MIDDLE SCHOOL STUDENTS' MATURE NUMBER SENSE IS UNIQUELY ASSOCIATED WITH GRADE LEVEL MATHEMATICS ACHIEVEMENT

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Students exhibiting mature number sense make sense of numbers and operations, use reasoning to notice patterns, and flexibly select the most effective and efficient problem-solving strategies (McIntosh et al., 1997; Reys et al., 1999; Yang, 2005). Despite being highlighted in national standards and policy documents (CCSS, 2010; NCTM, 2000, 2014), students' mature number sense and its nomological network are not yet well specified. For example, how does students' mature number sense relate to their knowledge of fractions and their grade-level mathematics achievement? We analyzed 129 middle school students' scores on measures of mature number sense, fraction and decimal computation, and grade-level mathematics achievement. We found mature number sense to be measurably distinct from their fraction and decimal knowledge and uniquely associated with students' grade-level mathematics achievement.

Keywords: Number Concepts and Operations; Cognition; Assessment; Rational Numbers

Over the last three decades, "number sense" has emerged as a central goal of mathematics education. While high quality mathematics teachers were certainly already teaching in ways that helped students make sense of number and operations before then, "number sense" was not yet a commonly used term in mathematics education. In the late 1980's, government commissioned reports (e.g., National Research Council, 1989) and national curriculum frameworks (e.g., National Council of Teachers of Mathematics (NCTM), 1989) highlighted "number sense" as a core objective of K-12 mathematics education. Students showing strong mature number sense exhibit the disposition to *make sense of numerical situations* and use a rich conceptual understanding of number and operations to flexibly solve problems.

Despite being a central objective in mathematics education, there is little evidence in the literature on how students' mature number sense relates to other important psychological constructs in mathematics education. For example, how do students' levels of mature number sense relate to their grade-level mathematics achievement or their understanding of fractions? Without an understanding of the nomological network, it is harder for researchers to explain how students construct mature number sense, leaving educators without rigorous evidence on how best to address it within their own classrooms. In this study, we aimed to begin to address this gap in the literature by examining how middle school students' mature number sense relates to other theoretically related constructs, testing the hypothesis that mature number sense is both distinct from fraction and decimal computation and grade-level mathematics achievement and uniquely associated with students' grade-level mathematics achievement.

Background and Theoretical Framework

Number sense is a difficult construct to define (e.g., Berch, 2005). How the construct is operationalized and assessed looks very different across the disciplines of cognitive and developmental psychology, mathematics education, and special education (summarized in Whitacre et al., 2020). In this project, our construct of interest is what Whitacre et al. (2020) have termed *mature number sense*, which we define in line with McIntosh et al. (1992) as "a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways." This focus is distinct from research on approximate number sense (e.g., Dehaene, 2001) or early number sense (e.g., Jordan et al., 2009).

Perhaps to help facilitate operationalizing it in empirical research, mature number sense has often been subdivided into components (McIntosh et al., 1997; NCTM, 1989; Reys et al., 1999; Yang & Lin, 2015). Building from this prior work and in consultation with expert mathematics teachers, teacher educators, and mathematics education researchers, we specified a four-component framework of mature number sense (Kirkland et al., *under review*) as follows:

1. Understanding the effect of operations on numbers: Students demonstrating strong mature number sense recognize how the four core operations affect whole numbers as well as fractions and decimals. They understand that patterns observed with operations with whole numbers may not hold true for fractions and decimals between 0 and 1. Students use this understanding to efficiently estimate computational results and ensure the results make sense given the relationship between operations and rational numbers.
2. Using multiple representations of a number: Students demonstrating strong mature number sense are able to translate among multiple representations of rational numbers efficiently and flexibly to solve problems. Students recognize that fractions are numbers, for example, rather than separately perceiving the numerator and denominator. Students are able to use this understanding to translate between representations to make sense of numerical situations.
3. Understanding mathematical equivalence: Students demonstrating strong mature number sense understand the equal sign as a relational symbol, reflecting that the two sides of an equation are equal and interchangeable. Rather than approaching equations with an "operational" approach, students understanding equivalence recognize patterns across the equal sign and use this relational thinking to flexibly solve problems (c.f. Jacobs et al., 2007).
4. Understanding basic number concepts and number magnitude: Students demonstrating strong mature number sense have a rich conceptual understanding of fractions, decimals, and whole numbers. Students use their understanding of place value and rational number magnitude to efficiently estimate results using concepts such as unit fractions.

We hypothesized, similar to Yang (2019), that mature number sense would be an overarching hierarchical latent construct, with the four components each theoretically related to mature number sense. That is, a student's understanding of the effect of operations on a number reflects their mature number sense as well as a more specific understanding of how operations affect computational results. In our initial development of a brief assessment of mature number sense (Kirkland et al., *under review*), the structure of the response data reflected this hypothesized framework. A bifactor model with each component as a specific factor best fit the response data over other theoretically related models. We now aimed to use this brief assessment to further knowledge about mature number sense's nomological network, including other important psychological constructs in mathematics education.

Rationale for Current Study

Mature number sense has been measured and studied internationally over the last twenty-five years (summarized in Whitacre et al., 2020), most frequently in a single time point. Most of these studies comment on the "low" levels of students' mature number sense observed. However, few studies have analyzed its potential overlap with other related psychological constructs in mathematics education. Understanding how mature number sense fits together with constructs such as fraction computation and school mathematics achievement is important for establishing the construct validity of measures of mature number sense (Cronbach & Meehl, 1955). Throughout the literature, mature number sense is often contrasted with standardized school mathematics achievement and the algorithmic, overly procedural mathematics instruction common in schools (Whitacre et al., 2020). On the other hand, it is reasonable to hypothesize that students with strong mature number sense are flexible problem solvers with a deep understanding of number and operation and, thus, that they might do well on more "traditional" school assessments. Yang et al. (2008) is the only study we are aware of that examined this association. They found 5th grade Taiwanese students' number sense were positively associated with school achievement, but there was no detail on how math achievement was measured (e.g., school grades, standardized tests, etc.). In addition, given Taiwanese students' strong performance on international standardized mathematics tests (ranked 5th in world in PISA 2018) and its centralized education system (Lin et al., 2021), Taiwan may be a unique context to compare students' mature number sense and school math achievement.

To begin to address this gap in the literature, we aimed to study how US middle school students' mature number sense related to their grade-level mathematics achievement, controlling for a variety of other potentially related factors, including fraction knowledge and addition fluency. Our research questions were as follows: 1) Is the mature number sense of middle school students measurably distinct from their fraction and decimal computation skills as well as their grade-level mathematics achievement? and 2) Is mature number sense uniquely associated with students' standardized grade-level mathematics achievement, even after controlling for their fraction and decimal computation?

Methods

Participants

One hundred twenty-nine middle school (6th-8th grade) students from schools surrounding a university in the midwestern United States participated (45% identified as female; 67% identified as White, 13% Multiracial, 8% Black, and 4% Hispanic). Sessions took place after school hours in a local middle school *(N = 36)* or university building *(N = 93)*. Students were participants in timepoint 1 of a larger (still ongoing) longitudinal study. They completed eight measures over the course of two 45-minute sessions, scheduled about a week apart (median of 7 days between sessions). For this analysis, we focused on the four measures described below, but note that the results reported hold up to robustness checks that include the additional measures.

Measures and Procedure

Brief Assessment of Mature Number Sense (Kirkland et al., *under review*). This is an electronic, 24 item multiple-choice test of students' mature number sense, aligned with our theoretical framework detailed above. Items differ from a traditional curriculum and are designed to specially assess students' number sense. Each item has a time limit of 60 seconds and students are not allowed to use paper and pencil to discourage the use of traditional algorithms. Student's total sum score is used in the analyses. Figure 1 includes an example item for each of the four components in our mature number sense framework. In the validation analysis (Kirkland et al., *under review*), student scores on the brief assessment were reliable over time (r = 0.83) and had evidence from expert reviews, factor analyses, student think-alouds, and item response theory analyses to support our validity argument.

Massachusetts Comprehensive Assessment System (MCAS) Grade-Level Mathematics Test (2019). Students completed the released 2019 MCAS paper test appropriate for their grade level. This is a freely available standardized test designed to assess student proficiency with grade-level mathematics standards. Student scores are converted to percent correct because the maximum possible correct differs by grade level. Students had no time limit to complete each section of the test and could use scratch paper but not calculators.

Rational Numbers Measure (Powell, 2014). This is a 35-item paper and pencil test of students' computational skills with fractions, decimals, and percent. Students are asked to perform the four operations with both fractions and decimals, find common denominators of fractions, generate equivalent fractions, and connect representations of fractions, decimals, and percent (e.g., "Convert 2.08 to a percentage"). Students had 20 minutes to work. They could use scratch paper but not calculators. Students received 1 point for each correct response.

Addition Fluency Task (Geary et al., 1996). This measure includes all combinations of randomly presented single digit addition facts with the numbers 1-9. Students were tasked with solving as many correctly as they could in 1 minute. The order of the facts was predetermined randomly and then kept standard for all participants. Students received 1 point for each correct response.

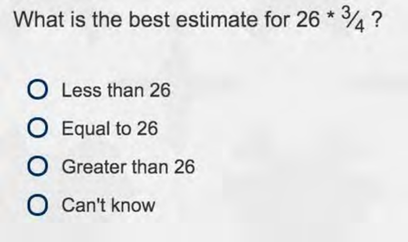
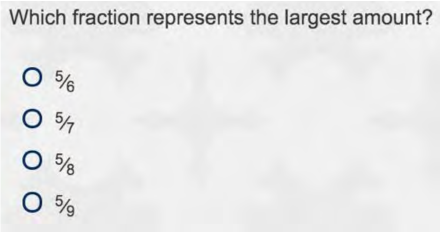
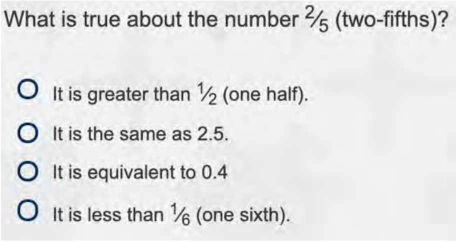
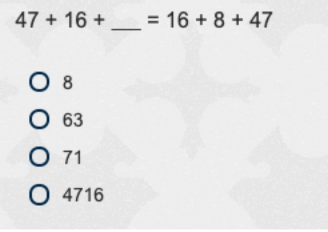
Understanding the Effect of Operations on Numbers	**Number Concepts and Magnitude**
 What is the best estimate for 26 * $\frac{3}{4}$? ○ Less than 26 ○ Equal to 26 ○ Greater than 26 ○ Can't know	 Which fraction represents the largest amount? ○ $\frac{5}{6}$ ○ $\frac{5}{7}$ ○ $\frac{5}{8}$ ○ $\frac{5}{9}$
Multiple Representations of a Number	**Mathematical Equivalence**
 What is true about the number $\frac{2}{5}$ (two-fifths)? ○ It is greater than $\frac{1}{2}$ (one half). ○ It is the same as 2.5. ○ It is equivalent to 0.4 ○ It is less than $\frac{1}{6}$ (one sixth).	 47 + 16 + ___ = 16 + 8 + 47 ○ 8 ○ 63 ○ 71 ○ 4716

Figure 1: Sample Items Assessing Mature Number Sense Organized by Component

Data Analyses

To address the research question on whether mature number sense is measurably distinct from students' general mathematics achievement, we first calculated the zero-order correlations between the constructs. We predicted mature number sense to be most closely related to students' grade-level math achievement on the MCAS and scores on the Rational Numbers Measure. We expected mature number sense to be least closely related to addition fluency. We then ran a series of partial correlation analyses. We analyzed the correlation between mature number sense and both other constructs after controlling for students' addition fluency scores. We then analyzed the associations between students' grade-level math achievement, fraction and decimal computation, and mature number sense, controlling for each in turn. In each case, we predict the relationship between mature number sense and the other construct to remain significant, even after controlling for a strongly related third variable.

For additional evidence that mature number sense and fraction and decimal computation are distinct, we then conducted a series of factor analyses (Shaffer et al., 2016). We used a common method where we tested a constrained model with 2 latent factors set with a covariance equal to 1 and an unconstrainted model where the 2 latent factors are allowed to freely covary with each other. If the unconstrained model provides a better fit through a RMSEA test that is significant, then the two measures of interest can be said to be distinct. We tested these two models using a chi-squared, $\chi 2$, difference test. Due to each grade's different test for the MCAS, we are unable to perform this same analysis on grade-level mathematics achievement.

To address research question 2, we ran a partial correlation test on the relationship between grade-level mathematics achievement and fraction and decimal computation, controlling for mature number sense. We then ran a linear regression model, regressing mature number sense and other predictors measured in the study on students' grade-level mathematics achievement.

Findings

We first begin with the overall student performance on the measures in the study. Students solved on average 14.3 (*60%)* items correctly on our brief assessment of mature number sense (*SD* = 5.65). Overall, students performed slightly better on the brief assessment than in the original validation study (*M* = *56%* correct). Across the three grade levels, students solved on average 50% of the items correctly on the MCAS. Out of a maximum possible of 35 correct, students answered on average 9.74 items correct on the Rational Numbers measure. While this number may appear very low as a percentage correct (*28%*), this measure was designed by Powell to include many problems to help differentiate students' fraction and decimal computation up to the college level. Finally, on average, students answered correctly 19.48 (*SD* = 6.72) single-digit addition problems on the addition fluency task.

To begin to address research question 1, the zero-order correlations between the measured constructs are summarized in Table 1. As predicted, students' mature number sense scores correlated very highly with their fraction and decimal computation skills (*r* = 0.80) and grade-level mathematics achievement (*r* = 0.76), and this was significantly higher than the correlation with addition fluency (*r* = 0.49).

Table 1: Correlation Between Measures in Study

Construct	*M*	*SD*	1	2	3	4
1. Mature Number Sense	14.34	5.65	-			
2. MCAS – Grade-Level Achievement	50.00%	24%	0.76	-		
3. Fraction and Decimal Computation	9.74	7.21	0.80	0.66	-	
4. Addition Fluency	19.48	6.72	0.49	0.36	0.53	-

Controlling for students' addition fluency scores, students' mature number sense still correlated highly with their grade-level mathematics achievement (r =0.72, $t(126)$ = 11.52, p < .001) and their fraction and decimal computation (r =0.73, $t(126)$ = 11.90, p < .001). We then further examined the associations between each pair of these three mathematics-specific constructs. Controlling for students' fraction and decimal computation, their mature number sense scores still were significantly positively related with their grade-level achievement (r =0.52, $t(126)$ = 6.77, p < .001). The same was true when controlling for grade-level achievement and examining mature number sense's relationship with fraction and decimal computation (r =0.61, $t(126)$ = 8.59, p < .001). We would predict that if mature number sense and grade-level mathematics achievement are redundant constructs, this relationship would no longer be significant. Together, this all provides initial evidence that mature number sense is distinct from their grade-level mathematics achievement.

As discussed above, we then ran a series of confirmatory factor analyses for evidence on if mature number sense and fraction and decimal computation are distinct (Shaffer et al., 2016). These analyses provided evidence that the Rational Numbers Measure and the Brief Assessment of Mature Number Sense measure distinct constructs, χ^2 difference = 56.65, p < 0.001; *CIF* difference = 0.02 compared to benchmark of 0.002. This suggests the constructs, as measured in this study, are highly related, but not redundant.

To further examine the unique importance of mature number sense, we first tested the partial correlation between fraction and decimal computation and grade-level achievement. Interestingly, when controlling for students' mature number sense, there is no longer a significant correlation between the two other constructs (r =0.13, $t(126)$ = 1.51, p =0.13). We then ran a linear regression on students' grade-level mathematics achievement (Model $R^2 = 0.59$). Results from the model are summarized in Table 2. Students' mature number sense was significantly positively related ($B_{NS} = 0.03$, p < .001) to students' grade-level mathematics achievement. However, students' fraction and decimal computation ($B_{RN} = 0.005$, p =0.11) and addition fluency ($B_{Add} = -0.002$, p =0.54) were not significantly related to achievement. To compare the relative importance of each regressor, we examined the semipartial correlation of each predictor in the model (Darlington & Hayes, 2016; Hayes & Rockwood, 2017). The semipartial correlation coefficient for mature number sense was significant (sr = 0.39, $t(126)$ = 4.74, p < .001). However, the semipartial correlation for fraction and decimal knowledge (sr = 0.09, $t(126)$ = 0.98, p = 0.33) was not significant. We can interpret the ratio of the two semipartial correlations (0.39/0.09 = 4.33) as a measure of the relative importance of each in explaining students' grade-level achievement. As another way to quantify mature number sense's importance, we ran an additional linear regression model with mature number sense removed from the model. The model R^2 dropped from 0.59 to 0.43 (adjusted R^2 from 0.58 to 0.42).

Table 2: Summary of Regression Model Predicting Grade-Level Mathematics Achievement

Variable	*B*	*SE*	*t*	*p*
(Intercept)	0.08	0.051	1.645	0.103
Mature Number Sense	0.03	0.004	6.78	<0.001
Fraction and Decimal Computation	0.005	0.003	1.61	0.11
Addition Fluency	-0.002	0.002	-0.62	0.54

Discussion and Implications

From the evidence presented here, students' mature number sense is distinct from their general grade-level mathematics achievement and fraction and decimal computation skills. It is not the same to say, for example, that a student has "strong number sense" and is "good with fraction operations." We also found students' mature number sense to be predictive of their mathematics achievement, above and beyond their computation skills with fractions and decimals. Mature number sense explained a significant portion of that association, suggesting that mature number sense is uniquely associated with students' math achievement.

It is perhaps surprising that mature number sense was a stronger predictor of grade level mathematics achievement than students' fraction and decimal computation, given the significance fraction knowledge in particular has been shown to play in students' mathematics achievement (e.g., Barbieri et al., 2021; Booth et al., 2014; Booth & Newton, 2012; Siegler et al., 2013). Fractions are often seen as a "gatekeeper" to success in algebra or later mathematics courses (e.g., Powell et al., 2019; Siegler & Pyke, 2013). However, we see in this study that middle school students' mature number sense was a more important factor than their knowledge of fractions and decimals. Perhaps students' ability to make sense of numbers and operations and use that knowledge flexibly to solve problems is a better indicator of future mathematics achievement than fraction or decimal computation skills. Further research is needed to understand the nomological network more fully, especially in a longitudinal setting where the potential mediating role of mature number sense can be accurately tested.

In reviewing the item level data from the brief assessment of mature number sense, students performed better overall on whole number items than those involving fractions and decimals. For example, 79% of students solved this whole number distributive property item (4 x 36 = 4 x (___ + 6)) correctly. However, only 36% of students correctly identified that the answer to $12 \div \frac{1}{5}$ would be greater than 12 whereas 50% of students said it would be less than 12. Across this and other items, students struggled estimating the effect of operations on fractions and decimals less than one, displaying a continued bias toward "rules" learned with whole number operations, such as multiplication always makes the answer bigger, division makes the answer smaller, and each number has a unique predecessor and successor (Fazio et al., 2016; Siegler, 2016). Greater attention is required to address these misconceptions that are often learned as "rules" in early elementary classrooms (Karp et al., 2015). Encouragingly, a greater number of students displayed accurate understanding of fraction and decimal magnitude. For example, 72% of students correctly identified $\frac{5}{6}$ as the largest among the following fractions: $\frac{5}{6}, \frac{5}{7}, \frac{5}{8}, \frac{5}{9}$. Such "common numerator" problems have been shown to be significantly more difficult for students than the much more frequently seen "common denominator" comparisons (Fazio et al., 2016).

Overall, we have initial evidence here to start to build a nomological network for mature number sense and its associated psychological constructs in mathematics education. This is an important step forward to future research more fully examining how mature number sense might lay a foundation for future mathematics achievement or whether improving mature number sense might be a more important target for instruction than, say, computational skills with fractions. This is, however, only one initial study with a sample of students that may not be wholly representative of middle school students more broadly. In addition, we only have evidence from a single time-point rather than longitudinal data to test if mature number sense precedes and predicts grade level mathematics achievement. By continuing to characterize mature number sense's nomological network, researchers can more fully explain how students develop understanding of important psychological constructs in mathematics education and provide rigorous evidence to educators on how to help ensure all students have the disposition to *make sense of numerical situations*.

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