# A Laplace-Domain Circuit Model for Fault and Stability Analysis Considering Unbalanced Topology

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***Abstract*—For systems subject to unbalanced faults, analytical model building for stability assessment is a challenging task. This letter presents a straightforward modeling approach. A generalized dynamic circuit representation is achieved by use of the Laplacian transform variable $s$. We translate the voltage and current relationship at the fault location into the relationship of three subsystems. The final circuit model is an interconnected sequence network with impedances in the Laplace domain. This circuit can be directly converted from a steady-state sequence network. This modeling procedure is illustrated by an example case of an induction motor served by a grid through a series compensated line. Electromagnetic transient simulation results demonstrate that sub-synchronous oscillations can be mitigated when a single-line to ground fault is applied at the motor terminal. Stability analysis results based on the dynamic circuit corroborate the simulation results. What's more, the derived circuit effortlessly reveals why unbalance can enhance stability.**

***Index Terms*—Dynamic circuit representation, fault analysis, stability analysis, unbalance**

## I. INTRODUCTION

STABILITY analysis considering a system with unbalanced topology is a challenging task. For a balanced system, a three-phase voltage can be converted to two constant voltage sources in the $dq$ frame at steady state. While for an unbalanced system, multiple coordinates are necessary to model the positive-, negative-, and zero-sequence components as state-space variables which are constant at steady state. Dynamic phasor-based modeling, has been adopted by Stankovic to study asymmetry faults in [1], for which tremendous efforts are required to deal with calculus and frame conversion.

In this letter, we present a different approach to come up with a dynamic circuit representation. The approach is straightforward, yet leads to great insights. We use an example case —an induction motor served by an unbalanced network —to illustrate the method and the application of the circuit representation. In the following, Section II presents the test case and the simulation results. Section III presents the circuit representation and frequency-domain analysis results. Section IV concludes the letter.

## II. THE EXAMPLE AND THE SIMULATION RESULTS

Fig. 1a presents a test bed of a 200 hp 460-V induction motor (IM) connected to a series compensated network. The motor speed is fixed at 0.70 pu. At $t = 1.5$ s, the parallel RL circuit is tripped leaving the motor radially connected to the RLC circuit. This RLC circuit has 50% compensation level. At $t = 2.1$ s, phase $a$ is connected to the ground to emulate a single-line to ground (SLG) fault. Fig. 1b presents the simulation results. It can be clearly seen that once the IM is radially connected to the capacitor, 26-Hz oscillations in the torque and 34-Hz oscillations in the stator currents become undamped. This 26-Hz mode is due to the LC resonance. After the phase $a$ of the terminal bus connects to the ground, the system quickly recovers stability. This example shows that the unbalanced topology helps mitigate the subsynchronous resonance (SSR).

Indeed, in the literature of SSR control, phase imbalance has

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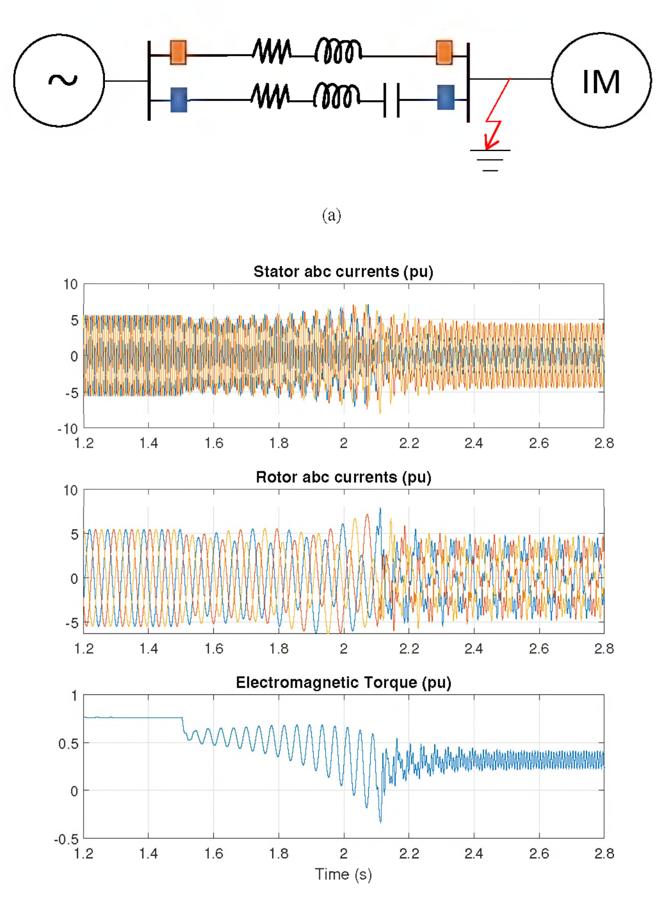


(b)

Fig. 1: (a) An induction motor connected to a series compensated network is subject to an SLG fault. (b) Simulation results. Before $t = 1.5$ s, the motor is connected to both the RL and the RLC circuits. At $t = 1.5$ s, the parallel RL circuit is tripped leaving the motor radially connected to the RLC circuit. At $t = 2.1$ s, phase $a$ of the motor's terminal bus is connected to the ground. Parameters in p.u.: the RLC circuit $R = 0.02$, $X_L = 0.2$, $X_c = 0.1$. IM: $R_s = 0.01282$, $L_{ls} = 0.05051$, $R_r = 0.00702$, $L_{lr} = 0.05051$, $L_m = 2.503$.

been pointed out to have positive impact [2] by Edris in 1993. In a 2011 paper [3], the dynamic phasor modeling approach was adopted for model derivation and finding eigenvalues under unbalance. The paper points out that large levels of phase imbalance can cause a significant movement in the subsynchronous network modal frequencies.

We pose the following question. Can a more insightful explanation be offered as why imbalance can improve damping of the subsynchronous mode? This letter sets to fill the gap.

## III. LAPLACE-DOMAIN CIRCUIT

For an induction machine, its dynamic circuit in $s$-domain has been derived by the authors in [4], [5]. Such a circuit for type-3 wind turbines has been used for SSR analysis in [4], [5]. The dynamic circuit of an IM extends the steady-state circuit of IM developed by Charles Steinmetz in 1900s by replacing $j\omega$ using $s$. Furthermore, the slip of the motor has been found to have the following expression: slip $= \frac{s-j\omega_m}{s}$ (where $\omega_m$ is the rotor speed).

In steady-state analysis, unbalanced fault conditions can be efficiently dealt with by use of symmetric component theory and sequence network interconnection technique. For example, for an SLG fault, the three-phase fault current and the fault bus voltage have the following characteristics: $\overline{V}_a = 0,\ \overline{I}_b = \overline{I}_c = 0$. In turn, the sequence domain voltages and currents have the following relationship:

$$\overline{V}_1 + \overline{V}_2 + \overline{V}_0 = 0,\ \overline{I}_1 = \overline{I}_2 = \overline{I}_0 = \frac{\overline{I}_a}{3}. \tag{1}$$

Thus, the sequence networks can be interconnected in series. The fault current in sequence can be found by circuit analysis of the sequence network interconnection.

Can we expand such techniques for dynamic analysis?

### A. Circuit 1

In order to check the possibility, we examine the same SLG fault using time-varying space vectors, instead of phasors at the nominal frequency. The boundary conditions are expressed in time domain as follows: $v_a(t) = 0,\ \ i_b(t) = i_c(t) = 0$.

The space vector aggregates the three-phase variables to form a single variable. In addition, we bring into the picture the conjugate of the space vector and the zero-sequence component in time domain:

$$\frac{\vec{v}(t)}{2} = \frac{1}{3}\left(v_a(t) + e^{j\frac{2\pi}{3}}v_b(t) + e^{-j\frac{2\pi}{3}}v_c(t)\right), \tag{2}$$

$$\frac{[\vec{v}(t)]^*}{2} = \frac{1}{3}\left(v_a(t) + e^{-j\frac{2\pi}{3}}v_b(t) + e^{j\frac{2\pi}{3}}v_c(t)\right), \tag{3}$$

$$v_0(t) = \frac{1}{3}\left(v_a(t) + v_b(t) + v_c(t)\right) \tag{4}$$

The above relationship leads to the expression of $abc$ variables in terms of the space vector, its conjugate and the zero-sequence component:

$$v_a(t) = v_0(t) + \frac{\vec{v}(t) + [\vec{v}(t)]^*}{2}, \tag{5}$$

$$v_b(t) = v_0(t) + \frac{e^{-j\frac{2\pi}{3}}\vec{v}(t) + e^{j\frac{2\pi}{3}}[\vec{v}(t)]^*}{2}, \tag{6}$$

$$v_c(t) = v_0(t) + \frac{e^{j\frac{2\pi}{3}}\vec{v}(t) + e^{-j\frac{2\pi}{3}}[\vec{v}(t)]^*}{2}. \tag{7}$$

Thus, the boundary condition leads to the following relationship in both time domain and frequency domain.

$$0.5\vec{v} + 0.5(\vec{v})^* + v_0 = 0, \tag{8}$$

$$0.5\vec{i} = 0.5(\vec{i})^* = i_0 = \frac{1}{3}i_a. \tag{9}$$

If the space vector $\vec{v}$ and $\vec{i}$ are viewed to be related with an impedance $Z(s)$, then their conjugates $(\vec{v})^*$ and $(\vec{i})^*$ are related with an impedance $(Z(s^*))^*$, based on the rule of Laplace transform. For an RLC circuit, both $Z(s)$ and $(Z(s^*))^*$ are the same: $R+sL+1/(sC)$. However, if there is a complex coefficient, the two impedances are not the same. For example, the induction machine's rotor equivalent resistance is expressed as $\frac{s}{s-j\omega_m}R_r$, its expression for the conjugate should be $\frac{s}{s+j\omega_m}R_r$.

With the zero-sequence circuit available, the three circuits can be interconnected at the faulted bus to have a series connection. Fig. 2 presents the interconnected circuit model.

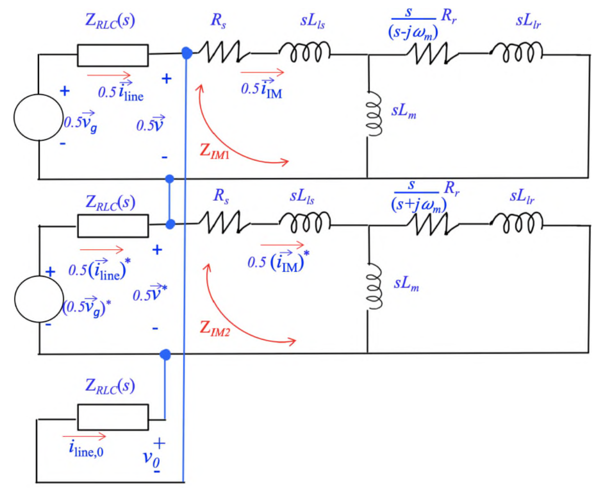


Fig. 2: Circuit 1: The example system subject to an SLG fault.

### B. Circuit 2

The circuit in Fig. 2 is viewed based on the space vectors and the conjugates. At unbalanced conditions, the space vector has both positive- and negative-sequence components. To be able to associate with the steady-state interconnected sequence network, we further derive the circuit for sequence components. The boundary conditions are re-examined. A space vector can be expressed as the sum of the positive- and negative-sequence components:

$$\vec{v}(t) = \vec{v}_1(t) + [\vec{v}_2(t)]^* = \overline{V}_1(t)e^{j\omega t} + \left[\overline{V}_2(t)e^{j\omega t}\right]^* \tag{10}$$

where the subscript 1 and 2 notate positive- and negative-sequences, respectively. Hence the first boundary condition $0.5\vec{v} + 0.5(\vec{v})^* + v_0 = 0$ is equivalent to

$$0.5(\vec{v}_1(t) + [\vec{v}_2(t)]^*) + 0.5([\vec{v}_1(t)]^* + \vec{v}_2(t)) + v_0(t) = 0,$$
$$\Longrightarrow \vec{v}_1 + \vec{v}_2 + v_0 = 0. \tag{11}$$

Another boundary condition $0.5\vec{i} = 0.5(\vec{i})^* = i_0 = \frac{1}{3}i_a$ is equivalent to

$$0.5\vec{i} = 0.5(\vec{i})^* = i_0 \Longrightarrow \vec{i}_1 + [\vec{i}_2]^* = [\vec{i}_1]^* + \vec{i}_2 = 2i_0,$$
$$\Longrightarrow \vec{i}_1 = \vec{i}_2 = i_0. \tag{12}$$

The relations in (11) and (12) hold in both time domain and frequency domain. Based on (11) and (12), the interconnected network is built and shown in Fig. 3. Note for $\vec{v}_1$ and $\vec{i}_1$, the impedance is the same as those for $\vec{v}$ and $\vec{i}$. For $\vec{v}_2$ and $\vec{i}_2$, the impedance is the same as those for $(\vec{v})^*$ and $(\vec{i})^*$. The advantage of the circuit in Fig. 3 is that it can be directly related to the steady-state sequence network. The balanced source voltage only appears in the positive-sequence network.

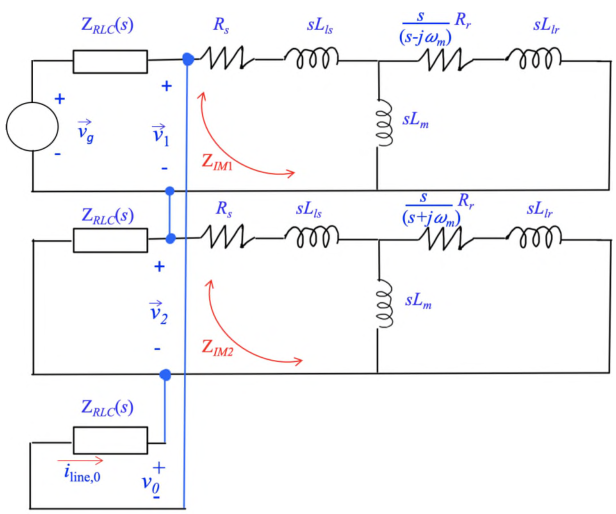


Fig. 3: Circuit 2: The example system subject to an SLG fault.

**Remarks:** (i) The circuit presented in Fig. 3 is a dynamic circuit with the unbalanced topology modeled. If we substitute $s$ by $j\omega$ where $\omega$ is the synchronous frequency, the resulting circuit is the same steady-state circuit for SLG faults. (ii) Indeed, for unbalanced systems that can be represented by a steady-state phasor/impedance-based sequence network, we may directly come up with the corresponding dynamic circuit by replacing $j\omega$ using the Laplace transform variable $s$.

### *C. Stability Analysis*

Compared to the steady-state sequence network which is mainly used for fault analysis, the dynamic circuit is capable of stability analysis. Below is a demonstration.

We ignore the shunt magnetizing branch $sL_m$ for simplicity since its impedance magnitude is one order greater than the rotor impedances in 20-40 Hz range. The total positive-sequence impedance of the IM is

$$Z_{\rm IM1} = R_s + \frac{s}{s - j\omega_m} R_r + s(L_{ls} + L_{lr}). \tag{13}$$

The total negative-sequence impedance is

$$Z_{\rm IM2} = R_s + \frac{s}{s + j\omega_m} R_r + s(L_{ls} + L_{lr}). \tag{14}$$

For the balanced system, the loop gain is

$$L_1(s) = \frac{Z_{\rm IM1}(s)}{Z_{\rm RLC}(s)} = \frac{R_s + \frac{s}{s-j\omega_m} R_r + s(L_{ls} + L_{lr})}{R + sL + \frac{1}{sC}} \tag{15}$$

For the SLG case, the loop gain is

$$L_2(s) = \frac{Z_1(s) + Z_2(s)}{Z_{\rm RLC}(s)}, \tag{16}$$

where $Z_1(s)$ and $Z_2(s)$ are

$$Z_1(s) = \frac{Z_{\rm IM1} Z_{\rm RLC}}{Z_{\rm IM1} + Z_{\rm RLC}}, \quad Z_2(s) = \frac{Z_{\rm IM2} Z_{\rm RLC}}{Z_{\rm IM2} + Z_{\rm RLC}}.$$

Fig. 4 presents the Bode diagrams of the two loop gains. It can be clearly seen that for the balanced system, at about 34 Hz when the phase shifts from $180^\circ$ to $-180^\circ$, $L_1$'s gain is at 0 dB, indicating instability. On the other hand, for the unbalanced system, the loop gain's phase keeps in the range of 0 to 180 degree in the 0-80 Hz range. At -26 Hz when $L_2$'s gain is 0 dB, the phase margin is about 30 degree. Hence, Bode diagram shows no stability issue for the SLG case.

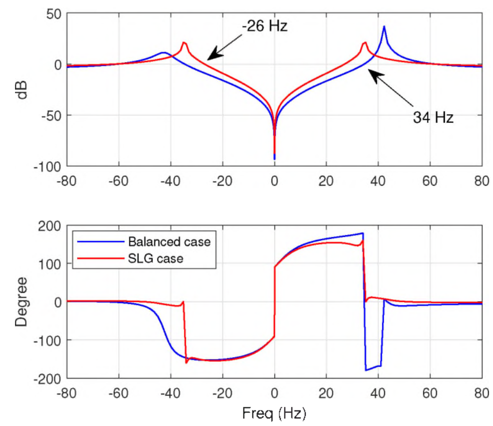


Fig. 4: Loop gains for stability check.

**Remark:** The dynamic circuit in Fig. 3 also reveals that unbalanced topology mitigates the effect of the equivalent rotor resistor $sR_r/(s - j\omega_m)$, which is negative if the excitation frequency is less than 42 Hz (corresponding to 0.7 pu rotating speed). This leads to the improvement of SSR stability.

## IV. Conclusion

In this letter, we demonstrate a concise procedure to construct two dynamic circuits for unbalanced topology. The second circuit is a generalized circuit for unbalanced systems and the well-known steady-state sequence network is a special realization when it is evaluated at the nominal frequency. This research connects dynamic modeling and unbalanced treatment. It further shows that the steady-state sequence network can be directly converted to an $s$-domain dynamic circuit suitable for both fault analysis and stability analysis.

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