Efficient Simulation of Broadband Non-Gaussian Quantum Optics Using Matrix Product States

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Abstract: We realize efficient full-quantum simulations of pulse propagation in highly nonlinear waveguides using matrix product states. As a demonstration, we study the quantum dynamics of an optical soliton, highlighting the emergence of non-Gaussian quantum features. © 2022 The Author(s)

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Harnessing the quantum nature of photons holds the key to performance beyond classical limitations in various applications [1], for which strong optical nonlinearity is a vital resource. To this end, ultrafast pulses propagating in highly nonlinear waveguides provide an all-optical pathway to strong nonlinearity by simultaneously leveraging spatial and temporal field confinements. Notably, recent advances in the fabrication of nanophotonics suggest that experimental capabilities are approaching the regime where even significant non-Gaussian quantum features, a necessary resource for continuous-variable (CV) quantum information processing, for instance, could emerge during the *coherent* dynamics of pulse propagation [2]. Understanding such quantum dynamics of photons is, however, a highly nontrivial task because of the exponentially large Hilbert space naïvely required to describe multimode non-Gaussian quantum states. Therefore, to fully leverage the potential of the emerging quantum photonic devices, it is essential to develop novel model reduction techniques to enable tractable numerical studies.

As illustrated in Fig. 1(A), the complexity of a CV quantum system is composed of three ingredients: (i) quantum non-Gaussianity, (ii) a large number of modes, and (iii) strong long-range entanglement, and the system dynamics can become intractable only when all of them coexist. Without (i), polynomial-time simulation is possible within the Gaussian quantum optics. Without (ii), we can fully diagonalize the low-dimensional Hamiltonian numerically. Unfortunately, quantum pulse propagation in a waveguide does not belong to either of these scenarios due to the involvement of many frequency modes and non-Gaussian quantum dynamics. Notably, however, condition (iii) does not necessarily hold for photons in a 1D waveguide because of the locality of interactions. In fact, it

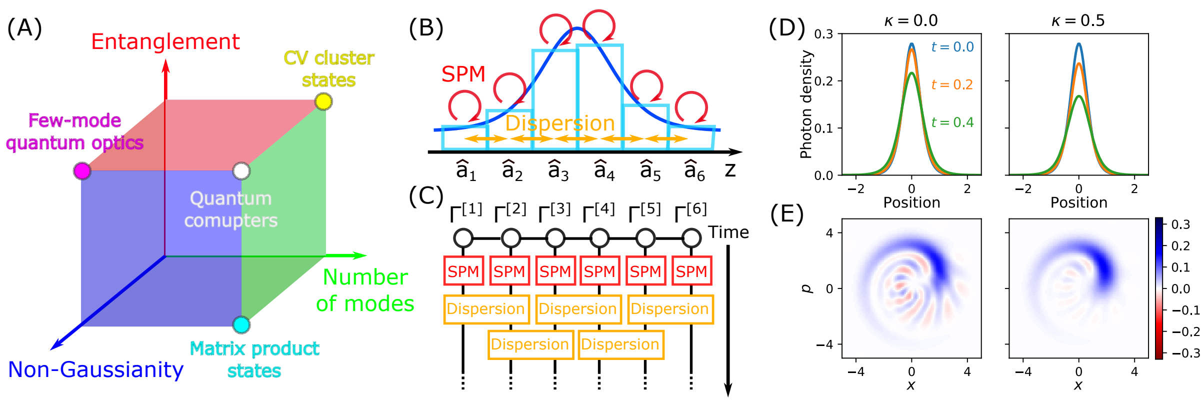

Fig. 1. (A) Three ingredients constituting the complexity of CV quantum systems. At the corners marked by circles, we show constituent quantum states and systems with corresponding ingredients. (B) Schematics of how discretized optical fields interact. (C) An example implementation of the TEBD. (D) Time evolution of the pulse envelope and (E) Wigner functions of the quantum state in the solitonic mode at $t = 0.4$ simulated for a soliton propagation with $\bar{n} = 5.0$. Simulation results with homogeneous linear loss with power attenuation rate $\kappa = 0.5$ are shown as well.

is heuristically known that 1D many-body quantum systems exhibit a limited amount of entanglement [3], which we exploit in our recent work with the use of matrix product states (MPS) to realize efficient simulations [4].

Discussions below follow Ref. [4]. For the purpose of concreteness, we consider a $\chi^{(3)}$ nonlinear waveguide under appropriate normalization of time and space, whose Hamiltonian takes a form

$$\hat{H} = -\frac{1}{2}\int \mathrm{d}z \left(\hat{\phi}_z^\dagger \partial_z^2 \hat{\phi}_z + \hat{\phi}_z^{\dagger 2}\hat{\phi}_z^2\right), \tag{1}$$

where $\hat{\phi}_z$ is a local field annihilation operator with commutation relation $[\hat{\phi}_z, \hat{\phi}_{z'}^\dagger] = \delta(z-z')$. Notice that classical dynamics under (1) follow canonical nonlinear Schrödinger equation (NLSE) $\mathrm{i}\partial_t \phi_z = -\frac{1}{2}\partial_z^2\phi_z - |\phi_z|^2\phi_z$. For numerical evaluations, we consider a limited spatial interval $-L/2 \leq z \leq L/2$ and discretize the space into N spatial bins. In this picture, as depicted in Fig. 1(B), nonlinear interactions act as local self-phase modulation (SPM), while the dispersion becomes linear couplings between nearest-neighbor bins. A generic quantum state is written as $|\psi\rangle = \sum_J c_J |J\rangle$ with $J = (j_1, j_2, \cdots, j_N)^\intercal$ and $|J\rangle = |j_1\rangle \otimes |j_2\rangle \otimes \cdots |j_N\rangle$, where each local Hilbert space is spanned by Fock states $|j_m\rangle$ $(j_m \geq 0)$. Up to this point, our treatment is exact given large enough N, but as a result, our state representation c_J retains the exponential complexity. To realize efficient polynomial-time simulation that benefits from the limited system entanglement, we introduce an MPS representation of c_J as [3]

$$c_J \approx \sum_{\alpha_1,\cdots,\alpha_{N-1}} \Gamma^{[1]j_1}_{1\alpha_1}\lambda^{[1]}_{\alpha_1}\Gamma^{[2]j_2}_{\alpha_1\alpha_2}\lambda^{[2]}_{\alpha_2}\cdots\lambda^{[N-1]}_{\alpha_{N-1}}\Gamma^{[N]j_N}_{\alpha_{N-1}1}. \tag{2}$$

Here, $\Gamma^{[m]}$ is a rank-3 tensor, $\lambda^{[m]}$ is a vector, and α_m runs from 1 to χ. Notice that parameters required to describe (2) have a polynomial scaling $\mathscr{O}(\chi^2 N)$, making it an ideal state representation if reasonably small bond dimension χ suffices. Generally, quantum states with longer range entanglement require larger χ for their accurate description. Notably, it is heuristically known that 1D quantum many-body systems exhibit a limited entanglement, making MPS an ideal state representation for quantum pulse propagation. Time evolution of an MPS under the system Hamiltonian can be implemented using time-evolving block decimation (TEBD) [3], whose machinery is shown in Fig. 1(C); Unitary evolution under the Hamiltonian is Trotter decomposed into one-mode and two-mode quantum gate operations, whose actions on an MPS can be efficiently evaluated with a numerical cost of $\mathscr{O}(\chi^3)$.

As a demonstration, we perform full-quantum simulations of the dynamics of optical solitons. The classical NLSE supports a coherent state soliton solution $\phi_z^{(\mathrm{sech})}(t) = \frac{\bar{n}}{2}e^{\mathrm{i}\bar{n}^2 t/8}\mathrm{sech}\frac{\bar{n}z}{2}$, where $\bar{n}$ is the average photon number under the envelope. While the soliton is completely stationary under the classical dynamics, quantum mechanical effects can lead to various non-classical dynamics, including squeezing and diffusion of the optical fields [5]. In Fig. 1(D), we show the time-evolution of the a pulse instantiated in the form of a few-photon coherent-state soliton, where we clearly observe the "quantum-induced" diffusion of the pulse envelope. To further unravel the quantum features of the soliton dynamics, we consider the solitonic supermode defined by a supermode operator $\hat{A}^{(\mathrm{sech})} = \int \mathrm{d}z\, f_z^{(\mathrm{sech})}\hat{\phi}_z$ with $f_z^{(\mathrm{sech})} \propto \phi_z^{(\mathrm{sech})}(0)$. To access the non-local information about the supermode encoded in an MPS, we develop a novel quantum circuit to demultiplex constituent supermodes to local spatial bins [4]. In Fig. 1(E), we show the Wigner function of the quantum state in the solitonic supermode, where we observe a significant amount of Wigner function negativities, indicating that the soliton physics in this highly nonlinear regime is beyond the conventional framework of Gaussian quantum optics. Such genuine quantum dynamics of solitons, when harnessed properly, can be used to implement non-Gaussian quantum gates for CV quantum computation [6], for instance. Also, our approach can readily incorporate realistic optical loss by means of quantum trajectory theory, and as shown in the figure, such dissipation critically diminishes the non-classical features, e.g., Wigner function negativity.

In this work, we have proposed and demonstrated the use of MPS to realize efficient full-quantum simulations of pulse propagation in highly nonlinear waveguides. We expect our work to establish MPS as an essential toolbox for the emerging field of broadband non-Gaussian quantum optics.

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