Measuring Topological Invariants within Dissipatively-Coupled Lattices

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Abstract: We demonstrate Bloch oscillations in a dissipatively-coupled network of time-multiplexed resonators, and experimentally measure the topological invariants in open systems. This reveals the complex interplay between topology and dissipation with potential applications in quantum/classical photonics.

Recently, there has been a great deal of interest in the emerging field of topological physics, which aims to understand and harness unique properties originally encountered in condensed matter physics. While topological phenomena are typically studied in closed systems, recent works on open systems has led to an array of intriguing effects ranging from robust topological lasing [1,2] to the emergence of non-Hermitian bulk-edge correspondence [3]. Despite growing theoretical efforts on dissipative systems, experimental measurements of geometric phases and their associated topological invariants have almost exclusively [4] occurred in systems with conservative couplings. Given the fundamental significance of such invariants in proving topologically nontrivial behaviors in open systems, experiments for the direct measurement of such quantities are of great importance.

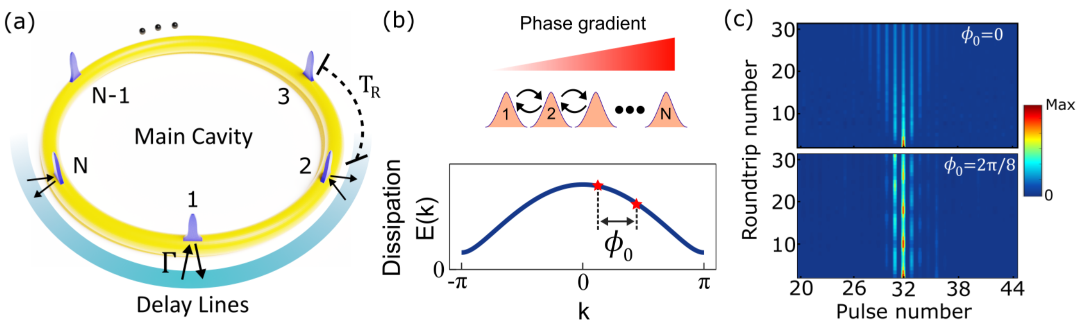

Fig. 1: **Experimental demonstration of Bloch oscillations (BO) in a network of time-multiplexed resonators (a)** Schematic diagram of a network of time-multiplexed resonators, where each resonator is represented by an individual pulse. **(b)** By imposing a uniform phase gradient between successive pulses (pulse-to-pulse phase shift of ϕ_0), equivalent to an effective force, the Bloch momentum shifts by $\Delta k = \phi_0$ per cavity roundtrip, leading to BOs. **(c)** Experimental measurements where in the absence of the effective force ($\phi_0 = 0$) light undergoes dissipative discrete diffractions in this synthetic lattice (top). However, when a nonzero phase gradient is established, optical power exhibits an oscillatory pattern with a Bloch period equal to $N_B = 8$ (bottom).

Here, we use a network of time-multiplexed optical resonators to implement photonic lattices with purely dissipative couplings. In accordance with our theoretical predictions, we experimentally demonstrate Bloch oscillations (BOs) in such open systems. We then use these BOs to directly measure the geometric Zak phase and the winding number associated with the different dimerizations of a dissipative Su-Schrieffer-Heeger (SSH) model.

Our time-multiplexed photonic resonator network, as shown schematically in Fig. 1(a), consists of a main fiber loop (the "Main Cavity"), which supports $N = 64$ resonant pulses separated by a repetition period, T_{R}. Each pulse represents an individual resonator, the dynamics of which is governed by the following Lindblad master equation:

$$\frac{d}{dt}\hat{\rho} = \mathscr{L}\hat{\rho} \equiv -i[\hat{H},\hat{\rho}] + \sum_j \mathscr{D}[\hat{L}_j]\hat{\rho}, \qquad (1)$$

where $\hat{\rho}$ denotes the system density operator, $\hat{H}$ is the system Hamiltonian and $\mathscr{D}[\hat{L}_j] = \hat{L}_j\hat{\rho}\hat{L}_j^\dagger - 1/2\{\hat{L}_j^\dagger\hat{L}_j,\hat{\rho}\}$ is the dissipator resulting from the nonlocal jump operators $\hat{L}_j$ acting on the lattice site j. By properly choosing

$\mathscr{D}[L_j]$, the Lindbladian of Eq. 1 supports Bloch eigenstates exhibiting bands of dissipation rates in the reciprocal space [5].

To experimentally demonstrate BOs, we construct a 1D lattice with uniform nearest-neighbor dissipative couplings (Fig. 1(b)) using jump operators $\hat{L}_j = \sqrt{\Gamma}(\hat{c}_j + \hat{c}_{j+1})$, where $\hat{c}_j^{(\dagger)}$ represents annihilation (creation) operators associated with site j. To implement the jump operators $\hat{L}_j$, we use delay lines which are equipped with intensity modulators that control the strengths of the dissipative couplings (Fig. 1(a)). We incorporate a phase modulator (PM) in the main cavity to realize the Hamiltonian $\hat{H}_{\mathrm{BO}} = \vec{F}.\vec{r}$, where $\vec{F}$ represents a constant effective force along the lattice, necessary for BOs. We excite a single lattice site in the network and monitor its evolution under different pulse-to-pulse phase gradients ϕ_0, corresponding to different BO periods (see Fig. 1(b)). As shown in Fig. 1(c), upper panel, when $\phi_0 = 0$, we observe in the experiment that the initial excitation undergoes dissipative discrete diffraction, corresponding to the spread of the energy in the lattice. However, when $\phi_0 = 2\pi/8$ which corresponds to a Bloch period of 8 network roundtrips, the initial excitation experiences periodic spreading and refocusing (Fig. 1(c) lower panel), which is the hallmark of BOs.

We use BOs to measure the topological winding number associated with an open SSH lattice. For this purpose, the intensity modulators in the delay lines are programmed to implement the staggered couplings of the SSH model $\hat{L}_{A,j} = \sqrt{\Gamma_A}(\hat{c}_{A,j} + \hat{c}_{B,j})$ and $\hat{L}_{B,j} = \sqrt{\Gamma_B}(\hat{c}_{A,j+1} + \hat{c}_{B,j})$, where A and B represent the two sublattices in the structure. The resulting Lindbladian exhibits a dissipative band structure that features a topologically nontrivial bandgap (Fig. 2(a)). We examine the geometric phase acquired by a Bloch eigenstate as it moves across the Brillouin zone in the upper band with the intracavity PM programmed to impart a pulse-to-pulse phase shift of $\phi_0 = 2\pi/8$. After a complete Bloch period, we measure the amplitudes and phases of the lattice sites using a 90° optical hybrid. Since the dissipative dynamics of our system do not impart a dynamical phase to the eigenstates, the relative phases between the final state and that of the originally launched pulses provide a direct measurement of the upper-band Zak phase. Figure 2(b) shows experimentally measured values of the Zak phase for different dimerizations of the SSH model. Based on these results, when $\Gamma_A = 2\Gamma_B$, we observe $\phi_{Z1} \approx 0.49\pi$ and $\phi_{Z2} \approx -0.46\pi$ for the two possible dimerizations D_1 and D_2 depicted in Fig. 2(b), respectively. Therefore, the absolute value of the Zak phase in this open topological system is measured to be $\phi_Z = \phi_{Z1} - \phi_{Z2} \approx 0.95\pi$, which is in agreement with the theoretically expected value of $\phi_Z = \pi$ for a topologically nontrivial SSH model, corresponding to a winding number of $W = 1$.

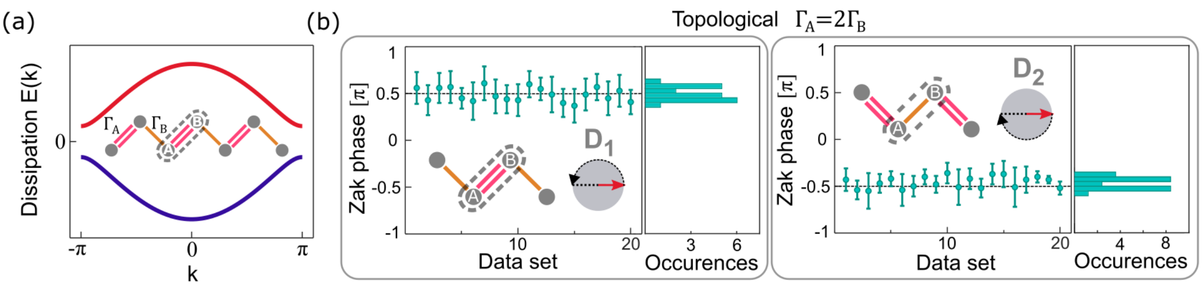

Fig. 2: **Experimental measurement of the geometric Zak phase in a dissipative SSH model using Bloch oscillations (a)** Schematic diagram of an SSH lattice with two different couplings $\Gamma_A = 2\Gamma_B$ together with its associated dissipation bands. **(b)** Experimentally measured Zak phases associated with the upper band when $\Gamma_A = 2\Gamma_B$ and for two different dimerizations shown in each case. Each data set represents various unit cells (shown in dashed boxes) within a single measurement.

In summary, we have experimentally demonstrated Bloch oscillations and the direct measurement of the geometric Zak phase and the winding number associated with a dissipative SSH model using a network of dissipatively coupled time-multiplexed resonators. Our results can open new avenues for implementing topological phases in open, dissipative systems in the quantum and classical regimes.

The authors gratefully ackowledge the support of ARO Grant No. W911NF-18-1-0285, NSF Grants No. 1846273 and 1918549, and NTT Research Inc.

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