

Intrinsic Lateral Inhibition Facilitates Winner-Take-All in Domain Wall Racetrack Arrays for Neuromorphic Computing

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Abstract—Neuromorphic computing is a promising candidate for beyond-von Neumann computer architectures, featuring low power consumption and high parallelism. Lateral inhibition and winner-take-all (WTA) features play a crucial role in neuronal competition of the nervous system as well as neuromorphic hardware. The domain wall - magnetic tunnel junction (DW-MTJ) neuron is an emerging spintronic artificial neuron device exhibiting intrinsic lateral inhibition. In this paper we show that lateral inhibition parameters modulate the neuron firing statistics in a DW-MTJ neuron array, thus emulating *soft-winner-take-all* (WTA) and firing group selection.

Index Terms—magnetism, spintronics, lateral inhibition, winner-take-all, domain wall racetrack, spin-transfer torque, neuromorphic computing

I. INTRODUCTION

Inspired by the signal processing of the brain, neuromorphic computing exceeds classical von Neumann computers in speed and power efficiency, particularly in data-intensive artificial intelligence applications [1] [2]. In the brain, neurons communicate through spike trains and the neuron connectivity are stored in synapses that are adjusted by learning. Numerous CMOS-based neuromorphic hardware have been proposed [3] [4] [5], but they lack key biological features for learning in the brain that would realize the full potential of neuromorphic circuits and networks [6].

Spintronic devices known for their small footprints, high endurance, low power consumption and highly tunable spin

dynamics are promising candidates for neuromorphic computing hardware [7]. Spintronic neuromorphic devices are closely modeled after their biological counterparts in both structure and functionality, leveraging non-volatility, non-linearity, stochasticity, and phenomena such as synchronization for efficient computing [8] [9] [10]. They have been shown to emulate the integrate-and-fire (IF) neurons [11], resonate-and-fire neurons [12] and memristive synapses [13] [14] [15].

However, unsupervised learning algorithms require additional functions, such as winner-take-all (WTA) [16], to be incorporated into spintronic neuromorphic devices. Here, we present a simulation study of an important mechanism of WTA, namely *lateral inhibition*, in an array of domain wall-magnetic tunnel junction (DW-MTJ) neurons. While it has been proposed that an array of co-integrated DW-MTJ synapses and neurons can perform online learning by combining plastic synapse updates with the behavior of interacting neurons [17], and that this approach can achieve natural clustering or unsupervised learning on small tasks [18], these results assumed a coarse-grained model of lateral interaction. In this study, we expand our understanding of lateral inhibition, informing new directions for optimized DW-MTJ neuron arrays. We will show that lateral inhibition strength and neuron array layout can modulate neuron firing statistics, which is potentially useful in implementing unsupervised learning algorithms.

II. WINNER-TAKE-ALL AND LATERAL INHIBITION

In the nervous system, it is of great importance that neuronal activities generate meaningful representations of external stimuli. This requires the selective firings of only a subset of the neuron population; otherwise, an explosive epileptic state will occur [19] [20]. One mechanism to regulate neuron firing is neuronal competition under the winner-take-all (WTA) rule, which dictates that only the most strongly stimulated member(s) of the neuron ensemble can win out and fire. Competitive learning algorithms incorporate the WTA function

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to choose one or more neurons that are best matched with the input stimuli to participate in synaptic weight updates [21].

The various mathematical WTA models are categorized based on the number of winners and the form of output: *Hard*-WTA selects only one neuron as the winner; two other models, namely the *k*-WTA and *soft*-WTA, are shown to have superior computational power [22]. *k*-WTA chooses $k > 1$ neurons as winners and is a closer emulation of the result of biological neuron competition than is *hard*-WTA, since it supports “distributed representation” which encodes sensory information not in one but in a group of neurons [19] [23]. Like *k*-WTA, *soft*-WTA produces multiple winners; but their outputs are analog and proportional to the strengths of stimuli. *Soft*-WTA allows all competitors to be updated based on their performances, and was shown to achieve higher accuracy in classification task than *hard*-WTA [24]. In unsupervised learning, *soft*-WTA are shown to enable efficient Hebbian learning [21] and autonomous pattern recognition [25].

One biologically plausible mechanism of facilitating WTA is *lateral inhibition*, by which the most strongly stimulated neurons win out by suppressing the activities of the less active competitors. In visual, auditory and somatosensory cortices, lateral inhibition enhances the contrast of neighboring cells in the receptive fields [26]. In WTA circuits, lateral inhibition can be implicitly modeled by properly biasing the transistors, or be explicitly implemented by an inhibitory interneuron [27] [28]. The WTA-via-lateral inhibition feature has been realized in CMOS VLSI circuits [29] [30] [31] as well as in hybrid CMOS-memristor crossbar arrays [32]. However, these implementations require complex peripheral circuitry that is area- and energy-expensive especially for large-scale networks. To attempt a solution to this problem, we next show that in a spintronic spiking neuron, intrinsic lateral inhibition can be efficiently facilitated by magnetostatic interaction, thereby simplifying neuromorphic hardware design and fabrication.

III. DW-MTJ INTEGRATE-AND-FIRE NEURON WITH INTRINSIC LATERAL INHIBITION

The domain wall - magnetic tunnel junction (DW-MTJ) neuron is an artificial IF neuron based on the three-terminal MTJ logic device [33]. It consists of a magnetic racetrack for DW motion and an MTJ for spike readout (Fig. 1). DW position and velocity encode neuron activity and the MTJ position defines the neuron firing point. During the *integration* phase (Fig. 1(a)), the current-driven DW propagates towards the MTJ due to spin-transfer torque (STT) or spin-orbit torque (SOT); once the DW passes under the MTJ, its magnetoresistance (MR) is switched low and an output current spike is produced, emulating the *firing* of the neuron (Fig. 1(b)). Therefore, the neuron with higher DW velocity fires first and is more active. The DW-MTJ neuron has high energy efficiency [11] [34], and simulation have also shown that leaking [35] and lateral inhibition [36] [37] can be implemented with little energy overhead.

Lateral inhibition of the DW-MTJ neuron is manifested in an enhanced DW velocity contrast: an active neuron delays or

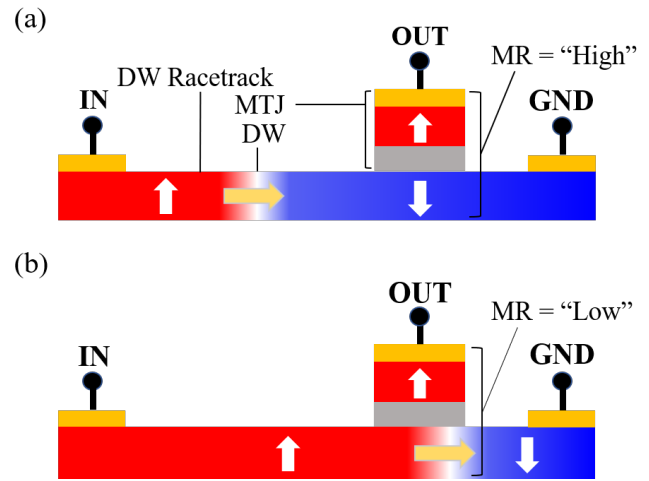


Fig. 1. Structure and functioning of the domain wall - magnetic tunnel junction (DW-MTJ) integrate-and-fire (IF) neuron. (a) integration: DW propagates towards but has not yet reached the firing point (MTJ) and the magnetoresistance (MR) state remains “high”. (b) fire: the DW reaches the firing point and the MR is switched “low”, generating an output current spike.

prevents the firing of its less active neighbor by further decreasing its DW velocity. The Landau-Lifshitz-Gilbert (LLG) equation and Walker’s formulation of DW motion [38] dictate that DW velocity can be controlled by external magnetic field. The magnetostatic interaction between a pair of DW-MTJ neurons is shown in Fig. 2. Here, DW_I and DW_N propagate along $+x$ with velocities $v_{DWI} < v_{DWN}$. Neuron N exerts a magnetic stray field along $-z$ originating from its $+z$ domain on DW_I ; reciprocally, DW_N experiences a stray field along $+z$ originating from the $-z$ domain of Neuron I. It has been shown that only the $-z$ stray field is critical to lateral inhibition, and by optimizing the stray field magnitude as well as the device geometrical and material parameters, an up to 90% reduction of DW_I velocity (*i.e.* 90% lateral inhibition) is achieved [37]. Since the DW-MTJ neuron is capable of performing lateral inhibition without electrical connections, this implementation is scalable and energy-efficient.

IV. LATERAL INHIBITION IN DW-MTJ NEURON ARRAYS

We next extend the discussion to the lateral inhibition in a one-dimensional DW-MTJ neuron array. Although the magnetic field acting on a neuron is the sum of stray fields of all other neurons, here we only consider the contribution from its two immediate neighbors. This simplification is justified by the rapidly decreasing stray field amplitude with increasing distances, validated in [37]. We focus on two types of neuron array layouts: (a) the neurons are evenly arranged with nearest neighbor lateral distance s (Fig. 3(a)) and (b) the neurons are arranged with alternating lateral distances s_1 and s_2 ($s_1 < s_2$) (Fig. 3(b)). For layout (a), the two nearest neighbors contribute equally to the net stray field exerted on the center neuron. In this case, there is only one inhibition condition (Case A) when both neighbors are more active than the center neuron; otherwise, the net stray field is either zero or along $+z$ because

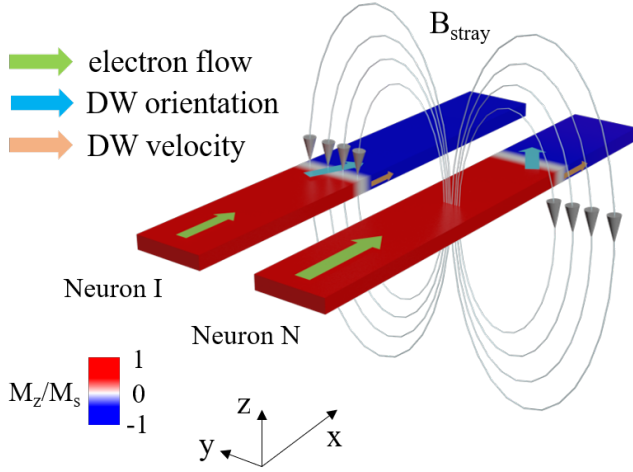


Fig. 2. Magnetostatic interaction and lateral inhibition of a pair of side-by-side DW-MTJ neurons (only DW racetracks are shown). The more active Neuron N impedes the motion of DW₁ by a magnetic stray field in -z, thereby mimicking lateral inhibition.

of the symmetry of the neighbor locations. Layout (b), on the other hand, allows for two inhibition conditions: Case B, the center neuron is less active than both neighbors, similar to Case A; Case C, the center neuron is less active than its close neighbor N_1 but more active than its far neighbor N_2 . As discussed in [37], inhibition strengths of Case B and C are generally different, and only one of them can be optimized by choosing s_1 and s_2 .

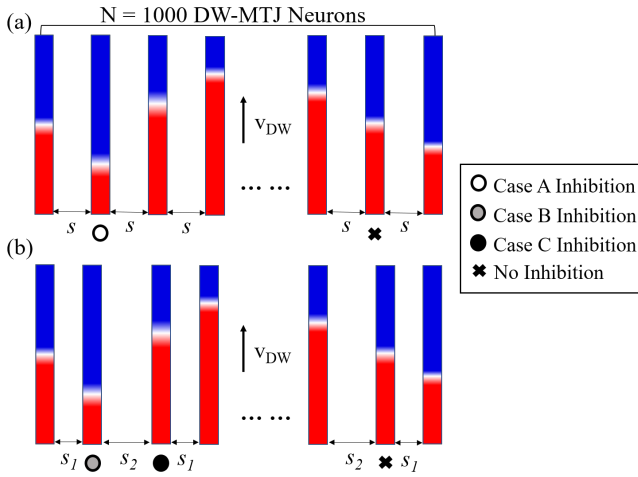


Fig. 3. Two layouts of an one-dimensional array of $N = 1000$ DW-MTJ neurons and lateral inhibition conditions. (a) Nearest neighbors are arranged with lateral distance s ; (b) nearest neighbors are arranged with alternating lateral distances s_1 and s_2 . Neurons with Case A, B, C inhibition and no inhibition are marked.

The DW-MTJ array investigated consists of $N = 1000$ neurons. We focus on the change of DW velocity distribution in the array due to lateral inhibition: $\{v_i\} \mapsto \{v'_i\}$, instead of tracking the states of any specific neurons. A series of 1000

uniformly distributed DW velocity values $\{v_i\}$ in the range $[v_{\min}, v_{\max}] = [1, 3]$ (a.u. and proportional to input charge current densities) are randomly assigned to the neurons. For array layout (a), the inhibition strength of Case A is parameterized by the velocity reduction Δv_A ; for array layout (b), inhibition strengths are parameterized by a pair $(\Delta v_B, \Delta v_C)$ corresponding to Case B and C, respectively. $\{v'_i\}$ is calculated as follows:

$$\{v_i\} \mapsto \{v'_i\} : v'_i = \begin{cases} v_i - \Delta v_A, & \text{for Case A inhibition} \\ v_i - \Delta v_B, & \text{for Case B inhibition} \\ v_i - \Delta v_C, & \text{for Case C inhibition} \\ v_i & \text{otherwise} \end{cases}$$

The calculated $\{v'_i\}$ are ranked in ascending order to yield the modified velocity distribution due to lateral inhibition. Noting that sufficiently strong inhibition may change the signs of DW velocity, we further categorize the inhibition into *weak* and *strong* inhibition based on the values of Δv :

$$\begin{aligned} \forall \Delta v \in \{\Delta v_A, \Delta v_B, \Delta v_C\} < v_{\min}, & \text{ weak inhibition} \\ \exists \Delta v \in \{\Delta v_A, \Delta v_B, \Delta v_C\} \geq v_{\min}, & \text{ strong inhibition} \end{aligned}$$

We first study the *weak* inhibition. $\{v'_i\}$ are calculated with inhibition strengths $\Delta v_A, \Delta v_B, \Delta v_C$ summarized in Table. 1. Fig. 4(a) compares the $\{v'_i\}$ due to lateral inhibition of neuron layouts (a) (solid lines) and (b) (dashed lines). For both types of array layouts, lowering of DW velocities from $\{v_i\}$ to $\{v'_i\}$ becomes more significant with larger inhibition strength. Notably, layout (a) results in non-linear $\{v'_i\}$: the low-velocity range shows the largest overall reduction, while the high-velocity range remains largely unchanged. This is due to the larger inhibition probability of the low-velocity (inactive) DWs, consistent with the inhibition mechanism described above. The non-linearity of the $\{v'_i\}$ increases with stronger inhibition. Layout (b), on the other hand, largely maintains the linearity of $\{v'_i\}$, since it improves the uniformity of inhibition probability across the whole DW velocity range as compared to layout (a). As visible, the weak inhibition does not strictly prohibit the firing of any members of the neuron array, but instead delays the firing of its inactive members.

TABLE I
INHIBITION STRENGTHS

Weak inhibition (Fig. 4(a))		
Layout (a)	Δv_A	{0.9, 0.7, 0.5, 0.3, 0.1}
Layout (b)	Δv_B	{0.9, 0.7, 0.5, 0.3, 0.1}
	Δv_C	{0.8, 0.6, 0.4, 0.2, 0}
Strong inhibition (Fig. 4(b))		
Layout (a)	Δv_A	{2.5, 2.2, 1.9, 1.6, 1.3, 1.0}
Layout (b)	Δv_B	{2.5, 2.2, 1.9, 1.6, 1.3, 1.0}
	Δv_C	{2.4, 2.1, 1.8, 1.5, 1.2, 0.9}

We next study the *strong* inhibition. We again calculate $\{v'_i\}$ with inhibition strengths summarized in Table. 1 and results plotted in Fig. 4(b). In this case, the distribution of $\{v'_i\}$ of both layouts (a) and (b) are highly non-linear due to the large inhibition strength, and as in the *weak* inhibition case, layout

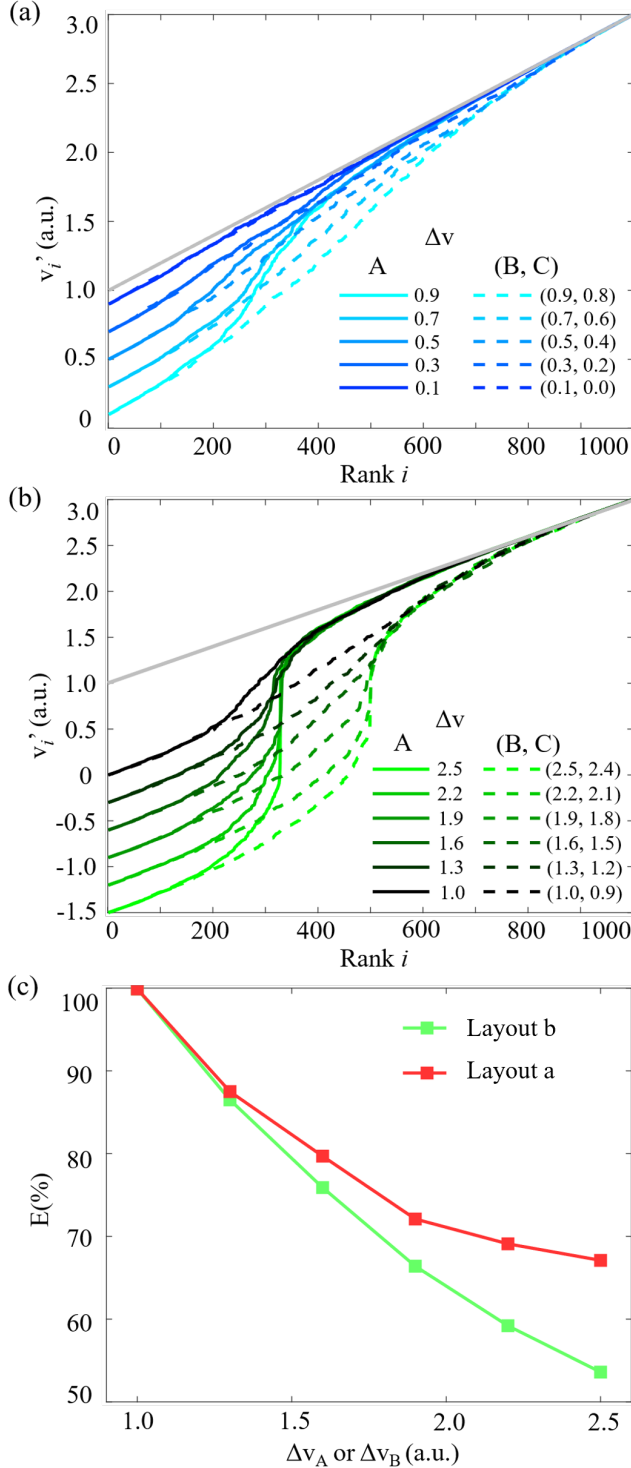


Fig. 4. DW velocity distribution change $\{v_i\} \mapsto \{v_i'\}$ due to lateral inhibition in an array of $N = 1000$ DW-MTJ neurons. (a) weak inhibition and (b) strong inhibition. Solid lines: layout (a); dashed lines: layout (b). $\{v_i\}$ are plotted in grey solid lines in both (a) and (b). (c) Neuron firing proportion $E\%$ versus inhibition strength, calculated from (b). Red: layout (a); green: layout (b). For the definitions of parameters Δv_A , Δv_B and Δv_C see text.

(b) more effectively reduces the overall DW velocities than does layout (a). Negative v_i' 's indicate that the corresponding neurons are prohibited from firing. We calculate the neuron firing proportion E of the array from the $\{v_i'\}$ of Fig. 4(b), shown in Fig. 4(c). The firing proportion E is less than unity when $\Delta v_A(\Delta v_B)$ is larger than v_{\min} and monotonically decreases as inhibition strength becomes larger. The effect is more prominent for neuron layout (b), with E as low as 53%.

Using a simple inhibition model with nearest neighbor interaction assumption, we are able to show the relation between neuron activity, firing statistics and lateral inhibition strengths. We can draw a direct comparison between *weak* inhibition and *soft*-WTA, and between *strong* inhibition and *k*-WTA. When inhibition is weak, all neurons in the array are allowed to fire, but the DW velocity contrast in the array is enhanced by lateral inhibition. In this case, the activities or performances of the neurons can be inferred from their firing times. When inhibition is strong, besides delaying the firing of the inactive members of the array, it can forbid some of them from firing and thus control the neuron firing proportion of the array. It is also worth noting that the lateral inhibition of DW-MTJ neuron can be effectively tuned by field, current and device materials [37], which endows the neuron array with additional tunability that may be explored in future works.

V. PERSPECTIVES AND CONCLUSIONS

Here we comment on the fabrication feasibility of the proposed DW-MTJ neuron WTA array. The fabrication procedure of the DW-MTJ neuron array is a straightforward extension of that of a single neuron, as described in [39]. The rich palette of magnetic thin film materials further allows us to tune important magnetic parameters, *e.g.* magnetic anisotropy, to meet the device specifications for particular applications. Besides using MTJ for neuron spike readout, the DW positions in the racetracks can be imaged in real time with magneto-optic Kerr effect (MOKE) microscopy to reveal the details of DW dynamics [40].

We show that lateral inhibition facilitates winner-take-all (WTA) in a DW-MTJ neuron array. Lateral inhibition of the DW-MTJ neuron arises from magnetostatic interaction and modulates the WTA behaviors of the neuron array: a small inhibition strength lowers the overall DW velocities of the neuron members but does not prohibit their firing, mimicking the *soft*-WTA; a large inhibition strength reverses the motion of some DWs and strictly prohibits the corresponding neurons from firing, reducing the group firing proportion E down to 53% in our simulations, mimicking *k*-WTA. In addition, the WTA characteristics are also dependent on neuron array layout: the non-evenly arranged neuron array yields a stronger overall inhibition than does the evenly-arranged array. Our proposed lateral inhibition model provides a novel mechanism for implementing *soft*-WTA and group selection in spiking neural networks, thus potentially powering efficient competitive learning algorithms.

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