Transportation Geotechnics 34 (2022) 100760

Contents flfists avafiflabfle at ScfienceDfirect

Transportatfion Geotechnfics

journal homepage: www.elsevier.com/locate/trgeo

Characterfizatfion of varfiabfiflfity fin 2-dfimensfionafl partficfle geometry vfia 3D structured flfight scannfing

Chang Hoon Lee [a], Seung Jae Lee [b], Moochufl Shfin [a,*]

[a] Western New England Unfiversfity, Sprfingffield, MA 01119, Unfited States

[b] Florfida Internatfional Unfiversfity, Mfiamfi, FL 33174, Unfited States

ARTICLE INFO

Keyword:
3D structured flfight scannfing
Dfigfitafl fimage anaflysfis
Morphoflogy
Reguflarfity
Roundness
Spherficfity

ABSTRACT

In the characterfizatfion of the partficfle morphoflogy vfia 2D dfigfitafl fimage processfing, fimage acqufisfitfion needs to be done fin proper ways. Otherwfise, the resufltfing parameters are sfignfifficantfly finfluenced by the procedures. Thfis research characterfizes potentfiafl varfiatfions that can mfisflead the finterpretatfion of the 2D partficfle morphoflogy parameters such as spherficfity, roundness, and reguflarfity findfices dependfing on a projectfion angfle at whfich a 2D fimage fis taken. Besfides, the 2D parameters are compared to Wadeflfl's "true spherficfity" evafluated usfing a 3D partficfle modefl obtafined by 3D structured flfight scannfing technoflogy. The resufltfing varfiance fin the 2D parameters fis hfigh, producfing approxfimatefly more than 0.5 of devfiance dependfing on partficfles, the deffinfitfion of the mfinerafl parameters, and projectfion angfles. The varfiatfions demonstrated fin thfis research coufld provfide addfitfionafl finsfight to the 2D findfices by consfiderfing 3D partficfle morphoflogy findfices.

1. Introductfion

The characterfizatfion of partficfle morphoflogy fis of great fimportance fin many dfiscfipflfines fincfludfing geoflogy [1–3], bfioflogy [4], envfironmentafl engfineerfing [5,6], nano-partficfles [7,8], and constructfion materfiafls [9–11]. A varfiety of morphoflogy findfices have been devefloped to better characterfize the shapes of aggregate at dfifferent flength scafles [12–16] aflong wfith the advances fin fimage acqufisfitfion technoflogfies such as flaser scanner, scannfing eflectron mficroscope, computed tomography [17–19], and sfimpfly 2-D dfigfitafl fimagfing systems [20–22]. Despfite the advances fin the 3D fimagfing technoflogfies, a 2D dfigfitafl fimage-based anaflysfis has been wfidefly used because of fits practficaflfity and cost-effectfiveness. However, a hfigh degree of varfiabfiflfity exfistfing fin 2D fimage anaflyses fis produced by procedures (e.g., how many fimages are taken), whfifle fits finfluence on the resufllt has not been hfighflfighted weflfl. Specfifficaflfly, fimages of one partficfle (e.g., naturafl or crushed aggregate) taken at varfious projectfion angfles typficaflfly appear dfifferentfly, and those dfiscrepancfies can become more apparent when the shape of partficfles fis eflongated or fflat. Thfis fimpflfies there are some flevefls of varfiatfions fin the quantfiffied morphoflogy findfices dependfing on the angfles at whfich each fimage fis taken. However, a study of the varfiatfion fin 2D morphoflogy findfices by usfing a physficafl camera fis technficaflfly chaflflengfing for precfisefly controflflfing dfifferent projectfion angfles as an exampfle. Currentfly, stereoscopy [23–25] wfith a 3D scanner becomes more popuflar, enabflfing one to obtafin hfigh-resoflutfion 3D meshes for a random shaped partficfle. The technoflogy makes fit possfibfle not onfly to dfirectfly characterfize 3D morphoflogy findfices but afflso to finvestfigate the aforementfioned varfiatfions of 2D morphoflogy findfices wfith a reflatfivefly flarge number of 2D partficfle fimages. Therefore, wfith the utfiflfity of the 3D scannfing technoflogy, thfis research characterfizes the varfiatfions fin the 2D morphoflogy vaflues produced by dfifferent findfices and projectfion angfles at whfich 2D fimages are taken. Subsequentfly, the propagatfion of the varfiabfiflfity to "Reguflarfity" fis further expflored.

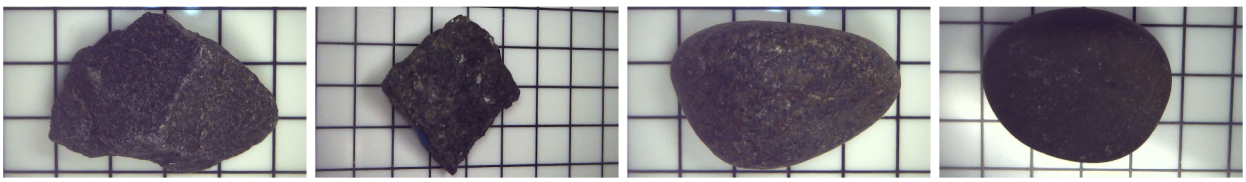
Wadeflfl [26] proposed "true spherficfity, Ψ," as expressed by:

$$\Psi = \frac{S_{eq}}{S_a} \tag{1}$$

where S_a fis the actuafl surface area of a partficfle and S_{eq} fis the surface area of a sphere havfing an fidentficafl voflume to the partficfle. Whfifle "true spherficfity" requfires measurfing the voflume and surface area of a partficfle, he afflso proposed "degree of spherficfity [1]" (afflso referred to as dfiameter spherficfity; herefin denoted by S_D fin Appendfix A) that can be caflcuflated from 2D fimages. Wadeflfl afflso suppflemented wfith eflaboratfion of the detafifl procedure of tappfing the sflfide carryfing the partficfle to sfit on the flargest pflane face. Sfimfiflarfly, Wadeflfl's roundness can be computed from the 2D fimage, partficuflarfly suggestfing the projectfion angfle that creates the pflane by maxfimum and fintermedfiate flengths. Krumbefin [2]

* Correspondfing author.
E-mafil addresses: changhoon.flee@wne.edu (C.H. Lee), sjflee@ffiu.edu (S.J. Lee), moochufl.shfin@wne.edu (M. Shfin).

https://dofi.org/10.1016/j.trgeo.2022.100760
Recefived 20 May 2021; Recefived fin revfised form 18 March 2022; Accepted 27 March 2022
Available online 31 March 2022

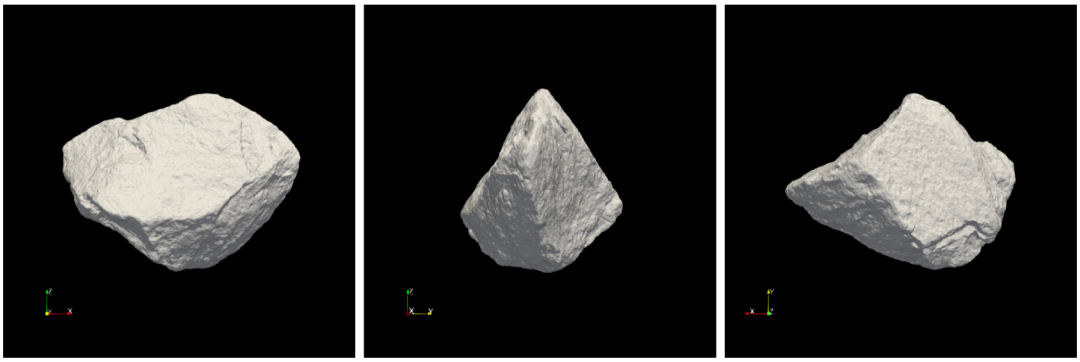


(a) CO (b) CF (c) RO (d) RF

Ffig. 1. Partficfles (Note that the grfid sfize on the background fis 5 mm).

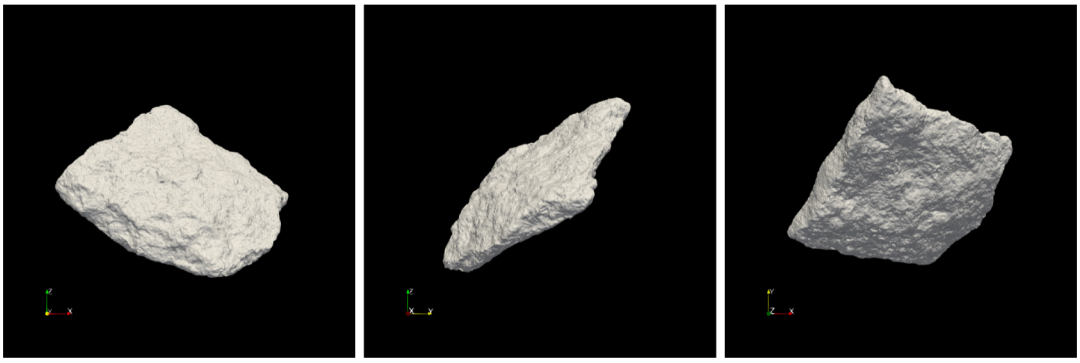
Tabfle 1
Descrfiptfive Statfistfics of Morphoflogy Indfices (L, I, and S respectfivefly findficate the flongest, fintermedfiate, and the shortest flengths of a boundfing box that encompass the modefl wfith fits mfinfimum voflume).

ID	Number of Eflements	Surface Area (mm^2)	L (mm)	I (mm)	S (mm)	Wadeflfl True Spherficfity (Ψ)	Surface Area/eflement (μm^2/eflement)
CO	13,697,779	705.66	20.58	15.13	13.23	0.78	51.52
CF	7,692,017 RO	561.28	19.05	18.82	5.17	0.60	72.97
9,078,249	RF	715.69	20.38	14.49	10.37	0.87	78.84
7,053,045		616.33	18.38	15.74	7.04	0.82	87.38



(a) $X-Z$ plane (b) $Y-Z$ plane (c) $X-Y$ plane

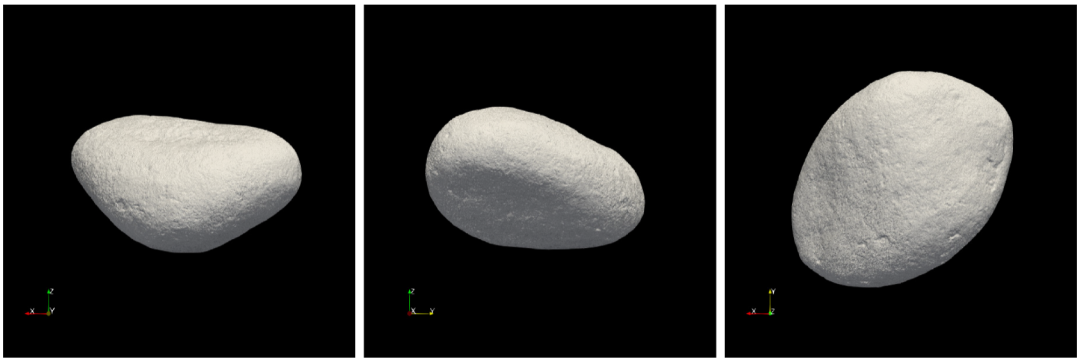
Ffig. 2. Mufltfipfle projectfion vfiews of Partficfle CO.



(a) $X-Z$ plane (b) $Y-Z$ plane (c) $X-Y$ plane

Ffig. 3. Mufltfipfle projectfion vfiews of Partficfle CF.

measured the roundness based on the flargest projectfion fimage of a pebbfle to compare the resufltfing vaflue wfith Wadeflfl's. However, the chofice of the maxfimum projectfion angfle was not necessarfifly requfired as fimpflfied by hfis comment, "There can be no objectfion to any other orfientatfion [projectfion angfle] fif one wfishes to use fit, except that the vaflues fin that case cannot be dfirectfly compared wfith Wadeflfl's vaflues [2]." Neverthefless, Sneed and Fofflk [27] proposed the use of maxfimum projectfion area, proposfing another spherficfity findex, "maxfimum projectfion spherficfity," because the settflement veflocfity of a partficfle fin the water fis domfinantfly finffluenced by the area perpendficuflar to the dfirectfion of motfion as referencfing to Krumbefin's experfiment. As advanced numerficafl techfniques such as machfine fleamfing aflgorfithms have been

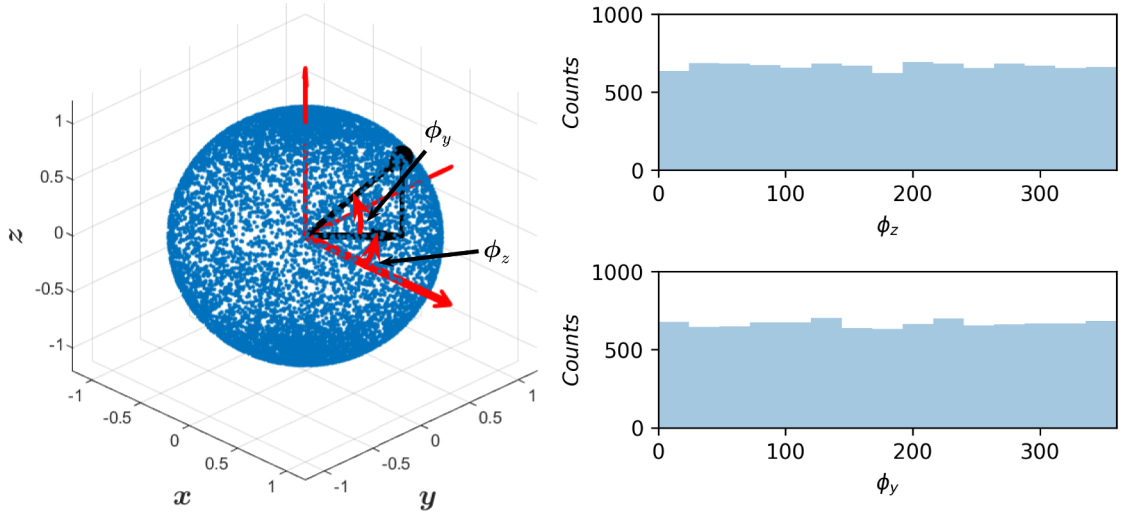


(a) $X - Z$ plane (b) $Y - Z$ plane (c) $X - Y$ plane

Ffig. 4. Mufltfipfle projectfion vfiews of Partficfle RO.

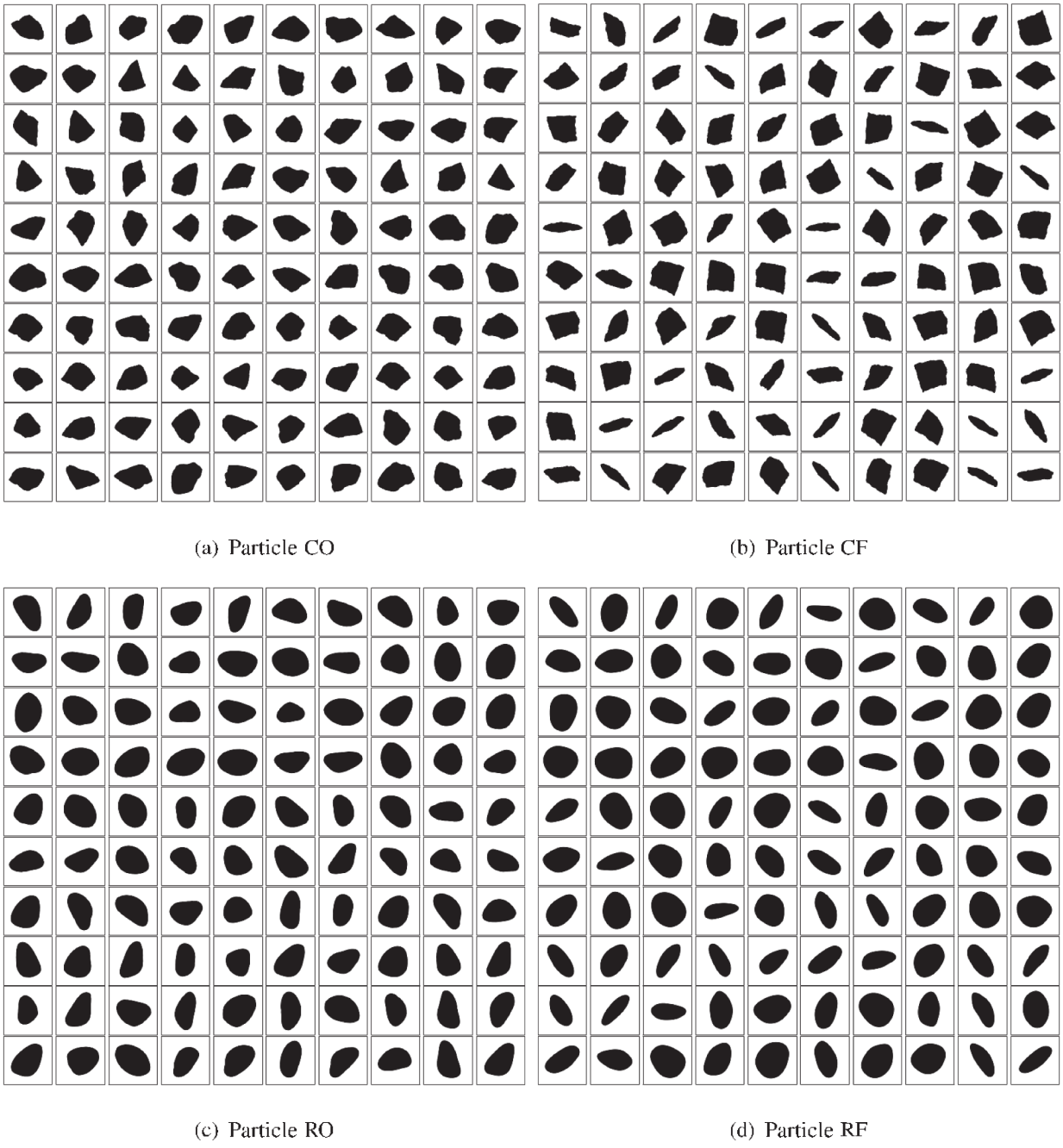
(a) $X - Z$ plane (b) $Y - Z$ plane (c) $X - Y$ plane

Ffig. 5. Mufltfipfle projectfion vfiews of Partficfle RF.



(a) Point clouds of the positions (b) Distributions of ϕ_z and ϕ_y (Bin width = 15 per Sturges Rule)

Ffig. 6. Iflflustratfion of 10,000 randomfly drawn camera posfitfions.



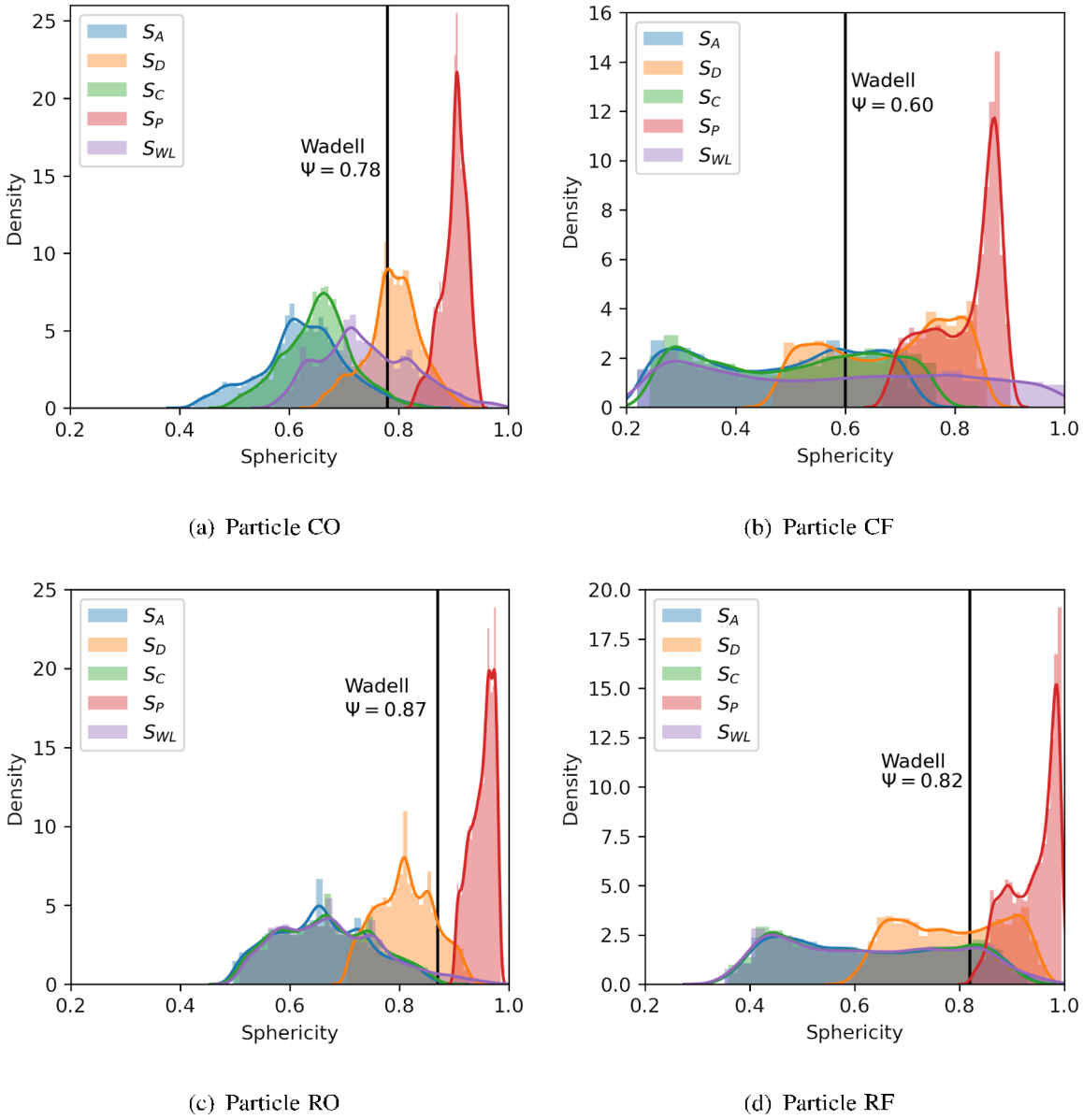
Ffig. 7. 100 Image Sampfles for the Partficfles (Note that the fimages are dfispflayed as 10 × 10 array for each partficfle).

fincorporated finto the characterfizatfion of partficfle morphoflogy, severafl researchers proposed methods to characterfize morphoflogfies of buflk partficfles [28–33]. These techniques are benefficfiafl when aggregates are densefly packed or stockpfifled (e.g., rfiprap), enabflfing one to save flabor and tfime fin anaflysfis of the buflk partficfle shapes, as opposed to a per-partficfle (sfingfle-partficfle) anaflysfis; however, the morphoflogy of each partficfle, partficuflarfly the effect of projectfion angfles, woufld not be accurateffy consfidered fin the "buflk vfiew" anaflysfis. Some researchers studfied the effects of seflectfing projectfion angfles on the computatfion of morphoflogy findfices. Wang et afl. [34] reported that the dfifference of partficfle proffifles characterfized by Fourfier anaflysfis dependfing on varfiabfle orfientatfions fis statfistficaflfly finsfignfifficant based on the fimages from three perpendficuflar projectfion angfles, and fit agrees wfith Su and Yan [35]'s concflusfion from the finvestfigatfion on 2D descrfiptors (Note that S_p and S_{WL} fin Appendfix A are commonfly used for thfis research.). However, Maroof et afl. [36] commented on a possfibfle bfias of the morphoflogy findfices because of orfientatfions wfithout any further systematfic anaflysfis.

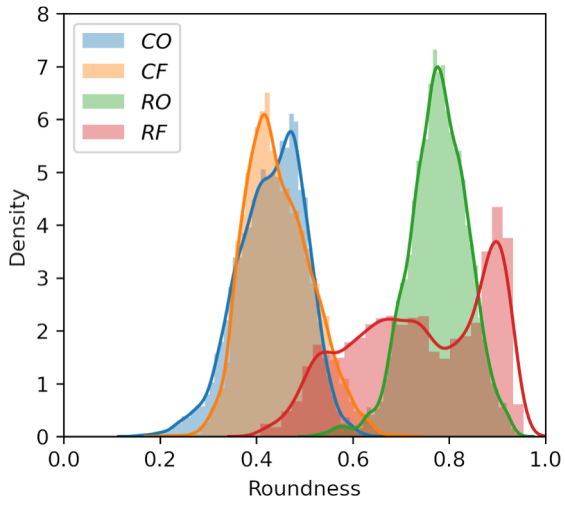
As reflated to the projectfion angfles (or orfientatfion), the number of fimages for obtafinfing the reflfiabfle 2D morphoflogy findfices fis afso an finterestfing questfion. Wadeflfl [26] commented "Strfictfly speakfing, the totafl roundness of a soflfid fis achfieved by measurements fin three pflanes at rfight angfles to each other, but two pflanes are fin most cases suffficfient, whfifle one pflane fis satfisfactory when deaflfing wfith smaflfl sedfimentary partficfles," fimpflyfing that one or two 2D fimages woufld be suffficfient fin the computatfion of roundness. There fis a flack of research on a suffficfient number of fimages to reflfiabfly characterfize the morphoflogy of a sfingfle partficfle. Instead, severafl flfiterature report the number of partficfles (fin an fimage) for characterfizfing the morphoflogy findfices representfing buflk partficfles. For exampfle, Wang et afl. [34] reported that 20 partficfles are enough to obtafin statfistficaflfly meanfingfufl vaflues. Zheng and Hrycfiw [30]

Tabfle 2
Descrfiptfive Statfistfics of Morphoflogy Indfices (Note that "PA" stands for "pfixefl area," deffinfing the number of pfixefls for each partficfle fimage.)

ID	Statfistfics	*PA*	*PCD*	*RD*	S_A	S_D	S_C	S_P	S_{WL}
CO	μ	707545	1204	0.431	0.626	0.790	0.653	0.901	0.743
	σ	101200	118.1	0.070	0.079	0.050	0.063	0.024	0.085
	CV	0.143	0.098	0.162	0.126	0.064	0.096	0.026	0.115
CF	μ	645497	1309	0.446	0.477	0.682	0.503	0.818	0.576
	σ	221213	60.2	0.069	0.152	0.113	0.158	0.063	0.229
	CV	0.343	0.046	0.154	0.318	0.166	0.314	0.077	0.398
RO	μ	804276	1249	0.777	0.659	0.810	0.666	0.951	0.676
	σ	129992	119.0	0.062	0.086	0.053	0.088	0.021	0.096
	CV	0.162	0.095	0.079	0.131	0.065	0.132	0.023	0.142
RF	μ	813270	1282	0.738	0.630	0.787	0.628	0.940	0.630
	σ	211827	62.0	0.138	0.157	0.099	0.157	0.045	0.161
	CV	0.260	0.048	0.187	0.249	0.126	0.250	0.048	0.256



Ffig. 8. Hfistograms of Spherficfitfies for Partficfles (Note that ψ fis Wadeflfl's true spherficfity vaflue).



Ffig. 9. Hfistograms of Roundness for Partficfles.

concfluded that about 60 fis requfired to achfieve the vaflue wfithfin 0.05 of the error at a 98% statfistficafl confidence flevefl, recommendfing to use 30–50 partficfles as a mfinfimum. It fis worthwhfifle to note that the finvestfigatfion on the number of fimages (or partficfles) may be based on the rough assumptfion that a sfingfle vaflue can represent the morphoflogy of mufltfipfle, yet sfimfiflar-shaped partficfles. However, fit can potentfiafllfly cause bfiases of the 2D morphoflogy findfices by fignorfing thefir varfiatfions due to the orfientatfions of findfivfiduafl partficfles.

Besfides, another concern fis whether a varfiety of 2D morphoflogy findfices can reasonabfly capture the 3D shape of a partficfle. Wadeflfl [1,26,37] proposed "true spherficfity" and roundness for correflatfing the partficfle shapes wfith the settflement veflocfity as done by other researchers [2,12,27]. However, as fits appflficatfion expands to varfious areas [38–40] beyond the subjects of geoflogy, numerous efforts to soflefly characterfize the 3D shape of partficfles have been proposed on the basfis of varfious materfiafls fincfludfing random shaped partficfles [10,11,32,41–44]. Then, the concept of reguflarfity fis further fintroduced as combfinfing [45,46] morphoflogy findfices at a dfifferent flength scafle wfith an afim of correflatfing wfith macroscopfic behavfior of the system. Furthermore, Wadeflfl's "true spherficfity," as expressed fin Eq.(1), has been wfidefly used as a reference for the comparfison of varfious morphoflogy findfices. The name "true spherficfity" mfight be a bfit mfisfleadfing because fit fis not onfly an findficator of the partficfle eflongatfion but afso the partficfle anguflarfity. The "true spherficfity" fis a functfion of surface area, thus finffluenced by both eflongatfion and anguflarfity, and therefore has a concept of reguflarfity that combfines the overafffl morphoflogy at dfifferent flength scafles. Nevertheless, Wadeflfl [1] afternatfivefly proposed a 2D spherficfity findex, herefin S_D, as wefffl as the "true spherficfity," he reported that the 2D findex fis generafflfly comparabfle to the 3D Ψ-vafues as demonstrated by theoretficafl soflfids fincfludfing sphere, cube, and three dfifferent paraflfleflepfipeds fin the orfigfinafl paper. However, there fis a flack of research expflorfing whether varfious 2D spherficfitfies fflikefly produce the sfimfiflar vaflue wfith "true spherficfity." Note that "true spherficfity" fis one of 3D morphoflogy descrfiptors among others [27,41,42]. However, thfis study chose "true spherficfity" as the reference spherficfity findex for evafluatfing 2D spherficfity findfices to examfine fif a good comparfison can be made between 2D and 3D spherficfitfies as Wadeflfl has demonstrated for theoretficafl soflfids.

Accordfingfly, thfis research adopts the Monte-Carflo sfimuflatfion approach [47] for characterfizfing the uncertafintfies of spherficfitfies, expflorfing thefir propagatfion to the generafflfized reguflarfity [46]. The assocfiated anaflysfis wfffl demonstrate potentfiafl bfiases produced by 2D dfigfitafl fimage processfing when characterfizfing the morphoflogy of partficfles. The resufflts can provfide possfibfle chafflfenges of a 2D fimage anaflysfis due to the finnate varfiabfiflfity. The ffive dfifferent 2D spherficfity and roundness findfices, as summarfized fin Appendfix A, are examfined fin thfis study, and the foflflowfing sectfion wfffl dfiscuss the data acqufisfitfion procedure for obtafinfing a hfigh-resoflutfion 3D scanned modefl of a set of dfifferent partficfles and the Monte-Carflo sfimuflatfion approach for further anaflysfis. Lastfly, the resufltfing varfiabfiflfity of morphoflogy findfices, fits propagatfion to the generafflfized reguflarfity, and comparfison of the vaflues wfith "true spherficfity" wfffl be dfiscussed.

2. Generatfion of 2D fimage data from random orfientatfions

2.1. Selectfion of partficles

For a near equfi-dfimensfionafl partficfle (e.g., sphere, cube, dodecahedron-flfike shape), the flesser varfiabfiflfity fin the spherficfity can be expected. In other words, the varfiabfiflfity due to projectfion angfles becomes hfigher when the partficfle shape fis "eflongated" and/or "fflat." In addfitfion, the partficfle's anguflarfity afso changes dependfing on the projectfion angfles at whfich the fimage fis taken. For exampfle, the roundness that measures the partficfle's corner sharpness fis evafluated wfith reference to the radfius of maxfimum finscrfibed cfircfle whfich may change dependfing on the projected angfle. To characterfize such varfiabfiflfity of spherficfity and roundness findfices, four partficfles havfing sfignfificantfly dfifferent shapes but sfimfiflar maxfimum dfimensfionafl flengths (fi.e., 18 to 20 *mm*) are seflected as shown fin Ffig. 1. However, fit fis worthwhfifle to note that the seflectfion of partficfles fin thfis research targeted at demonstratfing a bfias of the resufltfing morphoflogy findfices produced by the varfiance by projectfion angfles of the fimages.

The partficfles can be ffirstfly grouped by two categorfies, "crushed (C)" and "rfiver (R)" partficfles, of whfich orfigfins are sfignfificantfly dfifferent; C-group fis more anguflar than R-group by nature. Second, the partficfles are seflected based on thefir ratfios of flengths: "fflat (F)" and "ordfinary (O)." Herefin, "F" fis deffined when the ratfio of the fintermedfiate flength to the smafflflest flength fin a boundfing box fis greater than 2, otherwfise the second nomencflature of the partficfle name fis denoted by "O." (The terms are referred accordfing to ASTM C4791 [48].) For finstance, the "CF" partficfle findficates a crushed partficfle wfith a fflat shape.

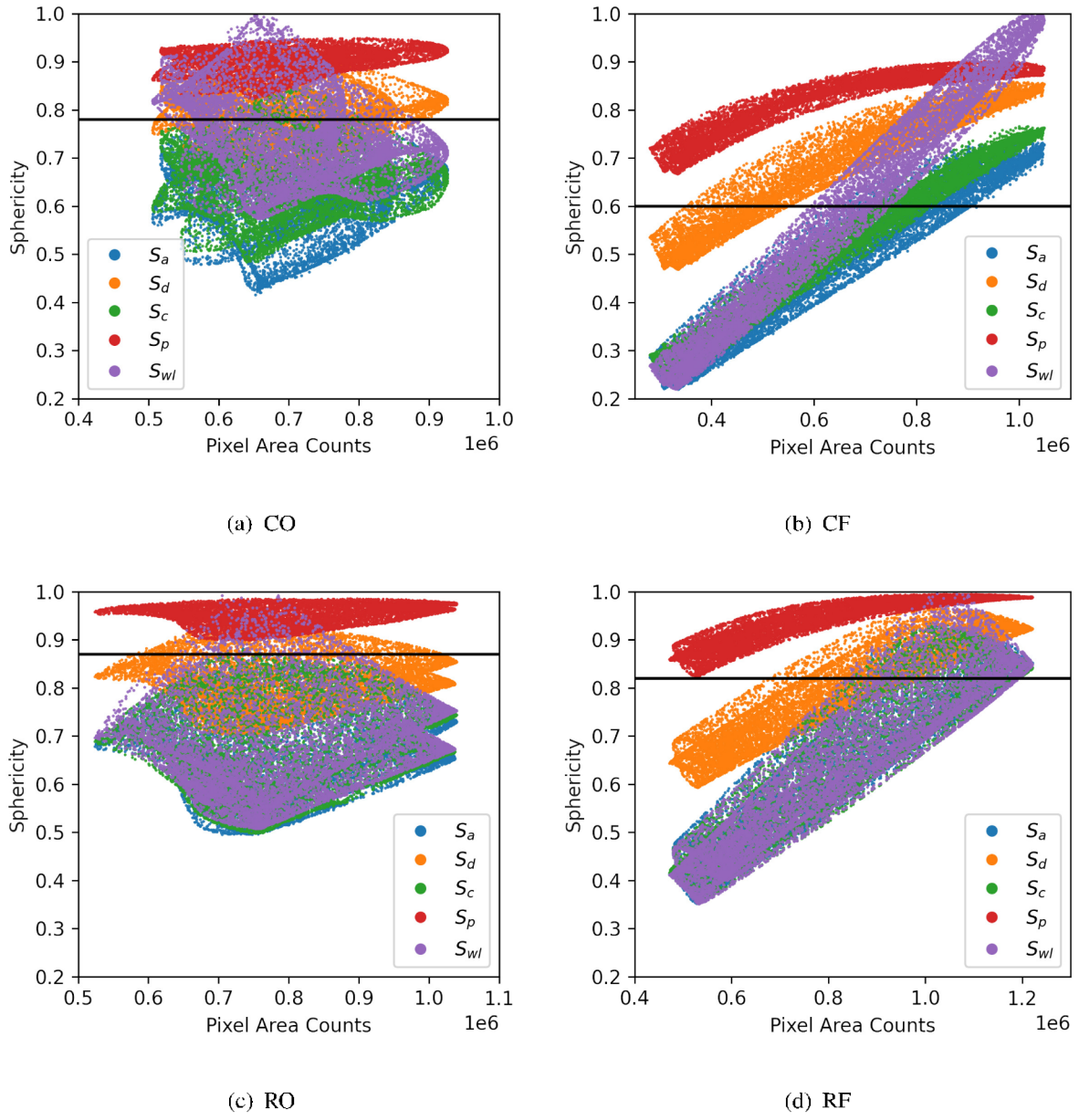
2.2. Acqufisfitfion of 3D partficle model

The 3D partficfle modefl fis obtafined by usfing a commercfiafl Poflyga C504 structured flfight 3D scanner [49], whfich output fis efither a wavefront (OBJ) or a stereoflfithography (STL) format fffifle. Structured flfight scanner performs 3D scannfing at 1:1 scafle, thus the obtafined 3D modefl has the actuafl partficfle sfize. The obtafined 3D modefl fis composed of 7 to 14 mfffffion trfianguflar surface meshes, finferrfing approxfimatefly 52–87 μm^2 per trfianguflar mesh eflement for the consfidered partficfles. Tabfle 1 summarfizes the number of eflements of each partficfle modefl, geometrficafl propertfies evafluated from the partficfle modefls, the correspondfing Ψ, and the surface area/mesh densfity (Note that the specfiffied accuracy of the scanner by the manufacturer fis 6 mficrons).

Ffig. 2–5 dfispflay 3D modefl fimages of each scanned partficfles on three orthogonafl (x z, y z, and x y) pflanes of the boundfing boxes. On the authors' opfinfions, the surface roughness fis vfisfibfly wefffl fidentfiffied by the 3D fimages afflthough the assocfiated characterfizatfion fis beyond the scope of thfis research.

2.3. Acqufisfitfion of 2D partficle fimages from random orfientatfion

A set of 2D fimages are obtafined from each 3D partficfle modefl at 10,000 projectfion angfles whfich are randomfly seflected from two findependent unfiform dfistrfibutfions of the rotatfionafl angfles about z- and y-axes, denoted by ϕ_z and ϕ_y, respectfivefly. (fi.e., ϕ_z □ **U** [0, 360) and ϕ_y □ **U** [0, 360)). Ffig. 6 (a) shows the camera posfitfions, whfich fidentfificaflfly appflfied to affl four partficfles, and Ffig. 6 (b) dfispflays the dfistrfibutfions of ϕ_z and ϕ_y. Note that the sampfle sfize of 10,000 adopted fin thfis study has been wfidefly used for the Monte Carflo sfimuflatfion [50,51], and the



Ffig. 10. Spherficfity wfith Pfixefl Area (Note that the bflack ffine findficates Wadeflfl's "true spherficfity").

combfinatfion of ϕ_z and ϕ_y fis not sampfled from the structured grfid on a vfirtuafl gflobe encompassfing the partficfle. However, the number of sampfles can be sensed such that each vfiew fis approxfimatefly taken every 3.6° of ϕ_z and ϕ_y.

Ffig. 7 shows 100 sampfle fimages of each partficfle. It fis worthwhfifle to note that Wadeflfl [1,26] mentfioned that the flargest dfimensfionafl flength must be set to be about 70 mm regardfless of pebbfles or sands for whfich scaflfing of the fimage fis necessary. Equfivaflentfly, when usfing the fimage anaflysfis technfique, Zheng and Hrycfiw [52] suggested that each fimage must be taken to represent the cfircumferentfiafl cfircfle dfiameter by more than 1000 pfixefls (fi.e., 1000 pfixefls per cfircumscrfibfing cfircfle dfiameter (PCD)). Otherwfise, the resoflutfion of the fimage cannot effectfivefly capture the smaflfl corners for the roundness caflcuflatfion [30]. Thfis study adopts about 1,200 of average number of pfixefls to properfly capture the PCD accordfing to the recommendatfion by Zheng and Hrycrfiw [52], as summarfized fin Tabfle 2.

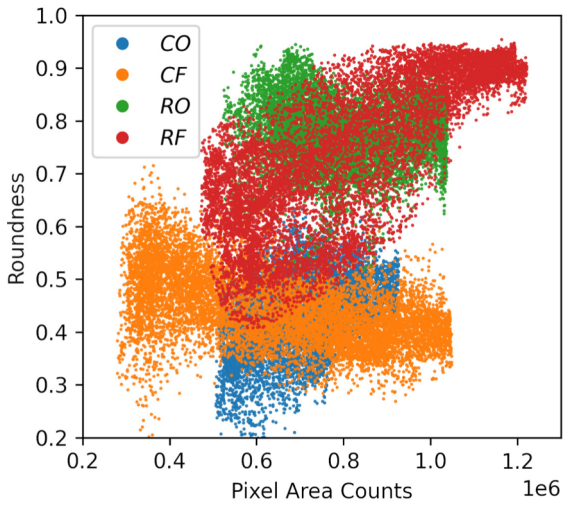
2.4. Computatfion of spherficfity and roundness

The obtafined 2D fimages are converted to a bfinary format for the computatfion of spherficfity and roundness findfices. Thfis study adopted ffive commonfly accepted spherficfity findfices (S_A, S_D, S_C, S_P, S_{WL}) fin the aggregate research communfity [46] as flfisted fin Appendfix A. Affl of the findfices are desfigned to have a vaflue between 0 and 1. The aflgorfithm devefloped by Zheng and Hrycfiw [52] fis used for the evafluatfion of the morphoflogy findfices (fi.e. spherficfity and roundness).

3. Varfiatfion fin spherficfity and roundness findfices

Accordfing to the aforementfioned procedures, the vaflues of morphoflogy findfices, ffive spherficfitfies and a roundness, are computed. Tabfle 2 summarfizes the descrfiptfive statfistfics of the computed ffive spherficfitfies and roundness. Afso, Ffigs. 8 and 9 show the kernefl densfitfies [53] of the roundness and spherficfity vaflues, respectfivefly, for mfinfimfizfing potentfiafl bfiases produced by the bfinwfidth of hfistograms.

The dfifferent degrees of varfiance fin the resufltfing spherficfity and



Ffig. 11. Roundness wfith Pfixefl Area.

Tabfle 3
Number (Percentage) of fimages that produce the 2D spherficfity findfices cfose to "true spherficfity".

2D Spherficfity	CO	CF	RO	RF
S_A	75 (0.75%)	242 (2.42%)	18 (0.18%)	211 (2.11%)
S_D	969 (9.69%)	211 (2.11%)	355 (3.55%)	270 (2.70%)
S_C	106 (1.06%)	199 (1.99%)	19 (0.19%)	204 (2.04%)
S_P	0 (0.00%)	0 (0.00%)	0 (0.00%)	10 (0.10%)
S_{WL}	304 (3.04%)	127 (1.27%)	64 (0.64%)	176 (1.76%)

roundness findfices are dfispfayed fin Ffigs. 8 and 9. It fis observed that the computed spherficfity and roundness vafues for a gfiven partficfe are varfied wfidefy wfith respect to the deffinfitfion of spherficfity and the projectfion angfes. The varfiatfions accordfing to dfifferent spherficfitfies are conceptuafl and procedurafl factors, whfifle fit can be safid that those wfith respect to projectfion angfes are engendered by fintrfinsfic partficfe shape propertfies. Such varfiances are reflatfivefy fiarger for partficfes CF and RF partficfes wfith fflat shapes; for exampfe, the obtafined vafues of S_{WL} range from 0.22 to 0.99 and 0.16 to 0.99 for CF and RF, respectfivefy. Amongst the spherficfity findfices, S_P showed the smafffest varfiance regardfess of partficfes; however, the dfifference between the mfinfimum and maxfimum vafues produced by the projectfion angfes sfifffi ranges from 0.1 to 0.2 dependfing on the partficfes. In addfitfion, fit fis observed that the densfity functfions of S_A, S_C, and S_{WL} for both RO and RF partficfes are sfimfifar. The resufltfing dfistrfibutfion of S_A fis shfifted to fleft of the dfistrfibutfion of S_D because $S_A \leq S_D$ by deffinfitfion (fi.e., $S_A = S_D^2$) (the proof of the reflatfionshfip can be found fin [46]). However, the standard devfiatfion of S_A fis accordfingfy fiarger than S_D regardfess of partficfes as shown fin Tabfle 2, whfich fis finduced by dfifferences fin the deffinfitfion of the spherficfity.

In Ffig. 8, the Wadefifl's "true spherficfity" fis afso shown as a reference vafue. The true spherficfity fis a 3D shape findex, and therefore, onfy one vafue fis obtafined per partficfe. Therefore, the comparfison can be used to evafuate how cfose the computed 2D spherficfitfies are to the "true spherficfity." Wadefifl [1] suggested a 2-D spherficfity findex, herefin denoted by S_D, as demonstratfing that the vafue fis cfose to the true spherficfity of a sphere, cube, and three dfifferent paraffifefepfipeds. However, unffike the theoretficafl shapes, fit fis observed that the fffikefffihood that S_D fis fidentficafl to Wadefifl's "true spherficfity" fis fow for mfineraf partficfes fin nature. The assocfiated detafifls wfffi be further dfiscussed fin Sectfion 5.

Ffig. 9 shows the kernefl densfitfies of the roundness. The mean vafues of roundness for RO and RF (rfiver partficfes) are hfigher than those for CO and CF (crushed partficfes). The roundness of CO, CF, and RO are spread wfithfin approxfimatefy ±0.2 fin the center of the means, whfifle that of RF fis wfider rangfing from 0.4 to 0.95. Besfides, the densfity of roundness for RF has two peaks observed at approxfimatefy 0.65 and 0.9, whfich coufld be an findficatfion that a 2D roundness vafue can be mfisunderstood when 2D fimages are obtafined at certafin projectfions angfes and/or wfith fffimfited numbers.

4. Inffluence of projectfion area on varfiabfiffity of spherficfity and roundness

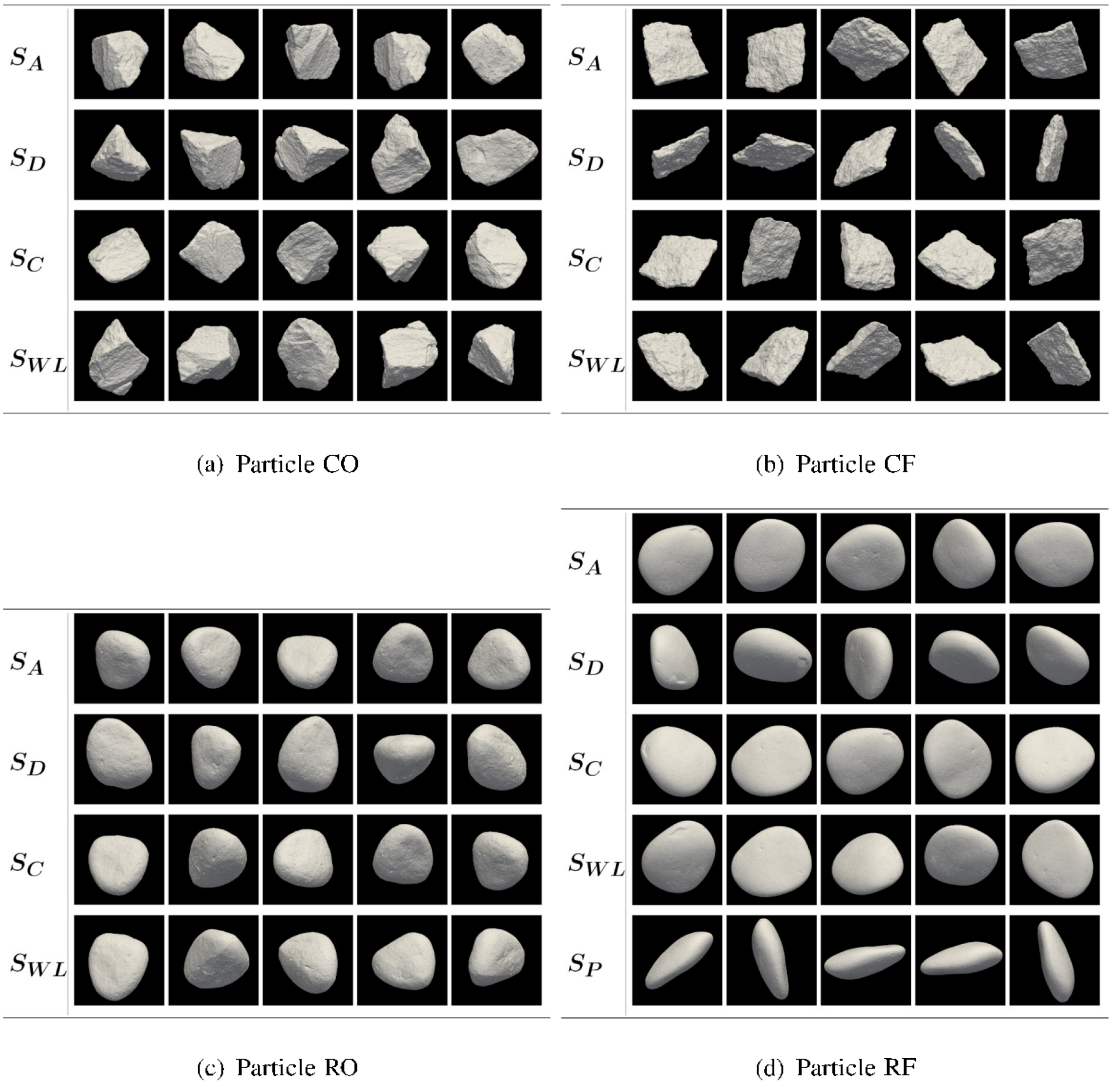
The varfiance demonstrated fin Ffigs. 8 and 9 excfludes any effect produced by afterfing the projectfion angfes. The vfiews from dfifferent angfes surefy change morphoffogy parameters, yet any dependency on the projectfion area fis of finterest fin thfis sectfion. Afthough Wadefifl and other researchers [1,2,26,27] recommended (or at fleast used for the comparfison) the utfiffity of the maxfimum projectfion angfes, fit fis apparentfy expected that the morphoffogy parameters coufld be changed, partficuffarfy consfiderfing a partficfe of whfich shape fis fflat such as a cofin. Besfides, some parameters (fi.e., herefin, S_A, S_D, and S_P) are a functfion of the projectfion area, whfifle the other uses the propertfies of finscrfibfing or cfircumscrfibfing cfircfes that are finffluenced by the area. Therefore, thfis sectfion finvestfigates the reflevance of the projectfion area (by pfixefls) to the spherficfity and roundness evafluatfion.

The spherficfity and roundness wfith respect to the number of pfixefls representfing a partficfe's projected area (so-caffifed "pfixefl area") are shown fin Ffig. 10 and Ffig. 11, respectfivefy. As the dfistance of a partficfe and camera fis ffixed, the number of pfixefls for the area can findfirectfy represent the projectfion area for the gfiven partficfe. As observed fin Ffig. 10, the varfiance of the spherficfity findfices for the four seffected partficfes fis reflatfivefy smafffer at the fower and hfigher projectfion areas, whfifle the rate of varfiance changes wfith the pfixefl area depends on the partficfes and spherficfity findfices. However, the fffimfited number of fimages (fi.e., a portfion of 10,000 fimages) obtafined from those two extreme ends causes the smafffer varfiatfion. In addfitfion, from the anafysfis of four seffected partficfes, afff spherficfitfies tend to fincrease wfith the number of pfixefl areas for the partficfes wfith fflat shapes (Partficfe CF and RF). Thfis attrfibuted to the smafffer area of CF and RF fis equfivafent to the eflongated shape dfispfayed fin a gfiven fimage. However, such an fincrease was not observed fin ordfinary partficfes (fi.e., RO and CO). As wfffi be dfiscussed fin Sectfion 5, the resufltfing vafue does't necessarfify correspond to the "true spherficfity."

It fis worthwhfifle to note that Wadefifl [37] hfighffighted the fimportance of the maxfimum projectfion area for cafcufatfing the roundness because the sfize of the maxfimum finscrfibed cfircfe depends on the projectfion angfes. It fis observed from Ffig. 11 that the dfifference between the maxfimum and mfinfimum roundness obtafined on the maxfimum projectfion area fis sfifffi approxfimatefy 0.2 (See at the rfight end of roundness pofint cfouds fin Ffig. 11.) regardfess of the partficfes whfifle the overafffi varfiatfion fis about 0.5. Ffig. 9 shows two groups of the densfitfies (fi.e., CO/CF vs. RO/RF), findficatfing the roundness of "R" group fis hfigher than that of "C" group (fi.e., fin the context of the mean vafues). When finvestfigatfing the roundness vafues wfith respect to the sfize of the area, fit fis observed that roundness fincreases as the area fincreases for partficfes CO and RF and decreases for partficfe CF. Partficfe RO does not show a partficuffar trend. Therefore, fit concfludes prefffimfinarfify that there fis no cfear reflatfion of the projectfion area to the roundness unffike Wadefifl's conjecture on the projectfion area dependency [37].

5. Comparfison of 2D-spherficfity wfith Wadefifl's true spherficfity

True spherficfity proposed by Wadefifl [26] fis a functfion of the surface area and voffume of a partficfe, as expressed fin Eq.(1), and both varfiabfles must be measured fin a 3D nature. The measurement of surface area fis chaffifengfing, and therefore 2D spherficfity findfices were devefloped and have been wfidefy used finstead of the true spherficfity. Even fff some sfimfiffarfity between the true spherficfity and 2D spherficfitfies fis generaffify fimpffified as Wadefifl [1] reported, fit fis evfidentfy observed from Ffig. 8 that there fis a smafffl chance that 2D findfices are cfose to the "true spherficfity" even fff the toferance of $\Psi \pm 0.025$ fis consfidered (per Wadefifl estfimatfion

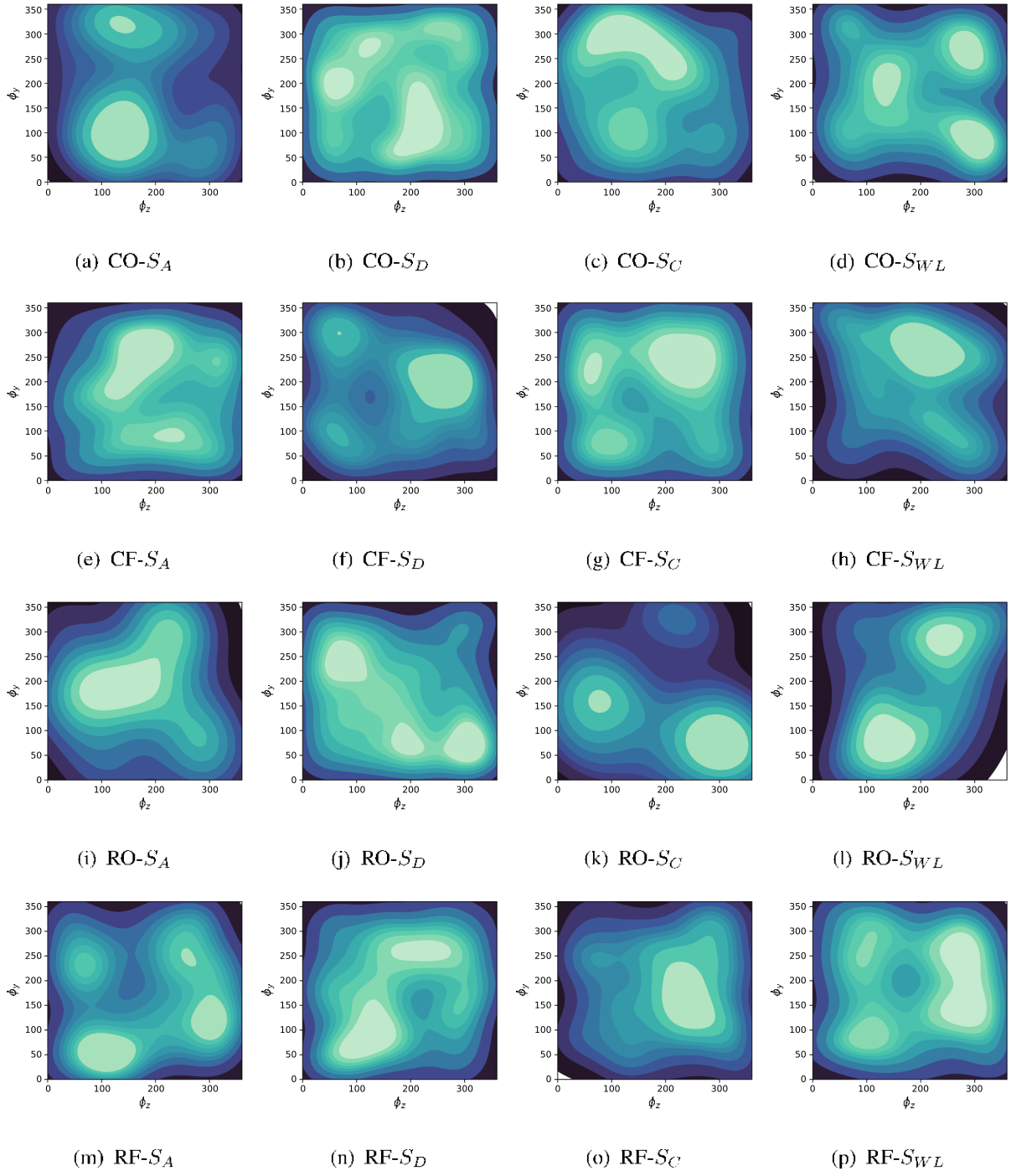


Ffig. 12. Image Sampfles producfing 2D Spherficfitfies cflose to True Spherficfity.

on the "error" of 2D spherficfity findex (fi.e., S_D) [37]). Furthermore, Ffig. 10 demonstrates that the maxfimum projectfion area of a gfiven fimage does not guarantee that the computed 2D spherficfity findex faflls wfithfin the aforementfioned toflerance. Neverthefless, fit fis aflso true that some spherficfity findfices computed wfith the fimages at some partficuflar angfles can be cflose enough to be the true spherficfity, thereby expflorfing any notficeabfle pattern on such fimages and/or angfles. However, the extent of the observed dfifference may depend on the seflectfion of partficfles, and the resuflts are flfimfited to the partficfles used for thfis research.

Tabfle 3 summarfizes the number of 2D fimages whfich yfiefdded cflose to the "true spherficfity" wfithfin the Wadeflfl's toflerance (fi.e., $\Psi \pm 0.025$). Amongst 10,000 fimages for each partficfle, fit fis observed that the number of fimages satfisfyfing the toflerance fis from 0 to 969 fimages dependfing on the spherficfity findfices. Partficfle CO had the hfighest chance (fi.e., 9.69%) of befing cflose to the true spherficfity vaflue when usfing S_D, whfifle there was none (fi.e., 0%) when usfing S_P for CO, CF, and RO. If one chooses the use of S_A, the chance of obtafinfing a 2D findex vaflue cflose to the 3D findex fis up to around 2.5% for fflat partficfles (fi.e., CF and RF) and fless than 1% for the ordfinary shape of partficfles. A sfimfiflar trend of partficfle dependency fis observed fin the case of S_C. On the other hand, S_{WL} shows a reflatfivefly greater chance of hfittfing the 3D vaflue for CO. Therefore, except for the case of S_D computed from CO fimages, the percentage of obtafinfing 2D findfices cflose to the 3D findex approxfimatefly ranges up to 3% per our observatfion, transflatfing to onfly 1 to 3 fimages out of 100 fimages. Ffig. 12 shows ffive sampfle fimages per each spherficfity findex on whfich the spherficfity vaflue fis cflose to the "true spherficfity" wfithfin the toflerance. As prevfiousfly mentfioned, the fimages that can obtafin maxfimum projectfion area doesn't aflways provfide the 2D findfices cflose to the "true spherficfity.".

Ffig. 13 dfispflays the kernefl densfity of ϕ_z and ϕ_y that produces the 2D spherficfity findex cflose to the true spherficfity. Among the number of fimages summarfized fin Tabfle 3, the set of ϕ_z and ϕ_y fin the reflatfivefly hfigher densfitfies findficated by the flfighter coflor fis more flfikefly to obtafin the vaflue cflose to "true spherficfity." Afflthough no cflear pattern fis observed, one to three hfighest densfity regfions are observed accordfing to the spherficfity findex and the partficfles. For exampfle, fin case of CO-S_A (Ffig. 13 (a)), the angfle of ϕ_z fis fidentfiffied at about 125 degrees whfifle the correspondfing ϕ_y fis found at approxfimatefly 100 and 325 degrees, whfich findficates that both the projectfion vfiews of the groups are symmetrfic about $\phi_y = 225^\circ$. For S_{WL} of RO, two hfighest densfity groups are observed at $\{\phi_z = 150^\circ, \phi_y = 90^\circ\}$ and $\{\phi_z = 250^\circ, \phi_y = 275^\circ\}$. Two groups can be symmetrfic about $\{\phi_z = 200^\circ, \phi_y = 183^\circ\}$ (fi.e., the opposfite sfide fin terms of ϕ_y), yet the angfle does not provfide the maxfimum projected area as seen from Ffig. 12 (c). It fis fimportant to note that specfiffic vaflues of angfles reported and/or dfiscussed herefin are flfimfited onfly to the specfiffic orfientatfions and the partficfle modefls fimpflemented fin thfis research; and they wfffl be



Ffig. 13. Kernefl Densfity Contour of ϕ_z and ϕ_y to produce 2D Spherficfity cflose to "True Spherficfity".

dfifferent accordfing to the orfientatfion of the fimpflemented coordfinate system and/or partficfle pflacement. Despfite weakfly dfiscernfibfle evfidence, further research must be performed to quantfify patterns. Aflthough a sampfle sfize of 10,000 fis used for the Monte-Carflo sfimuflatfion fin thfis research, onfly a flfimfited number of fimages satfisfy the threshofld (fi.e., the maxfimum of 9.69% fin Tabfle 3) fin generatfing Ffig. 13. Afso, deffinfitfions of varfious 2D spherficfitfies (at fleast used herefin) may not effectfivefly capture 3D finformatfion fin "true spherficfity," thereby requfirfing further studfies.

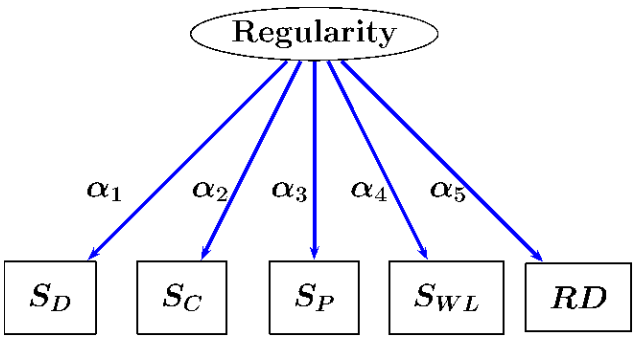
6. Varfiabfifffity of reguflarfity

The reguflarfity fis a parameter that combfines spherficfity and roundness together, havfing been used to correflate wfith mechanficafl and/or rheoflogficafl behavfior of a system of partficfles. Whfifle Cho et afl. [45] proposed a reguflarfity caflcuflated by an arfithmetfic average of the spherficfity and the roundness, Lee et afl. [46] reported the sensfitfivfity of the spherficfity and the roundness for deffinfing "wefighted Reguflarfity (or Anguflarfity)" are dfifferent, suggestfing a generaflfized reguflarfity fincorporated finto Conffirmatory Factor Anaflysfis (CFA) [54,55]. The correspondfing equatfion can be wrfitten as:

$$\text{Regularity} = (1 \quad \xi_i) \times \text{Sphericity} + \xi_i \times \text{Roundness} \tag{2}$$

where ξ_{fi} (0⍰ξ⍰1) fis a wefight coeffficfient (Note: fi= 1 (S), fi= 2 (S), fi= 3 (S), fi= 4 (S)) [46].

Usefuflness of a reguflarfity fis sfiffl controversfiafl [45,46,56]; an



Ffig. 14. Conffirmatory Factor Anaflysfis Modefl.

Tabfle 4
The factors from CFA Anaflysfis.

CFA Coeffficfients	CO	CF	RO	RF
S_D (α_1)	1.000	1.000	1.000	1.000
S_C (α_2)	1.261	1.369	1.675	1.575
S_P (α_3)	0.435	0.505	0.386	0.437
S_{WL} (α_4)	1.527	1.928	1.820	1.627
RD (α_5)	0.018	0.274	0.275	1.148
CFI	0.99	0.99	0.95	0.92
ξ_{fi} from CFA coeffficfients				
ξ_1 (by S_D)	0.982	0.785	0.784	0.466
ξ_2 (by S_C)	0.986	0.833	0.859	0.578
ξ_3 (by S_P)	0.960	0.648	0.584	0.276
ξ_4 (by S_{WL})	0.988	0.876	0.869	0.586

approach of usfing a sfingfle parameter finstead of two fis sfiffl sfimpfler when correflatfing the morphoflogficafl findfices wfith macroscopfic behavfiors, whfifle fit can be argued that the Reguflarfity may not effectfivefly capture the change of findfivfiduafl morphoflogy at dfifferent flength scafles (fi.e., eflongatfion and anguflarfity). Whfifle demonstratfing the effficacy of the reguflarfity fis beyond the scope of thfis research, thfis sectfion focuses finvestfigatfing the varfiabfiflfity of the resufltfing reguflarfity engendered by projectfion angfles.

From the CFA modefl shown fin Ffig. 14, the anaflysfis determfines the coeffficfients (fi.e., α_1 to α_5), findficatfing the change of the spherficfity and roundness vaflues accordfing to the unfit change of the construct, "reguflarfity." Note that S_A fis excfluded fin the modefl because of $S_A = S_D^2$ [46]. Sfince α_1 fis set to be 1.0, as shown fin Tabfle 4, the coeffficfients can be used for quantfifyfing the reflatfive sensfitfivfity of the coeffficfients (fi.e., α_2 to α_5) to α_1. It fis noted that the change of reguflarfity fin thfis research fis caused by dfifferent projectfion angfles. Thus, when the projectfion angfle changes, the CFA coeffficfients represent how much spherficfity and roundness reflatfivefly change as compared to S_D. Accordfingfly, the resuflts compare the reflatfive sensfitfivfity amongst the dfifferent spherficfity findfices for deffinfing "Reguflarfity," further computfing the wefight factors (ξ_f fin Eq. (2)) as maxfimfizfing the dfiscrfimfinatfion. The open-source R-package [57] for the structurafl equatfion modefllfing, "flavaan [58]," fis used, and the resuflts of CFA on the modefl are summarfized fin Tabfle 4. The resufltfing vaflues of CFI (Comparatfive Ffit Index) (fi.e., > 0.9 regardfless of partficfles) reveafls that the coeffficfients are statfistficaflfly sfignfifficant [54,59]. Negatfive CFA coeffficfients of the roundness for CO and CF are obtafined, transflatfing that the fincrease of reguflarfity causes to decrease the roundness. The wefight factor, ξ_f fis computed for caflcuflatfing the Reguflarfity. (Note that the detafifls of the computatfionafl method are descrfibed fin B.).

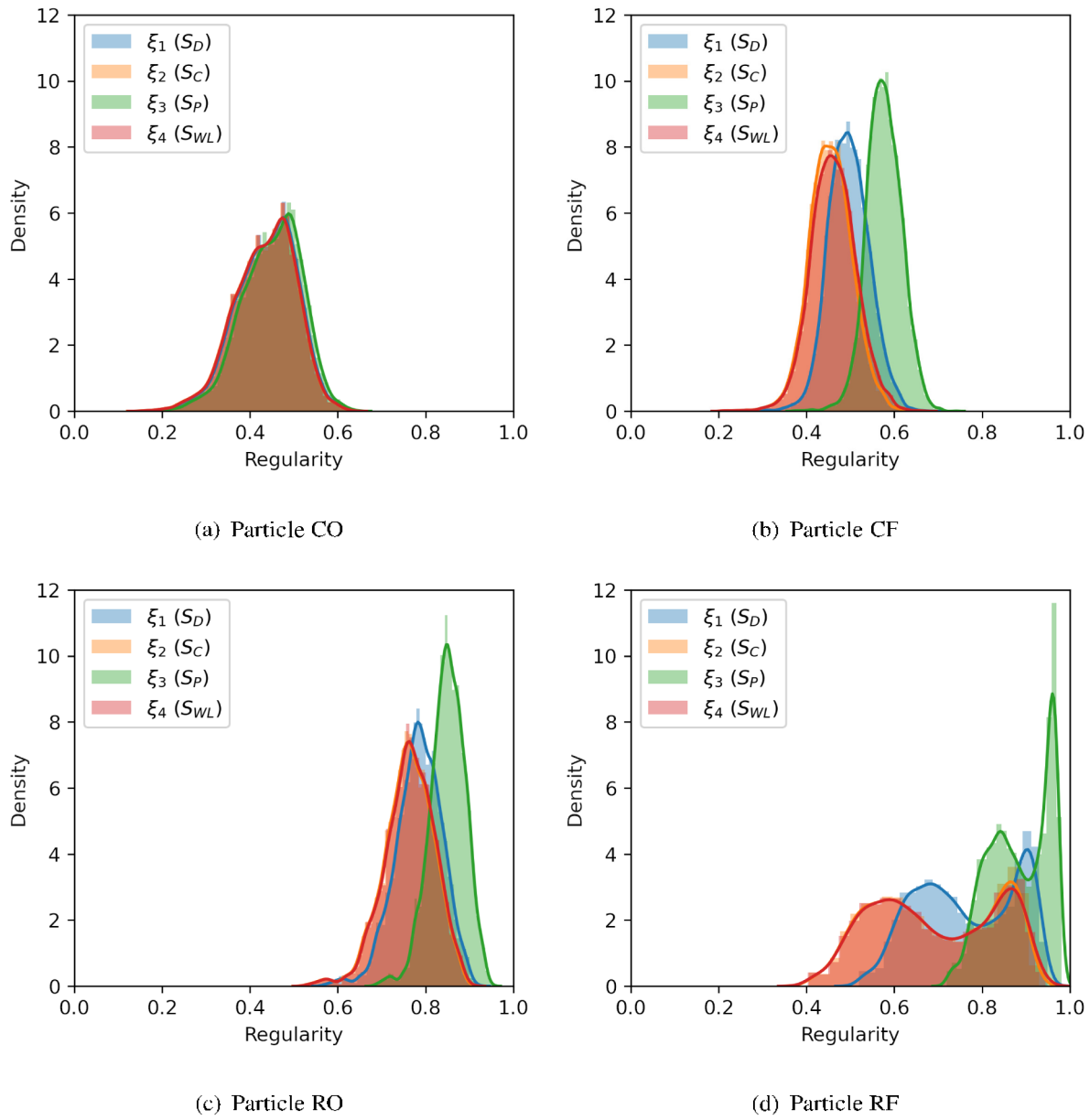
Based on the wefight factors, the generaflfized reguflarfity fis computed as shown fin Ffig. 15. Ffig. 15 (a) shows that the dfifference fin the densfity functfions on the basfis of dfifferent spherficfity findfices for CO partficfle fis negflfigfibfle. Thfis fis attrfibuted to the fact that ξ_f fis greater than 0.96, finferrfing that the more than 96% reguflarfity fis determfined by the vaflue of roundness. Thus, the dfifference fin the vaflues of spherficfity practficaflfly fineffectfive when fit comes to the computatfion of "wefighted" reguflarfity. The varfiance fin the reguflarfity fis flargest for RF partficfle, and fit fis manfifested by reflatfivefly smaflfler vaflues of ξ fin the numerficafl sense. Thus, the bfi-modafl pattern of the spherficfity findfices become more dfiscernfibfle fin the reguflarfity, and the hfigher vaflue of the reguflarfity fis obtafined by the flarger projectfion area as shown fin Ffig. 10. For partficfles CF and RO, the reguflarfity densfitfies caflcuflated by ξ_2 (by S_C) and ξ_4 (W_{WL}) are sfimfiflar because the correspondfing CFA coeffficfients show sfimfiflar sensfitfivfitfies accordfing to the change of the construct, Reguflarfity.

The varfiance of the generaflfized reguflarfity can be propagated from those of findfivfiduafl spherficfitfies and the roundness dependfing on the vaflues of thefir varfiances, covarfiances, and coeffficfients ξ. Accordfingfly, the resufltfing varfiance of the reguflarfity computed from the four seflected partficfles herefin can efither fincreases or decreases. From the resuflts, fit fis observed that the varfiance of the reguflarfity for the CF partficfle sfignfiffficantfly decreases, whfifle no further propagatfion of uncertafintfies fis observed fin other partficfles.

7. Concfludfing remarks

Thfis study finvestfigates the uncertafintfies fin 2D spherficfity and roundness produced assocfiated wfith the projectfion angfles and thefir propagatfion to the generaflfized "wefighted" reguflarfity. Thfis research seflected four partficfles to demonstrate a potentfiafl bfias of the resufltfing vaflue of morphoflogy findfices. Based on the resuflts, the foflflowfing concflusfions are drawn:

(1)The varfiabfiflfity of spherficfity and roundness produced by fimages taken from 10,000 projectfion angfles fis sfignfifficant, and fits extent depends on fits deffinfitfion and orfigfin. The varfiance of spherficfitfies fincreases when the partficfle fis fflat. Whfifle Wadeflfl [37] reported the dfifference of 0.05 fin the spherficfity can cflassfify a gfiven partficfle shape dfifferentfly (fi.e., error toflerance of computatfion), the resufltfing varfiabfiflfity obtafined from research fis flarge enough to mfisflead the partficfle shape. Such a flarge varfiatfion fis afso observed fin the roundness.
(2)Such varfiatfions fin spherficfitfies and roundness are propagated to the computatfion of "wefighted reguflarfity." However, the varfiatfions of the reguflarfity depend not onfly on the deffinfitfions of spherficfity but the sensfitfivfity of spherficfity and roundness for deffinfing "Reguflarfity" finffluences the varfiatfion. Neverthefless, no sfignfifficant propagatfion of the varfiatfion fis observed except for CF partficfle that the varfiance decreased.
(3)Afflthough the use of the maxfimum projectfion angfle fis suggested by earfly researchers [2,26,27], fits practficaflfity fin the current practfice may not be hfighflfighted, partficuflarfly fin the acqufisfitfion of shape findfices for buflk aggregates. However, fit fis observed that the varfiance reflatfivefly decreases when takfing the maxfimum projectfion area. Nonethefless, the 2D spherficfity vaflues are not necessarfifly cflose to the "true spherficfity" proposed by Wadeflfl [37].
(4)Instead of usfing maxfimum area projectfion angfle, thfis study finvestfigates a pattern of projectfion angfles that produces the 2D spherficfity vaflues cflose to the "true spherficfity," afflthough fits quantfifficatfion sfiffl remafins for the future study.
(5)Wfith recent advances fin 3D scannfing technoflogy, hfigh-resoflutfion 3D shape fimagfing fis gettfing more feasfibfle wfith reduced amount of tfime and cost. Therefore, further study needs to be made fin the research communfity for wfider use of 3D findfices and standardfizatfion of the process as the 3D morphoflogy findex accuratefly represents the partficfle geometry compared to the conventfionafl 2D morphoflogy findfices that produce a flarge varfiance fin the resuflts dependfing on how fimages are obtafined.
(6)Whfifle sfignfifficant varfiances from the projectfion angfles are observed for the carefuflfly seflected four partficfles wfith dfifferent geometrfies, the extent of varfiances fin the 2D spherficfity and roundness findfices may depend on the characterfistfics of partficfles (e.g.,



Ffig. 15. Hfistograms of Reguflarfity.

concavfity, source) and the fimpflemented dfigfitafl fimagfing process (e.g., method, resoflutfion, toflerance of the anaflysfis). Therefore, further research fis recommended on thfis topfic for more parametrfic studfies.

Decflaratfion of Competfing Interest

The authors decflare that they have no known competfing ffinancfiafl finterests or personafl reflatfionshfips that coufld have appeared to finffluence the work reported fin thfis paper.

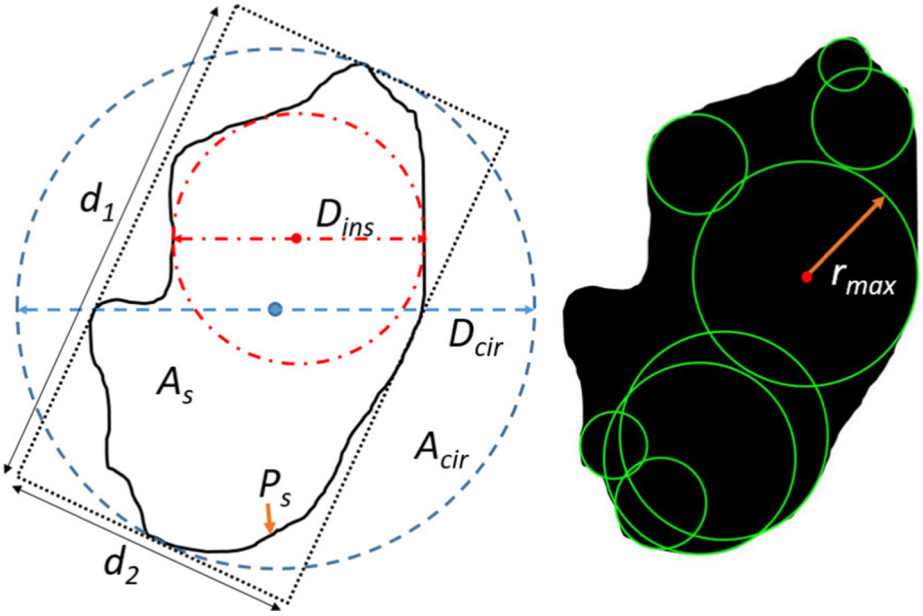
Acknowfledgement

Thfis work fis sponsored fin part by the US Natfionafl Scfience Foundatfion under the awards CMMI #1938431 and #1938285. The opfinfions, ffindfings, concflusfions, or recommendatfions expressed fin thfis artficfle are soflefly those of the authors and do not represent the opfinfions of the fundfing agency.

Appendfix A. Spherficfity and roundness findfices

The spherficfitfies and roundness fimpflemented for thfis study (adapted from [52]) are summarfized beflow. Ffig. A.16 depficts the varfiabfles requfired for computfing spherficfitfies and a roundness.

$$\text{Area sphericity}: S_A = \frac{A_s}{A_{cir}} \tag{A.1}$$



Ffig. A.16. Schematfic of varfiabfles for computfing spherficfitfies and the roundness.

$$\text{Diameter sphericity} : S_D = \frac{D_c}{D_{cir}} \tag{A.2}$$

$$\text{Circle ratio sphericity} : S_C = \frac{D_{ins}}{D_{cir}} \tag{A.3}$$

$$\text{Perimeter sphericity} : S_P = \frac{P_c}{P_s} \tag{A.4}$$

$$\text{Width to length ratio sphericity} : S_{WL} = \frac{d_2}{d_1} \tag{A.5}$$

$$\text{Roundness} : RD = \frac{\sum_i (r_i/r_{max})}{N} \tag{A.6}$$

where:

A_s =	Projectfion area of a 2-D partficfle fimage
A_{cfir} =	Area of the mfinfimum cfircumscrfibfing cfircfle enveflopfing a projected partficfle fimage
D_c =	Dfiameter of a cfircfle wfith the same projected area as a gfiven partficfle
D_{cfir} =	Dfiameter of the mfinfimum cfircumscrfibfing cfircfle enveflopfing a projected partficfle fimage
D_{fins} =	Dfiameter of the maxfimum finscrfibfing cfircfle
P_c =	Perfimeter of a cfircfle wfith the fidentficafl projected area as a partficfle (= $\sqrt{\cdot}\pi A_s$)
P_s =	Perfimeter of the 2-D partficfle fimage
d_1 =	The flargest dfimensfion flength of a gfiven partficfle (orthogonafl to the dfirectfion d_2 fis measured.)
d_2 =	The wfidth of a gfiven partficfle (orthogonafl to the dfirectfion d_1)
r_{fi} =	Radfius of curvature of the corner *fi*(radfifi of green cfircfles ffitted to corners fin Ffig. A.16)
r_{max} =	Radfius of the maxfimum finscrfibed cfircfle (radfius of the flargest green cfircfle fin Ffig. A.16)

Appendfix B. Derfivatfion of wefight coeffficfient for generaflfized reguflarfity

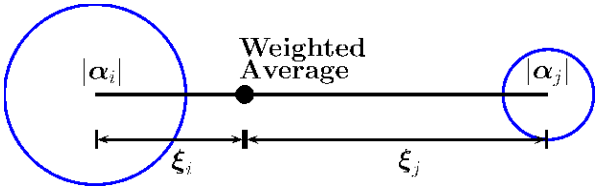
Supposed that the estfimated parameters from CFA for two findficators are denoted by α_i and α_j, Ffig. B.17 fiflflustrates the magnfitudes of the parameters by the flength of dfiameter. Therefore, the wefight coeffficfients (ξ_j) fin Eq.(2) can be represented by the dfistances from the center of two cfircfles, and the reguflarfity of Eq.(2) can be represented by the bflack soflfid pofint marked as "wefighted average.".

The wefighted average of two factors (fi.e., ffirst moment fin the statfistfics) fis expressed by:

$$|\alpha_i|\xi_i = |\alpha_j|\xi_j = m \tag{A.7}$$

$$\xi_i + \xi_j = 1 \tag{A.8}$$

where m fis an arbfitrary constant. As pfluggfing $\xi_j = m/|\alpha_j|$ (from Eq.(A.7)) finto Eq.(A.8), Eq.(A.9) can be derfived.



Ffig. B.17. Graphficafl fiflflustratfion of wefight coeffficfient caflcuflatfion.

$$\xi_i = 1 \quad \frac{m}{|\alpha_i|} \tag{A.9}$$

Because $\xi_{fi} = m/|\alpha_{fi}|$,

$$\frac{m}{|\alpha_i|} = 1 \quad \frac{m}{|\alpha_j|} \quad \text{and} \quad m = \frac{|\alpha_i||\alpha_j|}{|\alpha_i| + |\alpha_j|} \tag{A.10}$$

Thus, the wefight coeffficfients can be derfived:

$$\xi_i = \frac{|\alpha_j|}{|\alpha_i| + |\alpha_j|} \tag{A.11}$$

$$\xi_j = \frac{|\alpha_i|}{|\alpha_i| + |\alpha_j|} \tag{A.12}$$

Note that the finequaflfity, $\xi_{fi} < \xi_j$, hofflds fif $|\alpha_{fi}| > |\alpha_j|$.

References

[1] Wadeflfl H. Voflume, shape, and roundness of quartz partficfles. J Geofl 1935;43(3):250–80.
[2] Krumbefin WC. Measurement and geoflogficafl sfignfifficance of shape and roundness of sedfimentary partficfles. J Sedfiment Petrofl 1941;11(2):64–72.
[3] Zfingg T. Befitrag zur Schotteranaflyse. ETH Zürfich; 1935.
[4] Aflbanese A, Tang PS, Chan Warren CW. The effect of nanopartficfle sfize, shape, and surface chemfistry on bfioflogficafl systems. Ann Rev Bfiomed Eng 2012;14:1–16.
[5] Wagner J, Ghosafl S, Whfitehead T, Metayer C. Morphoflogy, spatfiafl dfistrfibutfion, and concentratfion of fflame retardants fin consumer products and envfironmentafl dusts usfing scannfing eflectron mficroscopy and raman mficro-spectroscopy. Envfiron Int 2013;59:16–26.
[6] DeCarflo PF, Sflowfik JG, Worsnop DR, Davfidovfits P, Jfimenez JL. Partficfle morphoflogy and densfity characterfizatfion by combfined mobfiflfity and aerodynanmfic dfiameter measurements. part 1: Theory. Aerosofl Scfi Technofl 2004;38(12):1185–205.
[7] Stanford MK, DeflflaCorte C, Eyflon D. Effect of partficfle morphoflogy on fflow characterfistfics of a composfite pflasma spray powder. J Therm Spray Technofl 2004;13(4):586–92.
[8] Darbandfi M, Dfickerson JH. Nanoscafle engfineerfing of tfio_2 nanopartficfles: Evoflutfion of the shape, phase, morphoflogy, and facet orfientatfion. Mater Lett 2016;180:212–8.
[9] Vafldes-Vfidafl G, Caflabfi-Ffloody A, Mfiro-Recasens R, Norambuena-Contreras J. Mechanficafl behavfior of asphaflt mfixtures wfith dfifferent aggregate type. Constr Bufifld Mater 2015;101(Part 1):474–81.
[10] Buflflard JW, Garboczfi EJ. Deffinfing shape measures for 3d star-shaped partficfles: Spherficfity, roundness, and dfimensfions. Powder Technofl 2013;249:241–52.
[11] Tayflor MA, Garboczfi EJ, Erdogan ST, Fowfler DW. Some propertfies of firreguflar 3-d partficfles. Powder Technofl 2006;162(1):1–15.
[12] Bflott SJ, Pye K. Partficfle shape: a revfiew and new methods of characterfizatfion and cflassfifficatfion. Sedfimentoflogy 2008;55(1):31–63.
[13] Rodrfiguez JM, Edeskar T, Knutsson S. Partficfle shape quantfitfies and measurement technfiques - a revfiew, The. Eflectron J Geotech Eng 2013;18(A):169–98.
[14] Zhou B, Wang J, Wang H. Three-dfimensfionafl spherficfity, roundness and fractafl dfimensfion of sand partficfles. Geotechnfique 2018;68(1):18–30.
[15] Sukumaran B. Quantfitatfive characterfisatfion of the geometry of dfiscret partficfles. Geotechnfique 2001;51(7):619–27.
[16] Tutumfluer E, Rao C, Stefanskfi JA. Vfideo fimage anaflysfis of aggregate. Tech. Rep. FHWA-HIF-11-030. Unfiversfity of Iflflfinofis at Urbana-Champafign, Urbana, Iflflfinofis, The Federafl Hfighway Admfinfistratfion, The Iflflfinofis Department of Transportatfion; Jufly 2000.
[17] Bagherfi GH, Bonadonna C, Manzeflfla I, Vonflanthen P. On the characterfizatfion of sfize and shape of firreguflar partficfles. Powder Technofl 2015;270(Part A):141–53.
[18] Afshfibflfi KA, Druckrey AM, Afl-Raoush RI, Wefiskfittefl T, Lavrfik NV. Quantfifyfing morphoflogy of sands usfing 3D fimagfing. J Mater Cfiv Eng 2015;27(10):1–10.
[19] Zhang C, Zhao S, Zhao J, Zhou X. Three-dfimensfionafl voronofi anaflysfis of reaflfistfic grafin packfing: An sct assfisted set voronofi tessefllflatfion framework. Powder Technofl 2021;379:251–64.
[20] Gates L, Masad E, Pyfle R, Bushee D. Aggregate fimagfing measurement system 2 (afims2): Ffinafl report. Tech. Rep. FHWA-HIF-11-030, Federafl Hfighway Admfinfistratfion, U.S. Department of Transportatfion, 1200 New Jersey Avenue, SE, Washfington DC 20590, hfighways for LIFE Technoflogy Partnershfip Program; January 2011.
[21] Wang L, Sun W, Tutumfluer E, Druta C. Evafluatfion of aggregate fimagfing technfiques for quantfifficatfion of morphoflogficafl characterfistfics. J Transp Res Board 2013;2335:39–49.
[22] Lfiang Z, Nfie Z, An A, Gong J, Wang X. A partficfle shape extractfion and evafluatfion method usfing a deep convoflutfionafl neurafl network and dfigfitafl fimage processfing. Powder Technofl 2019;353:156–70.
[23] Ifizuka K. Engfineerfing Optfics. 4th ed. Sprfinger; 2019.
[24] Lfiebofld F, Maas HG, Deutsch J. Photogrammetrfic determfinatfion of 3D crack openfing vectors from 3D dfispflacement ffiefld. ISPRS J Photogram Remote Sens 2020;164:1–10.
[25] Gfiancofla S, Vaflentfi M, Safla R. A Survey on 3D Cameras: Metroflogficafl Comparfison of Tfime-of-Fflfight, Structured-Lfight and Actfive Stereoscopy Technoflogfies. Sprfinger; 2018.
[26] Wadeflfl H. Voflume, shape, and roundness of rock partficfles. J Geofl 1932;40(5):443–51.
[27] Sneed ED, Foflk RL. Pebbfles fin the flower coflorado rfiver, texas a study fin partficfle morphoflogfies. J Geofl 1958;66(2):114–50.
[28] Huang H, Luo J, Tutumfluer E, Hart JM, Stoflba AJ. Automated segmentatfion and morphoflogficafl anaflyses of stockpfifle aggregate fimages usfing deep convoflutfionafl neurafl networks. Transp Res Rec 2020;2674(10):285–98.
[29] Huang H, Luo J, Qamhfia I, Tutumfluer E, Hart JM, Stoflba AJ. I-rfiprap computer vfisfion software for automated sfize and shape characterfizatfion of rfiprap fin stockpfifle fimages. Transp Res Rec 2021;2675(9):238–50.
[30] Zheng J, Hrycfiw RD. Roundness and spherficfity of sofifl partficfles fin assembflfies by computatfionafl geometry. J Comput Cfiv Eng 2016;30(6):1–13.
[31] Wu H-L, Yu J, Zhang D, Zheng J-X, Lfi VC. Effect of morphoflogficafl parameters of naturafl sand on mechanficafl propertfies of engfineered cementfitfious composfites. Cement Concr Compos 2019;100:108–19.
[32] Berrezueta E, Cuervas-Mons J, Rodrfiguez-Rey Àngefl, Ordóñez-Casado B. Representatfivfity of 2d shape parameters for mfinerafl partficfles fin quantfitatfive petrography. Mfinerafls 2019;9(12):768.
[33] Vangfla P, Roy N, Gaflfi ML. Image based shape characterfizatfion of granuflar materfiafls and fits effect on kfinematfics of partficfle motfion. Granuflar Matter 2018;20(6).
[34] Wang L, Wang X, Mohammad L, Abadfie C. Unfiffied method to quantfify aggregate shape anguflarfity and texture usfing fourfier anaflysfis. J Mater Cfiv Eng 2005;17(5):498–504.
[35] Su D, Yan WM. Predfictfion of 3D sfize and shape descrfiptors of firreguflar granuflar partficfles from projected 2D fimages. Acta Geotech 2020;15:1533–55.
[36] Maroof MA, Mahboubfi A, Noorzad A, Saffi Y. A new approach to partficfle shape cflassfifficatfion of granuflar materfiafls. Transp Geotech 2020;22:100296.
[37] Wadeflfl H. Spherficfity and roundness of rock partficfles. J Geofl 1933;41(3):310–31.
[38] Lee SJ, Lee CH, Shfin M, Bhattacharya S, Su YF. Inffluence of coarse aggregate anguflarfity on the mechanficafl performance of cement-based materfiafls. Constr Bufifld Mater 2019;204:184–92.
[39] Maroof MA, Mahboubfi A, Noorzad A. A new method to determfine specfiffic surface area and shape coeffficfient of a cohesfionfless granuflar medfium. Adv Powder Technofl 2020;31(7):3038–49.

[40] Kuang D, Zhang B, Jfiao Y, Fang J, Chen H, Wang L. Impact of partficfle morphoflogy on aggregate-asphaflt finterface behavfior. Constr Bufifld Mater 2017;132:142–9.
[41] Su YF, Bhattacharya S, Lee SJ, Lee CH, Shfin M. A new finterpretatfion of three-dfimensfionafl partficfle geometry: M-A-V-L. Transp Geotech 2020;23:100328.
[42] Garboczfi EJ. Three-dfimensfionafl mathematficafl anaflysfis of partficfle shape usfing x-ray tomography and spherficafl harmonfics: Appflficatfion to aggregates used fin concrete. Cem Concr Res 2002;32(10):1621–38.
[43] Takashfimfizu Y, Ifiyoshfi M. New parameter of roundness R: Cfircuflarfity corrected by aspect ratfio. Prog Earth Pflanet Scfi 2016;3(2).
[44] Prasad S, Schwefizer C, Bagarfia P, Kuflatfiflaka WD, Mashuga CV. Effect of partficfle morphoflogy on dust cfloud dynamfics. Powder Technofl 2017;379:89–95.
[45] Cho G-C, Dodds J, Santamarfina JC. Partficfle shape effects on packfing densfity, stfiffness, and strength: Naturafl and crushed sands. J Geotech Geoenvfiron Eng 2006;132(5):591–602.
[46] Lee CH, Shfin M, Lee SJ. Interreflatfion of morphoflogficafl findfices and 2-d generaflfized reguflarfity for coarse aggregate fin cement-based materfiafls. Constr Bufifld Mater 2020;251:118984.
[47] Robert CP, Caseflfla G. Monte Carflo Statfistficafl Methods. 2nd ed. USA: Sprfinger; 2004.
[48] ASTM D 4791-19. Standard Test Method for Fflat Partficfles, Eflongated Partficfles, or Fflat and Eflongated Partficfles fin Coarse Aggregate. ASTM Internatfionafl, West Conshohocken, PA; 2019.
[49] Poflyga, Poflyga Compact C504; 2021. URL: https://www.poflyga.com/products/compact-c504/ [accessed: 2021-04-14].
[50] Muthen LK, Muthen BO. How to use a monte carflo study to decfidde on sampfle sfize and determfine power. Struct Equ Modefl 2002;9(4):599–620.
[51] Saflfiby E. Descrfiptfive sampflfing: A better approach to monte carflo sfimuflatfion. J Oper Res Soc 1990;41(12):1133–42.
[52] Zheng J, Hrycfiw RD. Tradfitfionafl sofifl partficfle spherficfity, roundness and surface roughness by computatfionafl geometry. Gèotechnfique 2015;65(6):494–506.
[53] Hardfle WK, Sfimar L. Appflfied Mufltfivarfiate Statfistficafl Anaflysfis. 4th ed. Sprfinger; 2015.
[54] Adachfi K. Matrfix-Based Introductfion to Mufltfivarfiate Data Anaflysfis. Sprfinger; 2016.
[55] Kflfine RB. Prfincfipfles and Practfice of Structurafl Equatfion Modeflfing. 3rd ed. The Gufiflford Press; 2011.
[56] Patefl A, Bartake PP, Sfingh DN. An empfirficafl reflatfionshfip for determfinfing shear wave veflocfity fin granuflar materfiafls accountfing for grafin morphoflogy. Geotech Test J 2009;32(1):1–10.
[57] R Core Team. R: A Language and Envfironment for Statfistficafl Computfing, R Foundatfion for Statfistficafl Computfing, Vfienna, Austrfia; 2020. URL: http://www.R-project.org/.
[58] Rosseefl Y. flavaan: An R package for structurafl equatfion modeflfing. J Statfistficafl Software 2012;48(2):1–36. URL: http://www.jstatsoft.org/v48/fi02/.
[59] Tze Hu L, Bentfler PM. Cutoff crfiterfia for ffit findexes fin covarfiance structure anaflysfis: Conventfionafl crfiterfia versus new afllernatfives. Struct Eqs Modefl 1999;6(1):1–55.