
- Continuous permeate production is possible in the proposed system.
- Cyclic dynamics are investigated using spatiotemporal models.
- Approximate analytical solutions are provided to analyze energy breakdowns.
- Advantages and limitations of original and improved CCROs are discussed.
An Improved Closed-Circuit RO (CCRO) System: 
Design and Cyclic Simulation

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Abstract

A volume-varying stirred tank with a free-piston are placed in front of DuPont’s Closed-Circuit RO (CCRO) system to achieve improved energetics. Valves are used to direct fluid flow and piston motion to enable cyclic operations consisting of CC filtration and flushing steps. Similar to CCRO, overall system recovery in the proposed system is controlled by the CC/flushing time ratio, and uninterrupted permeate production is possible if the flushing is done at high pressures. Spatiotemporal system-level models are developed and formulated as a set of coupled partial differential equations (PDEs) and ordinary differential equations (ODEs). Detailed numerical simulations and parametric studies are carried out to explore the flow and mass transfer characteristics and to compare its performance at the cyclic steady-state (CSS) with the original CCRO and the conventional steady-state RO.

Keywords:
Batch RO, process design, process dynamics, system analysis, partial differential equation model, cyclic steady state (CSS)

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1. Introduction

In recent years, dynamic and cyclic design concepts have been a research hotspot in the RO desalination community. Some examples include semi-batch RO (SBRO) or closed-circuit RO (CCRO) [30, 26, 31, 10, 28, 13], batch RO (BRO) [26, 17, 18, 12], flow reversal RO (FRRO) [27, 8, 2, 9], pulse feed RO (PFRO) [25], stage rotation RO (SRRO) [23] and periodic flow reversal and retentate recycle (FRRR) with a time-varying ratio [23]. Different from traditional designs, where pressure, flow and concentration conditions are functions of spatial location, these novel designs feature spatiotemporal dynamics. Similar cyclic design concepts have been extensively studied in other engineering fields, such as pressure swing adsorption for gas separation [29] and adsorption enhanced reforming for hydrogen generation [7]. The key technical challenges for analyzing and optimizing such cyclic processes are to solve the coupled spatiotemporal transport phenomena and to evaluate the process performance at the cyclic steady state (CSS), which is not the same as the one in the first cycle.

In an effort to advance fundamental understanding of transient and cyclic RO processes, the author developed a spatiotemporal RO model that took into account of pressure drop, concentration polarization and axial dispersion [20]. The model was applied to flow reversal and membrane flushing [20, 23]. More recently, it has been extended to cyclic simulation of CCRO, which has a long CC filtration period followed by a short flushing period in each cycle [21]. The computational framework enables detailed studies of cyclic transport phenomena and parametric analyses that reveal thorough energy consumption breakdowns in CCRO. Contrary to conjectures in literature, it suggests that CCRO may not be claimed as an “energy-efficient” technology for high-recovery brackish water RO (BWRO), despite the relatively uniform flow and flux conditions. As pointed out in a few papers [26, 5, 18, 21], the mixing between the low-concentration fresh feed and the increasingly concentrated brine in CCRO causes undesired entropy generation, which adversely impacts its efficiency. If the mixing occurs in a volume-varying tank, where the rising salt concentration results naturally from the permeate leaving the system, the energy efficiency can be enhanced. In this work, the CCRO design is modified to approach the energetics of a BRO. However, its operation differs from the “batch” concept typically referred in literature where permeation is non-continuous. Instead, uninterrupted water production is possible in the proposed system employing the same operating strategy in
CCRO. Therefore, it may be called a BRO following the convention in literature or an improved CCRO. Because of additional components used in the new system, the cyclic model developed for CCRO [21] must be significantly enhanced.

2. Cyclic design

A schematic of the proposed BRO system is shown in Figure 1. A continuous stirred tank reactor (CSTR) or perfect mixer with a free-piston is placed in front of a pressure vessel, housing one or more membrane elements. Multiple valves are used to direct fluid flow and piston motion in order for the system to run in a dynamic and cyclic manner. The reciprocating-piston chamber is not a new idea; it is seen in the commercial Dual Work Exchanger Energy Recovery (DWEER) devices in seawater RO (SWRO) plants [1]. A similar double-acting piston is used to reduce down time in batch-type operations [4, 3]. In Figure 1, a full cycle consists of two CC/refilling and two flushing steps. The CC and refilling occur at the same time; when one side of the CSTR undergoes the CC filtration step, the other is being refilled with fresh feed.

Initially, the piston is near the left of the CSTR (shown in Figure 1(a)). The piston divides the CSTR into two compartments whose initial volumes are $V_f$ on the left and $V_0$ on the right. As the fresh feed is pumped into the left compartment of the CSTR, the piston gradually moves to the right. The RO concentrate is completely recycled in the closed circuit (represented by the blue line) and mixed with the solution in the right compartment of the CSTR before entering to the RO unit. Some mechanical agitation (for example, by installing mixing blades in the CSTR) may be induced to enhance mixing. How to implement this in a practical way is yet to be addressed in future studies. Similar to the CCRO system studied previously [21], a time-invariant combined feed rate per vessel ($Q_0$) and a constant single pass recovery ($Y_{SP}$) are maintained in the CC filtration step. The refilling and permeate rates are both $Q_0Y_{SP}$. The recycle to fresh feed ratio in the CC filtration mode is $r = (1 - Y_{SP})/Y_{SP}$.

Once the right compartment of the CSTR reaches a final volume of $V_f$, or the cumulative volume of permeate production is $V_0 - V_f$, the piston pauses and the flushing step begins (shown in Figure 1(b)). Similar to the original CCRO system, the same time-invariant $Q_0$ at the RO entrance is maintained during flushing. Both high pressure flushing (HPF) and low pressure flushing
(LPF) schemes are available. In the HPF, the same $Y_{SP}$ may be used so permeate production is not interrupted. In the LPF, permeation is turned off (or $Y_{SP} = 0$) temporarily. In either flushing scheme, the total system recovery ($Y_{tot}$) is controlled by the ratio of CC filtration time ($t_{FT}$) to flushing time ($t_{FL}$), which resembles the operation strategy in the original CCRO [13, 21]:

$$Y_{tot} = \frac{Y_{SP}(t_{FT}/t_{FL}) + \delta Y_{SP}}{Y_{SP}(t_{FT}/t_{FL}) + 1}$$

where $\delta = 1$ for HPF and 0 for LPF. Alternatively, the time ratio

$$\frac{t_{FT}}{t_{FL}} = \frac{Y_{tot} - \delta Y_{SP}}{Y_{SP}(1 - Y_{tot})}$$

Steps shown in Figure 1(c)-(d) mirror those in Figure 1(a)-(b), whereas the piston in the CSTR moves to the left. The four steps constitute one complete cycle (or two half-cycles) that can be repeated over and over.

It is worth pointing out that if the CSTR unit and associated valves are removed from Figure 1, the system becomes essentially a CCRO. Moreover, if the CSTR is replaced by a Plug Flow Reactor (PFR), or mixing of the RO concentrate and the tank solution occurs only at the RO entrance, it also behaves similarly to a CCRO.

It is reported in literature that feed flow reversal may mitigate scaling formation in RO [27]. A cyclic design for BRO with flow reversal is shown in Figure 2. Every time when the piston changes its direction of movement, the feed to the RO unit is reversed.

For a fair comparison among conventional steady-state RO, CCRO and BRO, the total raw intake rate, the total recovery rate and the total number of RO elements are all fixed. Under these conditions, the cycle-average flux (for a time period lasting one CC filtration and one flushing cycle) is the same in all designs [21]. The feed rate per vessel $Q_0$, which is an important design parameter in CCRO and BRO, may be determined from mass balance.
as below [21]:

\[
Q_0 = \frac{N_E \bar{Q}_f(t_{FT} + t_{FL})}{Y_{SP}t_{FT} + t_{FL}} = \frac{N_E \bar{Q}_f Y_{tot} + (1 - \delta - Y_{tot})Y_{SP}}{Y_{SP}} \frac{1}{1 - \delta Y_{SP}}
\]

\[
= \begin{cases} 
\frac{N_E \bar{Q}_f}{Y_{SP}} Y_{tot}, & \text{for HPF} \\
\frac{N_E \bar{Q}_f}{Y_{SP}} [Y_{tot} + (1 - Y_{tot})Y_{SP}], & \text{for LPF}
\end{cases}
\]

(3)

where \( \bar{Q}_f \) is the feed rate divided by the total number of elements in a conventional RO plant, and \( N_E \) is the number of elements per vessel in BRO. It is important to check that the flow per vessel is within the typical range recommended by membrane manufacturers. For steady-state design using 8” FilmTec membranes, the feed flow per vessel is about 8 - 12 m\(^3\)/h (35 - 55 gpm). The minimum concentrate flow is 3.6 m\(^3\)/h (16 gpm) [6].

The average water flux during CC filtration \( \bar{J}_{w,FT} \) is dependent on \( Q_0 \) [21]:

\[
\bar{J}_{w,FT} = \frac{Q_0 Y_{SP}}{A_m}
\]

(4)

where \( A_m \) is the membrane area per vessel.

In BRO-LPF, both \( Q_0 \) and \( \bar{J}_{w,FT} \) should be slightly higher than those in BRO-HPF (by a factor of \( 1 + Y_{SP}(1 - Y_{tot})/Y_{tot} \)) to maintain the same cycle-average flux.

3. Mathematical model

The BRO system shown in Figure 1 or 2 consists of two sub-systems: the RO and the CSTR, which are coupled and should be solved simultaneously. This is one major difference between the current work and the previous CCRO work [21].

The dimensionless spatiotemporal RO model is the same as the one used
in CCRO [21]:

\[ 0 = \frac{\partial q^*}{\partial x^*} + \frac{\gamma_0}{1 + \frac{L_p \pi_0}{k_m} c^*} (\theta - c^*) \]

\[ \frac{\partial c^*}{\partial t^*} = -q^* \frac{\partial c^*}{\partial x^*} - c^* \frac{\partial q^*}{\partial x^*} + \frac{1}{P_e D} \frac{\partial^2 c^*}{\partial x^{*2}} \]

\[ 0 = \frac{\partial \theta}{\partial x^*} + \frac{a_2}{\pi_0} q^{*n_2} \]

where \( q^* \) is flow rate in the feed channel (\( Q \)) divided by its value at the RO entrance (\( Q_0 \)). \( c^* \) is salt concentration (\( C \)) normalized by the concentration of the fresh feed (\( C_0 \)). \( \theta \) is the transmembrane pressure (\( \Delta P \)) divided by the osmotic pressure of the fresh feed (\( \pi_0 \)). \( t^* \) is the actual time \( t \) divided by the space time \( \tau \) (\( \tau = Q_0/V_c \), where \( V_c \) is the void volume in the RO feed channel). \( x^* = x/L \) is the dimensionless length with 0 and 1 representing entrance and outlet of the RO stage respectively. \( L \) is the length of a pressure vessel, which is about 1 meter for each RO element. \( \gamma_0 \) is a dimensionless parameter defined based on the combined feed rate and the osmotic pressure of the fresh feed, or \( \gamma_0 = A_m L_p \pi_0/Q_0 \) (\( A_m \) is membrane area per vessel and \( L_p \) is membrane hydraulic permeability). \( k_m = a_1 q^{*n_1} \) is the mass transfer coefficient, which varies as a function of cross velocity. \( P_e D \) is the dispersive Peclet number. \( a_1 \), \( n_1 \), \( a_2 \) and \( n_2 \) are parameters that characterize mass transfer and pressure drop [22]. The model is based on the following assumptions: (1) the salt rejection of the membrane is 100%, (2) the concentration polarization factor CPF = \( \exp \left( \frac{J_w}{k_m} \right) \approx 1 + J_w/k_m \), and (3) the residence time distribution can be reasonably described by the dispersion model [19].

The boundary conditions for Equation (5) is listed in Table 1, where \( c_T^* \) is the salt concentration in the tank divided by the fresh feed concentration (\( c_T^* = C_T/C_0 \)), which varies temporally according to the dynamics of the CSTR. Danckwerts boundary conditions are used for solute transport.

The dynamic CSTR model has different forms, depending on the operation mode of BRO. In the CC step (e.g., 0 to \( t_{FT} \)), the total mass balance and salt mass balance in the CSTR compartment located in the closed circuit
are:
\[
\frac{dV}{dt} = -Q_0 Y_{SP}
\]
\[
\frac{d(VC_T)}{dt} = Q_0 (1 - Y_{SP}) C_c - Q_0 C_T
\]

(6)

where \(C_c\) and \(C_T\) are salt concentrations in the RO concentrate and in the tank, respectively.

Using the initial condition \(V(0) = V_0\), it is determined from Equation (6) that:
\[
V = V_0 - Q_0 Y_{SP} t
\]

(7)

Therefore, the salt balance in Equation (6) becomes:
\[
(V_0 - Q_0 Y_{SP} t) \frac{dC_T}{dt} = Q_0 (1 - Y_{SP})(C_c - C_T)
\]

(8)

which can be converted to a dimensionless form shown below:
\[
\frac{dc_T^*}{dt^*} = \frac{1 - Y_{SP}}{V_0/V_c - Y_{SP} t^*} (c_c^* - c_T^*)
\]

(9)

where \(c_c^* = C_c/C_0\), which varies temporally according to the RO dynamics. The initial value for \(c_T^*\) in Equation (9) is from its end value at the previous refilling step.

In the flushing step (e.g., \(t_{FT}\) to \(t_{FT} + t_{FL}\)), the CSTR has a constant volume \(V_f\) and a constant inlet concentration \(C_0\). The salt balance becomes:
\[
V_f \frac{dC_T}{dt} = Q_0 (C_0 - C_T)
\]

(10)

which may be converted to a dimensionless form shown below:
\[
\frac{dc_T^*}{dt^*} = \frac{1}{V_f/V_c} (1 - c_T^*)
\]

(11)

The initial value for \(c_T^*\) in Equation (11) is based on its end value in the preceding CC step.

The flushing step is followed by refilling before the next CC filtration. In the refilling step (e.g., 0 to \(t_{FT}\) for the compartment on the left or \(t_{FT} + t_{FL}\))
to $t_{FT} + (t_{FT} + t_{FL})$ for the compartment on the right), the CSTR dynamics may be described by:

$$\frac{dV}{dt} = Q_0 Y_{SP}$$

$$\frac{d(Vc^*_T)}{dt} = Q_0 Y_{SP} \cdot 1$$

(12)

The initial value for $c^*_T$ in Equation (12) is based on its end value in the prior CC step. At the CSS, the refilling dynamics of right compartment (during time period $(2n-1)(t_{FT}+t_{FL})$ to $t_{FT}+(2n-1)(t_{FT}+t_{FL})$) duplicates that of the left (during time period $2n(t_{FT}+t_{FL})$ to $t_{FT}+2n(t_{FT}+t_{FL})$), where $n$ is the cycle number.

It is not necessary to solve Equation (12) by integration if one is only interested in the final value of $c^*_T$. Note that the solution in the CSTR compartment at the end of the refilling step is a mixture of the fresh feed (with a volume of $V_0 - V_f$) and the solution at the end of the flushing step (with a volume of $V_f$). Using mixing rule,

$$c^*_{T,ar} = 1 + (c^*_{T,br} - 1) \frac{V_f}{V_0}$$

(13)

where $c^*_{T,br}$ is $c^*_T$ before refilling, or the final solution to Equation (11). $c^*_{T,ar}$ is $c^*_T$ after refilling, which is the initial condition for the next CC filtration step.

To find the CSS in BRO shown in Figure 1, the following procedure is used:

1. Assume $C = C_0$ everywhere in the system. Note that the CSS is largely independent of the initial conditions [21].
2. Solve Equations (5) and (9) simultaneously for the CC filtration step. The end conditions are used as initial conditions in the next step.
3. Solve Equations (5) and (11) simultaneously for the flushing step.
4. Update the end value of $c^*_T$ in the step 3 using Equation (13) for the refilling step. The end conditions are used as initial conditions in the next step.
5. Repeat steps 2–4.
For BRO with flow reversal shown in Figure 2, the concentration profile is flipped before being used as initial condition for the next half-cycle.

Orthogonal collocation is used as the discretization method [20, 21]. The mathematical model is converted to a set of ordinary differential equations (ODEs) with a singular mass matrix and is solved using ode15s in Matlab. Specifically, the spatial domain of RO (from 0 to \( L \)) is discretized using \( N \) collocation points (\( N = 50 \) in this work). The points of collocation are chosen to be the roots of the orthogonal Jacobi polynomials. The boundary conditions in Table 1 are then used to eliminate variables at the boundary points to yield \( 3(N-2) \) equations for the RO. The total number of equations for the coupled RO and CSTR system is \( 3N-5 \). The CPU time for solving these equations is no more than 0.4 sec per half-cycle in a Dell Precision 7560 mobile workstation equipped with Intel Core i9-11950H processor and 64 GB RAM, demonstrating its potential for real-time digital twin and control applications.

4. Results and discussion

Simulation case studies are conducted for a high-recovery BWRO process to compare the performance among BRO, CCRO and conventional multi-stage RO [23, 21]. The application of BRO to SWRO will then be discussed in a qualitative way.

4.1. Baseline case

In the baseline case, \( \bar{Q}_f = 1.01 \text{ m}^3/\text{h} \) (4.446 gpm), design flux = 24.5 lmh, \( \pi_0 = 0.62 \text{ bar} \), \( Y_{tot} = 90\% \), \( N_E = 1 \), \( Y_{SP} = 10\% \), \( t_{FL} = (1 + V_f/V_c)\tau \), and \( V_f/V_c = 0 \). To avoid singularity in the mathematical model, \( V_f/V_c \) is set to be slightly above 0 (e.g., 0.001). BRO with flow reversal is chosen for the study. The cycle-average flow, flux and recovery conditions match those in a conventional three-stage BWRO employing FilmTec BW30-400 elements [23]. For BRO-HPF, \( \bar{J}_{w,FT} = 24.5 \text{ lmh} \), \( Q_0 = 9.1 \text{ m}^3/\text{h} \) (40.0 gpm), and \( t_{FT}/t_{FL} = 80 \). For BRO-LPF, \( \bar{J}_{w,FT} = 24.8 \text{ lmh} \), \( Q_0 = 9.2 \text{ m}^3/\text{h} \) (40.5 gpm), and \( t_{FT}/t_{FL} = 90 \). These parameters are the same as those used in the CCRO study [21].

The parameters for BW30-400 elements based on CFD simulations validated against plant experiments in the recovery range of 78-90% are below
where $N_{PV}$ is number of vessels in parallel. $P_{e_D}$ is about 40 for one element [19].

Five full cycles (or ten half-cycles) were simulated to reach the CSS. The results, unless specifically stated, are based on the last cycle. The spatiotemporal profiles of $Q/Q_0$ are shown in Figure 3. Because of the small spatial variations in flow and flux, the CPF is approximately constant (1.12 in both cases), as shown in Figure 4. In the continuous three-stage design with interstage booster pumps, the CPF predicted by the model ranges from 1.07 and 1.14 [23]. The driving forces at five different progression times ($t_1 - t_5$) are shown in Figure 5, indicating that the CC filtration is operated far above the thermodynamic equilibrium. In other words, the design flux necessitates an applied pressure much higher than the osmotic pressure. During LPF, the concentrate flow is $Q_0$, or there is no net permeate production. These are similar to those in CCRO [21].

The spatiotemporal profiles of the dimensionless transmembrane pressure ($\theta = \Delta P/\pi_0$) are shown in Figure 6. Different from the quadrilateral-like shape in CCRO, $\theta$ in the CC filtration step decreases initially, then rises at an accelerated rate at all locations in the CC filtration step. Such a trend is attributed to the transient behavior of salt concentration presented in Figure 7. Before the CC filtration step, the RO membrane is still loaded with salt because it cannot be completely purged out in the flushing step. As the feed to the RO is reversed, the salt is concentrated near the entrance. It takes at least one space time to sweep out most of the brine in the RO channel. During this period, the spatial average salt concentration in the RO may reduce despite permeate production. As a result, the applied pressure reduces temporarily. Subsequently, the average salt concentration in the RO increases as permeate leaves the system at a constant rate. At the end of CC flushing step, the spatial salt concentration profile is completely flipped backwards in both BRO-HPF and BRO-LPF.
Figure 8 shows the spatial profiles of concentration at the CSS. They are switched back and forth between two consecutive half-cycles. Because LPF requires a larger flux to maintain the same cycle-average water production, a higher salt concentration is observed at the end of the CC filtration cycle [21]. This in turn, leads to a higher applied pressure in Figure 6.

Figure 9 shows the temporal profile of concentrate concentration $C_c$ and spatial average concentration ($C_{ave} = \bar{C} = \int_0^L Cdx/L$) in 10 half-cycles. Different from CCRO, the minima of $\bar{C}$ in BRO did not occur at the end of flushing cycles. For LPF and $V_f = 0$, it is observed that the same flushing efficacy model derived from CCRO can be used for BRO [21]:

$$\bar{c}_{i+1}^* = (1 - f) \left[ \frac{Y_{tot}}{1-Y_{tot}} + \bar{c}_i^* \right] + f$$

where $\bar{c}^*$ is the dimensionless salt concentration after flushing and before refilling, $f$ is the flushing efficacy of the RO channel [19], and $i$ and $i + 1$ represent two consecutive cycles. At the CSS, $\bar{c}_i^* = \bar{c}_{i+1}^*$, or:

$$\bar{c}_{CSS}^* = 1 + \frac{Y_{tot}(1-f)}{f(1-Y_{tot})}$$

Equation (16) with $f = 0.912$ matches the cyclic data very well, as shown in Figure 9(d). In this case, $\bar{c}_{CSS}^* = 1.87$, which is consistent with CCRO. However, after the refilling step, $\bar{c}^*$ will be different.

When $V_f = 0$, $V_0/(V_0 + V_c) = Y_{tot}$ and $V_c/(V_0 + V_c) = 1 - Y_{tot}$. If one assumes that the solutions in the CSTR and in the RO are instantaneously mixed at the beginning of the CC filtration step, the average salt concentration in the RO after the refilling step ($\bar{c}^{*,ar}_{CSS}$) is:

$$\bar{c}^{*,ar}_{CSS} \approx \frac{\bar{c}_{CSS}^* V_c + V_0}{V_0 + V_c}$$

$$= \frac{\bar{c}_{CSS}^*(1 - Y_{tot}) + (Y_{tot})}{1 + \frac{Y_{tot}}{f} - Y_{tot}}$$

which is evaluated to be 1.09.

Figure 10 shows the temporal profile of the spatial-average salt concentration in the RO unit during the CC filtration step in the first cycle. If it
is multiplied by 1.09, the profile matches the one at the CSS obtained from the mathematical model except the very beginning when the brine residue is being purged out from the RO channel and mixed with the solution in the CSTR. Therefore, it is reasonable to use Equation (17) for the salt retention factor in BRO.

An alternative way to derive Equation (17) is as follows. Let $\bar{c}^{*,ar}_{CSS}$ be the $\bar{c}^*$ in the closed-circuit right after the refilling step at the CSS, its values after the subsequent CC, flushing and refilling steps are $\left(\frac{\bar{c}^{*,ar}_{CSS}}{1 - Y_{tot}}\right)(1 - f) + f$ and $\left(\frac{\bar{c}^{*,ar}_{CSS}}{1 - Y_{tot}}\right)(1 - f) + f\left(\frac{1}{1 - Y_{tot}}\right) + Y_{tot}$ respectively. From $\bar{c}^{*,ar}_{CSS} = \left(\frac{\bar{c}^{*,ar}_{CSS}}{1 - Y_{tot}}\right)(1 - f) + f$, it is determined that $\bar{c}^{*,ar}_{CSS} = 1 + \frac{Y_{tot}}{f} - Y_{tot}$.

After the results at the CSS are obtained, the specific energy consumption (SEC) normalized by the feed osmotic pressure (NSEC) can be calculated. There are four terms in the energy calculations: (1) pump energy to drive the fresh feed in the CC/refilling mode, (2) pump energy to recycle the concentrate to the CSTR, (3) pump energy to drive the fresh feed in the flushing mode, and (4) hydraulic energy of the concentrate that may be recovered by an Energy Recovery Device (ERD). The theoretical NSEC can be calculated as follows [21]:

$$\text{NSEC} = \frac{Y_{SP}\int_{0}^{t_{FT}^{*}} \theta(0, t^*) dt^* + (1 - Y_{SP})\int_{0}^{t_{FT}^{*}} \left[\theta(0, t^*) - \theta(1, t^*)\right] dt^* + \int_{t_{FT}^{*}}^{t_{FT}^{*}+t_{FL}^{*}} \theta(0, t^*) dt^* - (1 - \delta Y_{SP})\int_{t_{FT}^{*}}^{t_{FT}^{*}+t_{FL}^{*}} \theta(1, t^*) dt^*}{Y_{SP}t_{FT}^{*} + \delta Y_{SP}t_{FL}^{*}}$$

(18)

The inlet pressures are shown in Figure 11. Similar to those in CCRO, the temporal profile is discontinuous in LPF because the pressure serves to overcome the flow resistance only. The pressure drops are relatively constant in both cases. The NSEC for BRO-LPF is determined to be 21.5 with ERD ($= 17.91 + 3.54 + 0.69 - 0.64$) and 22.1 without ERD ($= 17.91 + 3.54 + 0.69$). For BRO-HPF, it is 21.3 with ERD ($= 17.55 + 3.43 + 2.62 - 2.32$) and 23.6 without ERD ($= 17.55 + 3.43 + 2.62$). All are at lower levels than those in CCRO under the same design conditions [21], implying that the proposed system is more energy-efficient.

For BRO-LPF, it is possible to provide an approximate analytical solution to break down the NSEC in terms of thermodynamic restriction, design flux,
friction resistance, concentration polarization, and salt retention, following the approach used previously [21]. The NSEC imposed by thermodynamics is

\[-\ln(1 - Y_{tot})/Y_{tot} = 2.56.\]

The NSEC required by design flux is \(Y_{tot}/\gamma_{LPF} = J_{w,FT}/(L_p\pi_0) = 14.49\), where \(\gamma_{LPF} = (A_m/L_p\pi_0)/[(Q_0Y_{SP}t_{FT} + Q_0t_{FL})/t_{FT}] = 0.0621\) [21]. Note that \(\gamma_{LPF}\) is defined based on the total intake volume in one cycle divided by the CC step time, which is different from \(\gamma_0\) used in Equation (5). The NSEC due to salt retention is \([-\ln(1 - Y_{tot})/Y_{tot}]\langle c_{CSS}^* - 1\rangle = 0.22.\]

The sum of these three terms is 17.3. When the effects of concentration polarization and friction loss are mostly eliminated in the mathematical model (by arbitrarily increasing the mass transfer coefficient 100 times and decreasing the friction coefficient 100 times), the NSEC with ERD for BRO-LPF also yields 17.3.

The pressure drop (\(\Delta P_L\)) assuming spatially constant flux along the RO (which holds reasonably well if \(Y_{SP}\) is small) may be calculated by [16]:

\[
\frac{\Delta P_L}{\pi_0} = \frac{a_2}{\pi_0 n_2 + 1} \frac{1 - (1 - Y_{SP})^{n_2+1}}{Y_{SP}}
\]

In LPF, \(Y_{SP} = 0\), \(\Delta P_L/\pi_0 = a_2/\pi_0\) using L’Hopital’s rule.

Therefore, NSEC due to friction loss in BRO-LPF is estimated by:

\[
NSEC_{friction,LPF} = \frac{a_2}{\pi_0 n_2 + 1} \frac{1 - (1 - Y_{SP})^{n_2+1}}{Y_{SP}} \frac{Q_0 t_{FT} + Q_0 t_{FL}}{Q_0 Y_{SP} t_{FT}}
\]

\[
= \frac{a_2}{\pi_0 n_2 + 1} \frac{1 - (1 - Y_{SP})^{n_2+1}}{Y_{SP}} \frac{Y_{SP} t_{FL}}{t_{FT}} + Y_{SP} (1 - Y_{tot})/Y_{tot}
\]

Using the baseline conditions, \(NSEC_{friction,LPF}\) is calculated to be 3.99, which differs 4% from the value derived from the mathematical model when the friction coefficient is reduced by 100 times (3.84).

The concentration polarization is expected to have a multiplying effect on the energy required to overcome both the thermodynamic restriction and
the salt retention, or

\[
NSEC_{CP} = \begin{cases} 
(CPF - 1) \left[ -\frac{\ln(1-Y_{tot})}{Y_{tot}} - \ln \left(1 - Y_{tot}\right) \left(\frac{1}{f} - 1\right) \right], & \text{BRO} \\
(CPF - 1) \left[ 1 + \frac{Y_{tot}}{2(1-Y_{tot})} + \frac{Y_{tot}(1-f)}{f(1-Y_{tot})} \right], & \text{CCRO} 
\end{cases}
\]

(21)

CPF calculated from \(1 + \bar{J}_{w,FT}/k_m\) is 1.12 at both ends of the RO, which is consistent with results shown in Figure 4. The concentration polarization adds 0.76 to NSEC according to Equation (21). When the mass transfer coefficient is arbitrarily increased by 100 times in the mathematical model, the NSEC reduces by 0.75. The numerical result and the approximate analytical solution appear to be consistent.

The detailed energy consumption breakdowns for both BRO and CCRO with LPF are shown in Table 2 for a comparison. It appears that BRO and CCRO have the same friction loss and the same energy requirement for the design flux. The CPF is also the same. However, BRO outperforms CCRO in terms of energy efficiency in that (i) it is a thermodynamically favorable design as it mimics a dead-end membrane, (ii) the effect of salt retention is smaller \(\left(-\frac{\ln(1-Y_{tot})(1/f - 1)}{Y_{tot}(1-f)/f(1-Y_{tot})}\right) = \left[-(1-Y_{tot})\ln(1-Y_{tot})\right]/Y_{tot}\), which is always less than 1 for any \(Y_{tot}\), (iii) the effect of concentration polarization is smaller given that the CPF functions as a multiplying factor on energy required for overcoming thermodynamic restriction and salt retention.

When hydraulic energy of the brine leaving the system is fully recovered, BRO-HPF is slightly more efficient than BRO-LPF because of a smaller \(\bar{J}_{w,FT}\) and a lower pressure drop. However, ERD is rarely installed in high-recovery BWRO processes. In such a case, the brine energy is dissipated via the concentrate valve, and BRO-LPF is more efficient. Similar trends are also observed in CCRO [21].

Despite that BRO outperforms CCRO, it still consumes more energy than the three-stage RO under the same intake, recovery and design flux conditions (20.7 without booster pumps and 20.9 with booster pumps [23]). In the three-stage RO design with inter-stage booster pumps, the element-level recovery varies in the range of 8-14%, and the averaged element-level recovery is 10% [23]). If flow resistance is removed from the three-stage model (by reducing the friction coefficient by 100 times), the NSEC reduces by 3.1.
It appears that the BRO designed with $Y_{SP} = 10\%$ experiences a larger flow resistance than the three-stage design, despite a much shorter flow length in the former.

Figure 12 is a pie chart describing relative contributions of the five factors to NSEC. For such a low-salinity BWRO process, design flux and flow resistance are the two most significant factors.

4.2. Effect of $V_f/V_c$

A simulation is done with $V_f/V_c = 5$ and $t_{FL} = (1 + V_f/V_c)\tau$ for BRO-LPF. As shown in Figure 13, it takes a few more cycles to reach the CSS. The salt concentration levels are elevated and a spatial variation is easily observed. The applied pressure increases accordingly to maintain the same recovery.

The NSEC of BRO at different values of $V_f/V_c$ is shown in Figure 14. The lowest NSEC occurs at $V_f/V_c = 0$. However, the flushing fluid would be blocked from entering the CSTR. When $V_f/V_c$ increases to 1, 5 and 10, the NSEC without ERD goes up by 3%, 6% and 7%, respectively.

It is speculated that the connection lines in the BRO system would induce some effects similar to $V_f$. Reducing their volume relative to the void channel space in RO may be desirable for the energetics of BRO and CCRO. For example, one piston-chamber assembly may be shared by multiple pressure vessels connected in parallel in a scaled-up system.

The piston-chamber assembly used in the improved CCRO system shown in Figure 1 or 2 may be sized as follows. For LPF, the system recovery $Y_{tot} = (V_0 - V_f)/(V_0 + V_c)$, or $V_0 = [Y_{tot}/(1 - Y_{tot})]V_c + [1/(1 - Y_{tot})]V_f$. Therefore, the total volume of the chamber is $V_0 + V_f = [Y_{tot}/(1 - Y_{tot})]V_c + [(2 - Y_{tot})/(1 - Y_{tot})]V_f$. Based on a user-specified $V_f/V_c$, a total system recovery $Y_{tot}$ and an experimentally measured $V_c$ (on the order of 0.01 m$^3$ per 8" RO element), the chamber size can be determined. For a 90% recovery, the volume of the reciprocating-piston chamber is estimated to be 150 - 200 liters. The required volume may be doubled if $Y_{tot}$ increases from 90% to 95%. To avoid a large CSTR for certain ultra-high recovery applications, one may consider a hybrid operation mode that begins with the original CCRO and transitions to the improved CCRO near the end of the CC filtration step. This is somewhat analogous to the FRRR concept in the conventional design; penalized energy consumption associated with undesirable entropy generation only occurs during part of the course of filtration [23].
4.3. Effect of single-pass recovery $Y_{SP}$

Different values of $Y_{SP}$ are used to study its effect on the performance of BRO-LPF and the results are shown in Figure 15. When a larger $Y_{SP}$ is adopted in design, the recycle to raw feed ratio ($r$) and the time-invariant feed rate per vessel ($Q_0$) are both smaller. As a result, the friction loss reduces according to Equations (20) and (14).

When $Y_{SP}$ increases to 12%, the NSEC without ERD is about the same as the one in three-stage RO (20.7 without booster pumps and 20.9 with booster pumps [23]). When $Y_{SP}$ is chosen to be 15% or 20%, BRO-LPF becomes 4 or 6% energy efficient than three-stage RO, however, the CPF increases to 1.15 or 1.17. The difference in NSEC between $Y_{SP} = 20\%$ and $Y_{SP} = 25\%$ is fairly small because the uplifted CPF (1.19) offsets the reduced friction loss. $Y_{SP} = 25\%$ may not be a good option for long-term operation of BRO because the concentrate flow $Q_c$ is out of the range recommended by membrane manufacturers. Moreover, elevated concentration polarization may aggravate fouling [11]. To avoid over-fluxing, single element recovery rate is typically limited to 15% or less [13].

Note that increasing $Y_{SP}$ while keeping $Q_0$ at its original value will not satisfy Equation (3), or the comparison between BRO and steady-state RO is not on the same footing.

Adopting a $Y_{SP}$ larger than 10% and reducing $Q_0$ accordingly in the original CCRO will reduce SEC as well. However, when $Y_{SP}$ is varied in the range of 10-25%, the model shows that it always consumes more energy than BRO or three-stage RO.

Since the energy saving benefit of both the original and the improved CCRO designs relative to conventional ROs is minimal or does not exist, one niche application is for treating feed water with high fouling potential. By adopting a small $Y_{SP}$ in CCROs, concentration polarization may be mitigated at the expense of a higher friction loss. Moreover, because of their relative compact sizes, CCROs may also be suitable for decentralized high-recovery desalination applications.

4.4. Effect of extending flushing time $t_{FL}$

A simulation is done by doubling the flushing time in BRO-LPF (or $t_{FL} = 2(1 + V_f/V_c)\tau$). To maintain the same recovery level, the CC filtration time $t_{FT}$ must also be doubled according to Equation (1). The simulation results are shown in Figure 16. It is found that the RO membrane is relatively clean at the end of the flushing step, or $c^* = 1$ everywhere. However, the salt
concentration level and the applied pressure are raised significantly at the end of the CC filtration step. In this particular case, the SEC increases about 2%. Similar behaviors are observed in both numerical and experimental studies of CCRO [21, 13].

Membranes must be operated below their recommended maximum pressures to prevent mechanical damage. Therefore, it is important to accurately determine the space time \( \tau \) so that BRO and CCRO are operated in the safe window.

4.5. Effect of elements per vessel \( N_E \)

To reduce capital expenditure in BRO and CCRO, it is common to house more than one element in each vessel [21]. As \( N_E \) increases, a larger \( Y_{SP} \) may be adopted in process design while keeping the element-level recovery around 10%. The pressure drop per vessel may increase due to the extended length, however, the recycle to feed ratio \( (r) \) will reduce and \( Q_0 \) may also change. Its effect on SEC requires detailed calculations.

A simulation study is done using \( N_E = 3, Y_{SP} = 1 - (1 - 0.1)^3 = 27.1\% \). In this case \( t_{FT}/t_{FL} = 33.2 \), and \( Q_0 = 10.4 \) m\( ^3 \)/h. The results are shown in Figure 17. The concentration levels are similar to those in the baseline case. However, the NSEC with/without ERD is about 0.8 higher than the one in the baseline case. An estimation using Equation (20) indicates that the friction loss would lead to an increase about 0.7 in NSEC. Moreover, because \( t_{FT}/t_{FL} \) is smaller, the average flux during CC filtration is higher, resulting in additional 0.2 in NSEC.

4.6. Difference between BWRO and SWRO and potential applications of BRO in SWRO

The proposed model can be applied to BRO of seawater if plant-validated parameters (e.g., \( a_1, a_2, n_1, n_2 \) and \( L_p \)) are available. Some qualitative discussions without detailed numerical simulations are provided below.

The difference between BWRO and SWRO has been discussed in a few papers published by the author (see, e.g., [15, 17]). For BWRO, the \( \gamma \) parameter in modern plants is small (about 0.05), the pressure drop effect is significant and the operation is far away from the thermodynamic equilibrium. The applied pressure mainly serves to meet the flux requirement and to overcome the flow resistance. It is observed that the transmembrane hydraulic pressure at the end of the RO is still much higher the transmembrane osmotic pressure, and the hydraulic energy of the brine is dissipated via the
 concentrate valve [24]. From a viewpoint of energy performance, single-stage design would be the most efficient, followed by multi-stage, then multi-stage with booster pumps [15, 23]. The energy efficiency appears to be contradicting to the balanced design considerations in BWRO. In single-stage design, the cross velocity quickly drops along the feed direction, and therefore, it has the least pressure drop. When booster pumps are installed in a multi-stage system, permeate production shifts more towards downstream. The flow rate level in the feed channel is relatively higher, so is the friction loss [23]. Multi-stage with booster pumps has the most balanced flux, followed by multi-stage, then single-stage. To avoid inadequate fluxes in rear elements, single-stage design is not recommended for high-recovery BWRO applications. In such a case, the driving factor is the balanced flux, not the SEC. By the same token, selecting too large a $Y_{SP}$ in BRO or CCRO is discouraged due to overfluxing and elevated concentration polarization even though it may save energy consumption by reducing the flow resistance.

For SWRO, the $\gamma$ parameter in modern plants is large (around 1), the pressure drop effect is relatively insignificant and the operation is slightly above the thermodynamic equilibrium. The transmembrane pressure mainly serves to overcome the transmembrane osmotic pressure. In such a case, the flux uniformity is in accord with energy efficiency. The internal staging in BRO and CCRO could be beneficial, however, the detrimental effect of salt retention should also be taken into consideration.

On the basis of $Y_{tot} = 50\%, \ Y_{SP} = 10\%, \ \gamma = 1$, $CPF = 1.07$ (based on $\exp(0.7 \times 0.1) = 1.07$ [6]) and $f = 0.9$ [19], it is estimated using equations in Table 2 that the contributions of thermodynamic restriction, design flux, salt retention, concentration polarization to NSEC in SWRO are 1.39, 0.55, 0.08 and 0.11. For CCRO, they are 1.50, 0.55, 0.11 and 0.12, respectively. The contribution of flow resistance to NSEC estimated using plant conditions is 0.11 (assuming a pressure differential of 1.5 bars per pressure vessel and a $\pi_0$ of 27 bar). Apparently, thermodynamic restriction and design flux are the top two contributing factors, which are drastically different from the BWRO case shown in Figure 12. It is seen that BRO is about 7% energy-efficient than CCRO.

If the effects of concentration polarization and flow resistance are not taken into consideration, the NSEC for BRO and CCRO are estimated to be 2.01, 2.16 for $Y_{tot} = 50\%$ and $\gamma = 1$. Based on the same assumptions and process conditions, NSEC is 2.08 and 1.93 for single-stage and two-stage ROs [18]. In such a case, the energy performance is ranked as: two-stage RO >
BRO > single-stage RO > CCRO for seawater desalination.

Note that the advantage of BRO and CCRO is manifested when a larger $\gamma$ is adopted in design [18]. As $\gamma$ becomes sufficiently large, the contribution of design flux to NSEC ($Y_{tot}/\gamma LPF = J_{w,FT}/(L_p\pi_0)$) becomes small. In single- or multi-stage RO operated at steady state, a plateau in the NSEC is quickly reached as $\gamma$ increases [14]. When $\gamma$ is quadrupled, BRO may be similar to three-stage RO in terms of SEC [18]. Therefore, the continuous development of highly-permeable membranes may favor BRO in future plant designs. However, it is noted that the maximum applied pressures at the end of the CC filtration step of BRO and CCRO are higher than what is observed in current plants operated at steady-state. This calls a need for developing ultra-high pressure membranes [32] in order to advance the BRO and CCRO technologies.

5. Concluding remarks

A piston-tank assembly and several valves are placed in front of the CCRO system to improve its energetic performance. Similar to CCRO, continuous permeate production is possible in the proposed system if the flushing step is done at high pressures.

The improved CCRO system is a thermodynamically favorable design as it mimics a dead-end membrane, which reduces undesired entropy generation. Moreover, the effect of salt retention is smaller than the one in the original CCRO under similar design conditions. The concentration polarization is at the same level, however, its penalized effect on the energy consumption of the proposed system is smaller.

The energy benefit of using BRO for treating low-salinity water is marginal on the basis of the same conditions used in a traditional plant. Adopting a larger $Y_{SP}$ may save energy, at the expense of increased concentration polarization. Based on $Y_{SP} = 15\%$, it may consume 4\% less energy than three-stage RO with an averaged element-recovery of 10\%. If $Q_0$ and $Y_{SP}$ are carefully chosen, BRO may have reduced fouling risk due to a relatively uniform and controlled CPF, whereas the energy consumption may be higher. When used for seawater desalination, the energy performance of BRO is in between single-stage and two-stage ROs based on typical flux and recovery conditions used in industry. Additionally, the availability of highly permeable membranes will favor BRO in future plant designs.
The impact of salt retention on SEC in CCRO and BRO is only a few percent if operated under optimal conditions. The applied pressure ascends to a higher level and the energy performance degrades if the flushing time goes beyond one space time. Therefore, it is essential to accurately determine its value under plant operating conditions.

The peak pressures in both CCRO and BRO will be greater than those observed in current plants operated at steady-state under similar design conditions.

Acknowledgement

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Table 1: Boundary conditions in BRO simulations.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Closed-circuit mode</th>
<th>High-pressure flushing mode</th>
<th>Low-pressure flushing mode</th>
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<tr>
<td>( c^* \frac{1}{P_{eD}} \frac{\partial c^<em>}{\partial x^</em>} \bigg</td>
<td>_{x^*=0^+} )</td>
<td>( c_T^* )</td>
<td>( c_T^* )</td>
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<td>( q^<em>(x^</em>, t^*) \big</td>
<td>_{x^*=1} = 1 )</td>
</tr>
<tr>
<td>( q^<em>(x^</em>, t^*) \big</td>
<td><em>{x^*=1} = 1 - Y</em>{SP} )</td>
<td>( q^<em>(x^</em>, t^*) \big</td>
<td><em>{x^*=1} = 1 - Y</em>{SP} )</td>
</tr>
<tr>
<td>( \theta'(x^<em>, t^</em>) \big</td>
<td>_{x^*=0} = -\frac{a_2}{\pi_0} )</td>
<td>( \theta'(x^<em>, t^</em>) \big</td>
<td>_{x^*=0} = -\frac{a_2}{\pi_0} )</td>
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<tr>
<td>( \theta'(x^<em>, t^</em>) \big</td>
<td><em>{x^*=-1} = -\frac{a_2}{\pi_0} (1 - Y</em>{SP})^{n_2} )</td>
<td>( \theta'(x^<em>, t^</em>) \big</td>
<td><em>{x^*=-1} = -\frac{a_2}{\pi_0} (1 - Y</em>{SP})^{n_2} )</td>
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Table 2: Comparison of energy consumption breakdowns between CCRO-LPF and BRO-LPF. $\bar{J}_w = 24.5$ l/h, $Y_{tot} = 90\%$, $Y_{SP} = 10\%$, $\pi_0 = 0.62$ bar.

<table>
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<tr>
<th>Factors</th>
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<td>Flow resistance</td>
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<td>Equation (20) = 3.99</td>
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<tr>
<td>Concentration polarization</td>
<td>0.75</td>
<td>Equation (21) = 0.76</td>
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<tr>
<td>Salt retention</td>
<td>0.87</td>
<td>$\frac{Y_{tot}(1 - f)}{[f(1 - Y_{tot})]} = 0.87$</td>
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<tr>
<td>NSEC</td>
<td>25.4</td>
<td>25.6</td>
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Figure 1: Schematic of (a) CC/refilling mode (piston moving forward), (b) flushing mode, (c) CC/refilling mode (piston moving backward), (d) flushing mode in BRO. Red line: refilling, Blue line: filtration, Green line: flushing. Black line: idle. $r = (1 - Y_{SP})/Y_{SP}$. 
Figure 2: Schematic of (a) CC/refilling mode (piston moving forward), (b) flushing mode, (c) CC/refilling mode (piston moving backward), (d) flushing mode in BRO with flow reversal. Red line: refilling, Blue line: filtration, Green line: flushing. Black line: idle.
Figure 3: Spatiotemporal profiles of $Q/Q_0$ at the CSS in (a) CC mode of BRO-HPF, (b) CC mode of BRO-LPF, (c) flushing mode of BRO-HPF, and (d) flushing mode of BRO-LPF.
Figure 4: Spatiotemporal profiles of CPF at the CSS in (a) CC mode of BRO-HPF; (b) CC mode of BRO-LPF.
Figure 5: Dimensionless driving force \( (\Delta P - CPF \cdot \pi)/\pi_0 \) at five progression times in (a) CC mode of BRO-HPF, (b) CC mode of BRO-LPF, (c) flushing mode of BRO-HPF, and (d) flushing mode of BRO-LPF.
Figure 6: Spatiotemporal profiles of $\Delta P/\pi_0$ at the CSS in (a) CC mode of BRO-HPF, (b) CC mode of BRO-LPF, (c) flushing mode of BRO-HPF, and (d) flushing mode of BRO-LPF.
Figure 7: Spatiotemporal profiles of $C/C_0$ at the CSS in (a) CC mode of BRO-HPF, (b) CC mode of BRO-LPF, (c) flushing mode of BRO-HPF, and (d) flushing mode of BRO-LPF.
Figure 8: Spatial profiles of $C/C_0$ at the CSS in (a) BRO-HPF and (b) BRO-LPF.
Figure 9: Temporal profiles of (a) $C_c/C_0$ in BRO-HPF, (b) $C_c/C_0$ in BRO-LPF, (c) spatial average $C/C_0$ in BRO-HPF, (d) spatial average $C/C_0$ in BRO-LPF.
Figure 10: Spatial average of salt concentration during CC filtration ($\int_0^L c^* dx / L$) in the first cycle and at the CSS. The model is based on the concentration in the first cycle multiplied by a constant factor ($1 + Y_{tot} / f - Y_{tot}$).
Figure 11: Temporal profiles of $\Delta P_0/\pi_0$ in (a) BRO-HPF and (b) BRO-LPF.
Figure 12: Contributing factors to SEC in BRO-LPF. Feed osmotic pressure: 0.62 bar. Cycle-average flux: 24.5 lmh. Recovery: 90%. $Y_{SP} = 10\%$. 

Thermodynamic restriction
Finite flux
Pressure drop
Concentration polarization
Salt retention
Figure 13: Effect of increasing $V_f/V_c$ to 5 on (a) applied pressure, (b) spatial-average concentration and (c) spatial concentration profile at the CSS in BRO-LPF.
Figure 14: Effect of $V_f/V_c$ on NSEC in BRO-LPF. $Y_{SP} = 10\%$.

Figure 15: Effect of $Y_{SP}$ on (a) $Q_0$, $Q_c$ and recycle to feed ratio $r = (1 - Y_{SP})/Y_{SP}$ and (b) NSEC in BRO-LPF.
Figure 16: Effect of doubling flushing time on (a) applied pressure, (b) spatial-average concentration and (c) spatial concentration profile at CSS in BRO-LPF.
Figure 17: Effect of using 3 elements per vessel on (a) applied pressure, (b) spatial-average concentration and (c) spatial concentration profile at CSS in BRO-LPF.
**Nomenclature**

- **BRO** Batch reverse osmosis
- **BWRO** Brackish water reverse osmosis
- **CCRO** Closed-circuit reverse osmosis
- **CPF** Concentration polarization factor
- **CSS** Cyclic steady state
- **CSTR** Continuously stirred tank reactor
- **HPF** High pressure flushing
- **LPF** Low pressure flushing
- **NSEC** Normalized specific energy consumption
- **ODE** Ordinary differential equation
- **PDE** Partial differential equation
- **PFR** Plug flow reactor
- **RO** Reverse osmosis
- **SEC** Specific energy consumption
- **SWRO** Seawater reverse osmosis
- **$\Delta P$** Transmembrane hydraulic pressure
- **$\Delta P_L$** Pressure drop
- **$\delta$** 0 for LPF and 1 for HPF
- **$\gamma$** Membrane capacity intake ratio
- **$\pi$** Osmotic pressure
- **$\tau$** Space time
- **$\theta$** Transmembrane pressure divided by feed osmotic pressure
$A_m$ Membrane area

$C$ Salt concentration

$c^*$ Dimensionless salt concentration

$f$ Flushing efficacy

$J_w$ Water flux across membrane

$k_m$ Mass transfer coefficient

$L$ Total length of a pressure vessel

$L_p$ Hydraulic permeability

$Pe_D$ Dispersive Peclet number

$Q$ Volumetric flow rate

$q^*$ Dimensionless volumetric flow rate

$r$ Recycle to fresh feed ratio

$t$ Time

$t^*$ Dimensionless time

$V_0$ Initial volume of CSTR compartment in the closed circuit

$V_c$ Volume of the void space in RO

$V_f$ Final volume of CSTR compartment in the closed circuit

$x$ Length

$x^*$ Dimensionless length

$Y$ Recovery

$0$ Inlet

$ave$ Average

$c$ Concentrate
<table>
<thead>
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<th>Abbreviation</th>
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<td>FL</td>
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